THE ECONOMETRICS OF DEMAND SYSTEMS
WITH SPECIAL REFERENCE TO COMMODITY
GROUP DATA FOR BAHRAIN

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Abstract

The main objective of this study is to estimate the demand relationships among commodity groups using Bahrain quarterly time series data for the period 1979-98. Three main demand systems are presented and estimated, namely the Linear Expenditure Demand System (LES) which was introduced by Kelin-Rubin (1946-47) and developed by Stone (1953-54), the Rotterdam System (RM) which was introduced by Theil (1965) and Barten (1966), and the Almost Ideal Demand System (AIDS) introduced by Deaton and Muellbauer (1980). Also, the variables reflecting the effects of habit on purchases are incorporated into three main demand systems.

Model selection procedures are applied to select the best model to reflect the Bahrain data. Based on procedures, the static Linear Almost Ideal Demand System (LAV/AIDS) is selected among other static demand systems, namely, the linear expenditure demand system and the Rotterdam system models. The selection is based on average information criterion. However the dynamic LAV/AIDS is selected over its static counterpart and recommended for the future application for Bahrain data. The selection is based on the likelihood ratio test. Further, the dynamic LAV/AIDS satisfies all the restrictions implied by demand theory. All the compensated own price elasticities are negative as expected and the expenditure elasticity classifies the food & beverage, clothing & footwear, housing, and transportation as necessity products, while the other commodity group is classified as luxury products.

The second main objective of this study is to test for unit roots and order of cointegration in Bahrain commodity group data. The results indicate that most of the time series that will be used in estimating the linear AIDS model, such as total real expenditures on various commodity groups, prices, budget shares are shown to be integrated of order one. The application of Johansen and Juselius
(1990) Full Maximum Likelihood approach in this study confirm that equilibrium relationships exist between the variables that make-up the LA/AIDS model. Second, the homogeneity of degree zero of price, postulated by consumer theory, is rejected by the data. The argument that time series issues are responsible for this rejection is not always true.

The study also focuses on a) Engel Curve and b) income distribution and poverty in Bahrain. The advantage of a cross-section study of the Engel Curve is that consumer demand theory is based on micro-demand data (individual's data). Income elasticities that are estimated from cross-section data are useful in predicting changes in aggregate time series data if the effects of price changes have to be eliminated from time series analyses. Therefore, seven Engel Curves for eight commodity groups are estimated using data from the Bahrain household survey for the period 1994-1995.

Studying income distribution and poverty in Bahrain is important. Inequality of income and poverty influences the pattern of household expenditure. It also influences the welfare of households in the country. Therefore, studying income distribution and poverty could guide the policy makers in Bahrain to improve the living standard of households. The analyses of income distribution and poverty in Bahrain are based on the household expenditure and income survey data for the period (1984-85 to 1994-95), involving Gini coefficient and other measures.
Chapter 1

Introduction and Overview

The principal object of this study is to analyze the pattern of consumer behavior in Bahrain, through an econometric analysis of systems of demand equations. The study makes reference to three main demand systems, namely, Linear Expenditure System (LES), Almost Ideal Demand System (AIDS), and Rotterdam Model (RM). The applications and analysis is for both static and dynamic versions of these models. The study also seeks to develop a dynamic form of an error correction model based on the AIDS flexible functional form. The study analyses the pattern of household consumption in Bahrain using household income and expenditure survey data for the year (1994-1995).

1. Why Study Consumer Behavior?

In microeconomics the theory of consumer behavior states that the consumption combinations of the individual depend upon the tastes and income of the consumer and the relative prices of all commodities that he/she faces. This theory is based on the assumption that a rational consumer will always try to maximize the utility function subject to a budget constraint. But when we take a household as a consumer unit, consumption depends not only on those variables, but also on the composition of the household. As the composition of household changes, the tastes and necessities of a household may also change and consequently, a change may occur in the demand for commodities. Household demand is a high percentage of final demand for any economy. In Bahrain it constitutes about 56 per cent of final demand.

The analysis of consumer behavior has always occupied a central position in research, both at the theoretical and at the empirical level. Moreover, demand analysis has been
one of the subjects where the inter-relations between theory and evidence have been strong and have proved to be very fruitful. The theory of consumer behavior and demand systems provides us with a framework well suited for model formulation and data analysis. Consumer theory encourages economists towards econometric application. It is such applications that form the basis for the current study.

New models and specifications in demand theory, together with earlier theory, directs this thesis into an examination of consumer demand for Bahrain. There have been improvements in the econometric techniques for testing and estimation. This study incorporates the latest developments in econometrics, to help estimate and evaluate demand systems.

A complete demand system stresses the inter-relationships within demand and provides a theoretical framework in order to give a better understanding of the effects of changes in real income, relative prices, and the time trend. Current consumption patterns need to be identified, and this requires reliable estimates of demand parameters. Disaggregated parameters for commodities in Bahrain help us understand the linkage to the international sector, as well as to domestic production. This facilitates the design of appropriate policies for influencing domestic production, consumption, and international trade. Investigation of consumption expenditure helps explain trends in consumption, and provides a basis for making estimates of likely future patterns of consumer expenditure.

Personal consumption comprises the largest share of gross national product (GNP) in Bahrain i.e. about 60%. It represents final demand on the productive sector. Meeting these demands generates wages and profits, thus completing the cycle. Household demand, as a link in this chain, is important for a number of reasons
• The commodity composition of personal demand varies with prices and income. An economy with growing per capita GNP will have a changing balance among its productive activities. Policy may need to take these changes into account. Important government and business decisions are often influenced by period-to-period changes in consumer expenditures or sales, and understanding the dynamics underlying the changes in demand can be helpful in making more informed decisions (Brown and Lee, 1992, p.9).

• Because the import and export content of consumer goods varies, a changing pattern of demand may have implications for external trade policy and for international financial management.

• Governments may wish to redistribute income to improve welfare. Such a change will affect the structure of aggregate consumer demand in ways that will need to be anticipated. Estimated elasticities form the basis for welfare analysis in evaluation of transfer and/or subsidy programs at the microeconomic level (Sabelhaus, 1990, p.1477).

• Domestic savings may need to be mobilized. Saving represent a surplus of income over consumption. A proper understanding of demand behavior may require an addition to knowledge concerning savings behavior.

• Commodity prices are known to be important in influencing consumer behavior. It may be necessary to examine the structure of demand (Lluch et al. (1977, pp. xxii, xxiii), in order to anticipate the effect of price changes. Demand elasticities may be necessary to determine the effects of an increase in VAT. Consumers could switch to un-taxed commodities or to cheaper substitutes, thus reducing consumption of the taxed good. Compensated cross-price elasticities of demand will indicate if there is a substitution effect between taxed and untaxed goods.
In macroeconomic modeling estimated price and income elasticities are extremely important for general equilibrium properties. (Almon, 1979).

Demand elasticities may be useful in determining the effects of exchange rate policies to assess the effects of subsidies (via the budget) on consumption (Balcombe, 1996, p.57).

Consumer expenditure studies also help identify investment opportunities in the economy in question. Studies may give information about consumers. Structural changes in consumer demand and the interrelation of demand among various commodities are also useful for market researchers in analyzing the demand for groups of goods and services. Income growth often brings dramatic changes in economic structure and consumption patterns. Finally, demand studies provide a reference point for the infrastructure program. An expenditure study will provide information, which can help improve the infrastructure of the economy, e.g. transport services for the benefit of consumers, producers, and investors.

2. Statement of the Research Problem as it Relates to Bahrain

In Bahrain, as in other oil exporting countries, there were greatly increased private consumption expenditures during the first and second oil shocks. Private consumption expenditure was a factor leading to instability of Bahrain economic growth during those periods. Therefore, the analysis of trends in personal consumption is an important issue for the Bahrain economy.

The government of Bahrain has developed a macroeconomic model, which is called Delmon-1: This macroeconomic model was designed by my own research center which is the Bahrain Center for Studies and Research (BCSR). The Delmon-1 model comprises 6 "blocks" namely the production block, domestic demand block, government block, monetary block, external trade block, and labor
block. Changes in consumption patterns will influence economic activities such as production, employment, and investment. The inclusion of an appropriate demand system in the macroeconomic model is necessary for the policy makers in Bahrain. I am a member of the research staff of the Bahrain Center for Studies and Researches.

This study contributes to the development of the macroeconomic model by providing information on domestic demand. The consumption model selected through this study is to be simulated within the Bahrain macroeconomic model Delmon-1 to study the effects of changes in real income and relative prices, thus providing information for demand prediction and for economic policy. The results of this study are important to the Ministry of Finance and Economy in Bahrain.

In Bahrain, which is one of the less developed oil exporting countries, there has been no work carried out on demand relationships in recent times and especially nothing on consumer behavior analyzed by complete systems of flexible functional forms. It is hoped, therefore, that the present study will be of use for policy making. The study will contribute to building information systems at national and sector levels, and will encourage more research in this area. In particular international organizations and foreign companies with branches or offices in Bahrain require information on the pattern of consumer expenditure in Bahrain so as to identify appropriate allowances for their staff.

I hope that this thesis makes a contribution to the literature on consumer demand, particularly to an understanding of consumer behavior in oil exporting countries. The Central Statistics Organization (CSO) in Bahrain may be encouraged to publish household data to permit further research in this field, in consequence of the research presented in this thesis.
3. Overview of the Study

Following on from this Introductory Chapter, Chapter (2) reviews consumer demand theory, the foundation of any study of the demand system. It covers the preference axioms of utility, properties of demand and cost functions, and the derivation of the demand system. The chapter focuses on three demand systems: Linear Expenditure System (LES), Almost Ideal Demand System (AIDS), and the Rotterdam models (RM). The chapter also covers demand elasticities. A review of existing empirical studies is provided. Various approaches are discussed. Demand elasticities and definitions of the price index are discussed. A comparison is made of different demand systems based on different measures.

Chapter (3) is devoted to data sources and explanations. It covers the definition of variables, data sources, sample size, and estimation methods.

Chapter (4) covers the specification of both the static and the dynamic LES models. The method of estimating the LES model is discussed. The parameter estimates obtained from the static LES and the dynamic LES models are presented. The chapter also contains elasticities estimates obtained from both the static LES and the dynamic LES model. A hypothesis test for "presence of habit" effect in the LES model is given in this chapter. Comparisons between the results obtained from static and dynamic versions are offered by way of conclusion.

Chapter (5) contains the estimation, discussions and analysis of the Rotterdam model and the dynamic Rotterdam version (Brown and Lee (1991). The model specification and estimation method is presented. This chapter includes estimates of the parameters of the model for each restriction implied by demand theory, estimation of prices, and expenditure elasticities for various commodity groups.

Chapter (6) presents the model specification and estimation method used for both the static and the dynamic version of the AIDS model. The chapter presents the main results,
discusses their implications, and tests the applicability of restrictions emanating from demand theory. It estimates elasticities, and interprets the results.

Chapter (7) provides a comparison of the models in the term of their empirical usefulness, basing the comparison on the data used in this study. The chapter outlines the different demand systems. Empirical results are reported. These include elasticity estimates for the various demand system models.

Chapter (8) contains three sections. In the first the concept of stationarity is presented. The statistical implications of findings a series to have non-stationarity are considered. Finally, there is an application of the Dickey-Fuller approach for testing for unit roots for each time series in the demand system. The second section of the chapter describes the concept of cointegration and its techniques. There is an application of the Engle Granger two steps. In the third section of the chapter a Full Information Maximum Likelihood (FIML) approach to cointegration is applied. This is followed by discussion and analysis of the results that have been obtained from these applications.

Chapter (9) is devoted to an estimate of the Engel Curve functional forms for Bahrain, using data from the household expenditure and income survey for the year (1994-1995). Classification and grouping of the data are discussed. Seven functional forms for estimating the Engel Curve are offered and briefly discussed. There is an examination of hypotheses, such as the effects of family composition and economies of scale on per capita consumption for each commodity group.

Chapter (10) discusses income distribution and poverty in Bahrain, with measures of inequality of income distribution. This is a little researched area, with data only now becoming available.

Chapter (11) provides a conclusion and wider implications of the thesis. This concluding Chapter is divided into 2 parts. In the first part I summarize the main findings of the
econometric tests carried out in Chapters 4, 5, 6, 7, 8, 9, and 10. In the second part I discuss the wider implications of the thesis.

I would argue that the work reported in this thesis adds to our understanding of consumer behavior in Bahrain. The work has been carried out with the support of the Bahrain Centre for Studies and Researches and should provide a valuable statistical resource for future activities.
References

Journals


Books

Chapter 2

Theoretical Approach and Demand Model Specification

2.1 Introduction

Since 1950's, the theory of consumer behavior has been widely accepted as a basic tool required for the construction and analysis of any demand system. The theory states that the consumer chooses a particular commodity basket yielding maximum satisfaction, subject to his spending ability (Theil (1975), Deaton and Muellbauer (1980a); Johnson, Hassan, and Green (1984)).

Demand theory suggests two approaches to demand analysis: direct specification demand models and utility-based demand models. The latter approach is preferred to directly specified demand models because the allocation issue can be presented within a utility maximization framework. (see Deaton and Muellbauer (1980a), Philips (1983), and Johnson, Hassan, and Green (1984)). The utility based demand approach “gives rise to elegant and intuitive interpretations of the coefficients of the demand equations in terms of the utility function” (Theil and Clements, 1987, p.2). This study focuses on utility-based demand models.

The chapter reviews consumer demand theory. First, the theory of utility maximization is presented. Duality in consumer demand theory is described. Second, restrictions on demand equations are put forward. Particular problems of aggregation over consumers and over commodities are discussed. Thirdly, various
functional forms of demand systems are presented. There are discussions of previous applications, followed by a conclusion.

2.2 The Microeconomic Theory of Consumer Behavior

The microeconomic theory of demand is a restricted approach to consumer choice. Given an opportunity set, there are specific preferences and tastes. Neo-classical theory states that from the opportunity set available to the consumer, he/she chooses the particular commodity basket highest in his ordering. Thus, consumer demand shows how people use their limited means to make purposeful choices. It assumes that consumers understand their choices (possibilities) and the prices (opportunity costs) associated with each choice. Moreover, it assumes that consumers consider the alternatives and choose the one that they like best.

The utility function is a theoretical tool and is not directly observable, but this choice becomes observable in the quantity demanded by consumer. Therefore, the theory of consumer demand tries to interpret the quantity in the basket purchased by the consumer. This interpretation can be realized by various techniques, which are as follows

2.2.1 Four Techniques for Acquiring Demand Functions

To acquire the demand functions there are four techniques, which are as follows:

2.2.1.1 The First Technique: Direct Utility Function

Consumer behavior theory is based on the assumption that a rational consumer will always try to maximize his utility function subject to the budget constraint, given income and prices.
"Rational" for the consumer means that all axioms of reflexivity, completeness, transitivity, continuity, non-satiation, convexity, differentiability hold constant for each consumer. The neoclassical axiomatic approach to the consumer problem gives a framework for explaining consumer preference and for the formulation of a consistent utility function of the form (2.1).

$$u = u(q_i) \quad \text{for } i=1, 2, \ldots, n.$$ (2.1)

where $$q_i$$ represents the quantity consumed of commodity i-th and $$u$$ is the total utility and reflects the preferences of an individual or household, and also it measures the level or degree of satisfaction that consumer achieves by consuming the bundle of goods and services ($$q_i$$). Equation (2.1) expressed utility in terms of quantities consumed and is called the direct utility function.

The consumer maximize his utility subject to the budget constraint

$$\sum p_i q_i = x$$ (2.2)

where $$p_i, q_i$$ are the price and quantity referred commodity i-th respectively, and $$x$$ is the total expenditure or income. The empirical studies of consumer behavior repeatedly use total expenditure as a proxy for income, because data on income are often not available or are subject to bias. Intriligator (1978, p.242) pointed out that "the elasticity for total expenditure with respect to income is close to unity"

Consequently, total expenditure is thought to be a good proxy of total income.

1 The definitions of axioms of choice are as follows: Reflexivity states that every bundle of goods is as good as itself. Completeness means that all bundles of goods are comparable. Transitivity, also called consistency, prevents the economic agent from making contradictory choices. If bundle of goods "X" is preferred to bundle "Y" and "Y" is preferred to "Z", then "X" is preferred to "Z". Continuity effectively rules out the use of a lexicographic scheme of preference ordering. An agent with lexicographic preferences would focus purely on the quantity of one or several highly valued goods (to the consumer and not to market) and would not consider the overall contents of the bundle. Non-satiation maintains that more of any good is better. Convexity maintains that the agent becomes less willing to give up goods that are relatively scarce within a bundle in order to maintain a constant level of satisfaction. Therefore convexity implies that as the economic agent moves down an indifference curve, the marginal rate of substitution, or rate at which the consumer is willing to give up one good for another, falls in absolute value. Differentiability, the utility function is assumed to be twice differentiable. This gives the utility function its smooth shape. The first derivative is the marginal utility or added satisfaction from consuming very small amount of particular commodity. The second-order partial derivatives of the utility function are the basis for the symmetric Hessian matrix. These axioms of choice lead to a set of preferences, which are the defining factors in the utility function.
Equation 2.2 is a linear budget constraint which implies that the consumer takes the prices of all goods as given and expenditure on \( n \) goods \( (p_1q_1 + p_2q_2 + \ldots + p_nq_n) \) must equal to a fixed total. In other words, the budget constraint measures the combinations of purchases that a person can afford to make with a given amount of monetary income.

The maximization of (2.1) subject to (2.2) leads to consumer demand function as follows.

\[
q_i = q_i(p_1, p_2, \ldots, p_n, x) \quad (2.3)
\]

The specification of this quantity \( q_i \) or the derivation of equation (2.3) can be done by the neo-classical approach, which involves the utilization of the lagrangean multiplier technique. To find the first-order conditions for maximum, we form the Lagrangean function.

\[
Max. L = L(q, \lambda) = u - \lambda(\sum_i p_iq_i - x) \quad (2.4)
\]

where \( \lambda \) is a Lagrangean multiplier and differentiation of (2.4) with respect to \( q_i \) and \( \lambda \), yields

\[
\frac{\partial L}{\partial q_i} = \frac{\partial u}{\partial q_i} - \lambda p_i = 0 \quad (2.4.1)
\]

\[
\frac{\partial L}{\partial \lambda} = x - \sum_i p_iq_i = 0 \quad (2.4.2)
\]

Simultaneously solving the above two equations (2.4.1, 2.4.2) will provide the \( n \) optimal values of \( q_i \) and the equilibrium value of \( \lambda \). Hence, the solution will provide us a complete system of demand equations. The resulting expressions are

\[
q_i = q_i(p_1, p_2, \ldots, p_n, x) \quad (2.4.3)
\]

\[
\lambda = \lambda(p_1, p_2, \ldots, p_n, x) \quad (2.4.4)
\]

The maximization of utility function (2.1) subject to budget constraint (2.2) is known as the primal approach to obtaining demand functions.
As can be seen from equations (2.4.3, 2.4.4) that these are functions of all prices and total expenditure or income. The lagrangean multiplier $\lambda$ is frequently explained as the marginal utility of total expenditure ($x$), or, in other words, it is the increase in maximum utility obtainable per unit in $x$.

Thus, the equation obtainable (2.4.3) is called Marshallian demand functions, and implies that the $i$-th commodity is a function of its own price, prices of other goods and income (Deaton and Muellbauer, 1980b). This equation describes the behavior of the consumer in the market.

To sum up, the neoclassical axiomatic approach to consumer problem provides a framework for describing consumer preference which leads to the formulation of consistent utility functions of the form of Equation 2.1 where the consumer demand functions are expressed as a system of all prices and total expenditure or income.

2.2.1.2 The Second Technique: Indirect Utility Function

The utility function (2.1) is formulated in terms of quantities. The demand equation (2.3) gives the optimal quantities in terms of total expenditure and prices. Substituting this demand equation (2.3) for the quantity in the utility functions (2.1) yields

$$\psi = u(q_i(p_1, p_2, \ldots p_n, x)) = \psi(p, x)$$

(2.5)

The function $\psi$, the argument of which is not quantities but expenditure and prices, is known as the indirect utility function. Equation (2.5) specifies the maximum level of utility, that can be attained for a given set of prices ($p$) and particular expenditure ($x$). This is the level of utility attained by the consumer when he purchased the quantities $q_i$, which are specified by the demand function (2.3).
To obtain Marshallian demand functions for the $i$-th commodity from a specified indirect utility function, it requires utilizing Roy's identity (1942), that involves only differentiating of indirect utility function with respect to total expenditure and prices. This can be seen as follows:

Firstly: Differentiating indirect utility function with respect to the total expenditure (income).

$$\frac{\partial u}{\partial x} = \sum_{i=1}^{n} \frac{\partial u}{\partial q_i} \frac{\partial q_i}{\partial x}$$  \hspace{1cm} (2.5.1)

As $\frac{\partial u}{\partial q_i} = \lambda p_i$ from the constrained utility maximization (2.4.1), and since

$$\sum_{i=1}^{n} p_i \frac{\partial q_i}{\partial x} = 1$$

from differentiating the budget constraint with respect to $x$.

$$(p_1 \frac{\partial q_1}{\partial x} + p_2 \frac{\partial q_2}{\partial x} + \ldots + p_n \frac{\partial q_n}{\partial x} = 1).$$

Therefore, Equation 2.5.1 can be written in the following form

$$\frac{\partial u}{\partial x} = \lambda \sum_{i=1}^{n} p_i \frac{\partial q_i}{\partial x} = \lambda$$  \hspace{1cm} (2.5.2)

Secondly: Differentiating indirect utility function with respect to the prices

$$\frac{\partial u}{\partial p_i} = \sum_{i=1}^{n} \frac{\partial u}{\partial q_i} \frac{\partial q_i}{\partial p_i}$$  \hspace{1cm} (2.5.3)

As $\sum_{i=1}^{n} p_i \frac{\partial q_i}{\partial p_j} = -q_j$ for $j=1, 2, \ldots, n$. This can be achieved by differentiating the budget constraint with respect to $p_j$:

$$(p_1 \frac{\partial q_1}{\partial p_j} + p_2 \frac{\partial q_2}{\partial p_j} = -q_j)$$

---

2 Roy derives his identity in the following way. At equilibrium, we must have $du = 0$ and $\sum_{i} x_i dp_i = dy$.

$$\frac{df}{dp_1} + \frac{df}{dp_2} + \ldots + \frac{df}{dp_n} dp_n = - \frac{df}{dy} dy$$

and

$$x_i dp_i + x_2 dp_2 + \ldots + x_n dp_n = dy$$

which implies

$$\frac{\partial f^*}{x_i} = \frac{\partial f^*}{x_2} = \ldots = \frac{\partial f^*}{x_n} = - \frac{\partial f^*}{dy} dy$$

or

$$x_i = - \frac{\partial f^*}{\partial p_i} \frac{\partial p_i}{\partial y}$$

Owing $f^*$ has been specified, it suffices to apply this identity to obtain the demand functions. Where $u^* = f^*(p_1, \ldots, p_n, y)$ is indirect utility function, has prices ($p_i$) and income ($y$) as arguments. $u^*$ represents the highest utility that may be obtained with alternative (givens) prices and incomes. $x_i^0$ being optimal quantities that maximize $u$. See Philips (1974, p.29).
Hence, \( \frac{\partial \psi}{\partial p_i} = -\lambda q_i \), for \( i = 1, 2, \ldots, n \) (2.5.4)

By dividing Equation 2.5.2 by 2.5.4 obtains,

\[ q_i = -\frac{\partial \psi / \partial p_i}{\partial \psi / \partial x} \] (2.5.5)

which is an explicit expression for optimal quantities \( (q) \). This result is due to Roy's identity.

2.2.1.3 The Third Technique: Cost Function

In production theory the cost function is utilized. This function has output and prices of inputs and states the minimum cost of producing a given output at those input prices. Selecting inputs to maximize output for any given input cost must produce that output in the cheapest possible manner. Hence, cost minimization and output maximization for a fixed outlay gives the same level of output.

The consumer choice problem can be treated in the same manner. So far, it has been assumed that the consumer's problem is that of choosing quantity \( q_t \) in order to maximize utility subject to the budget constraint. However, it may be reformulated by choosing quantities which to minimize the total expenditure necessary to acquire a given utility \( (u) \). These two problems are frequently referred to as the dual problem for the following reason. If the former is solved for given total expenditure \( (x) \), it leads to a set of quantities demand and a maximum utility level \( u \). Also, if the given utility level in the latter problem is set equal to \( u \), the solution of the latter problem guide, first to the same set of quantities demand (which implies the same choice) as does the solution to the former problem and secondly, to minimum total expenditure level \( (x) \) equal to the given \( x \) in total expenditure \( (x) \) in the former problem. This methodology is interpreted as the dual problem (Diewert, 1974).

The consumer cost function is dual to the (direct) utility function in that it gives the minimum expenditure required to reach a specific level of utility for given prices. That is
Minimize \( x = \sum_{i=1}^{n} p_i q_i \) \hspace{1cm} (2.6)

Subject to \( u = u(q_1, q_2, ..., q_n) \) \hspace{1cm} (2.7)

The solution of the dual problem is known as the Hicksian demand function or compensated demand function, written as \( h(u, p) \). It states how \( q \) is affected by prices with \( u \) held constant, hence the name compensated.

Again the neo-classical traditional approach which involves the application of the Lagrangean multiplier can be used to obtain the optimality level of \( q \). That is,

\[
L = \sum_{i=1}^{n} p_i q_i - \lambda (u(q_i) - u) \tag{2.8}
\]

\[
\frac{\partial L}{\partial q_i} = p_i - \lambda \frac{\partial u(q_i)}{\partial q_i} = 0 \tag{2.8.1}
\]

\[
\frac{\partial L}{\partial \lambda} = u - u(q_i) = 0 \tag{2.8.2}
\]

Simultaneously solving the Equation 2.8.1 and 2.8.2 lead to Hicksian demand function \( h(u, p) \); this can be done by inventing this solution obtainable into the objective function for the minimization problem. Equation 2.6 above gives the cost function, or expenditure function, that is

\[
q_i = h_i(u, p) \quad \text{for } i=1, 2, n \tag{2.8.3}
\]

and we have

\[
x = \sum_{i=1}^{n} p_i q_i \tag{2.8.4}
\]

Thus, solutions can be substituted back into their respective problems to give minimum attainable cost function. Hence,

\[
x = \sum_{i=1}^{n} p_i h_i(u, p) = C(u, p) \tag{2.9}
\]

The cost function (2.9) defined as the minimum cost of obtaining the utility level \( u \) at given prices \( p \), and is known as the cost or expenditure function.

Clearly, the cost function and indirect utility function are intimately related, reflecting alternative ways of expressing the same information. Therefore, since \( C(u, p) = x \), it can be rearranged or inverted to give \( u \) (indirect utility) as
function of $x$ and $p$, which is a function of expenditure and price (Deaton and Muellbauer, 1980b). Therefore, the cost function can be always be inverted to yield the indirect utility function and vice versa (Thomas, 1987). Once the indirect utility function is obtained it can be substituted into the Hicksian demand functions to obtain ordinary or Marshallian demand functions from this dual approach. Figure (1) summarizes the duality approach in consumer theory and a convenient way of deriving demand functions.

**Figure (1): Duality in Consumer Theory.**

Primal problem
\[
\max u = v(q) \text{ s.t. } pq = x
\]

Solve for first order condition

Marshallian demand functions
\[
q_i^* = g_i(x, p) \text{ for } i = 1, 2, \ldots n
\]

Indirect utility function
\[
u = \psi(x, p)
\]

Dual problem
\[
\min x = pq \text{ s.t. } u = v(q)
\]

Solve for first order condition

Hicksian demand functions
\[
q_i^* = h_i(u, p) \text{ for } i = 1, 2, \ldots n
\]

Expenditure function
\[
x = c(u, p)
\]

Invert

\(\text{A): Substitute } q_i^* \text{ into the utility function}\)

\(\text{B): Roy's identity } q_i = - (\frac{\partial u}{\partial p_i}) \frac{\partial u}{\partial x}\)

\(\text{C): Substitute } q_i^* \text{ into the dual objective function}\)

\(\text{D): Shepard's lemma } q_i = \frac{\partial c}{\partial p_i} .\)

The properties of defining this cost function should be given particular attention.

---

2.2.1.3.1 Properties of the Cost Function

Any expenditure function that is consistent with utility maximization has the following properties, namely:

Property (1): The cost function is homogenous of degree unity. A doubling of prices, will double the total expenditure necessary to attain a given utility level.

Property (2): The cost function is increasing in $u$ implies that the consumer has to spend more to attain a higher level of utility. Non-decreasing in $p$ means that the consumer will have to spend at least as much to maintain a level of utility if the prices increase.

Property (3): The cost function is concave in prices. Concavity implies that as prices increase, cost rises no more than linearly. This is because the consumer minimizes costs, rearranging purchases in order to take advantage of changes in the structure of prices (see Deaton and Muellbauer, 1980b).

Property (4): The cost function is continuous in $p$. Continuity of the cost function ensures that the first and the second derivatives with respect to $p$ exist.

Property (5): Several authors, including Deaton and Muellbauer (1980b) indicate that where they exist, the partial derivatives of the cost function with respect to prices are the Hicksian demand functions, that is

$$\frac{\partial C(u,p)}{\partial p} = h_i(u,p) = q_i \tag{2.10}$$

The properties of cost function have been explained and offered by Deaton and Muellbauer (1980b).

2.2.1.4 The Four Technique: Direct Specification of the Demand Function

A final approach for obtaining a demand function consists of establishing a complete system of demand equations or expenditure functions directly and imposing restrictions as constraints in regressions, as done by Barten (1967, 1969) and Byron (1970a and b). This approach has developed entirely since the 1960's, and it involves the simultaneous estimation of the complete systems. The contains demand equations for every commodity group purchased by consumers. The emergence of such an
approach can be explained in both theoretical and empirical terms. Theoretically, the demand for any commodity depends on the prices of all commodities. Data limitations, however, makes it impossible to include all prices in any empirical demand equations. However, the theory of consumer behavior provides a series of restrictions, which the equations of a complete system must theoretically satisfy.

The merit of the new approach is that no maximization is needed to be solved. The problem with such an approach is that, it requires consistency with utility maximization for each consumer.

2.3. General Restrictions on Demand Equations

This section is devoted to the properties of demand equations obtained by utility maximization. These properties take the form of mathematical restrictions on the derivatives of demand functions. They rely on these restrictions to reach a result.

The major advantage of applying these restrictions is to reduce the dimensionality of the problem and to deal with limitations of available data (Barten, 1977). In principle there are four basic restrictions on demand equations. These are as follows:

2.3.1 Adding-up Restriction

The adding up restriction is satisfied by the derived demand functions from particular utility function. Adding-up maintains that the total value of the Marshallian demand function is equal to the total expenditure, that is

\[ \sum_{i=1}^{n} p_i q_i = x \quad \text{for } i=1,2,\ldots, n \]  

(2.11)

The adding-up restriction can be classified into two components: - The first component is called the Engle aggregation condition which can be obtained by differentiating the budget constraint (2.11) with respect to the total expenditure, i.e.

\[ \sum_{i=1}^{n} p_i \frac{\partial q_i}{\partial x} = 1 \]  

(2.11.1)
The other component of the adding-up condition is called Cournot aggregation condition, which can be obtained by differentiating the budget constraint with respect to the price of the commodity while all other prices remain the same, that is

\[ \sum_{i} p_i \frac{\partial q_i}{\partial p_j} + q_j = 0 \quad \text{for } j=1,2,\ldots, n \]  

(2.11.2)

The changes in \( x \) and in \( p \) cause rearrangements in purchases that do not violate the budget constraint. Hence, the above two equations (2.11.1, 2.11.2) sometimes referred to as Engel and Cournot aggregation conditions respectively (Deaton and Muellbauer, 1980).

The two adding-up restrictions above arise mainly from the budget constraint \( \sum_{i=1}^{n} p_i q_i = x \) and these restrictions can be reformulated in terms of elasticities in order to utilize them as the restrictions on the parameterizes of demand equations. The procedure of obtaining these elasticities is as follows:

1. Differentiating the budget constraint with respect to \( x \) yields,

\[ \sum_{i=1}^{n} p_i \frac{\partial q_i}{\partial x} = \sum_{i=1}^{n} \left( \frac{\partial p_i q_i}{\partial x} \right) = 1 \]  

(2.11.3)

where \( p_i q_i \) is the expenditure on commodity \( i \)-th, \( \frac{\partial (p_i q_i)}{\partial x} \) is known as the marginal propensity to consume of commodity \( i \) or its marginal budget share. The marginal budget share (\( \theta \)), like the average budget share, \( w_i \), adding to unity. According to definition of income elasticity of demand which is, \( \eta_i = \frac{\partial q_i}{\partial x} \frac{x}{q_i} \), and hence, from this it can be shown that,

\[ w_i \eta_i = p_i q_i \frac{\partial q_i}{\partial x} \frac{x}{q_i} = p_i \frac{\partial q_i}{\partial x} = \frac{\partial (p_i q_i)}{\partial x} \]  

(2.11.4)

and hence,

\[ w_i \eta_i = \theta_i \]  

(2.11.5)

Since \( \sum \theta_i = 1 \), it follows from (2.10.5) that the sum of budget share weighted income elasticities is equal to unity. That is,
\[ \sum w_i \eta_i = 1 \]  

(2.11.6)

where \( w_i \) is the budget share of the i-th commodity \((w_i = \frac{p_i q_i}{x})\) and \( \eta_i \) is the elasticity of demand with respect to total expenditure or income. Thus, equation (2.11.6) implies that total expenditure (income) elasticities, weighted by their respective expenditure proportions (budget shares) add-up to unit.

The second restriction of adding-up is Cournot aggregation condition and it is concerned with the effect of a change in the price of the j-th commodity assuming other prices to be constant. This can be obtained by:

(2) Differentiating the budget constraint with respect to \( p_j \) gives,

\[ \sum_{i=1}^{n} p_i \frac{\partial q_i}{\partial p_j} = -q_j \]  

(2.11.7)

By multiplying the left hand side of equation (2.11.7) by \( q_i / q_j \), and both side by \( p_j / x \), yields,

\[ \sum_{i=1}^{n} p_i \frac{\partial q_i}{\partial p_j} \frac{p_j}{q_j} \frac{q_i}{x} + \frac{p_j}{x} q_j = 0 \]  

for \( i = 1, 2, \ldots, n \)  

(2.11.8)

And hence,

\[ \sum_{i=1}^{n} w_i e_{ij} = -w_j \]  

(2.11.9)

where \( e_{ij} \) is the cross elasticity of commodity i with respect to commodity j (which can be written as \( e_{ij} = \frac{\partial \log q_i}{\partial \log p_j} \)) and it is also known as uncompensated cross elasticities. Therefore, under uncompensated price changes, equation (2.11.9) states that the sum of all own and cross-price elasticities for the i-th commodity all weighted by the i-th budget shares, is equal to the negative of budget share of the j-th commodity. This is commonly known as the Cournot aggregation condition. Moreover, these above restrictions are obtained without any reference to the utility function and therefore must hold whether the consumer is a utility maximizer or not.
2.3.2 Homogeneity Restriction

The homogeneity restriction arises solely from the budget constraint. This restriction means that quantity demanded remains unchanged if all prices and income changes by the same proportion. This condition is sometimes referred to as the absence of money illusion (the consumer makes his decisions without regarding the monetary unit of account). Every demand equation must be homogenous of degree zero in income and prices. In other words, if all prices and income are multiplied by a positive constant \( k \), the quantity demanded must remain unchanged. In applied work, only those mathematical functions which have this property can be candidates for qualification as demand functions. It is easily seen that this restriction is the direct results of utility maximization. It suffices us to take a closer look at the first order conditions. Multiplying all prices by \( k \) in the n-1 equations

\[
\frac{\partial u}{\partial q_i} = \frac{p_i}{p}, \quad (2.12.a)
\]

we see that \( k \) drops out from the numerator and denominator of the price ration. As for the budget constraint, multiplying income and all prices by \( k \) we find (putting \( k \) before the summation sign).

\[
k\xi = \sum_i kp_i q_i = k\sum_i p_i q_i, \quad (2.12.b)
\]

Again \( k \) can be eliminated. All first order conditions remain unaffected. Phlips (1974, p.34) gave evidence from monetary history that illustrates the preceding exposition. Phlips (1974) mentioned that general De Gaulle's monetary reform, putting one hundred old French Francs equal to one new Franc. This is a change in the monetary unit and implies a multiplication of all prices and incomes by \( k = 1/100 \). The restriction says that the French should continue to consume exactly the same quantities after reform as though nothing happened. Consumers are supposed to have no 'money illusion': They should realize that a commodity priced '1 (new) French Francs' is not cheaper than a commodity priced '100 French Francs'. Phlips pointed out (1974, p.35) that on average this condition probably is realistic and has to be exactly fulfilled if we want to maintain that our computations produce estimates of derivatives or elasticities of demand functions. Clements et. al.
(1996, p.64) also reported that on the 13th February 1996 when decimal currency was introduced into Australia, all prices and money incomes doubled overnight; demand homogeneity assures us that this change in the unit of account would have had no effects on consumption. Further, the restrictions (adding-up, homogeneity, symmetry and negativity) of the demand theory are important in empirical studies ignoring these restrictions may result in failure of the empirical results to be consistent with the theory of consumer behavior. However, does observed demand behavior always satisfy the theoretical restrictions? Philips (1983, pp. 55-54) argues that the theory is a simplification of reality and statistical data always contain some measurement of error and, hence, they need not always satisfy these restrictions.

Formally, for positive number $k$ and, for all $i$ from 1 to $n$,

$$
g_i(kx, kp) = g_i(x, p) \quad (2.13)
$$

This restriction implies that the demand function is homogenous of degree zero, so equation (2.13) expresses the homogenous restriction and also the absence of money illusion, since the units in which prices and outlay are expressed have no effect on purchases. It is important to note that ordinary or Marshallian demand functions are homogeneous of degree zero in prices and income, while Hicksian demand functions are homogenous of degree zero in prices, that is for the same number $k > 0$

$$
h_i(u, kp) = h_i(u, p) = (g_i(kx, kp) = g_i(x, p) \quad (2.13)'
$$

The knowledge that all demand functions are homogenous of degree zero is not very useful as such. To make it operational, it requires to obtain it in term of elasticities, and this can be done by referring first to Euler's theorem (Layard and Walters 1978) which tell us that, if a function in form of $Z = k(x, y)$ is homogenous of degree $r$ then,

$$
x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = rz, \quad (2.14)
$$

Consequently, the derivation of demand function $q_i = q_i(p, x)$ can be achieved as follows.

$$
\sum_j p_j \frac{\partial q_i}{\partial p_j} + x \frac{\partial q_i}{\partial x} = 0 \quad (2.15)
$$
To express the above equation (2.10.2) in the terms of elasticities, it needs to divide equation (2.15) by $q_i$, which yields:

$$\sum_j \frac{p_j}{q_i} \frac{\partial q_i}{\partial p_j} + x \frac{\partial q_i}{\partial x} = 0$$
for $i=1, 2, n$  \hspace{1cm} (2.15.1)

$$\sum_{i=1}^n e_i + \eta_i = 0$$

The above equation (2.15.2) implies that the total of own price, all cross prices of commodity $i$-th, and the income elasticities must be zero. Therefore, homogeneity of demand functions implies that changes in relative prices are important in the consumer decision-making process, while proportional changes in the absolute level of prices and income should have no effect on the quantity demanded.

In conclusion, both the aggregation and homogeneity restrictions are properties of the budget constraint and are independent of whether the consumer maximizes utility.

2.3.3 Symmetry Restriction (Slutsky Symmetry)

The Symmetric restriction represents the stronger restriction of the demand equation. This restriction can be explained by first referring to the Slutsky equations. Slutsky (1915) pointed out that change in the price of any commodity could be divided into two main effects. First, there is the income effect and second there is substitution effect (Phillips, 1983). Slutsky studied the behavior of the consumer under prices changes. His requirement was to keep the consumer as before the price change. He expressed this by showing what happens when the price of one commodity changes, keeping all other prices and income constant, Slutsky indicated that the consumer’s real income would reduce in terms of that commodity. Accordingly, the consumer requires compensation in income so as to purchase the same commodity set as before the situation change (price). To simplify assume that the price of commodity $(i)$ rises by $dp_i$, therefore, the compensation is equal to

$$dy = q_i dp_i$$

(2.16)
where \( q_t \) is the quantity demanded before the price change, and \( dy \) is the change of income. According to this compensation the consumer will be overcompensated, due to the rise in the price of one commodity. The consumer might not buy the same set as before, but he would adjust to the new set of prices so as to rank highest in his ordering, in other words, to reach a higher level of utility than before. (Powell, 1974, p.5).

Under the compensated price change the variation is that,

\[
dq_t = \frac{\partial q_t}{\partial p_i} dp_i + \frac{\partial q_t}{\partial y} dy
\]

(2.16.1)

By inverting equation (2.16) into equation (2.16.1) and multiplying by \((1/ dp_i, )\) it gives:

\[
\left(\frac{dq_t}{dp_i}\right)_y = \frac{\partial q_t}{\partial p_i} + q_t \frac{\partial q_t}{\partial y}
\]

(2.16.2)

where \(\left(\frac{dq_t}{dp_i}\right)_y\) is the response of \( q_t \) to a compensated price change, evaluated at \( y' = y + dy \), (Phlips, 1974, p.41).

Hence, \(\frac{dq_t}{dp_i}\) is the substitution effect, which will be denoted to as \( S_u \) or \( S_y \).

Equation (2.16.2) can be rewritten as.

\[
\frac{\partial q_t}{\partial p_i} = \left(\frac{dq_t}{dp_i}\right)_y - q_t \frac{\partial q_t}{\partial y} = S_y - q_t \frac{\partial q_t}{\partial y}
\]

(2.16.3)

Equation (2.16.3) is the Slutsky equation, for the case where it is the commodity's own price changes. The term \(\frac{\partial q_t}{\partial p_i}\) is the uncompensated of a price change while \((- q_t \frac{\partial q_t}{\partial y})\) represents the income effect (the income compensation needed to maintain the given level of utility).

The Slutsky equation for the cross price change can be written as

\[
\frac{\partial q_t}{\partial p_j} = S_y - q_t \frac{\partial q_t}{\partial y}
\]

(2.16.4)

where \( S_y \) is defined as \( \frac{\partial q_t}{\partial p_j} + q_t \frac{\partial q_t}{\partial y} \)
Thus, the symmetry restriction may be obtained from Slutsky equations, since the matrix of substitution effects is, $S$, symmetric, that $S_{ij} = S_{ji}$, so this could be written as

$$
\frac{\partial q_i}{\partial p_j} + q_i \frac{\partial q_i}{\partial y} = \frac{\partial q_j}{\partial p_i} + q_j \frac{\partial q_j}{\partial y}
$$

all $i \neq j$ \hspace{1cm} (2.16.5)

Equation (2.16.5) implies that the total substitution effect of unit change in $p_j$ on $q_i$ hold utility constant is equivalent to the total substitution effect of unit change in $p_i$ on $q_j$, also keeping utility constant. The above restrictions can be rewritten in the terms of elasticities as follows

$$w_i(e_{ii} + \eta_i w_i) = w_j(e_{jj} + \eta_j w_i)$$

all $i \neq j$ \hspace{1cm} (2.17)

Thus, Slutsky symmetry means that the compensated cross substitution effects are equal. The symmetry condition means that the consumer makes a consistent choice. For instance, an income compensated increase in the prices of food should lead to an increase in the consumption of clothing equal to the amount that food consumption is increased when an equivalent income compensated price increase of clothing occur, provided the goods are substitutes.

2.3.4 Negativity Restriction

The final restriction is known as the negativity restriction. This restriction also relates to the substitution matrix and states that the elements should be such that the matrix as a whole should be negative semi-definite. Theoretically, this means that compensated price increases leads to lower demands for the goods involved. The condition is derived from the assumption of maximization of utility. This condition implies that the diagonal components of substitution Slutsky matrix are $(S_{ii} < 0)$ negative. The negativity restriction in terms of elasticities can be written as $e_{ii} < 0$, (where $i=1, 2, n$). This is the famous law of demand and implies that demand for a commodity always fall in response to a price increase which is accompanied by a compensating payment which maintains utility intact. A more, detailed discussion can be seen in Theil (1975), Deaton and Muellbauer (1980b), and Phlips (1982).
Thus, symmetry and negativity restrictions derived from the existence of consistent preferences, are not directly observable in Marshallian demand functions. Moreover, the negativity and symmetry properties of demand can be observed and examined only through the computation of the values in the Slutsky matrix. For instance, the value of $S_v$ is found to be negative, then a complementary relationship exists between goods $i$ and $j$. If $S_v$ is positive, then goods $i$ and $j$ are substitutes.

According to the above presentation, any specification of a demand equation system, which satisfies all the four basic general properties, must completely model consumer behavior. Philips (1983, pp.53-54) argues that the theory is a simplification of reality and statistical data always contain some measurement of error. Hence, the system does not always satisfy these restrictions.

Furthermore, it is clear that the imposition of these basic restrictions lead to a reduction in the number of independent parameters that have to be estimated. For instance, in an $n$ equation demand system without restrictions there will be $n^2$ price parameters and $n$ total expenditure parameters to be estimated. See Thomas (1993, p. 220). The aggregation restrictions reduce the number of independent response by $n+1$ The symmetry restrictions reduce the number by a further $n(n-1)/2$ (e.g. once $\delta_{33}$ has been estimated we also have to estimate of $\delta_{33}$). Thus, the number of independent responses can be reduced to $n^2 + n - (n+1) - n(n-1)/2 = (n(n+1)/2) - 1$. So, in the case of a 10 equations system, the number of independent responses is reduced from 110 to 54. Barten (1977) pointed out that $n$ must be small due to the imposition of symmetry restriction which creates $(n+1)(n+2)/2$ constraints, and hence when $(n)$ is big, this will lead to a greater number of constraints.

2.4 Remaining Issues.
In order to shift from theoretical concepts to empirical actuality, several adjustments to theory are frequently made. The empirical effects of such modifications should be
considered when analyzing consumer behavior. In this study two of these adjustments are used. These are as follows:

2.4.1 Aggregation over Consumers

The fact that available data usually refer to the combined expenditures of many households, raises two questions. The first is that: Does a 'macro demand equation' of exactly the same functional form as the micro-functions exist given a set of micro demand equations for individual consumers? The second question is that, Macro functions can be expected to satisfy restrictions derived from a theory that relates to a single consumer. Thus, even if the micro functions satisfy these restrictions, will such properties carry out to the macro-function? Only if the answer to this second question is 'yes' then the general theoretical restrictions on aggregate demand equations can be imposed or to aggregate data can be used to test the validity of such restrictions. In fact, the conditions under which the perfect aggregation of a system of demand equations is possible are the same as those derived for the single demand equation. Thomas (1993, p.228) notes that: "All households must have demand equations which are linear functions of total expenditure and the marginal propensity to spend on any good must be the same for all households"

That is, the \( hth \) household's demand for the \( ith \) good must be of the form

\[
q_{ih} = \alpha_{0h} + \alpha_{1h}x_h
\]

where \( x \) is the total expenditure of the \( hth \), while \( \alpha_{0h} \) and \( \alpha_{1h} \) are function of all prices. It assumed that all households face the same prices. The \( \alpha_{0h} \) functions vary across households in (2.18) but the \( \alpha_{1h} \) functions cannot as all households are assumed to have the same marginal propensities to spend, provided the general theoretical restrictions are satisfied by the equation (2.18). Thomas (1993) shows that they will also be satisfied by the macro equations obtained by aggregating the micro-equations. Thomas (1993, p.228) records: "Thus not only does linearity in total expenditures and identical marginal propensities enable sensible aggregate equations to be constructed but it also justifies the procedure of using such aggregate equations
to test theoretical restrictions which really only apply to the behavior of the individual household"  

Hence, the aggregation problem is obviously of a great importance in demand analysis. However the assumptions implied by equation (2.18) are very restrictive, implying that all Engel curves are linear and that the Engel curves of various households all have the same slope. However, most empirical studies of Engel curves have strongly suggested that they are non-linear, for example the Rotterdam model (see Barten, 1969). Moreover, with the exception of the LÉS, nearly all the demand systems that have been successfully fitted to data imply non-linear Engel curves. Blundell (1988) and Thomas (1993) pointed out if plausible forms for demand patterns could be found that can be sensibly aggregated, then not only can we deduce facts about individuals from aggregate data but we can expect aggregate models to make sensible prediction.  

Muellbauer (1975) and (1976) in fact demonstrated that it is possible to aggregate non-linear Engel curves, provided that aggregate demands are expressed as functions of "representative expenditure" rather than mean expenditure. An important special case of preference ordering which permit Muellbauer type aggregation is known as PIGLOG case. Muellbauer (1975, 1976) refers to the special case where representative expenditure is independent of prices and dependent only upon the distribution of household expenditures, as that of price independent, generalized linearity (PIGL) and a logarithmic version of PIGL is the PIGLOG case. Muellbauer records that if such aggregation is to be possible the Engel curves for the individual household must take the following form

\[ p_i q_i = \alpha_0 + \alpha_i \log(x_i/k_h) \]  

(2.19)

where \( k_h \) is a parameter that varies over households, it can be regarded as being determined by household's size and composition. The Engel curve represented by equation (2.19) may be aggregate to obtain a macro Engel curve expressing aggregate expenditure in terms of representative expenditure, defined as a weighted geometric
mean of individual \( (x_h/k_h) \). Provided the \( x_h's \) regularly change equiproporportionately, this representative expenditure will be proportionate to mean expenditure (Thomas, 1993, p.29). Therefore, the Muellbauer approach provides a manner of aggregating very plausible non-linear Engel curves.

2.4.2 Aggregation over Commodities

Econometric analysis of a complete demand system for whole items entering into a consumer's budget is not generally feasible because the number of parameters to be estimated becomes extremely large. Estimating such a great system causes problems of degrees of freedom and multicollinearity over price series. Thus, some type of aggregation of commodities into groups is needed. The concept of the separability of preferences clears up the problem of aggregation among commodities. This concept introduced independently by Leontief (1947) and Sono (1961) permits group of commodities to be considered separately in the utility function. The notation is that preferences provide a natural structuring of commodities so that preferences for some groups of commodities can be independent of preferences for other groups. The consumption set is partitioned into subsets, each including commodities that are closer substitutes or complements to each other than to commodities in other subsets. There are two kinds of separability; weak separability, and strong separability. Each kinds of separability can be stated in terms of marginal rate of substitution (MRS), the functional form of the utility function, or in terms of substitution matrices \( (S_y) \).

Preference structure has strong separability if the marginal rate of substitution (MRS) between two commodities \( i \) and \( j \) belonging to two different groups \( I \) and \( J \), respectively, is independent of the consumption of commodity \( k \) which does not belong to group \( I \) or \( J \). That is,

\[
\frac{\partial(u_i/u_j)}{\partial q_k} = 0 \quad \text{for all } i \text{ in } I, \ j \text{ in } J \text{ and } k \text{ is not in } I \text{ and } J
\]
where \( u_i \) and \( u_j \) represent the marginal utilities of commodity \( i \) and \( j \), respectively, in commodity group \( I \) and \( J \), and \( \frac{\partial u_i}{\partial u_j} \) is the MRS between commodities \( i \) in \( I \) and \( j \) in \( J \). It should be noted that under this kind of separability, \( i, j \) and \( k \) are all in different groups. A strongly separable utility function is of the form

\[
U = u_1(q_1) + u_2(q_2) + \ldots + u_n(q_n)
\]

This form is most popularly used and yet it is the most restrictive since it assumes that preferences are strongly separable. Equation (2.20) represents preferences that are strongly or additively separable and the latter would occur only when groups were broad aggregates. Such as food, health, clothing, etc. (Phillips, 1983, p.70).

Preferences, in general, are weakly separable if the MRS between any two commodities, \( i \) and \( j \) belonging to the same group is independent of the quantity consumed in any other group. That is:

\[
\frac{\partial (u_i / u_j)}{\partial q_k} = 0 \quad \text{for all } i, j \text{ in } I; k \text{ not in } I.
\]

where \( u_i \) and \( u_j \) represent the marginal utilities of commodity \( i \) and \( j \), respectively, in commodity group \( I \), \( q_k \) is the quantity of commodity \( k \) belonging to \( k \) and not belonging to \( I \).

Moreover, since commodities related in consumption as substitutes or complements are always grouped together, separability provides the justification for utilization of price or quantity index to represent a single price or quantity for the entire group. However, Thomas (1987, p.66) mentioned that assumption of strong separability has a restrictive implication to commodity groups. For example, commodity groups cannot be inferior and complementarily between groups, is ruled out.

### 2.5 Various Functional Form of Demand System

Economists have constructed demand relationships as utility functions with specific structures. These functions are frequently defined in light of axioms and properties.
This section is devoted to the different functional forms of demand system. The first sub-section presents the static demand system. The second-sub section considers the additive demand system. The final sub-section is devoted to the discussion of the dynamic demand system.

2.5.1 Static Demand System

This sub-section discusses three main demand systems, which can be examined according to the concepts expressed in the previous section. These are the Linear Expenditure System (LES), the Almost Ideal Demand System (AIDS), and the Rotterdam System (RS). These systems will be analyzed in a static context, and assume that the consumer adjusts immediately when prices and income change.

2.5.1.1 The Linear Expenditure System (LES)

The best known approach, based on a well-defined functional form for the utility function, is the Linear Expenditure System (LES). The LES was introduced by Kelin and Rubin (1947-48) in an attempt to construct a true cost of living index. Its economic interpretation was clarified by Samuelson (1947-48), and was developed by Geary (1950-51) and Stone (1953-54), who showed that the linear demand functions which make-up the LES can be integrated to obtain the unique utility function of the following form

\[ u = \sum B_i \log(q_i - \gamma_i) \]  

(2.21)

where the \( q_i \)'s are quantity flows and the \( B_i \)'s and \( \gamma_i \)'s are parameters such that \( 0 < B_i < 1 \) and \( \sum B_i = 1 \), and the function being defined only for \( 0 < (q_i - \gamma_i) \).

Equation (2.21) is called the Stone-Geary or Klein-Rubin utility function.

The LES is, however, derived from an additive utility function which means that the utility provided by the consumption of one commodity is not influenced by the consumption of any other commodity. This may also be unrealistic in general, but the additivity assumption might be a realistic assumption for large commodity groups such as, food, housing, health, etc. (Houthakker (1960) and Phlips (1974)).
The authors derived the LES from the maximization of the above utility function subject to the budget constraint, and this can be seen as follows:

Applying the Lagrangean functions that is,

\[ \text{Max}L = L(q, \lambda) = \sum B_i \log(q_i - \gamma_i) + \lambda(x - \sum p_i q_i) \]  

(2.21.1)

The first order conditions are

\[ \frac{\partial L}{\partial q_i} = \frac{B_i}{q_i - \gamma_i} - \lambda p_i = 0 \quad \text{for } i = 1, 2, n \]  

(2.21.2)

and

\[ \frac{\partial L}{\partial \lambda} = x - \sum p_i q_i = 0 \]  

(2.21.3)

From the Equation 2.21.2, \( B_i \) and \( \lambda \) can be derived as follows:

\[ B_i = \lambda p_i (q_i - \gamma_i) \quad \text{for } i = 1, 2, \ldots, n \]  

(2.21.4)

and hence,

\[ \lambda = \frac{B_i}{p_i (q_i - \gamma_i)} \]  

(2.21.5)

Summing \( B_i \) over \( i \) and incorporates the normalization condition \( (\sum B_i = 1) \), yields:

\[ \sum \lambda B_i (q_i - \gamma_i) = 1 \]  

(2.21.6)

and hence,

\[ \lambda = \frac{1}{x - \sum p_i \gamma_i} \]  

(2.21.7)

Solving Equation (2.21.5 and 2.21.7) together yields the LES, which could be written in the following form,

\[ p_i q_i = p_i \gamma_i + B_i (x - \sum p_i \gamma_i) \quad i = 1, 2, \ldots, n \]  

(2.22)

where \( q_i, \gamma_i > 0, \quad 0 < B_i < 1, \) and \( \sum B_i = 1 \)

Equation 2.22 describes the expenditure on the i-th commodity, as a linear function of income \( (x) \) and the \( n \) prices \( (p) \). An interpretation of Equation 2.22 as suggested by Samuelson (1947-48) is as follows: At the beginning of a period, the consumer purchases \( \gamma_i \) units of i-th commodity, this is the minimum purchase required of the commodity at each period without regard to prices.
For all $n$ commodities this amounts to $\sum_{k} p_k q_k$. The remaining amount of income after this expenditure is $x - \sum_{k} p_k q_k$, which is called supernumerary expenditure or income, and it is allocated among the $n$ commodities in fixed proportions $B_i (B_1, B_2, \ldots, B_n)$. Accordingly, at given prices, the sum of $p_i y_i$ measures subsistence income level and the $B_i$ is the marginal budget share $\left( \frac{\partial p_i q_i}{\partial x} \right)$.

The demand function can be obtained by dividing both sides of Equation 2.22 by prices ($p$), the result of which is

$$q_i = y_i + \frac{B_i}{p_i} (x - \sum_{k=1}^{n} p_k y_k) \tag{2.22.1}$$

The above function (2.22.1) is called ordinary Marshallian demand function and it is a function of all prices ($p$) and income ($x$) (Deaton, 1975b, ch.3).

The functional form of the LES, which was derived from the Stone-Geary utility function (2.21), automatically satisfies the demand properties. To achieve adding-up restriction, it requires multiplying both sides of Equation 2.22.1 by $p_i$ and the sum over $i$ yields,

$$\sum_{i} p_i q_i = \sum_{i} p_i y_i + \sum_{i} B_i (x - \sum_{k=1}^{n} p_k y_k) = x \tag{2.22.2}$$

Since, the normalization restriction ($\sum_{i} B_i = 1$) is obtained by equation (2.21), the adding-up restriction is satisfied automatically.

To satisfy the homogeneity property required to utilize the Euler's theorem, the following derivatives for Equation 2.14 could be considered

$$\frac{\partial q_i}{\partial p_i} = -\frac{B_i x}{p_i^2} + \frac{B_i}{p_i} \sum_{j=1}^{n} p_j y_j \tag{2.22.3}$$

$$\frac{\partial q_i}{\partial p_j} = -\frac{B_i}{p_i} y_j \tag{2.22.4}$$

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\[ \frac{\partial q_i}{\partial x} = \frac{B_i}{p_i} \]  

(2.22.5)

By substituting equations 2.22.3 to 2.22.5 into Equation 2.14 the homogeneity property is satisfied directly.

Lastly, the symmetry restriction of the Slutsky states that the cross substitution effects must be equal for any two commodities. The Slutsky equation requires

\[ \frac{\partial q_i}{\partial p_j} + q_i \frac{\partial q_j}{\partial x} = \frac{\partial q_i}{\partial p_i} + q_i \frac{\partial q_i}{\partial x} \quad \text{all } i \neq j \]  

(2.23)

Substituting Equation 2.22.4 and 2.22.5 into Equation 2.23 yields,

\[ -\frac{B_i}{p_i} \gamma_i + q_i \frac{B_j}{p_j} = -\frac{B_j}{p_j} \gamma_j + q_i \frac{B_i}{p_i} \]  

(2.24)

and hence from Equation 2.24, the substituting for \( q_j \) and \( q_i \) from Equation 2.22.1 result in the equality

\[ \frac{-B_i}{p_i} \gamma_i + \frac{B_j}{p_j} \gamma_j + \frac{B_i B_j}{p_i p_j} (x - \sum_k p_k \gamma_k) \]  

(2.24.1)

\[ \frac{-B_j}{p_j} \gamma_j + \frac{B_j}{p_j} \gamma_j + \frac{B_i B_j}{p_i p_j} (x - \sum_k p_k \gamma_k) \]  

(2.24.2)

Moreover, the LES is a very flexible structure; it can be obtained from the three main methodology, direct utility function, indirect utility function and cost function. These functions are as following (Deaton, and Mullbeauer, 1980b, p.65).

\[ v(q) = \prod_i (q_i - \gamma_i)^{k_i} \]  

(2.25)

\[ \psi(x,p) = \frac{(x - \sum_i p_i \gamma_i)}{\prod p_i^{r_i}} \]  

(2.26)

\[ C(u,p) = \sum_i p_i \gamma_i - u \prod p_i^{r_i} \]  

(2.27)

The derivation of the demand equations from the above three approaches can be seen clearly in appendix (A).
According to equation (2.25), the utility function is only defined for all values of \( q_i \geq \gamma_i \), and for the non-satiety axiom of demand theory to be satisfied, it is necessary that,

\[
\frac{\partial u}{\partial q_i} = \frac{B_i}{q_i - \gamma_i} > 0
\]

(2.28)

Hence, \( \frac{1}{q_i - \gamma_i} > 0 \) is positive, which means that \( q_i > \gamma_i \), it also requires \( B_i \) being positive. The positiveness of \( B_i \) result in other implications that are revealed and best interpreted through the elasticities of the system.

The demand elasticities are derived from Equation 2.22.3 to 2.22.4.

- The own price elasticity, \( e_u \), of the i-th commodity is given by:

\[
e_u = -1 + (1 - B_i) \frac{\gamma_i}{q_i}
\]

(2.29)

Since \( q_i > \gamma_i > 0 \) and \( 0 < B_i < 1 \), it follows \( -1 < e_u < 0 \) Thus, all price elasticities are negative.

- The cross-price elasticities, \( e_y(i \neq j) \) are given by the form

\[
e_y = -\frac{B_j \gamma_j}{p_j q_j}
\]

(2.30)

Since, \( B_i, \gamma_i, p_i, p_j, \) and \( q_i \) are all positive values, all uncompensated cross-price elasticities are negative. This implies that all commodities are gross complements.

However, the Slutsky cross-substitution terms are defined as

\[
\frac{\partial q_i}{\partial p_i} + p_i \frac{\partial q_i}{\partial x_i}
\]

and is equal to \( \frac{B_i}{p_i} (q_i - \gamma_i) = \frac{\partial q_i}{\partial p_i} \) which is positive in value and results in a compensated cross price elasticities.

\[
e_y = \frac{B_i p_j}{p_j q_j} (q_j - \gamma_j)
\]

(2.31)

\[
= \frac{B_i p_j q_j}{p_j q_i} + e_u = B_i \frac{w_j}{w_i} + e_u
\]

Or

\[
= \eta_i w_j + e_y \quad \text{for } i \neq j
\]

(2.32)
where, \( w_i, w_j \) represent the budget shares of commodity \( i \) and \( j \) respectively, and 
\( \eta \), refers to elasticity of total expenditure \( \left( \frac{B_i}{w_i} = \eta \right) \).

Since, \( B_i, w_i, \) and \( w_j \) are all positive and \( e_{ij} \) is negative, so as for \( e_{ii}^* \) to be positive, it requires the income effect \( B_i \frac{w_i}{w_i} \), must be bigger than substitution effect, \( e_{ii} \).

Moreover, the positive cross-substitution term implies that all pairs of goods (i and j) are substitutes for each other.

To sum up, the positiveness of \( B_i \)'s restricts all goods to be gross complements and net substitutes the LES. Moreover, the compensated cross price elasticities, \( e_{ij}^* \) are all positive, but the uncompensated cross-price elasticities, \( e_{ij} \) are all negative.

Added to that, all price elasticities of demand in the LES are less than unity in absolute value, unless some of the parameters \( \gamma_i \) are allowed to be negative.

- The compensated own-price elasticity is defined as

\[
\begin{align*}
\epsilon_i^* &= \left( \frac{(1-B_i)-q_i}{p_i} + q_i \frac{B_i}{p_i} \right) \frac{p_i}{q_i} \\
&= \epsilon_i + B_i \\
&= \epsilon_i + \eta_i w_i
\end{align*}
\]

(2.33) \hspace{1cm} (2.34) \hspace{1cm} (2.35)

Since, \( B_i \) is positive, the compensated own price elasticity will have smaller demand elasticity than the uncompensated own-price elasticity due to\( \epsilon_i^* \), which will be less negative than\( \epsilon_i \).

- Finally, the total expenditure elasticity is defined as

\[
\eta_i = \frac{\partial q_i}{\partial x} \frac{x}{q_i} = \frac{B_i}{p_i} \frac{x}{q_i} = \frac{B_i}{w_i}
\]

(2.36)

The income elasticities are all expected to be positive since the \( B_i \)'s are positive, so, any rises in income or total expenditure, is allocated to each commodity in constant proportion. Also, equation (2.36) states, that there is an inverse relationship between expenditure elasticity and budget share (\( w_i \)).
Moreover, there is an approximate proportional relationship between the own price elasticity and expenditure elasticity, that is known as Pigou's Law, since it was first put forward by Pigou (1910). It is defined as:

\[ e_u = \vartheta \eta_i \quad \text{for } i = 1, 2, \ldots, n \quad (2.37) \]

where \[ \vartheta = \frac{(x - \sum_{i=1}^{n} p_i q_i)}{x} \]

The derivation and description can be seen in Appendix (B).

This approximation arises from the assumption of additivity embodied in the LES. It implies that the response to variation in total expenditure is approximately proportional to the response of a change in prices for commodity. Since, there is an inverse relationship between the expenditure elasticity and the budget shares; this inverse relationship extends to the price elasticity. Deaton (1975b), mentioned that the existence and relative inflexibility of this relationship is limited to the LES more than other conditions.

LES implies strong separability and, therefore additive preferences. According to the theoretical aspect, the LES has the advantage of reducing the number of parameters to be estimated and also being a useful approach for estimating price elasticities on data with little or no price variation. However, the LES model has several disadvantages in using it as the specified demand model include the following: 1) \( B > 0 \), implies that no inferior goods are allowed; 2) all goods are Slutsky substitutes and 3) a proportionality between income and price elasticities as mentioned above are embedded in the specification. Notwithstanding the LES has been applied to many data sets representing consumption activities in various countries.

2.5.1.2 The Rotterdam System (RM).

The differential demand model, developed by Barten (1964) and Theil (1965), is based not on a particular utility function but more generally, on a first-order approximation to the demand functions themselves. Barten (1967) defined (RM), by
giving considerations to the Slutsky equations (2.16.3), and forming the following expression for the differential of the demand equation derived by the utility maximization, i.e. \( q_i = q_i(p_1, ..., p_n, x) \) yields

\[
dq_i = \sum \frac{\partial q_i}{\partial p_j} dp_j + \frac{\partial q_i}{\partial x} dx
\]  

(2.38)

Substituting Equation 2.16.3 into Equation 2.38 gives

\[
dq_i = \sum_j (s_{ij} - \frac{\partial q_i}{\partial p_j}) dp_j + \frac{\partial q_i}{\partial x} dx
\]  

(2.39)

Equation 2.39 can be rewritten in the following form:

\[
dq_i = \sum_j s_{ij} dp_j + \frac{\partial q_i}{\partial x} (dx - \sum_j q_i dp_i)
\]  

(2.40)

Utilizing the expression that \( dy = yd\log y \) yields

\[
q_i d\log q_i = \frac{\partial q_i}{\partial x} (xd\log x - \sum_j p_j q_i d\log p_j) + s_{ij} p_j d\log p_j
\]  

(2.41)

By multiplying both side of Equation 2.41 by \( \frac{p_i}{x} \) results Equation 2.42:

\[
\frac{p_i}{x} q_i d\log q_i = \frac{p_i}{x} \frac{\partial q_i}{\partial x} (xd\log x - \sum_j p_j q_i d\log p_j) + \frac{p_i}{x} \sum_j s_{ij} p_j d\log p_j
\]  

(2.42)

Since, \( w_i = \frac{p_i q_i}{x}, \beta_i = p_i (\frac{\partial q_i}{\partial x}), k_{ij} = s_{ij} (\frac{p_i p_j}{x}) \), then Equation 2.42 can be written in the following form:

\[
w_i d\log q_i = \beta_i (d\log x - \sum_k w_{ik} d\log p_k) + \sum_j k_{ij} d\log p_j
\]  

(2.43)

Equation 2.43 was developed by Barten (1967) (1969) and Theil (1965), (1975) and represents the Rotterdam system model and deals with the changes in, rather than the levels of demand.

The most practical form to utilize for estimation and the more recognizable form of the RE is the first difference form which shows the changes in the logarithms of quantities, prices and total expenditure from one period to the next, and hence, Equation 2.43 can be written in the following form,

\[
w_i \Delta \log q_i = \beta_i (\Delta \log x - \sum_k w_{ik} \Delta \log p_k) + \sum_j k_{ij} \Delta \log p_j
\]  

(2.44)
where $w'_u = \left( w_u + w_{u-1} \right) / 2$, $\Delta q_i = \log q_{it} - \log q_{i,t-1}$ and the $B_i$ and $k_v$ in Equation 2.44 have to be regarded as constants or parameters, and then the system has the greater advantage that all the general theoretical restrictions remain unchanged for all values of total expenditure, prices and hence for every observation in a sample. They are also of very simple form. The Rotterdam model, then, appears to be a convenient and powerful instrument for the testing of consumer theory. It has the ability to model the whole substitution matrix, the parameters of the model could be easily related to underlying theoretical restrictions, linear in parameters and, consequently, easy to estimate economically (Mountain). However, the weakness of the Rotterdam model that has constant marginal shares, a defect, which is shared with LES as discussed above.

2.5.1.3 The Almost Ideal Demand System (AIDS).

The AIDS was originated by Deaton and Muellbauer (1980a). This demand system was specified by starting from a specific class of preferences, which by the theorems of Muellbauer (1975, 1976) allows perfect aggregation over consumers. Basically, the perfect or exact aggregation property ensures that aggregate demand curves will be consistent with maximizing behavior by the representative consumer or optimizing behavior in the aggregate: the representation of market demands as if they were the outcome of decisions by a rational representative consumer (see appendix C). These preferences, known as the PICLOG class, are represented via the cost or expenditure function, which defines the minimum expenditure necessary to attain a specific utility level at given prices. This function $c(u,p)$ was denoted by Deaton and Muellbauer (1980a) and defines the PIGLOG class as a convex combination of the cost of subsistence and bliss. The authors have represented this cost function as follows

$$\log c(u,p) = (1 - u)\log(a(p)) + u \log(b(p))$$

(2.45)

where $u((0 \leq u \leq 1))$ represents utility, $p$ is the price vector, $a(p)$ and $b(p)$ are interpreted as the cost of subsistence and bliss respectively. In order to have a
flexible functional form, which must possess enough parameters, the following functional forms for \( \log a(p) \) and \( \log b(p) \) have been selected. Thus:

\[
\begin{align*}
\log a(p) &= a_0 + \sum_k a_k \log p_k + \frac{1}{2} \sum k \sum_j \gamma_{kj} \log p_k \log p_j \\
\log b(p) &= \log a(p) + B_0 \prod_k p_k^{B_k}
\end{align*}
\]  

(2.45.1)  

(2.45.2)

So that the AIDS cost function is written as:

\[
\log c(u, p) = a_0 + \sum_k a_k \log p_k + \frac{1}{2} \sum k \sum_j \gamma_{kj} \log p_k \log p_j + uB_0 \prod_k p_k^{B_k}
\]  

(2.46)

where \( a_i \), \( B_i \), and \( \gamma_{ij} \) are the parameters, \( p_i \) is price, and \( u \) is a specified utility level.

Equation 2.46 is known as AIDS which is representing consumer preferences by cost or expenditure function.

The demand functions can be derived directly from Equation 2.46. Deaton and Muellbauer (1980a) recorded that: "The flexible functional form of the AI demand system cost function implies that the demand functions derived from it are first order approximations to any set of demand functions derived from utility maximizing behavior."

It is a fundamental property of the cost function that its price derivatives are the quantities demanded.

Thus, \( \frac{\partial c(u, p)}{\partial p_i} = q_i(u, p) = q_i \) for \( i = 1, 2, ..., n \)  

(2.46.1)

In other words, the derivatives of the cost function with respect to prices will result in the optimal Hicksian quantity demand function.

Since the total cost \( c(u, p) \) is the same as the total expenditure, \( x \) for an optimizing individual and since the cost function is stated in logarithmic terms, and if, differentiated with respect to \( \log p \), obtains
\[
\frac{\partial \log c(u,p)}{\partial \log p_i} = \frac{\partial c(u,p)}{\partial p_i} \frac{p_i}{c(u,p)} = \frac{p_i q_i(u,p)}{c(u,p)} = \frac{w_i(u,p)}{x}
\]

for \( i = 1, 2, \ldots, n \) \hspace{1cm} (2.46.2)

where \( w_i \) is the budget share of commodity \( (i) \).

Moreover, Equation 2.46.2 states that cost elasticity of commodity \( i \) equals the budget share of commodity \( i \) and that, the budget shares are functions of prices and utility.

Hence, the logarithmic differentiation of Equation 2.46 with respect to \( \log p \), gives the budget shares as a function of prices and utility.

\[
\frac{\partial \log c(u,p)}{\partial \log p_i} = w_i = a_i + \sum_j \gamma_j \log p_j + B_i \sum_k p_k^h = w_i(u,p) \hspace{1cm} (2.47)
\]

where \( \gamma_j = \frac{1}{2}(\gamma_j^* + \gamma_j^* \cdot) \).

By inversion of \( \log c(u,p) = \log x \) in Equation 2.47, for the utility maximization, total expenditure \( x \) is equal to \( c(u,p) \). This results in the indirect utility function, which is a function of \( p \) and \( x \).

\[
v(p,x) = u = \frac{\log x - (a_o + \sum_k a_k \log p_k + \frac{1}{2} \sum_k \gamma_k^* \log p_k \log p_j)}{B_i \prod k p_k^h} \hspace{1cm} (2.48)
\]

By substituting Equation 2.48 into Equation 2.47 gives the budget share as a function of \( p \) and \( x \).

\[
w_i = a_i + \sum_j \gamma_j \log p_j + B_i (\log x - (a_o + \sum_k a_k \log p_k + \frac{1}{2} \sum_k \gamma_k^* \log p_k \log p_j)) \hspace{1cm} (2.49)
\]

Deaton and Muellbauer defined the price index as:

\[
\log p = a_o + \sum_k a_k \log p_k + \frac{1}{2} \sum_k \gamma_k^* \log p_k \log p_j \hspace{1cm} (2.50)
\]

Hence, the Equation 2.49 can be reduced to more simple form by inverting Equation 2.50, and that is,
Equation 2.51 states that the budget share of commodity \( i \) changes due to changes in relative prices and real expenditure. Hence, Equation 2.51 and Equation 2.49 represent the Almost ideal demand system approach (Deaton and Muellbauer, 1980a, p.316). Thus, the AIDS model is derived from an underlying structure of consumer preferences via a cost expenditure function of the form (2.49).

Stone's (1953) price index can be used because it is an excellent approximation to the Deaton price index (Deaton, 1980a, p.317). Stone's (1953) defined the price index as:

\[
\log p' = \sum x \log p_i
\]  

where \( p = \frac{x}{\xi} \) as an approximation for \( p \). Therefore, Equation 2.51 can be rewritten as follows:

\[
w_i = a_i + \sum y_i \log p_i + B_i \log\left(\frac{x}{p}\right)
\]  

where \((p')\) is the price index by this definition, and is equals to \( p \approx \xi p' \), and hence \( a_i = a_i - B_i \log \xi \). Equation 2.53 is known as linear approximate almost ideal demand system (LA/AIDS). According to Equation 2.53, the parameter \( y_i \) measures the change in \( i \)-th budget share corresponding to a percentage change in \( p_i \), holding all other variables, in case \( \left(\frac{x}{p}\right) \), constant, whereas, \( B_i \) measures the change in the budget share corresponding to a percentage change in \( \left(\frac{x}{p}\right) \) with \( p_i \) held constant.

Moreover, the intercept \( a_i \) represents the average budget share when the logarithm of prices and real expenditure is (0). The \( y_i \) represents the change in budget share for a given change in prices with income held constant. The \( B_i \) represent the response of budget shares to changes in income, these are add to zero and can assume either negative or positive values. Hence, the commodity \( i \)-th represents a necessary item if, \( B_i \) is negative, which means that \( i \)-th budget share decreases with the logarithm of
real expenditure, while, in the case of $B_i$ is positive, the $i$-th commodity is a luxury item, as result the $i$-th budget share increases with the logarithm of real expenditure.

In order to make AIDS consistent with the theory of demand the following restrictions on the parameters of the Equation 2.49 is required:

**Adding-up restriction:**
To obtain the adding-up restriction, $\sum w_i = 1$, it requires summing the Equation 2.52 over $i$ and, which will result in the following Equation

$$\sum w_i = \sum \alpha_i + \sum \gamma_i \log p_j + \sum B_i \log x
\tag{2.54}$$

This restriction will be met if $\sum a_i = 1$, $\sum B_i = 0$ and $\sum \gamma_j = 0$

**Homogeneity restriction:**
To meet the homogeneity restriction, the following derivatives are necessary, so that Euler's theorem may be utilized.

$$\frac{\partial q_i}{\partial p_i} = \frac{x}{p_i} (\gamma_i - B_i \alpha_i - B_i \sum \gamma_j \log p_j) - \frac{x}{p_i} w_i \tag{2.55}$$

$$\frac{\partial q_i}{\partial p_j} = \frac{x}{p_i p_j} (\gamma_i - B_i \alpha_j - B_i \sum \gamma_j \log p_k) \quad \text{for } i \neq j \tag{2.56}$$

and

$$\frac{\partial q_i}{\partial x} = \frac{1}{p_i} (w_i + B_i) \tag{2.57}$$

Substituting Equations (2.55, 2.56, and 2.57) into Equation 2.14 above yields

$$\sum p_i \frac{\partial q_i}{\partial p_i} + x \frac{\partial q_i}{\partial x} = \frac{x}{p_i} \sum \gamma_j = 0 \tag{2.58}$$

Therefore, homogeneity of degree zero will be satisfied if $\gamma$ is restricted, so that $\sum \gamma_j = 0$, since $\frac{x}{p_i} > 0$, this however, satisfies the homogeneity restriction.

**Slutsky symmetry restriction:**
Eventually, the Slutsky symmetry restriction is satisfied if
\[ y_i = y_j \quad \text{(for all } i, j = 1, 2, \ldots, n, \text{ and } i \neq j) \]  
(2.59)

**Negativity restriction**

As it is true for other flexible functional forms, negativity cannot be ensured by any restrictions on the parameters alone. It can however be checked for any given estimates by calculating the eigenvalues of the Slutsky matrix \( s_y \). According to Deaton and Muellbauer (1980b), in practice it is easier not to use \( s_y \) but \( k_y = p_s s_y / x \), the eigenvalues of which have the same signs as those of \( s_y \) and which are given by:

\[ k_y = y_y + \beta_i \beta_j \log(x/p) - w_i \delta_{ij} + w_i w_j \]  
(2.60)

where \( \delta_{ij} \) is the kronecker delta that is unity if \( i \neq j \).

The second-order condition of the equilibrium is fulfilled when the matrix \( k_y \) is negative semi-definite. Under that condition, the consumer is considered to be in stable equilibrium.

The above restrictions not only decrease the dimensionality of the parameter space but also ensure that own-price, cross-price, and expenditure elasticities are consistent with neoclassical theory.

Demand elasticities can be achieved throughout by utilizing the equations from 2.55 to 2.56. The uncompensated and compensated own price, cross prices, and expenditure or income elasticities are as follows:

For uncompensated own price elasticity is,

\[ e_u = -1 + \{ y_u - B_i (\alpha_i + \sum y_i \log p_j) / w_i \} \]  
(2.61)

and it is equivalent to

\[ e_u = \frac{1}{w_i} \left( \frac{\partial w}{\partial \log p_i} \right) - 1 = \frac{\gamma u - 1}{w_i} \]  
(2.62)

While, for uncompensated cross-price elasticity is,

\[ e_v = \{ y_v - B_i (\alpha_i + \sum y_i \log p_k) / w_i \} \]  
(2.63)
and it is equal to \( e_v = \frac{1}{w_i} \left( \frac{\partial w_i}{\partial \log p_j} \right) = \frac{\gamma_v}{w_i} \) (2.64)

However, for compensated price elasticities are as follows:

\[ e^*_v = \eta_i w_i + e_u \] (2.65)
\[ e^*_u = \eta_i w_j + e_v \] (2.66)

Finally, for expenditure elasticity is,

\[ \eta_i = 1 + \left( \frac{\beta_i}{w_i} \right) \] (2.67)

In case of utilizing Stone's price index, the uncompensated own price and cross price elasticities are as follows:

\[ e^*_u = \gamma_u \frac{w_i}{w_i} - (1 + B_i) \] (2.68)
\[ e^*_v = \frac{1}{w_i} - (\gamma_v - B_v w_j) \] (2.69)

and for compensated price elasticities are as following:

\[ e^*_u = \gamma_u \frac{w_i}{w_i} + w_i - 1 \] (2.70)
\[ e^*_v = \gamma_v \frac{w_i}{w_i} + w_j \] (2.71)

The main problem of utilizing AIDS model for estimating demand systems is the number of structural parameters, which must be estimated. According to AIDS Equation 2.49, the sum of unknown structural parameters is \( 2n + n^2 (n \alpha s + nB^s) \) and \( n^2 \gamma_y 's \). Homogeneity provides \( n \) restrictions (\( \sum \gamma_y = 0 \)), while symmetry provides \( n(n-1)/2 \) restrictions (\( \gamma_y = \gamma_y' \)), and for adding-up provides two restrictions (\( \sum B_i = 0, \sum \alpha_i = 1 \)). The total of these restrictions is \( (n^2 + n + 4)/2 \)

This decreases the number of free unknown structural parameters to be estimated to \( (n^2 + 3n - 4)/2 \). For instance, in case of six commodities, one requires to estimate 25 parameters. Therefore, this number of parameters is largely increased for more utilizing commodity groupings, and this could occur in significant estimation problems.
The main advantage of the LA/AIDS is that it gives an arbitrary first-order approximation to any demand system, which exactly satisfies the axioms of choice. It aggregates perfectly over consumers without invoking parallel linear Engle curves. It has a functional form, which is consistent with known household budget data. It represents an arbitrary first-order approximation to any demand system. Moreover, the theoretical restrictions can be imposed through linear restrictions on the parameters and tests of these restrictions. The AIDS model in its general form is nonlinear. In practice, however, by a suitable approximation to the price index, it is made linear.

To sum up, this study will focus on three utility-based demand models. The Linear Expenditure System (LES) is a demand system derived from the utility function subject to the budget constraints. The Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980a) is based on the consumer cost function approach. This approach utilizes the dual formulation of the consumer allocation problem to transform the consumer's problem from one of maximizing utility, given prices and income, to that of minimizing the cost of reaching a given level of utility with the same prices and income. Finally, the Rotterdam Model (RM) of Theil (1965) and Barten (1967) is an example of demand approach based on direct approximation of demand functions. It is not theoretically plausible in that it is not derived from a specific utility function. The model is one of the flexible demand systems that can be expressed as a system of linear equations in behavioral parameters. The negativity of own price substitution term and the relationships of net substitutes and complements can be immediately decided from parameter estimates, while income and price elasticities are linear in parameters. Variances of income and price elasticities are estimated from the variance-covariance matrix.
2.5.2 Additivity Demand Systems

One methodology of defining additivity is to say that the consumer utility function is such that marginal utility of any commodity is independent of the quantities consumed of all other commodities. This states that the utility function is of the form

\[ u = u_1(q_1) + u_2(q_2) + ... + u_n(q_n) \]  

(2.72)

The specific restriction, which can be obtained from this assumption can be expressed as follows:

\[ \frac{\partial q_i}{\partial p_k} = \frac{\partial q_i}{\partial x} \frac{\partial q_i}{\partial x} = \theta \text{ for } (i \neq k; j \neq k) \]  

(2.73)

Accordingly in Equation 2.73 the restriction states that the change in the demand for commodity \( i \) induced by a change in the price of commodities \( k \) is proportional to the change in the demand for commodity \( i \) induced by a change in total expenditure. The factor of proportionality \( \theta \) does not depend on the good \( i \) whose quantity response we are considering (but it does depend on the good \( k \) whose price has changed (Phlips, 1983, p.63).

Houthakker (1960) who supported the utilization of the additive utility function simplified the additivity restriction by explaining it in the terms of cross substitution effects which is in general is equal to.

\[ k_i = \frac{\partial q_i}{\partial p_k} + q_i \frac{\partial q_i}{\partial x} \]  

(2.74)

\[ = -\mu \frac{\partial q_i}{\partial x} \frac{\partial q_j}{\partial x} \]  

(2.75)

where \( \mu = \frac{\partial \lambda}{\partial \lambda / \partial x} \) is the flexibility of the marginal utility of income, \( k_i \) is the substitution effect between the \( i \)-th and \( j \)-th commodities, thus seen to be proportional to the income derivatives of both commodities and to the income flexibility.
The other possible assumption of additivity is called indirect additivity assumption, and the latter was attributable to Leser (1941-42) and Houthakker (1960). The derivatives of restriction which was obtained by Houthakker (1960) for indirect additivity assumption is as follows:

\[
\frac{\partial q_i}{\partial p_k} = \frac{q_i}{q_j} \quad \text{for} \quad (i \neq k; j \neq k)
\]  

(2.75)

The equation above states that the cross price derivatives are proportional to the quantities influenced. The other possibility of explaining the above restriction is by considering it in the terms of elasticities, and this can be achieved by dividing both sides of equation (2.75) by \( p_k \) and the finding is that

\[
\frac{\partial q_i}{p_k} \frac{p_k}{q_i} = \frac{\partial q_j}{p_k} \frac{p_k}{q_j} \quad \text{for} \quad (i \neq k; k \neq j)
\]  

(2.76)

Equation 2.76 states that all cross prices elasticities with respect to price of commodities \( (k) \) are equivalent. Based on these assumptions (direct and indirect additivity), Houthakker introduced two system of demand equations. The first system of demand equations which is derived from direct additivity utility function \( u(q) = \alpha_1 q_1^n + \alpha_2 q_2^n + \ldots + \alpha_n q_n^n \) is called the direct addilog demand function. It is as follows:

\[
\log \frac{x_i}{p_i} - \log \frac{x_j}{p_j} = \log \alpha_i - \log \alpha_j + (1 - \beta_i) \log q_i - (1 - \beta_j) \log q_j
\]  

(2.77)

where \( \alpha_i^* = \alpha_i \beta_i \).

The second demand system which is derived from an indirect additivity utility function \( \psi(x/p) = \alpha_1 (x/p_1)^h + \alpha_2 (x/p_2)^h + \ldots + \alpha_n (x/p_n)^h \) is known as the indirect addilog demand function. It is as follows

\[
\log q_i - \log q_j = \log \alpha_i^* - \log \alpha_j^* + (1 - b_i) \log(x/p_i) - (1 - b_j) \log(x/p_j)
\]  

(2.78)

Houthakker (1960) estimated both equations (2.77, 2.78) above (direct and indirect) by applying three types of data, long times series of consumption in current and constant prices for Sweden and for Canada and for a combination of short time series...
for thirteen countries. The estimated results confirmed the impression that direct
additivity is a better approximation to reality than indirect additivity.

Houthakker (1960) also estimated direct addilog and indirect addilog models but in
this case by applying a cross section of observations of U.K budget survey for the year
(1953). These models are presented below

**Direct addilog demand function**

\[
\log p_j q_j = \frac{1}{\beta_j} \left[ \log (\alpha_j^{*} / \alpha_j) + \beta_j \log p_j q_j \right]
\]  

(2.79)

**Indirect addilog demand function**

\[
\log p_j q_j - \log p_j q_j = \log \alpha_j^{*} - \log \alpha_j^{*} + (b_i - b_j) \log x
\]  

(2.80)

The above equations (2.79, 2.80) were estimated by dividing the expenditure into four
main groups. Houthakker (1960) concluded that the direct additivity assumption is
more flexible than indirect additivity.

However, these assumptions were criticized particularly by Deaton (1974b,p.338), by
saying that "the assumption of additive preferences is almost certain to be invalid in
practice and the use of demand models based on such an assumption will lead to
severe distortion of measurement" In other words, the utilization of demand systems
which is derived from direct and indirect additivity utility functions which is based on
these assumptions led to distorted the estimation of own-price and income elasticities
for demand functions due to the existence of approximation linear relationships
between these elasticities, which under direct additivity the ratio of own-price to
income elasticities is approximately constant, whereas, under indirect additivity the
sum is approximately constant (see Deaton, 1974b ,p.341)

Deaton (1974b, p.339) followed Houthakker (1960) in defining a direct additivity
utility function, maximized it subject to the budget constraint from which he derived
the first order maximization conditions in the logarithmic form. By differentiating these with respect to \( \log p \) and \( \log x \), Deaton reached to the following relation

\[ e_y = \phi e_i \delta_y - e_y w_j (1 + \phi e_i) \]  \hspace{1cm} (2.81)

where \( e_i \) is the income elasticity of the \( i \)-th good, \( e_y \) is the uncompensated cross-price elasticity of good \( i \)-th with respond to the price of good \( j \), and \( \delta_y \) is the kronecker delta, equal to unity if \( i=j \) and zero otherwise, \( w_j \) is the budget share of good \( j \) and \( \phi \) is a constant. According to Deaton, the left hand side of the equation above could be approximated for own-price elasticity by the first term of the right hand side, that is

\[ e_n = \phi e_i \]  \hspace{1cm} (2.82)

Also, the author mentioned that the above approximation will only be closed for a good, which occupies a very small fraction of the budget, and it will only be closed for all goods if only the level of disaggregation adopted is high. In order to prove these relationships Deaton (1974b) utilize the log linear demand models of the following forms

\[ \log q_i = \alpha_i + \beta_i \log (x/\pi) + \gamma_i \log (p_i/\pi) \]  \hspace{1cm} (2.83)

where \( \pi \) is the implicit price deflator of consumer expenditure.

Since the quality of the two approximations (2.81, 2.82) depends upon the degree of commodity disaggregation, studies were carried out by Deaton (1974b) on two different versions of the same data. For the first set, it was data on purchases of 37 distinct non-durable goods from 1954 to 1970, while for the second set; the goods were aggregated into eight broad groups. The resulting estimates of the price and income elasticities were plotted against one another in a figure which was presented very clearly. It indicated that there was no visual evidence for either of the relationships required by direct or indirect additivity, and that the relationships between these points were actually positive.

Also Deaton estimated the LES for the same set of data, and also plotted the estimated own-price and income elasticities for 1963 against each other. He concluded that the
proportionality approximation is very close, even for the broad groups. Accordingly, he suggested that if additivity is to be rejected the log linear or the Rotterdam model could be utilized. Both are fundamentally inconsistent with utility maximization. Deaton (1978) suggested utilizing the duality technique to derive a system of demand equations. A demand system, which has derived first order conditions of utility maximization frequently, cannot be solved explicitly for the demand functions, and if they can be resolved obtaining equations might be hard to estimate. Therefore, the direct utility approach has been ineffective to implement empirical analysis (Deaton, 1978, pp.524). As a result, he depended on the duality approach to derive a system of demand equations. He also suggested that a model in which budget shares are linearly related to the logarithm of total outlay provide an excellent fit to the data. He introduced another model based on Working (1943) and Leser (1976) model, which presented below

\[ w_i = \alpha_i + \beta_i \log x \]  \hspace{1cm} (2.84)

where \( w_i = p_i q_i / x \). This form is consistent with adding-up provided \( \sum \alpha_k = 1 \) and \( \sum \beta_k = 0 \).

In order to render equation (2.84) applicable for time series analysis, prices were introduced and the model of equation (2.84) can then be written in following form

\[ w_i = \alpha_i + \beta_i \log x + \sum \gamma_k \log p_k \]  \hspace{1cm} (2.85)

Deaton (1978) estimated another model, which was derived from the cost function of Muellbauer's (1975) PIGLOG form, which it is presented below

\[ w_i = \alpha_i + \beta_i \log \left( \frac{x}{p} \right) \]  \hspace{1cm} (2.86)

where \( p = \sum \alpha_i p_i \) is a price index.

Deaton (1978) estimated both equations (2.85, 2.86) by using U.K data for years 1954-1974, and by dividing expenditure into eight commodity groups. The major finding of Equation 2.86 was that the 5 out of 8 of the \( \beta_i \) were different from zero and
23 out of 56 of the $\gamma_i$ estimates were significantly different from zero at a 5% significance level. However, the estimated findings of Equation 2.86 have a worse fit as compared to Equation 2.85. Based on log likelihood test Equation 2.86 is rejected in favor of Equation 2.85. Deaton (1978) used the same data above and also estimated a model of LES in the following form

$$w_{it} = \frac{P_u Y_i}{x_i} + \eta_i \left( 1 - \sum_k \frac{P_u Y_k}{x_k} \right)$$

(2.87)

The major application of this model (2.87) was to compare findings of this equation with the PIGLOG model. Based on statistical results and on log likelihood test the PIGLOG was rejected. Accordingly, the author made another attempt to develop the cost function approach and finally Deaton and Muellbauer (1980a) established AIDS model.

2.5.3 Dynamic Demand System.

The traditional neo-classical theory of consumer behavior is static in nature, yielding equilibrium values for quantities demanded and implying an instantaneous adjustment to new equilibrium values in response to price or expenditure change. However, in reality, the consumer does not react immediately to price and income change. Altering consumption involves certain costs to the consumer and delayed reaction seems more reasonable. Philips (1974, p.149) emphasized this when he mentioned that "in fact consumers often react with some delay to prices and income changes, with the implication that the adjustments towards a new equilibrium situation is spread over several time period". In other words, the consumer in a static demand system does not behave optimally that is we ignore the dynamic specifications of demand behavior. Economists express the movement to a new equilibrium by including lag variables arising from durability of many consumer goods and the existence of habits developed as result of past period consumption. Hence, the main weakness of static demand systems is ignoring short run adjustments made in consumer behavior. Further, dynamics are also important in aggregate time series models, where Jorgenson et al.
(1986,p.1), for example, claim that for almost all demand functions estimated using aggregate time series data, there is evidence of persistence in the consumption behavior.

2.5.3.1 Ad Hoc Approaches

Economists tend to utilize ad hoc approaches to studying habit effects on consumer behavior and dynamic demand systems. Stone et al (1964) and Barten (1967) tried to incorporate the time trend variable in a demand system which derived mainly from the utility maximization hypothesis, to examine changes in tastes and others socio-economic factors. Kmenta (1986, p.568) mentioned that "the term trend is always a camouflage for factors that change over time, and it would certainly be preferable if these factors could be identified and measured” In general the incorporation of a time trend variable is not sufficient because it provide so little insight into the structure of a demand system. It states that demand will continue to increase even if the other factors such as prices and income remain constant over a long period of time. Adjustment needs to be made to allow variables other than prices and income to be incorporated into the demand system. There are dynamic approaches such as the state adjustment model suggested by Houthakker and Taylor (1970, ch.1) This model is based on the assumption that quantities purchased depend upon the existing stock, either physical stocks of commodities or psychological stocks of habits which implies that tastes and hence purchases are influenced by past consumption. Previous behavior is assumed to be represented completely by the current values of certain state variables. These variables are in turn altering current decisions and the net outcome of a distribution lag. Thus, the equation, which includes lagged dependent on variable accounting for a weighted distribution of the consumer’s behavior from the pervious period. Therefore, the state adjustment model is presented in that following form.

\[ q_t = \alpha + \beta S_t + \gamma y_t \]  

(2.88)

or can be written in the following form:
\[ S_t = \frac{1}{\beta} (q_t - \alpha - \gamma y_t) \]  

(2.89)

where \( q_t \) is the rate of demand at time \( t \), \( y_t \) is the rate of income at time \( t \) and \( (S_t) \) refers to the rate of the stock (state variable) of commodity at time \( t \).

Equation 2.88 implies that the current purchase of commodity depends upon both income and the stock of that commodity. The stock of commodity holds for either durable commodities or for habit formulation. The authors suggested excluding the state variable from Equation 2.89 due to difficulty of estimating a psychological stock of habit-forming commodities and a physical stock of durable commodities (since it was unobservable). Therefore, Equation 2.88 becomes as follows:

\[ S_t = q_t - \delta S_t \]  

(2.90)

where the dotted variable denoted the rate of change of that variable respecting to time. By substituting Equation 2.88 into Equation 2.90 yields the following form.

\[ \dot{S}_t = q_t - \frac{\delta}{\beta} (q_t - \alpha - \gamma y_t) \]  

(2.91)

Differentiate Equation 2.88 with respect to time yields

\[ \dot{q}_t = \beta S_t + \gamma \dot{y}_t \]  

(2.92)

By substituting Equation 2.91 into Equation 2.92 gives

\[ \dot{q}_t = \beta (q_t - \frac{\delta}{\beta} (q_t - \alpha - \gamma y_t)) + \gamma \dot{y}_t \]  

(2.93)

Or

\[ \dot{q}_t = \alpha \delta + (\beta - \delta) q_t + \gamma \dot{y}_t + \gamma \delta \dot{y}_t \]  

(2.94)

That is the first order differential equation which including only the observable quantities and income, and it represents short run equation, whereas, for long run the equation was represented by

\[ \ddot{q}_t = \frac{\alpha \delta - \gamma \delta}{\delta - \beta} \dot{y}_t \]  

(2.95)

where \( \dot{S}_t \) is equal zero in the long run, and assuming \( \beta = \delta \).
This model is a continuous model and must be approximated by one involving discrete time intervals. The approximation has been introduced by the authors, and then the estimating equation utilized was

\[ q_t = k_0 + k_1 q_{t-1} + k_2 \Delta y_t + k_3 y_{t-1} \]  (2.96)

where,

\[ \alpha = \frac{2k_0(k_2 - 1/2k_3)}{k_3(k_i + 1)} \quad \beta = \frac{2(k_i - 1)}{(k_i + 1)} + \frac{k_3}{k_2 - 1/2k_3} \]

\[ \gamma = \frac{2(k_i - 1/2k_3)}{(k_i + 1)} \quad \delta = \frac{k_3}{k_2 - 1/2k_3} \]

The equations presented above do not include a price variable because when the authors estimated the U.S data for groups commodity such food and clothing it was found to be statistically insignificant. Houthakker and Taylor estimated Equation 2.99 for the U.S.A. data for the eleven commodity groups, and their findings were that the estimated value of \( \beta \) was positive for food and negative for clothing. This implies that aggregate food purchasing is habit forming while \( \beta \) negative indicated that clothing is a durable commodity. This model was also estimated by Hassan, Johnson, and Green (1977) applied to Canadian data, their findings were the same as Houthakker and Taylor results.

Anderson and Blundell (1983) and (1984) introduced habit effects into the Deaton-Muellbauer AIDS model to explain long-run behavior but non-symmetric and non-homogeneous short-run behavior is permitted. The symmetry and homogeneity restrictions are only expected to hold in steady state. The authors utilized the following type of dynamic AIDS Equation for estimating:

\[ \Delta w_n = \sum_j c_j d \log p_{jt} + b_j d \log (x/p)_t - \lambda \Delta w_{t-1} \]

\[ - \sum_j \gamma_j \log p_{jt-1} - \beta_j \log (x/p)_{t-1} \]  (2.97)

Equation 2.97 implies that, the current changes in budget shares depend not only on current changes in the normal AIDS explanatory variables (equation 2.53 above) but
also on the extent of consumer disequilibrium in the previous period. In their 1983 study, Anderson and Blundell utilized their model to annual Canadian data on five categories of non-durable commodities for the period 1947-79, whereas in 1984 study deals with quarterly UK data on four categories of non-durable for the period 1955-81. Their findings are substantially the same in both situations, where as the static AIDS model was rejected in favor of the above dynamic version. Moreover, their findings also indicate that the restrictions of symmetry and homogeneity are not rejected when imposed on the long run or steady-state structure of the model. This suggested that the past rejections of symmetry and homogeneity be indeed due to mis-specifications that are inherent in purely static models.

Blanciforti and Green (1983), estimated a dynamic version of Deaton Muellbauer AIDS model, by introducing habit effects of the manner of Pollak Wales (1969), Manser (1976) which will be discussed later in next sub section of this chapter. The authors claim, on the basis of their results for the U.S., that the neglect of dynamic effects is the reason for auto-correlation found in the residuals of demand equations. Accordingly, the authors estimated the dynamic AIDS Equation of the following type:

$$w_i = \alpha_i + \alpha_i' q_{it-1} + \sum_j \gamma_j \log p_j +$$
$$B_i \left( \log x - (\alpha_0 + \sum_k (\alpha_k' + \alpha_k' q_{it-1} \log p_{it-1} + 1/2 \sum_j \gamma_j \text{log } p_k \log p_j) \right)$$

Equation 2.98 is the same as Equation 2.53 above. It is developed by specifying the \((\alpha_i = \alpha_i' + \alpha_i' q_{it-1})\) to be Linear A function of past consumption levels. The authors estimated both static Equation 2.53 above and dynamic approaches Equation 2.98 for annual United State data for the years 1948-78, which included 11 commodity groups. Their results were in fully presented account of dynamic models. For static models it was summarized briefly. For the static model, 9 of the 11 commodities are classified as necessities with 2 durable and other nondurables, which are classified as luxuries. The homogeneity condition was rejected for 5 of the commodities based on an F-test. In general, the income and own-price elasticity estimates in the static and dynamic
approaches are similar, while the estimated income elasticities are higher in 8 of 11 cases for the static model relative to the dynamic model when homogeneity is imposed. Moreover, 10 of the habit coefficients $\alpha_i$ are positive, the only exception being for the commodity group, "durable goods". However, 6 of these estimated habit coefficients are statistically significant at the 5% level. Furthermore, the homogeneity restrictions were rejected in 5 of the 11 cases.

Mergos and Donates (1989a) also estimated the dynamic model of the AIDS which was developed by Deaton and Mullbauer (1980a) and its dynamic version proposed by Blanciforti and Green (1983). The estimation was for Greek annual expenditure data for the years 1960-86. It was found that the homogeneity and symmetry conditions were rejected. Alessie and Kaptenyn (1991) also estimated the dynamic equation (2.98) and their findings suggested that equation (2.101) does not satisfy the theoretical requirement of the utility theory.

2.5.3.2 Dynamic Utility Approaches.

This sub-section will deal with dynamic approaches, which are directly derived from the utility function. There are great contributions in dynamic models, for instance; Philips (1972), Houthakker and Taylor (1970), Pollak (1970), Manser (1976), Pollak and Wales (1969). These authors developed their dynamic approaches by using utility function, including the habit effects to reflect changes in tastes. The dynamic utility function provides a complete system of demand equations. Properties must be consistent with the theoretical foundation.

2.5.3.2.1 A Dynamic Version of Linear Expenditure System (DLES)

Philips (1972) introduced a dynamic model by following the method of Houthakker and Taylor (1970, ch.5). However, Philips approach's is based on the assumption that the minimum required quantities, $y_i$, are related to the state variable according to the relationship as defined below.
\[ \gamma_i = \theta_i + \alpha_i S_i \]  

(2.99)

where \( \gamma_i \) is the minimum required quantity of \( i \)-th commodity, \( S_i \) is defined as the state variable and in this case represents non measurable stocks of habits or durable commodities, and \( \theta_i, \alpha_i \) are constants that might take positive or negative values. The sign of the \( \alpha_i \)'s is related to habit formation and inventory adjustment. When commodity \( (i) \) is habit formation, \( S_i \) has a positive influence on \( \gamma_i \): the more you smoke, the larger is the minimum daily number of cigarettes you required to buy. When commodity \( (i) \) is durable good, \( \alpha_i \) is negative (in the absence of habit formation): the better your library is furnished, the fewer books you have to buy per year, but admittedly buying books may become a habit, and \( \alpha_i \) may reflect the net combined effect of both phenomena.

Accordingly, the author redefines the Stone-Geary utility function as

\[ u(q_i) = \sum_i \beta_i \log(q_i - \theta_i - \alpha_i S_i) \]  

(2.100)

where \( q_i \) is defined in this case as measuring flows of quantities purchased but not consumed, due to durable goods purchased are different from consumption goods. As the restrictions on the utility equation above (2.100) to be defined, the author imposed such restrictions: \( \beta_i > 0 \) in order to have decreasing marginal utilities, and \( 0 < (q_i - \theta_i - \alpha_i S_i) \) for utility function to be defined.

By maximizing the utility above subject to budget constraints \( (\sum p_i q_i = x) \) and by applying the lagrangean multiplier, the first conditions are as follows

\[ \beta_i / (q_i - \theta_i - \alpha_i S_i) = \lambda p_i \]  

(2.100.1)

\[ \sum_i p_i q_i = x \]  

(2.100.2)

Solving for \( q_i \),

\[ q_i = \theta_i + \alpha_i S_i + \beta_i / \lambda p_i \]  

(2.101)

However, Equation 2.101 includes two un-measurable variables \( \lambda \) and \( \theta_i \), in order to eliminate them, Philips assumed that
\[ \dot{S}_t = q_t - \delta_t S_t \]  

(2.102)

where the dotted variables donated the rate of change of that variable with respect to time and represents as constant rate of depreciation.

Equation 3.102 states that the net stock is equivalent to quantities purchased minus depreciation. Since, in long run equilibrium \( \dot{S}_t = 0 \) (quantities purchased are those equal to consumption or depreciation \((\delta_t S_t)\)), therefore, Equation 2.102 could be written as

\[ \dot{S}_t = q_t - \delta_t S_t = 0 \]  

(2.103)

Accordingly, \( q_t = \delta_t S_t \) and hence, \( S_t = q_t / \delta_t \). Replacing the latter in the first order conditions (2.101.1) and taking account of the budget constraint, yields the long run system of demand equations

\[ q_t^* = \gamma_t^* + \frac{\beta_t}{p_t} \left( x - \sum_j p_j \gamma_j^* \right) \quad \text{for} \ j = 1, \ldots, n \]  

(2.104)

Which the familiar static linear expenditure system extended to \( n \) commodities. The reparametrization is as follows:

\[ \beta_t^* = \frac{\delta_t \beta_t / (\delta_t - \alpha_t)}{\sum_j \delta_t \beta_j / (\delta_t - \alpha_j)} \]  

(2.104.1)

\[ \gamma_t^* = \delta_t \theta_t / (\delta_t - \alpha_t) \]  

(2.104.2)

Philips also defined the short run demand function, which is in the following structure

\[ q_u = k_{10} + k_{11} q_{u-1} + k_{12} \left( \lambda p_{u-1} \right)^{-1} + k_{13} \left( \lambda_{u-1} p_{u-1} \right)^{-1} \]  

(2.105)

where:

\[ \delta_t = 2(k_{12} + k_{13}) / k_{12} - k_{13} \]

\[ \alpha_t = \{ 2(k_{12} + k_{13}) / (k_{12} - k_{13}) \} - 2(1 - k_{11}) / 1 + k_{11} \]

\[ \beta_t = (k_{12} - k_{13}) / 1 - k_{11} \]

\[ \theta_t = k_{10} (k_{12} + k_{13}) / (1 + k_{11}) (k_{12} + k_{13}) \]

For each commodity, the four structural coefficients can thus be computed from the regression coefficients.
Phlips estimated equation (2.105) for the United States by utilizing the 11 consumption series for the period 1929-1967 and by excluding the war period from 1942-45. The quantities purchased ($q_{ui}$) are measured by expenditure in constant (1958) prices, while prices are obtained by dividing constant-dollar into current dollar expenditures. However, income ($x_i$) is defined as total personal consumption expenditures in current prices - i.e. the sum of the 11 series in current prices, and that all expenditures are per capita. The main conclusion of the estimation was all regressions coefficients were significantly different from zero except the coefficient $k_3$ in the housing equation, also the value of $\beta$, were positive which means that marginal utility of every commodities was decreasing in the short run. The coefficients $\delta$ indicated that food and clothing were necessities. Finally, Phlips suggested that the dynamic model still includes many restrictive hypotheses (in addition to independence among state variables). The behavior of the representative consumer is myopic which means that the consumer does not take into account the effect of current consumption decisions on future tastes. In other words the consumer in these equations does not behave as "rational" consumer.

Pollak (1970) developed a model of consumer behavior based on studying the impact of habit formation. The main assumption of this model is that the necessary quantity of commodity, $y_i$, depends on previous consumption of that commodity. Accordingly, the author implied the necessary quantity of each commodity as a linear function of consumption of that commodity in the past period. In this case the consumer does not take account of the effect of current purchases on future preferences and future consumption. In other words, the consumer exhibits myopic habit formation. Pollak (1970) used the formulation Stone-Geary utility function (2.106) to develop his model.

$$u = \sum_B \log(q_i - y_i) \quad \text{for i=1, 2,..n}$$  \hspace{1cm} (2.106)
The utility function above is re-expressed by including habit effects, through allowing the $\gamma_u$, to be a linear function of past consumption levels of that commodity and the formulation for that necessary quantity can be defined as:

$$\gamma_u = \gamma_i^* + \Gamma_i q_{u-1}$$  \hspace{1cm} \text{(for } i = 1, 2, \ldots n \text{)} \hspace{1cm} \text{(2.107)}$$

where, $\gamma_i^*$ was defined as the "physiologically necessary" component and $\Gamma_i q_{u-1}$ was defined as the "psychologically necessary" component of $\gamma_u$. Accordingly, by substituting equation (2.107) into (2.106) and maximizing subject to the budget constraint yields

$$q_u = \gamma_i^* + \Gamma_i q_{u-1} + \frac{\beta}{p_i} \left( (x_i - \sum_k p_k (\gamma^* + \Gamma_k q_{u-k})) \right) + u_u$$  \hspace{1cm} \text{(2.108)}$$

Pollak (1970) hypothesizes that past consumption influenced current preferences and current demand. Moreover, the author developed a model to study the effect of habit formation in the long run. The long run equilibrium or "steady state" demand model of Pollak is based on the assumption that the optimal consumption vector for period (1) will be identical with the consumption of period (0), and if the income and prices remain constant among the time, the optimal consumption vector in each subsequent period would also be equal to the consumption vector of period (0). To obtain long run function of Pollak, it requires to maximization of the Stone-Geary utility function subject to the budget constraint, where $\gamma_i$ in this case is defined as;

$$\gamma_u = \gamma_i^* + \Gamma_i q_{u-1}$$  \hspace{1cm} \text{for } i = 1, 2, n \hspace{1cm} \text{(2.109)}$$

where $q_i$ refers to long run value of quantity. Pollak concluded after he had determined the properties of both short run and long run equations, that this dynamic approach was stable.

Pollak and Wales (1969) examined the aggregate demand function in post-war United State and considered various approaches including the impact of habits and various forms of the error structure for the linear expenditure system. These approaches were
concerned with the value of \( y_i \), the minimum required quantities of each commodity in period \( t \), \( Y_i \), and the assumptions made were:

1) \( y_i \) is constant in each period \( Y_i = y_i^* \)
2) \( y_i \) change by a constant amount every year, that is, by a linear function of time, \( y_i = y_i^* + \Gamma_i T \)
3) \( y_i \) is proportional to lagged consumption, \( y_i = y_i^* + \Gamma_i q_{i-1} \)
4) \( y_i \) is a linear function of past consumption \( y_i = y_i^* + \Gamma_i q_{i-1} \)
5) \( y_i \) is a linear function of the highest level of consumption in the three previous period, \( z_{i-1} \); where \( z_{i-1} \) was the highest level of consumption in three years prior to period \( t \)
6) \( y_i \) is a linear function of a three year moving average of past consumption, \( y_i = y_i^* + m_{i-1} \); where \( m_{i-1} \) was the average level of consumption during three years prior to period \( t \)
7) \( y_i \) is a linear function of the sum of the rate of growth of consumption, \( \Delta y_i = \Gamma_i g_{i-1} \); where \( g_{i-1} = (q_i / q_{i-1}) - 1 \)
8) \( y_i \) is a linear function of the sum of the period to period change in consumption, \( \Delta y_i = y_i^* + \Gamma_i g_{i-1} \).

The major findings of models (1) to (4) were presented. However, the results for (1) were considered inappropriate by Pollak and Wales since the estimates of the minimum quantities exceeded consumption values in each time period. The results for model (4), which \( y_i \) was a linear function of lagged consumption, yielded marginal budget shares that were all less than one and significant. That is, the marginal budget shares were more than twice their standard error. The \( \beta \)'s were also significant but the model was not acceptable because calculated \( y_i \) values exceeded the corresponding consumption values in every time period for all commodities. The findings for the approaches (5) to (8) were not been presented by the Pollak and Wales (1969) due to their inconsistency with the underlying utility maximizing framework.

2.5.3.2.2 A Dynamic Version of Almost Ideal Demand System (DAIDS)

associated with an item to depend not only on its own past consumption but also those of other items. The dynamic AIDS cost function proposed by author is given

\[
\log c(u,p) = a_0 + \sum_i \delta_i e_{u-1} + \sum_i a_i \log p_u + \frac{1}{2} \sum_i \sum_j (\delta_{ij} + \theta_{ij} x_{t-1}) \log p_u \log p_j + u B_0 \prod_i p_i^{\eta_i x_i}
\]  

(2.110)

where \( e_{u-1} \) denotes lagged purchased of item (i), \( x_{t-1} = \sum_i e_{u-1} \) is lagged aggregate purchase.

The same steps can derive the DAIDS to obtaining the static AIDS from the cost function, and these procedures are as follows:

By inversion of \( \log c(u,p) = \log x \) into Equation 2.110, doing so results in indirect utility function, which is a function of the following form.

\[
\frac{\log x - \left( \sum_i \delta_i e_{u-1} + \sum_i a_i \log p_u + \frac{1}{2} \sum_i \sum_j (\gamma_{ij} + \theta_{ij} x_{t-1}) \log p_u \log p_j \right)}{B_0 \prod_i p_i^{\eta_i x_i}}
\]  

Hence, the logarithmic differentiating of Equation 2.110 with respect to \( \log p_i \) gives the budget shares Equation

\[
\frac{\partial \log c(u,p)}{\partial \log p_i} = w_i = a_i + \sum_j (\gamma_{ij} + \theta_{ij} x_{t-1}) \log p_j + u_0 (\beta_i + \eta_i x_{t-1}) + \prod_i p_i^{\eta_i x_i}
\]  

(2.112)

By substitution Equation 2.111 into Equation 2.112 gives the following form

\[
w_i = a_i + \sum_j (\gamma_{ij} + \theta_{ij} x_{t-1}) \log p_j + (\beta_i + \eta_i x_{t-1}) \log(x_i / p_i^*) + u_0
\]  

(2.113)

where \( \log p_i^* = a_0 + \sum_i \delta_i e_{t-1} + \sum_i a_i \log p_i + \frac{1}{2} \sum_i \sum_j (\delta_{ij} + \theta_{ij} x_{t-1}) \log p_i \log p_j \) and \( u_0 \) denotes the stochastic error term, whereas the other variables as previously defined.

The following set of restrictions is implied on the DAIDS parameters:

1. Adding-up: \( \sum_i a_i = 1, \sum_j \gamma_{ij} = \sum \theta_{ij} = \sum \beta_i = \sum \eta_i = 0 \)
2. Homogeneity: \( \sum_j \gamma_{ij} = \sum \theta_{ij} = 0 \)
3. Symmetry: $\gamma_y = \gamma_p$, $\theta_y = \theta_p$

In addition to above, the author allows for serial correlation of the errors by assuming $u_u = \rho u_{u-1} + \epsilon_u$, where the $\epsilon$'s are independently and normally distribution random variables obeying classical assumption. Thus, Equation 2.113 can be rewritten in the following form

$$w_u = \rho w_{u-1} + \alpha_y (1 - \rho) + \sum_j (\gamma_y + \theta_y x_{t-1}) \log p_{i_t} + (\beta_i + \eta_i x_{t-1}) \log (x_i / p_{i_t}^*)$$

$$- \rho \sum_j (\gamma_y + \theta_y x_{t-2}) \log p_{i_{t-1}} - \rho (\beta_i + \eta_i x_{t-2}) \log (x_i / p_{i_{t-1}}^*) + u_u$$

(2.114)

Ray estimated the Equation 2.114 for the U.K. private sector for the years 1900-1980. The years affected by wars have been excluded namely, 1914 to 1921 and 1939 to 1953 inclusive. A four-item classification of consumer expenditure was used, namely, Food, drink and tobacco, footwear and clothing, fuel and light. Added to above, Ray set up the value of $\alpha_0$ by following Deaton and Muellbauer (1980). The major findings of the author are that, under the free model, the parameters estimated seem fairly plausible. Secondly, the dynamic adjustment parameters are mostly significant and point to dynamic AIDS representing a significant generalization of its static version. Thirdly, Ray also imposed and tested for homogeneity and symmetry restrictions, which were heavily rejected. Finally, Ray suggested that the dynamic version of AIDS instead of the static version should be used because of its superior performance.

2.5.3.3 Rational Behavior of Dynamic Demand Systems.

So far the presentation of dynamic demand analysis has been in the context of habit formation defined to be myopic when in each period the consumer takes into account his consumption history but does not recognize the impact of his present consumption decisions on his future tastes. However, the following presentation will focus on rational habit formation, which refers to the case of the consumer who is forward as well as backward looking. Spinnewyn (1981) and Philips and Spinnewyn (1982),
argued that myopic behavior is empirically indistinguishable from rational behavior, in the sense that a model of rational habit formation is equivalent to a model of myopic habit formation. Accordingly, Pashardes (1986) developed a model where myopic and rational specification is not empirically equivalent except under certain restrictions on preferences. The specification of the dynamic model is as follows:

$$z_n = q_n + dz_{n-1}$$ \hspace{1cm} (2.115)

where $z_n$ refers to stock of commodity $i$ which the consumer has at time $t$, $q_n$ is the quantity (purchases) of commodity $i$ in period $t$, and $d$ represents as a parameter. According to Equation 2.115, when $d$ defined to be $0 < d < 1$ for net durable commodity and $-1 < d < 0$ for habit forming ones.

In addition to that, Pashardes (1986) defined the rental price $P_y$ of commodity $i$ in time period $j$ (user cost) as follows

$$\hat{P}_y = \frac{1 + r - d}{1 + r} p_y = \lambda p_y$$ \hspace{1cm} (2.116)

where $r$ refers to the rate of interest and $p$, the current price of commodity $i$ and hence, $\lambda$ will be defined from Equation 2.116 as

$$\lambda = \frac{(1 + r - d)}{1 + r}$$ \hspace{1cm} (2.117)

where $\lambda > 1$ for habit forming commodities and $\lambda < 1$ for durable commodity.

Moreover, it was assumed that the decision as how to allocate the period to period budget $\hat{x}_t = \sum \hat{P}_u^i z_u$ can be made separately from the rest of the life cycle decisions. Correspondingly, the definitions of the period budget constraint for rational and myopic consumer were respectively as follows:

$$\hat{x}_t = \sum \hat{P}_u^i z_u = \lambda p_u z_u$$ \hspace{1cm} (2.118)

$$\bar{x}_t = \sum p_u z_u$$ \hspace{1cm} (2.119)

However, Pashardes defined the budget share for rational and myopic consumer respectively, as follows:

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According to Equation 2.120 and 2.121, we can write that,

\[ \hat{x}_t = \frac{\sum \lambda_i p_{it} z_{it}}{\bar{x}_t} \cdot \sum \lambda_i \bar{w}_it \]  

and hence,

\[ \hat{x}_t = \bar{x}_t \sum \lambda_i \bar{w}_it \]  

By substitution Equation 2.123 into Equation 2.120 gives

\[ \hat{w}_it = \frac{\lambda_i p_{it} z_{it}}{\bar{x}_t \sum \lambda_i \bar{w}_it} = \frac{\lambda_i}{\sum \lambda_i \bar{w}_it} \]  

Therefore, Equation 2.124 shows that one can distinguish the budget shares of the rational consumer from those of the myopic consumer by observing the variation in the term \( \frac{\lambda_i}{\sum \lambda_i \bar{w}_it} \) over \( t \). There are only two cases where this is not possible and they are as follows:

1) \( \lambda_i \) is constant over \( i \) (i.e. \( \lambda_i = \lambda \)) for all \( i \), so that \( \hat{w}_it = \bar{w}_it \)
2) \( \bar{w}_it \) is constant over \( t \) (i.e. \( \bar{w}_it = \bar{w}_i \) for all \( i \) and \( t \)) which implies that \( \hat{w}_it \) is all constant over \( t \), which equal to \( \frac{\lambda_i}{\sum \lambda_i \bar{w}_it} \). Since these cases imply several restrictions on preferences, it reasonable to suggest that is generally possible to distinguish empirical myopic from rational behavior in a model where consumer demands are expressed in a budget share equation. Moreover, Equation 2.124 can be generalized to include the myopic and rational specifications as special cases as follows,

\[ \hat{w}_it = \left[ \frac{\lambda_i}{\sum \lambda_i \bar{w}_it} \right] ^\theta \bar{w}_it \]  

where \( 0 < \theta < 1 \) can be thought as a parameter reflecting the extent to which consumer decisions are rational.
To distinguish between the myopic and rational budget shares, Pashardes developed a dynamic model for the rational consumer based on the structure of AIDS. This model can be defined as follows.

\[ \hat{w}_t = a_i + \sum_k \gamma_k \log p^*_t + \beta_i \log(\frac{\hat{x}_t}{p_t}) \]  

(2.126)

where

\[ \log p^*_t = a_0 + \sum_i a_i \log p^*_t + \frac{1}{2} \sum_k \gamma_k \log p^*_t \log p^*_t \]

\[ \hat{x}_t = \sum_i \hat{x}_u z^*_u = \sum_i \hat{\lambda}_i p^*_u z^*_u, \quad z^*_u = q^*_u + d^*_u z^*_{u-1} \]

Moreover, Pashardes utilizes the following relationship for empirical estimating of the dynamic AIDS.

\[ w^*_t = \frac{p^*_t q^*_t}{x_t} = \left[ p^*_t (z^*_t - d^*_t z^*_{t-1}) / x_t \right] \left( \hat{\lambda}^*_t / \lambda_t x_t \right) \]  

(2.127)

\[ = \left( \hat{w}^*_t \hat{x}_t / \lambda_t x_t \right) - \left( p^*_t d^*_t z^*_{t-1} / x_t \right) \]  

(2.128)

Which is estimated as

\[ \hat{w}_t = [a_i + \sum_k \gamma_k \log p^*_t + \beta_i \log(\frac{\hat{x}_t}{p_t})] \left( \frac{x_t}{\lambda_t x_t} \right) - p^*_t d^*_t z^*_{t-1} / x_t + \eta_t \]  

(2.129)

where \( \eta_t \) is an error term with mean zero. The other terms in Equation 2.129 are as previously defined.

Pashardes (1986) applied equation (2.129) to nine commodity groups by utilizing annual British expenditure data in constant price for period 1947-53. Pashardes pointed out that there was a strong habit forming feature of consumption and the only commodity group exhibiting net durability was the group of durable commodity itself. Moreover, the author also estimated both the static and dynamic model of AIDS and pointed out (based on the log likelihood value), that the static model was rejected in favor of the dynamic model.
2.6 Further Discussions and Applications of Demand Systems.

This section reviews earlier applications of demand systems. We include testing the properties of the demand function, different approaches to measuring demand elasticities, an alternative definition of the price index to Stone's price index, the methods of estimating the demand system under these definitions of price index (so as to avoid biased estimates of the parameters of the demand system model). The misapplication of demand restrictions also is discussed in this section.

2.6.1 Testing the Properties of Demand Function

This sub-section is focused on testing the properties of the demand function for various studies of the demand system. The discussions covers the following topics: types of demand system models, the types of data and estimation methods utilized in applications, the major findings of studies, reasons for rejecting demand theory properties, and types of statistical tests to evaluate the final results. Finally, the comparison between different demand systems is presented, method to select the best demand system describe the country’s consumer behavior.

The most popular demand model to represent additive preferences is the linear expenditure system (LES) which is derived mainly from Stone-Geary utility function. Stone (1954b) was the first to estimate the LES but the estimation was in different forms to the demand system shown below

\[ p'q = py + b(x - p'y) \]  \hspace{1cm} (2.129)

where:
- \( q \) is an \( n \) vector of price of goods and services.
- \( p \) is an \( n \) vector of quantities of goods and services.
- \( b \) is an \( n \) vector of marginal budget shares required to be estimated.
- \( y \) is an \( n \) vector of committed expenditure required to be estimated.
- \( x \) is Total expenditure.

Stone estimated the above model using U.K data for the years 1920-1938, in which expenditure \( (x) \) is classified into 6 commodity groups. Stone started by assuming initial values for \( b's \) so as to obtain the values of \( y \) and these values were achieved.
after appropriate transformation. The same procedure was used to yield the set values of $b$'s. After the iterative technique, Stone obtained appropriate values for both sets. But by applying this technique there were no standard errors (S.E) for the estimates and this was the main limitation of the technique. The major finding was that the demand system satisfies general demand restrictions in addition to additive restrictions.

Leser (1958) estimated LES for Australia using time series data and Leser's estimation was for a limited number of highly aggregated groups of commodities for a few numbers of years. The expenditure was divided into 9 commodity groups. The major results of this study were as follows: first, in general the findings of estimation were accepted as consistent with theoretical foundations. Secondly, the expenditure elasticities indicated that demands for clothing and "miscellaneous" were elastic with respect to the change in gross expenditure. While expenditure elasticities for food and housing were inelastic. Finally, price elasticities showed that demand in each commodity group was inelastic with respect to change in its price.

Powell (1966) estimated the LES for Australian private consumer commodity data for the period 1949-50 to 1961-62. The expenditure in this case was divided into 10 commodity groups. The main findings of this estimation were that: first, the estimation indicted that 15 of total of 20 coefficients differed significantly from zero at the 5% level, secondly; the own-price elasticities for all commodity groups were inelastic. It should be mentioned here that the author estimated the own-prices and income elasticities at mean prices and expenditure respectively. Finally, all commodities were gross complements due to great income effects. Added to above, Powell followed Leser (1960) in using the average quantity of the i-th commodity as the subsistence level $(\gamma_i)$ for the sample period under study rather than estimating the value of $\gamma_i$. 
Goldberger and Gamaletsos (1970) also estimated LES for five expenditure categories: Food, clothing, rent, durables and other goods. The authors provide an extensive comparison of consumption patterns across nations, using data from 13 OECD countries for the period from 1950-1961. They found that U.S consumers spent a smaller portion of their supernumerary income on food and durables than did their Canadian and British counterparts. Most supernumerary income in the U.S was spent on the rent. For all countries, as may be expected, food expenditures make up the greatest proportion of the base expenditure.

Byron (1970a) estimated the log linear equations for Dutch data for the period 1921-39 and 1948-62. All commodities were divided into 16 main commodity groups. Byron estimated the model twice, one with and one without restrictions of homogeneity, Engle aggregation, and Slutsky symmetry restrictions. The main conclusion was that all restrictions were rejected based on the likelihood test. Byron (1970b) made another attempt and the estimation results were the same.

Lluch (1973) introduced a new version of LES called Extended Linear Expenditure System (ELES), which is based on the assumption that household decisions are made on the basis of per capita income. Demand properties were tested in this application. The results indicated that demand properties were rejected in this study. The rejection can be related to the prior information is simply incorrect. Consumers do not attempt to maximize utility and do experience money illusion due to changing prices and income. In other words, prior information is not found to be equivalent to sample information.

Lluch, Powell and Williams (1977) estimated both LES and ELES models. The structure of the ELES model is as follows

\[ p_i q_i = p_i \gamma_i + c \beta_i (x - \sum_{j=1}^{n} p_j \gamma_j) + u_i \]  

(2.130)

where \( x \) represents income only and not expenditure, \( c \) is the aggregate marginal propensity to consume, where the remains as earlier defined. The major findings of
this estimation were as follows: first, there were values of $R^2$ which exceed 0.95 for 103 out of 134 equations. These results indicate that overall fit was very high. The D.W statistics pointed to positive first order serial correlation in the residuals of individual equations. Secondly, the estimations of $\beta$, (marginal budget share) were all positive as anticipated except for Jamaica which was negative and was not significant different from zero at the 5% level. Finally, those compensated and uncompensated own-prices elasticities were negative. Compensated cross price elasticities were positive (except for Jamaica where the results were the opposite for the "other services group" due to the negative value of $\beta_1$). Estimations where consistent with the theoretical requirements of the utility theory.

Stoker (1986) estimated a four good linear expenditure system (LES) using annual US data. The main result of this application was that a simple dynamic model with first-order autocorrelation fitted the data equally well. Stoker concluded that it was not possible to discriminate statistically between omitted variables in static model and simple dynamic models. Thus both factors have been suggested as possible reasons for failure of homogeneity.

Chang et al. (1999) estimated the LES model for Taiwan using annual data for the period from 1951 to 1990. Chang et al. (1999) used seven commodity groups to estimate the model, and these groups were as follows: (1) Food, beverage & tobacco, (2) Clothing & footwear, (3) Gross rent, fuel & power, furniture, furnishing & household, equipment & operation, (4) Medical care & health expense, (5) Transport & communication, (6) Recreation, entertainment, education & cultural services, (7) Miscellaneous good & services. The main results were as follows: all the budget share and subsistence expenditure parameters were significantly different from zero at 5% level. Second all the subsistence parameter was positive for all commodity groups, except for transportation and communication commodity group. These results imply that all commodity groups are price inelastic, except the transport and communication commodity group which is price elastic. Third, the results of price elasticities indicated that transport & communication commodity
group has higher price elasticities which indicate that price interventions for these products can have a significant impact on consumption, at least in the short run. Fourth, the results also indicate that commodity groups such as (2)-(7) are highly preferred items in the consumer's budget, and their consumption is fairly sensitive to changes in income. Five, the expenditure elasticities results indicate that all commodity groups were classified as luxurious products except for food and the beverage & tobacco commodity group. Finally, the transport and communication commodity group had the higher expenditure elasticities. These results indicated that the situation might improve automatically as income increases.

Pollak and Wales (1992) provide of the most recent and extensive contributions to apply demand analysis based on annual U.S. per capita data from 1948 to 1983. They consider three broad expenditure categories that do not include consumer durables, or non-durables that seem closely related to durables. The categories they use are food, clothing, and miscellaneous goods. Pollak and Wales specify and estimate four types of demand systems; the linear expenditure system (LES), the quadratic expenditure system (QES), the basic translog (BTL) and the generalized translog (GTL). The specific form of the LES in expenditure term is given above. The QES, BTL and GTL in budget share forms are specified as follows:

The QES demand system in budget share form is given by

\[ w_i = p_iγ_i / x + \alpha_i((1 - \sum_k p_kγ_k) / x) + \{ p_iδ_i / x - \alpha_i\sum_k p_kδ_k / x \} \prod_k (p_k / x)^{-2\alpha_k}(1 - \sum_k p_kγ_k / x)^2 \]

where \( \sum_k \alpha_k = 1 \)

The GLT demand system in budget share form is

\[ w_i = p_iγ_i / x + \alpha_i((1 - \sum_k p_kγ_k) / x)\{ \alpha_i + \sum_j β_j \log( p_j / (x - \sum_k p_kγ_k)) \} \]

\[ /\{ \sum_k \alpha_k + \sum_j β_j \log( p_j / (x - \sum_k p_kγ_k)) \} \]

The BTL demand system in budget share form is

\[ w_i = (\alpha + \sum_j β_j \ln( p_j / m)) / (-1 + \sum_k β_k \ln( p_k / m)) \]

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Pollak and Wales utilized dynamic structure-static (includes no lagged values), dynamic translating (implies the use of a lagged dependent variable) and dynamic scaling (includes the effects of consumption in previous periods on the independent variables) models, along with three representations for the error structure. Pollak and Wales conclude the best model of demand to be the QES with either dynamic translating or dynamic scaling, as well as with a stochastic specification of the error terms. This assumes a general first order autoregressive process. They found that the results of the expenditure systems were preferable to those of the translog systems.

Deaton and Muellbauer (1980a) specified and estimated the AIDS model utilizing annual UK data from 1954-1974, inclusive of eight non-durable groups of consumer's expenditure. Deaton and Muellbauer used the results obtained by estimation to test for the homogeneity and symmetry restrictions. Their findings seemed to be consistent with earlier results in that both sets of restrictions were decisively rejected. Deaton and Muellbauer also pointed out that the imposition of homogeneity generates positive serial correlation in the errors of those equations which most strongly rejected the restrictions.

Blanciforti (1982) adopted the static AIDS to analyze annual USA data for the period 1948-1978, in which eleven commodities groups were distinguished. The restrictions of demand were tested and rejected. Muellbauer and Pashardes (1982) estimated the static AIDS with and without the homogeneity and symmetry restrictions. The major result of this study was that the restrictions were rejected, based on the log likelihood test.

Anderson and Blundell (1983), Fulponi (1989) and others such as Mergos and Donatos (1989a and b) applied the static AIDS model. Based on their results they concluded that the theoretical restrictions were rejected.

There is also a tradition of testing the homogeneity restriction for AIDS, since the latter represents the weakest restrictions placed on estimated consumer demand.
systems. Deaton and Muellbauer (1980a) who introduced the AIDS, used an eight good model with annual UK data from 1954-74. They noted how homogeneity was less firmly rejected when their model was estimated in first differences. Utilizing the same data set, Deaton (1981) examined the role of housing, estimating an AIDS model conditioned on the quantity of housing which he assumed to be fixed, rather than freely chosen. The major finding was that the F-statistics for homogeneity (on a good-by-good basis) were lower following the inclusion of housing, as also was the system-wide test statistic.

Ray (1984) estimated the AIDS using Deaton and Muellbauer’s AIDS model generalized to account for factors other than prices and income to explain the demand for a given product. Ray used data from the U.K. from 1900 to 1980, excluding the war years, to estimate a linear habit formation version of the AIDS model. This model includes lagged values of past consumption for each of the goods in the system, along with lagged expenditure. By imposing the restrictions on demand functions, and by assuming that the error structure is $u_t = \rho u_{t-1} + e_u$, the estimated model becomes

$$w_t = \rho w_{t-1} + \alpha_t (1 - \rho) + \sum_j (\gamma_j + \theta_j x_{t-1}) \log p_{jt} + (\beta_t + \eta_t x_{t-1}) \log (x_t / p_{t^*})$$

$$\rho \sum_j (\gamma_j + \theta_j x_{t-1}) \log p_{jt-1} + (\beta_t + \eta_t x_{t-1}) \log (x_{t-1} / p_{t-1}^*) + \epsilon_u$$

(2.134)

where the dynamic price index is given by

$$\log p_t^* = a_0 + \sum_i \delta_i e_{ui-t} + e_{ui-t} \log p_{ui} + \frac{1}{2} \sum_i \sum_j (\delta_i + \theta_i x_{t-1}) \log p_{ui} \log p_{ui}$$

(2.135)

The estimated system contains four categories of commodities: food, alcohol and tobacco, footwear and clothing, and fuel and light. Most of parameters are found to be significant, and a huge drop in Likelihood occurs when Ray compared their work with the restricted (static) model. Ray reported that homogeneity and symmetry are heavily rejected in testing both the static and dynamic versions of the AIDS model. Ray (1980, 1982, and 1984) applied AIDS to time series and pooled cross-section
household budget survey data from India. In these studies, Ray extended the AIDS model by including explicitly family size.

Blanciforti and Green (1983) incorporated habit effects in an AIDS model after the manner of Pollak and Wales (1969) and estimated this model on an eleven good classification using annual US data for 1948-78. Their results suggested that habit-formation was the reason for the autocorrelation found in Deaton and Muellbauer's (1980a) results. However, incorporation of the habit formation term appeared to make only a small difference to tests for homogeneity.

Anderson and Blundell (1983, 1984) estimated AIDS for a five good model using annual Canadian data (1947-79) and a four good model using quarterly UK data (1955:1-1981:2). In both cases, they found that the long run homogeneity could not be rejected.

Sabelhaus (1990) applied ADDS using two data sets to estimate this expenditure system across eight commodity groups, namely, food, clothing and shoes, gasoline and oil, other goods, household operation, transportation services, personal services, and recreation. The first type of the data is aggregate data from U.S for the period 1959-1988. The second data set is derived from consumer expenditure survey (CES) for the period 1980-1986, and it represents a multiple cross-section. It should be noted here that Pollack and Wales (1978, 1979, 1980) have shown that if a multiple cross-section is used a complete expenditure system can be estimated. The main results reached by Sabelhaus are as follows: The aggregate data are inconsistent with the theory whereas the cross-section data are not. Pollack and Wales suggest several possible reasons for the failure of the theory, including shifts in tastes over the long time period being studied, and reliance on the average consumer being representative. However, Deaton and Muellbauer (1980) suggest several other reasons, including inflexible spending patterns in the short run, and changing expectations about future prices there are striking differences in estimated price and income elasticities for some of the eight commodities in the system. There are
positive compensated price elasticities for the two of the goods in the cross section data, and one good in the aggregate data. The negativity restriction fails in both data sets. The price elasticities vary in direction as well as magnitude in the two data sets.

Dynamic modeling was applied by Muellbauer and Pashardes (1992), who estimated an AIDS model incorporating both habit-formation and durability effects using the same breakdown of commodities as Deaton and Muellbauer (1980a) but with a long run of data. The main result obtained by Muellbauer and Pashardes is that homogeneity is strongly rejected for the static version of the AIDS model but accepted for the dynamic version.

Leybourne (1993) estimated a time-varying AIDS model using UK quarterly data (1958:1-1988:3) among five categories of expenditure. Leybourne found that tests for homogeneity were sensitive to the use of a time varying or constant coefficient model. However, in both cases the homogeneity restriction was rejected for one good, although the relevant good was sensitive to the model chosen.

Rougier (1997) derived a necessary condition for negativity in the AIDS when used with the Stone price index. The necessary condition was derived when the budget share of good (i) lies in the range (0, 1) is that $\gamma_u \leq 0$. Rougier suggests that this condition can be examined by simple t-test hypotheses on each equation within the framework of ordinary least squares (OLS) estimation.

$$H_0 : \gamma_u = 0 \quad H_A : \gamma_u > 0$$

The rejection of the null hypothesis in this test indicates that negativity is violated when the budget shares lies in the rang (0, 1). Therefore the rejection of the null hypothesis indicated a failure of that particular demand function to conform to consumer theory.

Song, et al. (1996) developed Ray's (1980) equation by including a family size variable, which allows for the effect of economies of household size:
Song, et al. (1996) estimated the unrestricted AIDS model above by using Netherlands time series data for the period 1960-1988. The system of equations above includes total household consumer expenditure \( x \), and expenditure on the following commodities: Food, Housing, Clothing, Hygiene and Medical Care, Education and other consumption. A time trend \( t \) is also added to the specification to capture the effect of changes in consumers' tastes. The main conclusions of the unrestricted model (2.140) are summarized as follows: First, the homogeneity restriction is accepted for demand equations for Food, Education, and other commodities only (these results were based on Wald test). Second, the family size variable is significant in the Food, Housing and other consumption equations, although the sign is negative in the last case. Third, the coefficients of own-price for Housing, Clothing, Education, and Other consumption are not significant and, apart from Hygiene and Medical Care, most of the cross-price coefficients are not significantly different from zero. Four, the expenditure coefficients are all significant except for that of Clothing. Five, the own-price and expenditure elasticities for all commodities have the expected signs and plausible magnitudes. Finally, the coefficients of family size \( \theta_i \) are positive for Food, Education, and Other commodities only, and these results are consistent with the interpretation (that, in order to feed extra mouths from a limited budget, spending patterns must be changed). Thus, as family size increases for a given level of expenditure and prices, families are forced to adjust their pattern of demand towards "essential commodities such as Food, Housing, and Clothing, and away from "less essential "commodities such as Hygiene and Medical Care, Education, and Other consumption.

Sasaki (1996) estimated the linear approximate of the AIDS model developed by Deaton and Muellbauer (1980a), using Japanese time series data for the period (1963-1986). The total expenditure is desegregated into 16 commodity categories. The LA/AIDS, which was estimated, is of the form:

\[
 w_a = \alpha + \sum_j \gamma_j \log p_a + \beta_i \log \left( \frac{x}{p_i} \right) + \theta_i \log \bar{n}, \quad (2.136)
\]
\[
\Delta w_u = \alpha_i + \beta_i Dq_i + \sum_j \gamma_{ij} Dp_j + \varepsilon_u \tag{2.137}
\]

where: 
\[
\Delta w_u = w_u - w_{u-1}
\]
\[
Dx_i = \log x_i - \log x_{i-1} \quad (x: \text{an arbitrary variable})
\]
\[
Dq_i = Dm_i - Dp_i \quad \{Dq_i = \sum_k w_{ik} Dq_k\}
\]
\[
\{Dp_i = \sum_k w_{ik} Dp_k\} \quad (k = 1, 2, ..., N) \quad \text{and} \quad w'_u = (w_u - w_{u-1})/2
\]
\[\varepsilon_u: \text{is the error term.}\]

The major findings are summarized as follows: First, the goodness of fit of the model to sample data is quite high in terms of information inaccuracy\(^4\). (i.e. information inaccuracy diminishes; the goodness of fit of the model improves). Second, the Durbin-Watson statistic centers on 2 of the 12 commodities indicating that the serial correlation problem is not serious as a whole. Third, of the 16 income parameters \(\beta^*\)'s, 8 parameter estimates are significant at 10%, and the derived marginal budget share, \(p_i (\partial q_i / \partial m) = (\beta_i + w_i)\), is significant for 13 commodities at a 5% level. Four, intercept \(\alpha_i\) is positive for electricity and gas, water charges and transportation which show an upward tendency of demand as time passes. For non-durables, clothing, and medical care are negative. Accordingly, Sasaki reported that changes in demand are for the most part explained by changes in income and prices for majority of the commodity categories. A few commodities exhibit a significant time trend effect. Five, own-price substitution terms are all negative while the number of negative eigenvalues amounts to 12 of the total of 15 values. Therefore, Sasaki reported that the second-order condition of the equilibrium is not fully satisfied at the sample means. Finally, the symmetry restriction is not rejected, given the homogeneity restriction. It should be noted here that Sasaki used expenditure, calculated at 1980 constant prices as the quantity data (see Sasaki, 1996, p.339).

\(^4\) Average information inaccuracy can be defined as: \(I = \sum_i w_i \log(w_i / \hat{w}_i)/T\), where \(w_i\), \(\hat{w}_i\), and \(T\) refer to the observed budget share, estimated budget share, and the number of observations respectively.
Madden (1993) estimated the AIDS and Rotterdam (RM) models for Ireland using annual data for the period 1958-1988. Total expenditures for this period were desegregated into ten commodities. Madden estimated each model (AIDS and RM) in three forms (unrestricted, with homogeneity imposition, with symmetry and homogeneity imposition). All the elasticities were evaluated at their average budget shares over the estimating period. The main results of these applications were as follows: First, the estimates from the AIDS model in levels with and without a time trend and the estimates obtained from AIDS in differences model are very close to those in the Rotterdam model. Second, the greater the degree of desegregation of commodities, the higher the values of own price elasticities one would expect as there is greater scope for substitution. Third, the expenditure elasticities results can be identified as food, alcohol, tobacco and fuel and power as "necessities" while clothing and footwear, transportation and equipment durables, other goods and services are "luxuries". Fourth, the results all indicate a firm rejection of homogeneity restriction under AIDS and Rotterdam as a system wide basis. The latter results were based on the Wald test and likelihood ratio test statistics. Five, the homogeneity was rejected for four cases only when Madden tested this restriction on equation by equation and when the system was on the level form. Six, under the different forms the homogeneity was rejected only for food which is less likely to reject homogeneity on both an equation by equation and system-wide basis. Seven, the rejection would be less when the model includes a time trend as suggested by Madden and in the equation by equation case.

Eight, AIDS in the differences form and Rotterdam models, under the imposition of the homogeneity and including a time trend as a wide system \((n-1\) equation) test is applied causing a sharp drop in the Wald statistic. Nine, the tests for symmetry in the presence of homogeneity are rejected less conclusively than are tests for homogeneity alone. Ten, the negativity approximately holds given the imposition of symmetry due to the matrices are not far from being negative semi-definite (that all the eigenvalues of that matrix should be less than or equal to zero (Johnston, 1984,p.151)). Finally Madden concludes by saying that stochastic and dynamic
rather than deterministic seems to matter more both for the plausibility of estimates and for the rejection or non-rejection of the restrictions implied by utility maximization.

Barten (1967) estimated the Rotterdam model for the Netherlands with and without constant terms so as to express the change in tastes. Barten applied time series data for 4 (food, pleasure goods, durables, and all other goods) commodity groups for the periods 1920-1939 and 1949-1961, and try to estimate models with and without restrictions. The major findings of these applications were as follows: First, the summation of the $\beta_i$ was approximately equal to unity. Secondly, the negatively condition was satisfied due to the $k_y$ coefficients being negative. Finally, the other condition homogeneity and symmetry Slutsk symmetries were not exactly satisfied.

Barten (1969) used Dutch data for 1921-63, and estimated RM one with and one without restrictions. In each case intercept terms were added to the basic RM type equation to allow for changes in taste. The main finding of the estimations when homogeneity was imposed was a large fall in the likelihood value obtained. Therefore, it seemed that Barten's data did not support the theoretical proposition that equiportionate changes in prices and total expenditure should leave demands unchanged. The symmetry restrictions were similarly rejected. However, as Deaton (1972) pointed out, if homogeneity is taken as given, it may be that the additional restriction of symmetry need not be rejected.

Lluch (1971) utilizes data from Spain to examine similar logarithmic and Rotterdam system. Lluch assumes that the findings of Barten and Byron, which contradict consumer theory, may be unique to Dutch data. However, Lluch achieved same contradictory results, which the homogeneity restriction does not hold, whereas the symmetry condition does.

Deaton (1974a) estimated a nine-equation RM, using UK data for 1900-70. Deaton obtained similar results to Barten. Intercept terms were significant, homogeneity
was rejected by the data and so was symmetry. However, when symmetry (which implies homogeneity if the aggregation restrictions are necessarily met) was imposed the likelihood value was not significantly lower than when was homogeneity alone was imposed. Thus, as seemed possible from Barten's study, it appears to be only the homogeneity aspects of symmetry that are rejected by actual data. These results confirm the earlier work of Brown and Deaton (1972), who encountered similar problems when testing the homogeneity restriction. There are also great contributions of estimation Rotterdam Model such Barten (1977), Theil (1975). These contributions indicated that the general demand restrictions were rejected.

Borooah (1985) estimated the RM for the U.K data for the period 1954-1981; Borooah classified 48 commodities into 11 broad groups and estimated the RM between the 11 commodity groups and within these groups. The main conclusion of this study was that the restrictions were rejected between the groups (broad groups), while within the groups there were 8 out of 10 groups, the restrictions of homogeneity, symmetry and non-negativity were accepted. Accordingly, Borooah mentioned that one reason for rejection of demand restrictions in previous study, it could be that the classification of consumption was so broad, which does not reflect the objective of utility maximization for the consumer.

Modesto et al. (1993) estimated RM for Portugal using the small sample period from 1987-1980. Modesto allocated total private expenditure over four groups, namely, non-durables, durables, energy and transport, and services and leisure. The author used FIML as the estimation method. The results obtained were satisfactory as indicated by Modesto (1993, p.347). Most of the coefficients were significantly estimated showing the correct sign and magnitudes and the findings were, in general, compatible with those obtained from previous studies on the Portuguese reality.
Brown and Lee (1992) extended the Rotterdam Model to include lagged consumption through translation parameters, allowing habit and inventory effects. The form of the Rotterdam model with translation is:

\[ w_u d \log q_u = z_t - \sum_j z_j + \beta_i d \log Q_i + \sum_j \gamma_j \beta \log p_j \]  

(2.138)

where \( z_t = \frac{p,y_i}{x} d \log y_i \), the log change in the translation parameters \( (\gamma_i) \) weighted by the share of total expenditure committed to the good, \( p_i \gamma_i / x \). The effect of past consumption is introduced into the model by letting the translation term \( z_t \) depend on lagged consumption. The hypothesis is that:

\[ z_t = a_t w_{u-t} d \log q_{u-t} \]  

(2.139)

The left-hand side variables in (2.139) can be viewed as the percentage change in demand weighted by the budget share. The difference between the usual Rotterdam model and Brown and Lee (1992) specification is in the first two terms on the right hand side of equation (3.138), which involve changes in the translation parameters. The first term is a direct effect \( z_t \) due to a change in the translation parameter for the good in question, while the second term is an indirect income effect \( d, \sum z_j \) due to a change in supernumerary income caused by an overall change in the translation parameters. Changes in the translation parameters can be viewed as preference changes, and the resultant direct \( (z_i) \) and indirect \( (d, \sum z_j) \) effects, which made the difference between (2.138) and usual model lead to re-allocation of income. In the equation (2.139) the parameters \( a_t \) is normally expected to be negative (positive) when inventory (habit) effects dominate. The shorter the time interval of an observation, the more likely inventory effects is to dominate habit effects.

Brown and Lee (1992) applied the model for the U.S. personal consumption expenditures, using two types of data annual and weekly data sets. The annual data runs from 1934 to 1989, and for four commodity groups, namely, food, alcohol, other non-durables and services. Brown and Lee applied the model in five versions, with constant terms but without homogeneity and symmetry imposed, without
translation lag variables, without constant terms, with homogeneity imposed and with homogeneity and symmetry imposed. The same methodology used to study the annual US expenditure data is used to study the weekly data (200 weekly observations for the period from the week ending November 14, 1987, through September 7, 1991) but in this case for five fruit juice demand model specification. FIML method was used to estimate these models. The aim of applications was to illustrate the importance of time interval of an observation on the relative strengths of the habit inventory. The main results of this study also show the importance of the time dimension in observing habit and inventory effects in systems of demand equations. The shorter the observation time interval, the more likely it is that inventories will dominate habits. Again both homogeneity, and homogeneity and symmetry were rejected at the 5 percent level of significance for the two types of data.

Laitinen (1978) Meinser (1979) Bera et al. (1981), Bewley (1983), Theil and Clement (1987) indicated from their studies that traditional asymptotic test was biased towards rejected homogeneity and symmetry. Accordingly, Theil et al. (1985), Taylor et al. (1986) and Selvanathan (1991) tended to apply Monte Carlo simulation test which was developed by Barnad (1963) in order to examine the validity of demand restrictions. The main conclusion reached by Theil et al. (1985), Taylor et al. (1986) and Selvanathan (1991) was that the Monte Carlo simulation test could be regarded as another option to the standard asymptotic test.

2.6.2 Other Discussions.

The empirical comparison of various demand system performances has also been taken place in the literature review. Parks (1969) examined three different functional forms of demand of systems in order to compare the theoretical properties and performance of these systems. These are as follows: the Rotterdam model the indirect addilog model, and the linear expenditure system. The empirical comparison was based on a set of long time series data for Sweden for the period 1861 to 1955, and that data related to demand for consumer goods for 8 commodity
The concept of average information inaccuracy was used to evaluate the overall fit of the model to the data of the entire budget. Theil (1967) introduced this technique and the latter can be measured by

$$I_i = \sum_{i=1}^{n} w_u \log \left( \frac{w_u}{\hat{w}_u} \right),$$

where $w_u$ is the actual share of the $i$-th commodity in the time $(t)$, $\hat{w}_u$ donated the predicted shares for period $(t)$. This measure gives each commodity its appropriate weight in the measure of fit and can be applied over the prediction period or over various sub-periods. The major findings were that the theoretical comparison failed to provide a basis for selecting the best model. The models satisfied the adding-up, homogeneity and Slutsky symmetry conditions. Finally, only the RM showed consistently better results for all commodities together on the basis of average information inaccuracy measured.

Yoshihara (1969) examined four major demand systems often used on empirical studies to see if they met the four properties for the demand system. Yoshihara concluded according to the examinations that Stone's linear expenditure system and Houthakker's indirect addilog system passed the theoretical test which means that they are consistent with utility maximization and are applicable for empirical studies, whereas the double log system and Theil's differential specifications were not. Accordingly, Yoshihara fitted the two theoretically satisfactory systems to Japanese data on per capita consumption for the period 1920-1960, and the major findings of these estimations were that the LES explains the pattern of Japanese demand much better than Houthakker's system (indirect addilog system) due to the sum of squared residuals for each group under the LES which is smaller than that under the indirect addilog system. Thus, Yoshihara suggested, based on estimation results, that LES shall be the demand system in future econometric models in Japan.

Deaton (1974a) also made a comparison between different functional forms of demand system to the United Kingdom time series from 1900-1970. The years affected by wars have been excluded, namely 1914 to 1921 and 1939 to 1953 inclusively. The major findings of testing these models on a nine-commodity model
using maximum likelihood estimation were as follows: First; the symmetric variant of the Rotterdam system dominated the direct addilog and the LES models. Second; the additive Rotterdam model which is a subcase of the symmetric model was rejected at high level of confidence. Finally, the addilog and the LES performed better than the additive Rotterdam system. The comparison between models was based on the concentrated likelihood value ration of the restricted and unrestricted system.

Green, Hassan, and Johnson (1995) estimated four popular demand equations systems, (LES, AIDS, indirect addilog system, and the indirect translog model of Christensen, Jorgenson and Lau (1975) using a set of seasonally adjusted quarterly Canadian data series for four commodity groups ranging the period 1947-1987. The main conclusion reached by Green, Hassan, and Johnson (1995) was that AIDS dominates various alternative demand systems. The results were based on the Likelihood Dominance Criterion (LDC). The latter represents a new method of selecting non-nested demand models which were developed by Pollak and Wales (1991) due to difficulties in estimating a nest model (general model) which contained the large number of parameters to be estimated and the usually associated high degree of multcollinearity. In addition, the researchers may not be interested in artificially constructed model, but rather in determining which of the two competing models is better according to some criterion. It should be noted here that the LDC can be used only for the models that have the same dependent variables.

2.6.3 Various Approaches and Discussions of Demand Elasticities.
The almost ideal demand system (AIDS) of Deaton and Muellbauer (1980a, b) has become popular in recent years. A variety of approaches to computing elasticities have been used and some of the approaches may lead to significant errors. This section presents the discussions and the major differences among alternative approaches to estimate demand elasticities in the AIDS model.
Green and Alston (1990) reported that all the previous studies of estimating the demand elasticities from the LA/DIDS were theoretically incorrect. Green and Alston (1990) compare the empirical results from the AIDS with four alternative approaches to estimate elasticities from LA/AIDS parameter estimates. The alternative measures of uncompensated elasticities are as follows:

### AIDS

\[ \varepsilon_y = -\delta_y + \frac{\gamma y}{w_i} - \frac{b_j}{w_i} (\alpha_j + \sum_k \gamma_{y_k} \ln p_k) \]  

(2.141)

### LA/AIDS:

**Model-1** \[ \varepsilon_y (AI) = -\delta_y + \frac{\gamma y}{w_i} (\alpha_j + \sum_k \gamma_{y_k} \ln p_k) \]  

(2.142)

**Model-2** \[ \varepsilon_y (LA'') = -\delta_y + \frac{\gamma y}{w_i} \]  

(2.143)

**Model-3** \[ \varepsilon_y (LA') = -\delta_y + \frac{\gamma y}{w_i} - \frac{b_j}{w_j} \]  

(2.144)

**Model-4** \[ \varepsilon_y (LA) = -\delta_y + \frac{\gamma y}{w_i} - \frac{b_j}{w_j} [w_j + \sum_k w_{y_k} \ln p_k (\varepsilon_{y_k} + \delta_{y_k})] \]  

(2.145)

Green and Alston (1990) applied the same data of U.S. food consumption, which were reported by Blanciforti, Green, and King (p.36, 1986) and used FIML estimates to compute the LA/AIDS formula model-4. The major findings of this application were as follows: First: the correct LA/AIDS model-4 provided similar elasticities to the AIDS. Secondly, the error from using model-3 which treats shares as exogenous was small but the large error was from using model-2 of LA/AIDS which assumed the price index is exogenous (this used the AIDS formula with LA/AIDS parameters). Finally, Green and Alston (1990) suggested employing the AIDS price elasticity formula only when estimating the AIDS and conclude that correcting for autocorrelation in this particular application essentially reduces the real income effect to zero and the differences in the various elasticity expressions vanish.

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5 AIDS formula was applied by Blanciforti Green, and King (1986) while the four LA/AIDS alternative were applied by (1) Anderson and Blundell (1983), (2) Eales and Unnevehr (1988) with the group price is constant, independent of individual goods' prices (1990), (3) Chalfant (1987) with expenditure shares are constant \( \left( \frac{d \ln P'}{d \ln P_j} = w_j \right) \), and (4) Green and Alston (1990).

6 The conventional AIDS elasticity \( \left( \frac{d \ln P'}{d \ln P_j} = 0 \right) \).
Hahn (1994) criticized the two articles of Green and Alston (1990, 1991) on the correct method of calculating elasticities for LA/AIDS. Hahn reported that Green and Alston's formulas are relevant only if the LA/AIDS is treated as a system in its own right and even as a system. Hahn pointed out the LA/AIDS itself lacks merit. The main problem with an approximate system is that the price index does more than just deflate expenditures and it also affects system performance. Hahn also criticized Green and Alston (1990, 1991) when they mentioned their strong statement, that it is not known whether the LA/AIDS has satisfactory theoretical properties, without proofing it. Accordingly, Hahn made an attempt to examine the properties of approximate systems, and the main results were that the adding-up conditions were met by approximates while the homogeneity condition was met only when the price index was constructed, so that was homogenous of degree one in prices. However, LA/AIDS is the symmetric if all prices are equal, but changing even one price can make all symmetry conditions invalid. Finally, Hahn (1994) suggested utilizing the Rotterdam model or a differential version of the AIDS which are presented in the Keller and Van Driel (1985) paper as an alternative to LA/AIDS. That is:

\[ \Delta w_i = \beta \bar{D}(m_i/p_i) + \sum_j \gamma_j \bar{D}p_j + \varepsilon_i \]  

where \( \Delta w_i = w_i - w_{i-1} \)
\( \bar{D}x_t = \log x_t - \log x_{t-1} \) (x: an arbitrary variable) and \( t = 2, \ldots, T \)

Alston, Foster and Green (1994) used the parameters estimated in linear approximate AIDS and compute the four alternative formulas for compensated demand elasticities, which were assumed to be of the AIDS form. The alternative measures of compensated elasticities are as follows:

\[ e^*_\delta (AI) = -\delta_y + \gamma_j \frac{w_i}{w_j} + w_j - \frac{\beta}{w_j} (\alpha_j + \sum_k \gamma_k \ln p_k - w_j) \]  

LA/AIDS:
Model-1 \[ e^*_\delta (AI) = -\delta_y + \gamma_j \frac{w_i}{w_j} + w_j - \frac{\beta}{w_j} (\alpha_j + \sum_k \gamma_k \ln p_k - w_j) \]  

(2.147)  

(2.148)
In this article Alston, Foster and Green (1994) utilize the Monte Carlo experiments to identify which formula provides accurate and similar result to true AIDS demand elasticity. The main results were as follows: First, the model-4 and model-3 formulas were consistently more accurate than model-1 and model-2 formulas. Second, as the degree of multicollinearity increases, accuracy decreases for all of the formulas, and especially for the model-2 and model-1 formulas. Third, as a preference departs further from homotheticity (as the \( \beta \)'s increase) precision falls and as the substitutability among the goods rises (as the \( \gamma \)'s increase) precision improves. Fourth, an increase in the simple size will improve precision slightly but did not change the overall picture. Five, the results were similar for uncompensated elasticities but in general the accuracy was much lower than for compensated elasticities. However, the formulas for the true AIDS model results in very poor estimates especially when multicollinearity among prices was high. The comparison between each formula was based on Mean Percent Error (MPE) and can be defined as:

\[
MPE = 100 \sum_{i=1}^{I} \frac{1}{J} \sum_{j=1}^{J} \left| \frac{\varepsilon_{ij} - \varepsilon_{ij}}{|\varepsilon_{ij}|} \right|
\]

Asche and Wessells (1997) study the effect of price normalization under the AIDS and LA/AIDS models. The major findings of Asche and Wessells (1997) were summarized as follows: First, when all the prices are normalized to unity, the two representations AIDS and LA/AIDS, are equal when evaluated at that point of normalization, and hence the reduced formula for both models can be written as: \( w_{it} = \alpha_i + \beta_i \ln x_i - \beta_i \alpha_o \). Secondly, the uncompensated elasticity for the AIDS model will be reduced to the new form \( \varepsilon_{u} = -\delta_y + \frac{\gamma y}{w_{it}} - \frac{\beta}{w_{it}} \). This
expression is identical to the formula used by Chalfant in the LA/AIDS model, and that the computed uncompensated elasticity of Green and Alston (1990) will also reduce to the same formula under price normalization. Third, Asche and Wessells (1997) indicated that the uncompensated elasticities were equal between the AIDS and its LA/AIDS if $\alpha_0$ was equal to expenditure in the base period, and also true if the Stone index was replaced by any of the indices suggested by Moschini (1995) to ensure consistent parameter estimates. Fourth, under the price normalization, the expenditure elasticities for the LA/AIDS, which were modified by Buse (1994)

$$\eta_{it} = 1 + (\beta_j / w_{it}) \left\{ 1 - \left( \sum_{j=1}^{n} \beta_j \ln p_{jt} / \bar{P} \right) \right\}$$

will reduce to AIDS formula that is:

$$\eta_{it} = 1 + (\beta_j / w_{it}).$$

Fifth, Asche and Wessells (1997) found that the expression of compensated price elasticities were also equal, at the point of normalization, and given by $\varepsilon_{jt}^* = -\delta_y + (\gamma_y / w_{it}) + w_{jt}$.

Finally, Asche and Wessells (1997) concluded by mentioning that the LA/AIDS will satisfy restrictions of demand theory locally only.

Buse (1994) discovered three features of the Green and Alston computations (elasticities) that require correction. First, the ML/AIDS elasticities were based on 31 observations for the period 1948-78, whereas the LAIDS elasticities were calculated for 32 observations for the period 1947-78. Second, Green and Alston did not follow conventional practice and normalize all the price indices at unity, but used the original series with a base of 100 in 1972. Third, in computing LAIDS elasticities using the AIDS definition given by model-1 above, Green and Alston used $\alpha_i$ instead of the correct estimate given by $\hat{\alpha}_i = \hat{\alpha}_i + \beta_i \ln \zeta_0$ for the equation (2.153) below

$$w_{it} = \alpha_{it} + \sum_{j=1}^{n} \gamma_{ij} \ln p_{jt} + \beta_i \ln (x_i / \bar{p}_i) + u_{it}$$  \hspace{1cm} (2.153)

Accordingly, the LAIDS model should be estimated under this normalization, as the estimated intercepts $\hat{\alpha}_i^*$, in equation above, depend on the choice of normalization. Buse (1994) reported that re-estimation was not required since the intercepts under
unit normalization can be recovered by subtracting $\beta, ln100$ from Green and Alston's $\hat{\alpha}^\ast$, then the LAIDS elasticities denoted by $\eta_i(A')$ change dramatically.

The main findings of the Buss were that there were two positive own-price elasticities, and the corrected elasticities deviate from the benchmark ML elasticities by even greater amounts than recorded in Green and Alston. Buse also followed Blanchforti, Green, and King (1986, p.40) chosen ln $586.90$, the log of per capita expenditure in the reference year 1972. and estimated $\alpha_i$ by $\hat{\alpha}_i = \hat{\alpha}^\ast + \beta, ln(5.869)$, and recalculated the elasticities. The major results of those applications show that those elasticities gave the best approximations to the benchmark ML elasticities of AIDS (Buse, 1994, p.786). Finally, Buse suggested using the conventional AIDS elasticity when estimating LA/AIDS and his modification $\eta_i(A_o)$ which is written as follows: $\eta_i(A_o) = -\delta + (\gamma / w_i) - (\beta, w_j / w_i) - \beta, \sum w_i \gamma_i ln p_j$ (2.154)

2.6.4 Various Definitions of Price Index

As result of existence of various definitions of price index which are used to linearize of AIDS model to easily estimated it (rather than using the original form which is made the model in non-linear form), some of these price indices lead to bias in the estimation of parameters, while the others provide good results and are closer to the original form of the AIDS model. Therefore, this section will focus on these definitions and review most of the discussions of previous studies.

Madden (1995) estimated seven models of AIDS, in each case, test for homogeneity: AIDS in the level (Model 1), AIDS in first differences (Model 2), AIDS in levels with a quadratic time trend added (Model 3), AIDS in levels conditioned on total employment (Model 4), AIDS in levels with habit-formation terms (Model 5) and two versions of a more general dynamic AIDS along the lines of Anderson and Blundell (Model 6a and 6b). The forms of these seven models of AIDS are as follows:
Model (1) \[ w_t = \alpha_t + \sum_j \gamma_{jt} \log p_{jt} + \beta_t \log (m / p), \] (2.155)

Model (2) \[ \Delta w_t = \alpha_t + \sum_j \gamma_{jt} \Delta \log p_{jt} + \beta_t \Delta \log (m / p), \] (2.156)

Model (3) \[ w_t = \alpha_t + \sum_j \gamma_{jt} \log p_{jt} + \beta_t \log (m / p), + \delta_t + \zeta_t t^2 \] (2.157)

Model (4) \[ w_t = \alpha_t + \sum_j \gamma_{jt} \log p_{jt} + \beta_t \log (m / p), + \eta_t N_t \] (2.158)

Model (5) \[ w_t = \alpha_t + \sum_j \gamma_{jt} \log p_{jt} + \beta_t \log (m / p), + \sum_j \theta_j w_{t-j} \] (2.159)

Model (6) \[ \Delta w_t = \alpha_t + \sum_j \phi_{jt} \Delta \log p_{jt} + \mu_t \Delta \log (m / p), \] (2.160)

\[ \prod \left[ w_{t-1}^* - \Gamma_t \log p_{t-1}^* - \beta_t \log (m / p) \right] \]

where \( p \) is a price index defined by \( \log p = \sum_k w_k \log p_k, N_t \) is total employment in period \( t \), \( \Delta \) is the first difference operator, in model 6, \( w_{t-1} \) and \( \log p_{t-1} \) are vectors and the asterisk indicates that the n-th row is omitted, while \( \prod \) is an appropriately dimensioned short-run coefficient matrix. The other variables are as previously defined. In all cases the models were estimated over annual Irish data from 1958 to 1988. Madden (1995) utilized six categories of goods: food, alcohol and tobacco, clothing and footwear, fuel and power, durables, and other goods and services, however Madden (1995) avoided singularity in the system, hence, the other goods and services equation was omitted. Madden (1995) results were based on Wald chi-square statistic. In general, the pattern of results was not entirely in accordance with intuition. Second, the AIDS model estimated in first differences, ad hoc specification which provides no information concerning the long run and shows the least rejection of homogeneity. Third, the addition of omitted variables (total employment) shows a slight improvement over the static model. Fourth, the habit formation model also shows only a bare rejection of homogeneity. Fifth, the homogeneity was not rejected for two categories: alcohol and tobacco and clothing and footwear for most of the AIDS model, however, durables and other goods and services show the most consistent rejection of homogeneity. Finally, homogeneity also cannot be rejected on good by good basis and the same is true for first differences of AIDS at the 95% significant level.

\[ \text{Pashardes (1993) suggested the utilization of this definition of price as an approximation will lead to bias and the problem is more serious for estimates from micro-based data, rather than aggregate data.} \]
Moschini (1995) made attempts in estimating AIDS by applying different price indices in linear forms such as Tomqvist \((\ln p^T_t = 1/2 \sum_{i=1}^n (w^T_{i}\cdot ln(\frac{p^T_t}{p^0}))\), the loglinear analogue to the Paasche index \((\ln p^*_t = \sum_{i=1}^n w_i \cdot \ln(\frac{p^*_t}{p^0}))\) which Moschini calls the "corrected" Stone price index, and Laspeyres price index \((\ln p^L_t = \sum_{i=1}^n w_i \cdot \ln(\frac{p^l_t}{p^0}))\), in addition to the Stone price index and translog price index in non linear of the AI system. Moschini (1995) estimated these five models with different price indices by using US annual data for the period 1958 to 1985 for four commodities namely: beef, pork, chicken and other food. Beef, pork, and chicken quantity data are expressed in pounds per capita while prices are in $/lb. Expenditures on these three meats are obtained as the product of the aforementioned prices and quantities. The expenditure on the other foods was obtained by subtracting the sum of the expenditure of these meats from the per capita expenditure on food, and hence, for this composite group, clearly prices are not in natural units but in price index for food (100=1967). The major findings were that the Tomqvist index \(p^T\) was the best in producing the same results as the true nonlinear AI model. On the other hand, the linear AI with the standard Stone index \(p^*\) performs rather poorly, in particular producing extremely biased estimates of income which will spill over to price elasticities as well through the homogeneity condition (where the Marshallinan price elasticities are related to income elasticities by the homogeneity condition \((\sum_{j=1}^n e_{ij} = -\eta_i)\). Accordingly Moschini (1995) suggested using the Tomqvist index or the modified Stone price index mentioned above to estimate a linear approximation to the AI demand model and that the standard Stone index should be avoided due to the latter being dependent on the nature of the data (units of measurements).

Pashardes (1993, p.913) found that an alternative approximate index for \(\ln p^*_t\) in Deaton and Muellbauer (1980) can be expressed as:
\[ \ln p_i = (\alpha_0 + \ln s_i - \ln y_i \sum_{j=1}^k \beta_j \ln p_{ij} - 0.5 \sum_{k=1}^k \sum_{j=1}^k y_{ij} \ln p_{ij} \ln p_{ij}) / (1 - \sum_{j=1}^k \beta_i \ln p_{ij}) \] 

(2.161)

where \( \ln s_i \) is Stone price index and is defined as \( \ln s_i = \sum_{j=1}^k w_{ij} \ln p_{ij} \). Deaton and Muellbauer (1980) suggested to set \( \alpha_0 \) in the above equation as the logarithm of total expenditure in the base period. See also Pashardes (1993, p.911). However, Wan (1998, p. 182) suggested an alternative manner to deal with this problem (\( \alpha_0 \)) is to: (i) transform both quantities and prices such that the first observation of quantities all equal one, but the expenditures on individual goods are kept unchanged; (ii) divide each price series by its first observation so the first price observations are also equal to unity, and the corresponding log expenditure is simply \( \ln k \). Thus, \( \alpha_0 = \ln k \). Moschini (1995) indicated such transformations do not alter the properties of AIDS estimates due to the AIDS model being invariant to measurement units. Wan (1998) indicated that the imposition of such restriction, (\( \alpha_0 = \ln k \)) is most likely to introduce bias into parameters estimates, but as usual it may lead to reductions in the variances of the estimates. On the other hand; Buse (1994, p.782) indicated that a prior choice of (\( \alpha_0 \)) must be made for utilizing the Non - linear Maximum Likelihood estimation approach since this procedure will not require the use of the Stone index.

Pashardes (1993) reported that iteration procedures (such as Zellner’s SUR technique) are performed to obtain better cross-equation covariance estimates. However, under estimating LA/AIDS and using SUR technique direct will provide biased estimates of the parameter in the LA/AIDS, since the budget shares in the Stone index are endogenous and thus correlated with the disturbance terms. To overcome this problem Pashardes (1993) suggested to first use OLS to estimate the equations with the Stone index in place and then construct a new Stone index using the predicated shares from the OLS equations and finally iterate within SUR.
Browning and Meghir (1991) and Fan et al. (1995) used another iterative procedure, starts with the commonly used SUR, which produces estimates of $\alpha$'s and $\gamma$'s. They are used to construct $ln p_t$ and further estimates can then be obtained. The iterations continue between SUR and $ln p_t$ until convergence will produce the Deaton and Muellbauer Iterative estimator (DMI). Wan (1998, p.185) pointed out after using a different procedures such as the linear or nonlinear (OLS, SUR, 3SLS, PI 8 and DMI) to estimate AIDS model. The main findings of Wan (1998) applications suggested that the Deaton and Muellbauer Iterative (DMI) procedure is the best to use in empirical studies, in particular, it produces unbiased estimates of $\beta$'s. Wan's (1998) results were based on the usage of the Monte Carlo experiments.

Taube, Huth, and MacDonald (1990) followed Barton's (1964) procedure (which allows for family characteristics to affect demand) and extend the AIDS framework by including consumer expectation in order to study the effects of this variable on budget allocations. Taube, Huth, and MacDonald (1990) explain that under the temporal structure, expectations form a linkage between the present and the future while habits link the past to the present. The structure of the extended model is:

$$w_i = \alpha_{10} + \alpha_{11} q_{i-1} + \alpha_{12} q_{i-2} + \beta_1 \log(x/R) \sum_j \pi_j \log p_j + \delta_i \log s$$

(2.162)

where $\log R = \sum_i w_i \log p_i$, and $s$ represents an index of consumer anticipations. For simplicity and tractability, the authors utilize $\log R^* = \log R - \sum_i w_i \log h_i$ (where $h_i$ be a commodity specific scale factor for good (I) defined by $\log h_i = \delta_i \log s$) as a modified Stone (1953) index as was used satisfactorily by Deaton and Muellbauer (1980) and others to linearize the AIDS model. Quarterly time series data for the period third quarter 1969 to second quarter 1988 were used to estimate the equation above. Eleven expenditure categories were analyzed. The Consumer Sentiment

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8 PI is Pashardes's (1993) procedure but in this case replacing the Stone index by his modified index (see Pashardes, 1993, p. 913) and then is called PI method.
Index (CSI)\(^9\) which contain information regarding respondents own current and expected future financial status current and expected business conditions and buying conditions for durable goods. Juster and Wachtel (1972) interpreted the CSI as reflecting not only the general state of consumer optimism and pessimism, but also uncertainty about economic events important to purchasing decisions. Taube, Huth, and MacDonald (1990) estimated the LA/AIDS for each equation separately by OLS, while in the presence of significant serial correlation, the affected equation should be re-estimated by two-stage least squares to obtain consistent parameter estimates. The demand system was estimated from the CSI. The main findings are as follows: First, six categories of expenditure were found to be necessities ($\beta < 0$). These were food, housing services, medical care, other non-durables, personal care services, and other services. However, alcohol, clothing, utilities, transportation services, and durable goods were found to be luxuries ($\beta > 0$). Second, the coefficient $\alpha_{it}$ on the lagged consumption was found to be statistically significant for 6 of the 11 expenditure categories. These results imply that short-term habit is a significant element in the demands for these goods. These findings are generally consistent with those of Blanciforti and Green (1983), which were obtained from annual expenditure data, with the exception that no significant habits were found for durable goods. Third, the coefficient on $\alpha_{t-2}$ measuring longer-term habits was significant for two expenditure categories, clothing and other non-durable goods. The signs of each were negative. Parameter estimates for $\delta$, were statistically significant at least at the 5% level for three expenditure categories (alcohol, housing services, and durable goods). Fourth, the sign of $\delta_i$ was negative for alcohol and housing services, implying that improved consumer sentiment will shift expenditure at the margin away from these items to durable goods and other services. The results demonstrate the model potential usefulness for both demand forecasting and policy analysis.

\(^9\) The consumer sentiment index is published monthly by the survey research center of the University of Michigan. The survey is conducted by using both personal and telephone interviews from a national sample. For more details about SCI see (Taube, 1990, p.235).
2.7 Conclusion.

In conclusion, this chapter focused was on three main demand systems: Linear Expenditure System (LES), Almost Ideal Demand System (AIDS), and the Rotterdam models (RM). We have also used the derivation of these systems and the different functional forms of other dynamic versions of demand systems. This chapter shows how habits can be included in models of consumer demand. Particular attention has been given to the dynamic versions of both AIDS and the LES models.

The review which I have undertaken of previous empirical work includes testing of the properties of the demand function, comparisons of different demand systems in terms of "best performance" discussions of demand elasticities and different definitions of the price index. We have also discussed problems of incorrect prior information of consumer behavior, data limitations, difficulties with aggregation over broad product categories, inability of consumer theory to explain aggregate data, and the problems of variables over long time periods when preferences are changing. We have seen the difficulty of expressing the consumption of durables and non-durables in the same system. The number of expenditure categories included in a demand system has been shown to depend on the rejection of theoretical restrictions on the demand system.
References

Journals


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Books


Blanciforti, L., R. Green and G. King (1986), U.S. Consumer Behavior Over the Postwar Period: An Almost Ideal Demand System Analysis, Monograph number 40, Giannini Foundation of Agricultural Economics, University of California.


3.1 Introduction.

Researchers in less developed countries often face difficulties in obtaining data whether from official sources, or by dealing directly with the people working in government offices. Collecting the data in the form and quality that researchers require takes time and effort on the part of the researcher. For this study it took more than six months to obtain and classify the data.

This chapter provides a brief discussion on the sources of the data used to estimate the demand system for the State of Bahrain. This chapter also provides information on the estimation method utilized in this study.

Efforts have been made to obtain quarterly time series data instead of annual time series data. There are two main reasons for using quarterly time series data instead of using annual data, which are as follows: Firstly I decided to close a gap in previous studies which have usually been based on annual time series data. There have been very few studies based on quarterly time series data. The second reason is that quarterly data are best for the discussion and investigation of properties of time series and to apply the Maximum Likelihood approach introduced by Johansen and Juselius (1990). There are heavy demands on the data, requiring a large number of observations for a complete dynamic specification. In Bahrain the annual data contain only 24 observations while for quarterly data the series contain 80 observations which are better for the applications in this study. Therefore, quarterly time series data are
more appropriate for this research topic. The period covered is from 1979-q1 to 1998-q4.

3.2 Data Sources and Explanations

The data used in this study are based upon the quarterly time-series data of consumption statistics obtained from various governmental statistical offices. To estimate demand systems, data on consumption are taken to be the sum of domestic production and imports, less exports, assuming the change in stock is zero. All the data calculated from the different sources are for the period 1979-q1-1998-q4.

Quarterly production time-series data has been obtained from the National Account Department of the Ministry of Finance and Economy, and it is unpublished data. However, the data on exports and imports are extracted from recorded foreign trade statistics which are published by the Central Statistics Organization (CSO) of the State of Bahrain.

It should be noted here, that the data for imports (c.i.f.), which are obtained from the Central Statistics Organization do not include profit and import duty. They are evaluated only at cost price. However, total personal consumption expenditure in the economy is evaluated at market price. Accordingly, imports must be evaluated at market prices. To translate to market prices imports at cost price are revalued to include import duty and a profit margin.  

The information on profit margin is obtained directly from Supply and Prices Control, Department of Ministry of Commerce. Information on import duty is obtained directly from the Customs Department of the Ministry of Finance and Economy. The data are

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1 A law decree No. (18) was issued in 1975 to set-up the profit margin for food goods and other goods as determined by the Minister of Commerce. The main objective of the decree was to prevent the prices from increasing. The value of the profit margin was determined as a percentage of total value of import cost of the concerned commodity. Table 3.1 shows the commodity item and the percentage profit margin, which was set-up by the Ministry of Commerce. Accordingly the profit margin is defined as: Total revenue - Total cost (import (c.i.f) + import duty + local transportation and inventory cost). Local transportation and inventory cost represents very small percentage of total cost of imports in Bahrain. Accordingly, it assumed that the transportation and inventory cost is equal to zero in this study. See (Al-Klaie, and Hermes (1989), p.101).
unpublished but are presented in this study according to the type of commodity. (see Table 3.1).

Table 3.1
Profit margin and import duty

<table>
<thead>
<tr>
<th>Type of commodity</th>
<th>Profit margin</th>
<th>Imports duties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Tobacco</td>
<td>30%</td>
<td>30% in (1979-1981) and 50% in (1982-1998)</td>
</tr>
<tr>
<td>Alcoholic drinks</td>
<td>50%</td>
<td>70% in (1979-1981) and 100% in (1982-1998)</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Motor Cars</td>
<td>16%</td>
<td>20%</td>
</tr>
<tr>
<td>Household equipment</td>
<td>15%</td>
<td>10%</td>
</tr>
<tr>
<td>Household appliances and utensils</td>
<td>15%</td>
<td>5%</td>
</tr>
<tr>
<td>Household Furniture</td>
<td>20%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Sources: Ministry of Commerce, Supply and Prices Control Department, State of Bahrain.

The data on gasoline are obtained directly from the Oil Department, Ministry of Oil and Development. Data on light and water are obtained from the Ministry of Electricity and Water for the State of Bahrain. Data on private telephone expenditure are obtained directly from the Bahrain Telecommunication Company (Batelco). The data are unpublished. People who work in government service can apply for the data and associated information.

The numbers of licenses and private cars owned by the people in Bahrain are obtained from the Traffic & Licensing Directorate. The fee for renewing the license in Bahrain is 15 B.D per five years, while the fee for the registration of the cars is 20 B.D per year. Fees are paid annually. The quarterly fee for each license is 0.75 B.D. The fee for registration or renewal for each car is 5 B.D per quarter. The data series for driving cars and registering cars is as follows: number of private cars in the quarter times the registration fee per quarter, and the car license fee per quarter times the number of licenses in the quarter.
Data on insurance premium for private cars are published annually in the statistical abstract for the period 1979-1998. It is difficult to obtain data per quarter for this variable in Bahrain. I contacted by phone most insurance companies in Bahrain to obtain data per quarter. The companies informed us that they do not have per quarter data for insurance premiums in Bahrain. They recommended us to divide the annual data by four to obtain per quarterly data. Accordingly the annual data is divided by four in order to translate it into quarterly data, taking in account the reasons mentioned above.

Total private final consumption expenditure for the period 1979-1998 has been obtained from the Ministry of Finance and Economy. It is annual data. Because no quarterly data are available on private consumption expenditure, the annual data of private final consumption expenditure was interpolated using the technique of Goldstein and Khan (1976, p. 223) [See appendix D]. This technique is also used by Al-Jarrah (1996) in a study involving cointegration and causality analysis of money, income and prices in Saudi Arabia. Al-Jarrah (1996) and also Weliwita *et al.* (1998) used this technique because there were no quarterly data available on real GDP. Accordingly annual data for private consumption expenditure was interpolated using the technique of Goldstein and Khan (1976, p. 223).

The consumption of each commodity is calculated by adding together domestic production and imports, less exports. The consumption of each group of commodities is in Bahrain million of Bahrain dinar (M.BD). However, the consumption of other commodities is calculated by subtracting total consumption expenditure for four items, namely Food and Beverages, Clothing and Footwear, Housing, and Transportation from total expenditure per quarterly for all commodity groups.

The series for the whole period (1979-q1:1998-q4) have been deflated by estimates of total population, so as to give per-capita expenditures for each of the five commodity groups at current prices. The income, or total expenditure variable, \( x \) has been computed by adding-up expenditure on each commodity group. Budget shares for
each category represent the portion of total expenditure allocated to that commodity. Moreover, it is assumed that each budget share is positive and all shares add-up to one.

The sum of all categories has been used to obtain the budget share $w_i$ for each commodity group, which in turn has been multiplied by the total of final private consumption expenditure so as to obtain expenditure on each commodity group.

The group index numbers of the consumer price index for Bahrain have been used as the price $p_i$ variables. Prices per quarter for commodity groups are published by the Bahrain Central Department of Statistics. Thus the price data for the five commodity groups were directly provided in the form of index numbers.

As is generally assumed in this kind of analysis savings were assumed to be exogenous. Total expenditure assumed to be equal to income. Quantity per capita ($q_i$) of commodity ($i$) can be obtained by dividing expenditure of commodity ($i$) on price corresponding to that commodity. Modesto (1993, p.344) and Saksati, (1996, p.339) used directly expenditure at constant prices as quantity data ($q_i$).

Data were aggregated into five categories, namely; Food and Beverages (1), Clothing and Footwear (2), Housing (3), Transportation (4), and Others (5). The expenditure items included in each aggregate group are presented in Table 3.2. Table 3.2 contains a detailed description of each category.

The classification of the five commodity groups above was performed utilizing the model specification in this study. Accordingly a quarterly time series data was constructed for these five commodity groups to estimate all the demand systems that have been specified in this study.
Table 3.2
Expenditure items in each of five aggregate commodity groups.

<table>
<thead>
<tr>
<th>Commodities groups (1)</th>
<th>Description of group (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Food and Beverages</td>
<td>Food and Beverages includes the aggregate of food and beverage plus tobacco products.</td>
</tr>
<tr>
<td>2 Clothing and Footwear</td>
<td>Clothing and Footwear includes the ready-made cloth (for men, women, girls, and boy, children clothing). Plus footwear.</td>
</tr>
<tr>
<td>3 Housing</td>
<td>Housing includes fees for gas, light, and water. Household utensils. It includes also household appliances, household equipment, and household furnishings. Expenditure on housing does not contain rent.</td>
</tr>
<tr>
<td>4 Transportation</td>
<td>Transportation includes motor cars, registration fees, license fees, gasoline, insurance's premiums, expenses on public transportation, and on telephone bills.</td>
</tr>
<tr>
<td>5 Others</td>
<td>Others include medical care and health expenditure, personal care, education and culture, recreation, other expenses.</td>
</tr>
</tbody>
</table>


3.3 Estimation Method

Full Information Maximum Likelihood (FIML) estimation was widely applied to estimate the demand system models. The FIML method gives estimates for the entire system of equations simultaneously by using all the information available for each of the equations of the system. In general, FIML are derived for all structural parameters of a system while maximizing the likelihood function and using a wide range of a priori information referring to both the individual equations and several equations simultaneously. The information can include constraints on the coefficients and certain restrictions on the error structure. FIML estimates are values associated with local maximum point. The FIML estimates are consistent, asymptotically efficient among estimates of the simultaneous equation model. It should be noted here, that if I use any other estimation method, it is indicated at its place in the thesis.

Using an Iterative Seemingly Unrelated Regression technique (ISUR) provides estimates, which are equal to (FIML) estimates (David et al., 1996, p.77). However, an unrestricted model such as a demand system can be estimated using Ordinary Least Square method (OLS). See (Judge et al., 1985, p.468). When symmetry and homogeneity restrictions are imposed the system cannot estimate equation by equation
and hence, an Iterative Seemingly Unrelated Regression technique (ISUR) is used in this study and the latter method leads to convergence upon estimates that are Maximum Likelihood (Judge, et al., 1985, p.420).

The theoretical approach which has been outlined in Chapter 2 of this study gives background to the ways in which the data can be organized and interpreted. The data explanations and estimation methods in this Chapter (3) aim to simplify the empirical applications allowing the researcher to estimate the demand system equations and to test the restrictions that implied by consumer demand theory. The results of the empirical work are presented and discussed in the following chapters.
References

Journals


Books


Officials


Chapter 4

An Empirical Analysis of Aggregate Consumer Demand in Bahrain: Linear Expenditure Systems (LES) Model

4.1 Introduction.

The demand systems discussed in Chapter 2 are either derived directly from utility function maximization, (the Linear Expenditure System (LES)) or are obtained from the cost function, (Almost Ideal Demand System (AIDS)) or are derived directly from demand specification, (the Rotterdam model (RM)). In this chapter an attempt is made to estimate the parameters of the static LES model and its dynamic version in the manner of the Pollak and Wales (1969). Chapters 5 and 6 focus on the application of RM and LA\AIDS models respectively. Chapter 7 is devoted to a comparison of the demand systems that have been used in this study. The objective is to choose the one which fits Bahrain data best.

Demand systems aim to test the validity of the demand theory, to analyze the preference structure of consumers, and to estimate income and price elasticities. These are important in order for the decision maker to adopt appropriate polices for the economy in question.

However, demand systems do not always provide us with good results. They depend first on the quantity and the quality of data that are used in the application. Secondly they depend on the type of commodity. Thirdly the time period of observation is important. The aggregation of commodities (high/low aggregated groups of commodities) is also a factor, as is the definition of the
price indices. Finally, we need to take into account the purpose of using the model.

The plan of this chapter is as follows: Section 4.2 contains a brief review of the LES model and estimation method. Section 4.3 presents the discussions and results of the static LES model giving an account of the overall performance of the static LES model. Section 4.4 provides the main findings of the dynamic LES model, and discusses the overall performance of the dynamic LES model. A hypothesis test for "presence of habit" effects in the LES model is given in section 4.5. Concluding remarks along with comparisons between the results obtained from both versions of the LES model are presented in section 4.6.

4.2 Model Specification and Estimation Method.

The linear expenditure system has been used in many empirical demand studies, for example, Stone (1953) and (1954), Parks (1969), Yoshihara (1969), Pollak and Wales (1969), Phlips (1974), Powell (1974) and Deaton (1975), Lluch and Williams (1975), Mackinnon (1976), Green, Hassan, and Johnson (1980), Sasaki (1987), Heien and Durham (1991), Fan et al. (1995), Chang et al. (1999) and others. All of these authors have estimated the static and simple habit formation variants of the linear expenditure system.

The LES was first introduced by Kelin and Rubin (1947-48) in an attempt to construct a true cost of living index. Its economic interpretation was clarified by Samuelson (1947-48), and was developed by Stone (1953-54), who showed that the linear demand functions which make-up the LES can be integrated to obtain the unique utility function of the following form:

$$u = \sum B_i \log (q_i - \gamma_i)$$  \hspace{1cm} (4.1)

where the $q_i$'s are quantity flows and the $B_i$'s and $\gamma_i$'s are parameters such that $0 < B_i < 1$ and $\Sigma B_i = 1$, and the function being defined only for $0 < (q_i - \gamma_i)$. Equation (4.1) is called the Stone-Geary or Klein-Rubin utility function.
The authors derived the LES from the maximization of the above utility function subject to a budget constraint. It provides the LES, which can be written in the expenditure form as:

\[ p_i q_i = p_i \gamma_i + B_i (x - \sum_{k=1}^{n} p_k \gamma_k) \quad i=1, 2, \ldots, n \quad (4.2) \]

where \( q_i > \gamma_i > 0 \), \( 0 < B_i < 1 \), and \( \sum_i B_i = 1 \)

Equation 4.2 describes the expenditure on the \( i \)-th commodity, as a linear function of income (\( x \)) and the \( n \) prices (\( p \)) but nonlinear in its parameters (\( B_i \), the marginal budget share and \( \gamma_i \), the minimum subsistence quantity).

An interpretation of Equation 4.2 as suggested by Samuelson (1947-48) is as follows: At the beginning of a period, the consumer purchases \( \gamma_i \) units of \( i \)-th commodity, this is the minimum purchase required of the commodity at each period without regard to prices.

For all \( n \) commodities this amounts to \( \sum_i p_k q_k \). The remaining amount of income after this expenditure is \( x - \sum_i p_k q_k \) which is called supernumerary expenditure or income, and it is allocated among the \( n \) commodities in fixed proportions \( B_i (B_1, B_2, \ldots, B_n) \).

The ordinary Marshallian demand function can be obtained by dividing both sides of Equation 4.2 by prices (\( p \)), and the result is the LES in terms of quantity.

\[ q_i = \gamma_i + \frac{B_i}{p_i} (x - \sum_{k=1}^{n} p_k \gamma_k) \quad (4.3) \]

The LES is not flexible, and hence would not be acceptable for testing the theoretical demand restrictions, since homogeneity and Slutsky symmetry conditions hold automatically for this LES model. However, the LES does
exactly aggregate over consumers and may be a viable model for highly aggregated groups of commodities.

Most economists would agree that past consumption patterns are an important determinant for present consumption patterns. Goods may be habit forming so that an individual's current preferences depend on his past consumption patterns. In this case a change in prices or income will cause a change in consumption which will induce a change in taste, which in turn causes a further change in consumption. Heien and Durham (1991, p.97) emphasized that habit effects are large and highly significant and play an important role in consumer behavior pattern.

Pollak (1970) developed a model of consumer behavior based on studying the impacts of habit formation. The main assumption of the Pollak's argument is that the necessary quantity of a commodity, $y_i$, depended upon previous consumption of that commodity. Accordingly, Pollak implied that the necessary quantity of each commodity is a linear function of past period consumption level of that commodity.

Pollak used the formulation of Stone-Geary utility function to develop his model:

$$u(q_i) = \sum_i \beta_i \log(q_i - y_i) \quad \text{(for } i=1,2,\ldots, n) \quad (4.4)$$

The utility function above is re-expressed by including the habit effects through allowing the $y_i$, to be a linear function of past consumption levels of that commodity and the formulation for that necessary quantity can be defined as:

$$y_i = y_i^* + \Gamma_i q_{i-1} \quad \text{(for } i=1,2,\ldots,n) \quad (4.5)$$

where, $y_i^*$ was defined as the "physiologically necessary" component and $\Gamma_i q_{i-1}$ was defined as the "psychologically necessary" component of $y_i$. Accordingly,
by substitution of Equation 4.5 into Equation 4.2 and then maximization subject to the budget constraint yields the dynamic LES

In the expenditure form:

$$p_u q_u = p_u (r^*_i + \Gamma_i q_{u-1}) + \beta \{ (x_t - \sum_k p_u (\gamma^*_k + \Gamma_k q_{u-1})) \} + u_u$$  \hspace{1cm} (4.6)

In quantity form that is:

$$q_u = r^*_i + \Gamma_i q_{u-1} + \frac{\beta}{p_u} \{ (x_t - \sum_k p_u (\gamma^*_k + \Gamma_k q_{u-1})) \} + u_u$$  \hspace{1cm} (4.7)

The hypothesis implies that past consumption influences current preferences and current demand. In Equation 4.6, \( p_u (r^*_i + \Gamma_i q_{u-1}) \) becomes committed expenditure on commodity (i), and \( p_u \Gamma_i q_{u-1} \) is the part of the minimum required expenditure consequent from the purchase in the current time period at the current market price of that similar amount of the commodity that was purchased in the past period, \( \Gamma_i q_{u-1} \). Following Samuelson's (1947-48) argument that the amount that involves, for all \( n \) commodities together is \( \sum_k p_u (\gamma^*_k + \Gamma_k q_{u-1}) \), while, the remainder amount of income after that expenditure is \( (x_t - \sum_k p_u (\gamma^*_k + \Gamma_k q_{u-1})) \) which is called supernumerary expenditure or income, and which is allocated among the \( n \) commodities in fixed proportions, \( B_t \).

The static LES and its dynamic version involves cross equation restrictions on the parameters such as each of the \( \gamma^*_s \) parameters takes the same value in each equation of a given model, and that the sum of \( B_t \) must be equal to one. This constraint needs the demand system to be estimated simultaneously by methods that allows for the cross equation constraints. Further, the system of equations has to be estimated by an appropriate regression technique given that the system has to be additive, in the sense that the sum of the estimated expenditure has to be equal to the sum of observed expenditure in each time period. Therefore, the error terms of the equations are not independent and hence Zellner's (1988)
method of "Iterative Seemingly Unrelated Regression (ISUR)" is more appropriate. This estimation method has been used by Chang et. al. (1999) when he applied the linear expenditure system model to Taiwan economy.

4.3 Overall Performance of the Static LES Model.

Private consumption expenditure was disegregated into five aggregate commodity groups: (1) Food & Beverages, (2) Clothing & Footwear, (3) Housing, (4) Transportation, and (5) Other. The model to be estimated is the LES in the expenditure form, using Bahrain quarterly time series data for the period (1979Q1-1998Q4). Data sources and explanations are discussed in Chapter 3 of this study.

The static LES (4.2) is fitted to Bahraini data, and the main results of this application are reported in Table 4.1. Table 4.1 presents the estimates of the marginal budget shares, $B_i$, and the committed commodities, $\gamma_i$, along with the computed values for $R^2$ and durbin-watson ($d.w$) statistics results. The estimated results indicate the following:

1. All the coefficients parameters of committed commodities, $\gamma_i$, are significantly different from zero at the 5% level of significance except the committed expenditure for the Transportation commodity group, which is found to be insignificant.

2. The coefficient parameter of the marginal budget share is found to be not significant for one commodity group namely Clothing & Footwear.

3. Overall goodness of fit is low, the value of the coefficient of determination, $R^2$, ranges from 0.88 for the Food and Beverages commodity group to 0.37 for the Housing commodity group. The low $R^2$ values could be related to the variables in the model which are not normally distributed, or improperly specified in the LES model (omitting factors that effect consumer expenditure patterns). The values of the $d.w$ statistics are low suggesting the presence of positive first order serial correlation in the residuals of the individual
equations. The low $R^2$ and $d.w$ values can be improved by including dynamic factors. These results can be seen in the estimation of the dynamic linear expenditure system in the next section of this chapter. Deaton (1978, p.533) also obtained very low $R^2$ when he applied the LES model to annual British data. Personal expenditure in that case was disaggregated into eight commodity groups, namely, Food, Clothing, Housing, Fuel, Drink & Tobacco, Transport & Communication, Other goods, and finally, Other Services. The $R^2$ result was for other services group (0.1560) as reported in Table 2 of the Deaton's article. Moreover, it should be noted here; that during the review of literature of the most of previous empirical studies of the LES; it has been found that the presentations focused only on the t-statistic (or standard error) results, and that the statistics results of the $R^2$ and the $D.W$ for each equation were totally ignored.

(4) The estimated budget shares, $B_i$, are all positive and satisfy a prior knowledge in the sense that the value of each $B_i$ estimate is less than unity as required by the model, and the summation of the marginal budget shares is equal one.

(5) The estimated values of, $\gamma_i$, are also positive as needed by the model, and at the average values of the $p_j \gamma_i$ estimates are less that the average of actual per capita expenditure for all commodity groups.

(6) The estimated total subsistence expenditure is almost B.D 295.02 per person. This indicates that over 63.96% of average total expenditure is utilized to satisfy only the basic needs in Bahrain. Lewis and Andrews (1989) reported that the estimated total subsistence expenditure for China accounts to about 42.4% of average total expenditure, while the subsistence expenditure on Food & Beverages commodity group accounts to about 50% of average Food & Beverages expenditure. According to Bahrain data it indicates that the subsistence expenditure on Food & Beverages commodity group accounts to 60.72% of average Food & Beverages expenditure. The ranges for other groups are Clothing & Footwear 81.90%, Housing 17.91%, Transportation 14.79%, and Other 75.95%. It appears from Table 4.1 that Other commodity group dominates
subsistence expenditure which accounting for 35.5% of total subsistence expenditure.

(7) The marginal budget shares, $B_i$, measures the proportion of extra BD's of discretionary which is spent on the group concerned.

Furthermore, Table 4.1 indicates that the expenditure on both Other and Housing commodity groups is important for Bahrain consumers as apparent from the results of this application. The values of the marginal budget shares are 0.580 and 0.208 respectively, while it represents 0.079, 0.008, and 0.125 for Food & Beverages, Clothing & Footwear, and Transportation respectively. This means that the Bahrain consumers allocate around 7.9% of supernumerary expenditure to Food & Beverages commodity group, 0.01% for Clothing and Footwear commodity group, 12.5% to Transportation commodity group, 20.8% for Housing commodity group, and finally, the average Bahrain consumer tends to allocate most non-committed expenditure for Other commodity groups (58.0%). (See Table 4.1.)

<table>
<thead>
<tr>
<th>Commodity group (i)</th>
<th>Marginal budget share ($B_i$)</th>
<th>The minimum subsistence level ($\gamma_i$)</th>
<th>Coefficient of determination ($R^2$)</th>
<th>Durbin-Watson ($d.W$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; Beverages</td>
<td>0.079 (0.027)</td>
<td>54.119 (5.918)</td>
<td>0.831</td>
<td>0.152</td>
</tr>
<tr>
<td>Clothing &amp; Footwear</td>
<td>0.008 (0.025)</td>
<td>29.648 (5.668)</td>
<td>0.391</td>
<td>1.010</td>
</tr>
<tr>
<td>Housing</td>
<td>0.208 (0.020)</td>
<td>13.288 (3.867)</td>
<td>0.376</td>
<td>1.080</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.125 (0.021)</td>
<td>7.385 (7.867)</td>
<td>0.532</td>
<td>0.820</td>
</tr>
<tr>
<td>Other</td>
<td>0.580 (0.051)</td>
<td>207.964 (11.307)</td>
<td>0.709</td>
<td>0.747</td>
</tr>
</tbody>
</table>

(*)-values in parentheses are asymptotic standard errors

$d_u$ and $d_f$ are 1.57 and 1.69 respectively and imply that there is a positive serial correlation in the residual of the equations

Parameters describing consumer's behavior, i.e. income and price elasticities, are not only helpful to predict future demand but are also useful for evaluating the effects of economic reform, changes in income and sales tax rates. Accordingly, Table 4.2 reports the estimation of uncompensated own and cross price
elasticities along with total expenditure elasticities evaluated at the mean for period under consideration.

The formulation for the expenditure elasticity for the i-th commodity is given by:

$$\eta_i = \frac{\beta_i}{w_i} \quad (4.8)$$

The expenditure elasticities will be classified as the demand for commodity as necessities products when \( \eta_i < 1 \) and as luxuries product when \( \eta_i > 1 \) The income elasticities are all expected to be positive since the \( \beta_i's \) are positive, so any rise in income or total expenditure is allocated to each commodity in constant proportion. Also, Equation 4.8 states that there is an inverse relationship between expenditure elasticity and budget share \( (w_i) \).

Two out of five commodity groups are classified as necessities due to expenditure elasticity of these commodity groups are less than one, which include: Food & Beverages, and Clothing & Footwear. While the remainder of aggregate commodity groups is classified as luxuries, namely Housing, Transportation, and Other commodity group. The classification of these commodities as luxuries is due to the expenditure elasticities for these groups being bigger than one \( (\eta_i > 1) \), and hence the demand for these commodity groups are elastic with respect to total expenditure (See Table 4.2).

The formulation for the own price elasticity, \( e_{ii} \), of the i-th commodity is given by:

$$e_{ii} = -1 + \left(1 - \beta_i\right)\frac{\gamma_i}{p_i} \quad (4.9)$$

Since \( q_i > \gamma_i > 0 \) and \( 0 < \beta_i < 1 \), it follows own-price elasticities is limited to value between \(-1 < e_{ii} < 0\) Thus, all price elasticities are negative.

However, the formulation for the compensated own-price elasticity is defined as:

$$e'_{ii} = e_{ii} + \beta_i \quad (4.10)$$

Or

$$e_{ii} + \eta_i w_i \quad (4.11)$$
Since, $\beta$, is positive, the compensated own price elasticity will have smaller demand elasticity than the uncompensated own-price elasticity due to $e^*, \text{will be smaller negative than } e_n$.

In the context of the static LES uncompensated cross-price elasticity $e_{ij}(i \neq j)$ is given by the form

$$e_{ij} = -\frac{\beta_i \gamma_i p_j}{p_i q_i}$$

(4.12)

Since, $\beta_i, \gamma_i, p_i, p_j,$ and $q_i$ are all positive values, all uncompensated cross-price elasticities are negative. This implies that all commodities are gross complements.

However, the definition of compensated cross price elasticities is given by

$$e^*_{ij} = \frac{\beta_i p_j q_j}{p_i q_i} + e_{ij} = \beta_i \frac{w_j}{w_i} + e_{ij}$$

(4.13)

Or $$= \eta_i w_j + e_{ij}$$ for $i \neq j$

(4.14)

where, $w_i, w_j$ represent the budget shares of commodity $i$ and $j$ respectively,

and $\eta$ refers to elasticity of total expenditure ($\frac{\beta_i}{w_i} = \eta$).

Since, $\beta, w_i, , \text{and } w_j$ are all positive and $e_{ij}$ is negative, so as for $e^*_{ij}$ to be positive, it requires the income effect $\beta_i \frac{w_j}{w_i}$, must be bigger than substitution effect, $e_{ij}$. Moreover, the positive cross-substitution term implies that all pairs of goods ($i$ and $j$) are substitutes for each other.

As appeared from Table 4.2 all uncompensated own-price elasticities are negative and less than one in absolute value at the mean values and for the period under consideration. This means that the demand is all inelastic. This result is expected since it is considered to be one characteristic of the LES model, given that all the $\gamma_i$ estimates are positive. Table 4.2 shows that uncompensated own price
elasticity estimate ranges from -0.139 for Clothing and Footwear to -0.865 for the Transportation commodity groups.

Furthermore, as indicated by Table 4.2 all cross-price elasticities estimates are negative and less than unity in absolute value and none of the cross-price elasticity estimates for the individual equations dominate the own price elasticity estimates. These findings, however, satisfy a prior knowledge in the sense that the demand for any specific commodity is most strongly affected by its own-price elasticity. Lastly, a non-positive signs of cross-price elasticities indicate complementarily between commodities. The uncompensated cross price elasticities denote that all commodities are gross complements, which is consistent with a prior expectation.

The estimated values for compensated price elasticities as evaluated at mean values are presented in Table 4.3. Table 4.3 shows that all compensated own price elasticities are negative, and less negative than uncompensated own-price elasticities (see Table 4.2). All compensated cross price elasticities are positive. Positive signs of cross price elasticities donate substitutability. Thus, all commodities are net substitutes as indicated by the compensated cross-price

### Table 4.2

**Total expenditure and uncompensated own-cross price elasticities of the static LES model classified according to five aggregate commodity groups.**

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>Total expenditure elasticity (*( \eta_1 ))</th>
<th>Elasticity of demand with respect to price of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Food &amp; beverages</td>
<td>Clothing &amp; footwear</td>
</tr>
<tr>
<td>Food &amp; beverage</td>
<td>0.465</td>
<td>-0.402</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.114</td>
<td>-0.012</td>
</tr>
<tr>
<td>Housing</td>
<td>1.486</td>
<td>-0.162</td>
</tr>
<tr>
<td>Transportation</td>
<td>1.563</td>
<td>-0.157</td>
</tr>
<tr>
<td>Other</td>
<td>1.074</td>
<td>-0.115</td>
</tr>
</tbody>
</table>
elasticities. Further, the only cross price elasticities estimates for other commodity group equation dominated own-price elasticity estimates.

Table 4.3
Compensated own-cross price elasticities of the static LES model classified according to five aggregate commodity groups.

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>Food &amp; beverages</th>
<th>Clothing &amp; footwear</th>
<th>Housing</th>
<th>Transportation</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverages</td>
<td>-0.323</td>
<td>0.006</td>
<td>0.054</td>
<td>0.032</td>
<td>0.067</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.007</td>
<td>-0.131</td>
<td>0.013</td>
<td>0.008</td>
<td>0.016</td>
</tr>
<tr>
<td>Housing</td>
<td>0.091</td>
<td>0.015</td>
<td>-0.643</td>
<td>0.100</td>
<td>0.180</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.109</td>
<td>0.023</td>
<td>0.183</td>
<td>-0.740</td>
<td>0.241</td>
</tr>
<tr>
<td>Other</td>
<td>0.068</td>
<td>0.012</td>
<td>0.124</td>
<td>0.072</td>
<td>-0.285</td>
</tr>
</tbody>
</table>

To sum up, the positiveness of $\beta_i's$ restricts all goods to be gross complements and net substitutions in the LES. Moreover, the compensated cross price elasticities, $e_{ij}'$, are all positive, but the uncompensated cross-price elasticities, $e_{ij}$, are all negative. Added to that, all price elasticities of demand in the LES are less than unity in absolute value, unless some of the parameters $\gamma_j$ are allowed to be negative.

The flexibility of money coefficient $\omega$ introduced by Frisch (1959) is defined as the elasticity of marginal utility of expenditure $\lambda$ with respect to expenditure. 

\[
\frac{\partial \lambda}{\partial x} = -\frac{1}{(x - \sum p_i y_k)^2} \left( \frac{x}{(x - \sum p_i y_k)} \right)
\]  

(4.15)

\[
\frac{\partial \lambda}{\partial x} = -\frac{x}{(x - \sum p_i y_k)}
\]  

(4.16)

where $\lambda$ is defined as the rate of change of utility with respect to a change in income. The coefficient $\lambda$ can be estimated by using the equation below.
In the context of the LES model, the definition of money flexibility is equal to the negative value of the expenditure ratio to supernumerary expenditures. That is:

\[ \lambda = \frac{1}{(x - \sum_k p_ky_k)} \]  

(4.17)

The value of \( \omega \) is used as an indicator of welfare. Bieri and Janvry (1972, p.44) reported that the value of \( \omega \) ranged from 0.61 for high income countries to 3.90 for low-income countries. In other words, for high-income countries the value will be smaller than for the low-income countries. The value of \( \omega \) will decrease (increase) in absolute value as per capita total expenditure increases (decreases) as suggested by Frisch. Theil (1975) pointed out that under the utilization of LES model this value will not exceed (-1) even for large income \( x \), which is not in agreement with Frisch’s suggestion. The results of the current study indicate that the flexibility of money for Bahrain evaluated at the mean value for the period under consideration is close to -2.543. With respect to Frisch’s (1959) suggestion, the result (-2.543) is classified for the rich part of the population with ambitions towards "conspicuous consumption"

Pigou’s law is an approximate proportional relationship between the own price elasticity of demand and expenditure elasticity, since it was first put forward by A.C. Pigou (1910). It is defined as:

\[ e_{i} = \theta \eta_{i}, \quad i=1, 2, ..., n \]  

(4.19)

where

\[ \theta = -\frac{(x - \sum_{k} p_k q_k)}{x} \]

This approximation arises from the assumption of additivity embodied in the LES. It implies that the response to variation in total expenditure is approximately proportional to the response of a change in prices for commodity. Since there is an inverse relationship between the expenditure elasticity and the budget shares, this inverse relationship extends to the price elasticity. Deaton
(1975b) reported that the existence and relative inflexibility of this relationship is limited to the LES more than other conditions. In this study the estimates value of \(\theta\) is -0.393 for the LES for all commodities in Bahrain data. The study indicates that the absolute value of the own price elasticity is approximately 0.393 of the expenditure elasticity.

Important government and business decisions in consumers’ expenditures or sales and understanding the dynamics underlying the changes in demand can be helpful in making more informed decisions. The purpose of this study is to introduce the dynamic effect in LES model in the manner of Pollak and Wales (1969). The term of past consumption represents the dynamic effect and is defined in this type of the analysis as the effect of habit on current consumption. Accordingly, the aim of the next section is to study the effect of past consumption patterns on current Bahraini consumers’ expenditure, and secondly, introduction of the dynamic factors that have any effect on improvement of the statistical results of both the \(R^2\) and the \(d.w\) values.

4.4 Overall Performance of the Dynamic (LES) Model

Total personal consumption expenditure were disaggregated into five commodity groups, namely (1) Food and Beverages, (2) Clothing and Footwear, (3) Housing, (4) Transportation, and (5) Other. The model to be estimated is the dynamic linear expenditure (DLES) model and is described in section 4.2. The dynamic LES model is fitted to the same set of data that was used in estimating the static of the LES in section (4.3). The model is estimated by the means of Seemingly Unrelated Regression (SUR) estimator developed by Zellner (1962). The estimates of the structural parameters along with the main summary of the statistics results of the model are presented in Table 4.4.

In Table 4.4 it is shown that all the marginal budget shares (\(\beta_i\)) estimates are positive and sum to one, the psychological aspects (\(\gamma_{tq_{it-1}}\)) are all positive and
are significantly different from zero at five percent level. This support the persistence consumption pattern is present in the Bahrain data. However, the marginal budget shares are significantly different than zero at 5% level for only three commodity groups (Food & Beverages, Housing, and Other group). The physiological aspect \( (\gamma^*_i) \) is also positive and significant for only the Housing and Transportation commodity group. See Table 4.4.

The results of the \( d.w \) statistics are high for all equations of the model and suggest that the absence of the first order autocorrelation in the residual of the error for all equations in the DLES model. However, the coefficient of the determination \( (R^2) \) is ranges from 0.96 for Food & Beverage commodity group to 0.41 for the Housing commodity group.

Table 4.4
The estimates of coefficients parameters of the dynamic LES model with the main statistics results classified according to five aggregate commodity groups.

<table>
<thead>
<tr>
<th>Commodity group (i)</th>
<th>Marginal budget share ( \beta_i )</th>
<th>Habit parameter ( \Gamma_i )</th>
<th>Physiological aspect ( \gamma^*_i )</th>
<th>Psychological aspect ( \gamma_i + \Gamma \delta_{i+1} )</th>
<th>Total ( \bar{\gamma}_i )</th>
<th>The effect of last period consumption ( \bar{\gamma}_i (1 - \beta_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; Beverages</td>
<td>0.180 (0.023)</td>
<td>0.942 (0.028)</td>
<td>1.192 (2.726)</td>
<td>78.52</td>
<td>79.71</td>
<td>0.772</td>
</tr>
<tr>
<td>Clothing &amp; Footwear</td>
<td>0.062 (0.045)</td>
<td>0.590 (0.089)</td>
<td>13.407 (74.001)</td>
<td>19.82</td>
<td>33.23</td>
<td>0.544</td>
</tr>
<tr>
<td>Housing</td>
<td>0.284 (0.081)</td>
<td>0.634 (0.078)</td>
<td>14.449 (7.486)</td>
<td>44.83</td>
<td>59.28</td>
<td>0.439</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.035 (0.048)</td>
<td>0.746 (0.065)</td>
<td>12.739 (4.120)</td>
<td>35.77</td>
<td>48.51</td>
<td>0.720</td>
</tr>
<tr>
<td>Other</td>
<td>0.416 (0.133)</td>
<td>0.958 (0.035)</td>
<td>5.412 (12.594)</td>
<td>249.46</td>
<td>254.88</td>
<td>0.559</td>
</tr>
</tbody>
</table>

The coefficients of determination \( (R^2) \) for Food & Beverage, Clothing & Footwear, Housing, Transportation, and Other are 0.96, 0.48, 0.41, 0.68 and 0.79 respectively. The Durbin Watson \( (d.w) \) Statistics for Food and Beverage, Clothing & Footwear, Housing, Transportation, and Other are (1.87), (2.17), (2.10), (2.07), (2.10), respectively.

* - coefficients are based on Bahrain quarterly data for the years (1979Q1-1998Q4).

** - values in parentheses are asymptotic standard error.

Further, in the DLES, the following evidence has been found: (1) the estimated marginal budget share, \( \beta_i \), for Other commodity group (0.416) is less than it was
in the static LES model (0.580). (2) The total estimated minimum subsistence level for the various commodity groups are all positive ranging from 33.23 per capita for the Clothing and Footwear commodity groups to 254.88 for the Other commodity group. In the static LES model, the total estimated minimum subsistence level for the various commodity groups are all positive but ranging from 7.38 per capita for the Transportation commodity group to 207.96 for the Other commodity group. (3) The Bahrain consumers allocate the largest of their committed expenditure and the largest part of their incremental expenditure for Other commodity group products.

In the DLES model, the formulation for the own-price elasticities \( e_n \) of the i-th commodity is given by:

\[
e_n = -1 + \{(1 - \beta_i)(\gamma_i' + \Gamma_i q_{n-1})\}/q_n
\]

(4.20)

Since \( q_i > \gamma_i > 0 \) and \( 0 < \beta_i < 1 \) It follows that \(-1 < e_n < 0\). Thus, all own price elasticities are negative. While the formulation for cross-price elasticities for DLES \( e_{ij} \) are given by the form:

\[
e_{ij} = -\beta_i/p_i(\gamma_i' + \Gamma_i q_{n-1})/p_i q_n
\]

(4.21)

Since, \( \beta_i, \gamma_i, p_i, p_j \), and \( q_i \) are all positive values, all uncompensated cross-price elasticities are negative. This implies that all commodities are gross complements.

However, the Slutsky cross-substitution term is defined as \( \beta_i (q_j - \gamma_j) = \frac{\partial q_j}{\partial p_i} |_{\mu} \) which is positive in value and results in compensated cross price elasticities.

\[
e_{ij}^* = \frac{\beta_i p_j}{p_i q_i} (q_j - \gamma_j)
\]

(4.22)

Table 4.5 contains total expenditure elasticities together with uncompensated own-cross price elasticities evaluated at sample mean values. Table 4.5 indicate
that total expenditure elasticity estimates suggest that the expenditure elasticity of demand for Food and Beverages and Housing group is elastic, which confirms that these commodity groups are luxuries. On the other hand, the demand for Clothing & Footwear, Transportation, and Other commodity groups are inelastic with respect to total expenditure. This result suggests that the latter commodity groups should be classified as necessities for Bahrain consumers. Moreover, the total expenditure elasticity classifies the demand for Food and Beverages products as luxury products for Bahrain consumers. According to DLES model, it shows that in spite of the rise of the cost of standard of living in Bahrain, the average per capita income has not increased during the period under study. Finally, the very high expenditure elasticity of the housing demand indicates that a further increase in the consumption of housing products should be expected in the future provided that consumer income increases.

As shown in Table 4.5 all uncompensated own-price elasticities are less than unity and all are negative. However, uncompensated cross price elasticity estimates are all negative and less than unity in absolute value except one which is found elastic and is between the demand for Housing commodity group with respect to the Other commodity group price. See Table 4.5.
The compensated own-cross price elasticities for each commodity group are presented in Table 4.6. Table 4.6 indicates that all compensated own-price elasticities are negative, except one, which is found to be positive and that for the Transportation commodity group. The positive sign of compensated own price elasticity for Transportation is due to the marginal budget share of the Transportation commodity group is bigger than uncompensated own price elasticity for that commodity group. This result is not considered to be a characteristic of the LES model. The positive compensated own price elasticity appears to violate the law of demand and hence the properties of demand theory. Finally, all compensated cross-price elasticities are positive, as expected.

Table 4.6
Compensated own -cross price elasticities of the dynamic LES model classified according to five aggregate commodity groups.

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>Elasticity of demand with respect to price of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Food &amp; beverages</td>
</tr>
<tr>
<td>Food &amp; beverage</td>
<td>-0.036</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.141</td>
</tr>
<tr>
<td>Housing</td>
<td>0.363</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.047</td>
</tr>
<tr>
<td>Other</td>
<td>0.125</td>
</tr>
</tbody>
</table>

4.5 A Hypothesis Test of the Habit Effects in the LES Model.

To test for the presence of habit effects in the LES, the likelihood ratio test procedure has been used. It can be shown that minus two times the logarithm of the ratio of the restricted to the unrestricted likelihood function \((-2 \ln \lambda)\) has an asymptotic chi-square distribution with degrees of freedom equal to the number of linearly independent restrictions. The computed value of \(-2 \ln \lambda\) was equal to 144.8 when testing the hypothesis. \(H_0 : \text{All } \Gamma_i = 0\). This value is greater than the critical value of \(\chi^2_{0.05} = 9.49\) suggesting the presence of habits.
4.6 Conclusion

The static LES and the dynamic LES models are fitted to Bahrain quarterly time series data for the period 1979Q1-1998Q4. The applications give the following results:

- In the static LES, all the coefficients parameters of the minimum subsistence level are significantly different from zero at 5% level except that for the transportation commodity group while the coefficient parameter of marginal budget is found to be not significant only for the Clothing & Footwear commodity group.

- In the DLES model, it is found firstly, that the psychological aspect \((T_i, q_{u_i})\) are positive and highly significantly different from zero at the 5% level. This finding supports the persistence of consumption patterns is present in the Bahrain data. However, the physiological aspect \((\gamma_i)\) is also positive and significant at 5% level for two commodity groups and that are for Clothing & Footwear and Transportation categories. Secondly, the marginal budget share of two commodity groups are not significant, and that for Clothing & Footwear and Transportation commodity group.

- The \(R^2\) and \((d.w)\) statistical results indicate that the system of equations is more fitted in the DLES than in the static LES model. The reason can be related to omitting the dynamic factors (lagged consumption expenditure) that mainly affects the consumer expenditure patterns.

- All the marginal budget shares are positive and sum to one in both models. The marginal budget share coefficients are considerably different from the static LES estimates. In the DLES model for example, it is found that the estimated marginal budget share for the three commodity groups, namely Transportation, Housing and Other is bigger to what it was in the static LES model.
• The Bahrain consumer’s largest value of estimated minimum subsistence level is the same in both models. With respect to the lowest value of estimated minimum subsistence level, it is found that in the LES model, the lowest is in the Housing commodity group, whereas in the DLES, the lowest is in the Clothing and Footwear commodity group.

• The largest part of incremental expenditure goes toward the Other commodity group in both versions of the LES model.

• The results of the static LES indicate that over 63.96% of the average total expenditure is used to satisfy the basic needs in Bahrain, whereas the results of the dynamic LES model indicates that over 90.79% of the average total expenditure is used to satisfy the basic needs in Bahrain. These results suggest that a substantial increase in income will be needed to raise living standards to a higher level above basic needs.

• Expenditure elasticities indicate that two commodity groups are classified as necessities (Food & Beverages and Clothing & Footwear) and three are classified as luxuries (Housing, Transportation, and Other) in the static LES model, while in the DLES three commodity groups are classified as necessities (Clothing & Footwear, Transportation, and Other) and two are classified as luxuries (Food & Beverages and Housing).

• The magnitude of the estimated Food and Beverages expenditure elasticity is higher in the DLES than in the LES (0.465 compared to 1.059), the magnitude is also higher for both the Clothing and Footwear, and Housing commodity group in the DLES than in the static LES model.

• Furthermore, the results of the LES model indicated that the demand for Food and Beverages commodity group represents necessity products, while in the DLES model it represents luxury products. According to DLES model, it shows that in spite of the rise of the cost
of standard of living in Bahrain, the average per capita income has not increased during the period under study.

- Estimated direct price elasticities are bigger for the five groups in the LES compared to the DLES mode in the absolute values. In the Food and Beverage groups, for instance, it is -0.402 in the static LES model compared to -0.216 in the DLES model.

- The relationship that total expenditure elasticities were greater than the absolute values own price elasticity estimates \( \eta_i > |\epsilon_i| \) is inherent in the LES. This relationship is held for DLES model while in the LES model it does not hold.

- Prior assumptions pertaining to gross complements and net substitutes hold for both models.

- It appears in this study that the strongest positive relationship between the current and past consumption occurs for Food & Beverages commodity and the weakest positive relationship between the current and past consumption occurs for the Housing commodity group.

- The results of the current study indicate that the flexibility of money for Bahrain, evaluated at the mean value for the period under consideration is determined to be close to -2.543. This result, according to Frisch (1959) classification reflects the rich part of the population with ambitions towards "conspicuous consumption".

- The study indicates that the absolute value of the own price elasticity is approximately 0.393 of the expenditure elasticity.

- A hypothesis test of the habit effects in the LES model suggests the presence of habits in the Bahrain data.
References

Journals


**Books**


Chapter 5
An Empirical Analysis of Aggregate Consumer Demand in Bahrain: The Rotterdam Demand System Model

5.1 Introduction

The Differential Demand Model or Rotterdam Model developed by Barten (1964) and Theil (1965), provides a first-order approximation of true demand. Analysis by Barnett (1984), Byron (1984), and Mountain (1988) shows that the approximation is comparable to other popular flexible functional forms. In order to allow for trends in consumption and changes in tastes, a constant term is sometimes included in the Rotterdam demand specification as a rough approximation (e.g. Theil (1976), Barten (1969), Deaton (1974), Deaton and Muellbauer (1980)). Recently, Brown and Lee (1992) have extended the differential demand system to include lagged consumption through translation parameters, allowing habit and inventory effects.

Accordingly, the current chapter provides an analysis of commodity groups demand in Bahrain using the Rotterdam Model and quarterly data for the period 1979Q1-1998Q4 in which five commodity groups were distinguished. Correspondingly, the similar set of data and the similar commodity groups that were fitted to LES model are also utilized to apply the Rotterdam Model to the data. The system of the Rotterdam Model was estimated in four versions. First, it was estimated to the data without imposing any of the restrictions implied by the demand theory on their values. Second, the homogeneity restriction was
imposed on the parameters of the system. Third, the symmetry restriction was imposed, and lastly both homogeneity and symmetry restrictions were imposed on the parameters of the system. However, the presentations of the structural parameters are only for the first three versions of the usual Rotterdam Model while the structural parameters for the restricted version are not presented, but is used only for the hypotheses test in section 5.4 of this chapter.

This chapter will also apply a dynamic differential demand system of Brown and Lee (1992) in order to study the impact of lagged consumption in the differential version of the translation model for alternative Rotterdam Model specifications. The application will be for quarterly data for the period from 1979Q1 to 1998Q4. It is worth noting, that there is no existing empirical work regarding analysis of quarterly data by applying this model.

The analysis will focus on the estimation of coefficients of the Rotterdam model and testing the restrictions of demand theory to obtain empirical evidences about commodity groups' consumption expenditure behavior for Bahrain. The chapter is organized as follows: a brief overview of the model and the estimation method is presented, followed by overall performance and discussions of their implications which include estimating the parameters of the model for every restriction implied by the demand theory, estimation of prices and expenditure elasticities for various commodity groups at every stage of analysis, followed by testing the restrictions of the demand theory for the Rotterdam demand system. Finally, a general conclusion is offered at the end of this chapter.

5.2 Model and Estimation Method.

The Rotterdam Differential Demand Model, developed by Barten (1964) and Theil (1965), is not based on a particular utility function but more generally, on a first-order approximation to the demand functions themselves. Barten (1967)
defined the RM by giving considerations to the Slutsky Equations, and forming the following expression for the differential of the demand equation derived by the utility maximization, i.e. \( q_t = q_t(p_1, ..., p_n, x) \) yields

\[
dq_t = \sum_j \frac{\partial q_t}{\partial p_j} dp_j + \frac{\partial q_t}{\partial x} dx
\]  

(5.1)

Substituting the Slutsky equation into Equation 5.1 gives

\[
dq_t = \sum_j (s - \frac{\partial q_t}{\partial p_j} q_t) dp_j + \frac{\partial q_t}{\partial x} dx
\]  

(5.2)

Equation 5.2 can be rewritten in the following form:

\[
dq_t = \sum_j s dp_j + \frac{\partial q_t}{\partial x} (dx - \sum_i q_i dp_i)
\]  

(5.3)

Utilizing the expression that \( dy = yd \log y \) yields

\[
q_t d \log q_t = \frac{\partial q_t}{\partial x} (x d \log x - \sum_i p_i q_i d \log p_i) + \sum_j s dp_j d \log p_j
\]  

(5.4)

By multiplying both side of Equation 5.4 by \( p_i/x \) gives

\[
\frac{p_i}{x} q_t d \log q_t = \frac{p}{x} \frac{\partial q_t}{\partial x} (x d \log x - \sum_i p_i q_i d \log p_i) + \frac{p_i}{x} \sum_j s dp_j d \log p_j
\]  

(5.5)

Since, \( w_i = \frac{p_i q_t}{x} \), \( \beta_i = p_i (\frac{\partial q_t}{\partial x}) \), \( k_y = s_y (\frac{\frac{p_i}{x}}{x}) \), then Equation (5.5) can be written in the following form:

\[
w_t d \log q_t = \beta_i (d \log x - \sum_i w_i d \log p_i) + \sum_j k_y d \log p_j
\]  

(5.6)

Equation 5.6 which was developed by Barten (1967) (1969) and Theil (1965), (1975), represents the Rotterdam Demand System Model and deals with the changes in, rather than the levels of demand.

*The Slutsky Equations are defined as: \( \frac{\partial q_t}{\partial p_j} = s - q_j \frac{\partial q_t}{\partial x} \)
Equation 5.6 is formulated in terms of infinitesimal changes. The Rotterdam model, however, is a finite change version of (5.6) which can be written for empirical purposes as (Theil et al. 1987).

\[ w_{n}^{-} Dq_{n} = B_{i} DQ_{i} + \sum_{j} \gamma_{y} Dp_{j} + u_{n} \]  

(5.7)

where:

- \( w_{n}^{-} = (w_{n} + w_{n-1}) / 2 \), \( D \) denotes the log difference operator,
- \( Dq_{n} = \log q_{n} - \log q_{n-1} \)
- \( DQ_{i} = \sum_{i} w_{ij} D \log q_{ij} \approx (d \log x_{i} - \sum_{i} w_{ij} d \log p_{j}) \) is a finite change version of the Divisia volume index, (Theil and Clements 1987, p.25).
- \( B_{i} \) are the marginal budget shares, while, \( \gamma_{y} \) are the parameters of the Slutsky matrix.
- \( u_{it} \) is a disturbance term with the following assumptions: \( E[u_{it}] = 0, E[u_{it}, u_{jt}] = 0 \), for \( i \neq j \) and \( E[u_{it}, u_{jt}] = \omega_{y} \) for \( t=t' (i,j=1,\ldots, n) \).

That is, it is assumed that the random disturbances are un-correlated across observations, but are correlated across equations for the same observations. The contemporaneous covariance matrix is singular. In the light of the equality

\[ \sum_{i} w_{n}^{-} Dq_{n} = DQ_{i} \]  

(5.8)

is a value weighted average of the logarithmic differences of the quantities demanded. It is thus a volume index of the change in total consumption and can be interpreted for our purpose as a measure of the change in real income.

One of the attractive features of the Rotterdam Model is the relative ease of imposition and testing of such restrictions as aggregation, homogeneity and symmetry. Adding up restriction implies \( \sum \beta_{i} = 1, \sum \gamma_{y} = 0 \) Simply dropping an arbitrary equation from the system can easily impose these two conditions. Homogeneity implies \( \sum \gamma_{y} = 0 \), while symmetry implies \( \gamma_{y} = \gamma_{y} (i \neq j) \). Unlike
the AIDS models, negativity can be imposed by the condition that the matrix 
\[ \{ Y_{ij} \} \] be negative \((Y_{ij} < 0)\) semi-definite of rank \(n-1\), where \(n\) is the number of commodity.

With respect to the adding-up conditions it will be fulfilled without imposing restrictions on the parameters since the sum for the dependent variables in Equation 5.7 be will equal to the first independent variable. Correspondingly, if classical Ordinary Least Square is used (OLS), the parameter estimates will automatically satisfy the adding-up property. The homogeneity and symmetry conditions could be imposed on the parameters of Equation 5.7 and hence can be tested.

Brown and Lee (1992) extended the usual Rotterdam model by including lagged consumption through translation parameters, allowing for habit and inventory effects. Brown and Lee stated that the demand impacts of lagged consumption could be specified through the translation parameters. The effect of past consumption is introduced into the model by letting the translation term \(z\) depend on lagged consumption. Brown and Lee hypothesized that:

\[
z_t = a_t w_{t-1} d \log q_{t-1}
\] (5.9)

and then the Rotterdam Model with translation is

\[
w_u d \log q_u = z_t - d \sum_i z_i + \beta_i d \log Q_i + \sum_j Y_{ij} d \log p_i
\] (5.10)

where \(z_t = \frac{p_t}{x} \frac{d}{d \log x} d \log y_t\), the log change in the translation parameters \((Y_{ij})\) weighted by the share of total expenditure committed to the good, \(p_t y_t / x\).

The left-hand side variables in Equation 5.10 can be viewed as the percentage change in demand weighted by the budget share. The difference between the usual Rotterdam model and the Brown and Lee (1992) specification is the first two terms on the right hand side of Equation 5.10, which involve changes in the
translation parameters. The first is a direct effect $z_t$ due to a change in the translation parameter for the good in question, while the second term is an indirect income effect $d_t, \sum z_j$ due to a change in supernumerary income caused by an overall change in the translation parameters. Changes in the translation parameters can be viewed as preference changes, and the resultant direct ($z_t$) and indirect ($d_t, \sum z_j$) effects, which made the difference between (5.10) and (5.7), lead to re-allocation of income.

The effects of past consumption can be introduced into the model by letting the translation term $z_t$ depend on the lagged consumption, as indicated by Equation 5.9.

Equation 5.9 states that the weighted log change in the translation parameters equals a constant times the weighted log change in lagged consumption. However, Equation 5.10 is not based on separability assumptions with attention focused on the conditional demand equations for separable groups of goods (Brown and Lee, 1992, p.2). In the Equation 5.9 the parameters $\alpha$ is normally expected to be negative (positive) when inventory (habit) effects dominate. The shorter the time interval of an observation, the more likely inventory effects are to dominate habit effects.

As the data add-up by construction- income in the model the total consumer expenditure on the five product categories- the error covariance matrix is singular, and the equation for other commodity groups is excluded (Barten 1969). Where errors across equations are assumed to be contemporaneously correlated, the (ISUR) method is used to estimate the dynamic model.

The Rotterdam Demand System Model of Equation 5.10 will be applied to the Bahrain quarterly time series data (1979Q1-1998Q4) in this study.
5.3 Overall Performance of the Usual Rotterdam Model

The empirical implementation of the model is accomplished by estimating the system of Equation 5.7. The unrestricted Rotterdam Model was estimated for each of five commodity groups separately by OLS. The adding-up restrictions were embodied into the unrestricted model and hence it will not be tested. Table 5.1 presents the marginal budget shares, $\beta_i$, and the $\gamma_y$ estimate together with the row sum of $\gamma_y$ estimates, the coefficients of determination, $R^2$, and the values of the Durbin-Watson statistics. The main results of the application are as follows: The t-ratios suggested that there are three of the marginal budget share estimates, $\beta_i$, which are significantly different from zero at the five percent level, for Food & Beverages, Clothing & Footwear, and the Other commodity group. This is true for only 3 out of 25 price coefficient estimates. For instance, Deaton (1974a) adopted the Rotterdam Model to U.K data and reported that only 7 out of 81 price responses were significantly different from zero. Similarly, Barten (1969), and Park (1969) had the same problem when applying the Rotterdam Model.

The coefficients of determination $R^2$ indicate that the equations are low fitted. For most equations in the system $R^2$ ranges between 66.1% for the Food & Beverage commodity group to 0.21% for Other commodity groups. Values of the Durbin-Watson ($d.w$) statistics indicate that there is no first-order serial correlation in the residuals for all equations of the system.

With respect to the sign of the own price coefficient estimates, it is found that only three of own-price coefficient estimates are negative, namely Clothing & Footwear, Housing, and Other commodity group equation (See Table 5.1). This implies that the estimates $[\gamma_y]$ matrix is not negative semi-definite as required.
by the second order conditions. The row sum $\sum_i \gamma_{yi}$ for each individual equation is very close to zero, which is suppose to be under the homogeneity condition.

The most interesting economic parameters for policy analysis are elasticities. Having estimated the Rotterdam Model, its coefficients $\beta_i$ and $\gamma_{yi}$ are obtained by using relations of elasticities of demand for commodity ($i$) with respect to total expenditure and prices. These relations are defined as follows.

**Expenditure elasticity** measures the change of expenditure on one commodity group when expenditure on all commodity groups changes by one unit and the formula for it is given by $\eta_i = \beta_i / w_i$.
Own price elasticity is the direct price elasticity of demand and is defined as the degree of responsiveness of the quantity demanded of a commodity to change in its price. Economic theory requires direct compensated own price elasticity to be negative. Compensated and uncompensated own price elasticities often have the same sign and the former is usually smaller than the latter in absolute value. And the relations of these are as follows:

For uncompensated own price elasticity is, $e_u = e_u^* - \eta_i w_i$.

while, for compensated own price elasticity is, $e_u^* = \gamma_i / w_i$.

Cross price elasticities may be either positive or negative depending on whether the two goods are substitutes or complements. However it is important to understand the sign which reflects consumer behavior regarding the importation of commodity groups. The relationships of these cross price elasticities under uncompensated and compensated are given by:

For uncompensated cross-priced elasticity: $e_y = e_y^* - \eta_i w_i$.

Whereas, for compensated cross price elasticities: $e_y^* = \gamma_y / w_i$.

Based on the results which are presented in Table 5.2, the elasticities of demand with respect to total expenditure and to own and cross-price for each commodity group are evaluated at the mean values of the corresponding budget shares. The evaluated demand elasticities with respect to total expenditure indicates that the demand for Food & Beverages, Clothing & Footwear, and Transportation commodity group are elastic and large while the demand for Housing and Other group are inelastic. This suggests that the latter groups be classified as necessities (which implies that these commodities are ranked highest in consumer total expenditure responses) and the former groups be classified as luxuries, in the sense of having expenditure elasticities bigger than one. As income increases, Bahrain consumers tend to buy relatively more from three commodity
groups products namely; Food & Beverages, Clothing & Footwear, and Transportation commodity group, and less from Other products. This also indicates that Bahrain is a potential for these product markets.

Table 5.2
Total expenditure elasticities, and Uncompensated own-cross price elasticities for unrestricted Rotterdam demand system - Usual model

<table>
<thead>
<tr>
<th>Commodity group</th>
<th>Total expenditure elasticity ($\eta$)</th>
<th>Elasticity of demand with respect to price of each commodity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Food &amp; beverage</td>
</tr>
<tr>
<td>Food &amp; beverage</td>
<td>2.988</td>
<td>-0.484</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>2.471</td>
<td>0.780</td>
</tr>
<tr>
<td>Housing</td>
<td>0.650</td>
<td>-2.189</td>
</tr>
<tr>
<td>Transportation</td>
<td>2.213</td>
<td>0.211</td>
</tr>
<tr>
<td>Other</td>
<td>0.093</td>
<td>0.271</td>
</tr>
</tbody>
</table>

The uncompensated own-price elasticities indicate that the demand for all groups of commodities (Food & Beverages, Clothing & Footwear, Transportation and Other commodity group) are inelastic, except for the Housing commodity group, which seems to be elastic. Finally, in terms of the cross price elasticities other commodity groups has an inelastic income and own price elasticities which indicate that this market is saturated and their relative budget share is losing importance as part of total expenditure.

Uncompensated own price elasticities are greater than compensated own price elasticities in the absolute values. Moreover, three of compensated own-price elasticity estimates are found negative and consistent with prior expectation regarding their sign. However, the own compensated price elasticities for Food & Beverages and Transportation commodity group are found positive as not expected in this type of analysis, and hence violated the demand theory law.
Most of the relationships appear to be net substitution, but there are complements relationships that have been found - i.e. between the Other commodity group and the Clothing & Footwear commodity group, etc. See Table 5.3.

<table>
<thead>
<tr>
<th>Commodity group</th>
<th>Elasticity of demand with respect to price of each commodity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Food &amp; beverage</td>
</tr>
<tr>
<td>Food &amp; beverage</td>
<td>0.024</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>1.200</td>
</tr>
<tr>
<td>Housing</td>
<td>-2.079</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.588</td>
</tr>
<tr>
<td>Other</td>
<td>0.287</td>
</tr>
</tbody>
</table>

Furthermore, as can be seen from Tables 5.2, 5.3, 5.6, 5.9, 5.10 and 5.13 of this chapter (5), that the total expenditure elasticities are bigger in the size (especially for Food & beverage and Clothing & footwear commodity groups) and are different in the classification of commodity groups between necessary and luxury products than those obtained from the linear expenditure system (LES) and Linear almost ideal demand systems (LA\AIDS). The main reason is that, the Rotterdam model was estimated by using the equation (5.7). This equation used the volume index of the change in total consumption ($DQ_t = \sum w_i d log q_i$) as independent variable, rather than using the change in real income ($d log x - \sum w_i d log p_i$) as given by equation (5.6). The utilization of the volume index of the change in total consumption ($DQ_t = \sum w_i d log q_i$) in estimating of the Rotterdam model because it assumed in the Rotterdam demand system that this index can be interpreted as a measure of the change in real
income \( (\sum w_i d \log q_i = (d \log x - \sum w_i d \log p_i)) \). This assumption is not true in the case of Bahrain quarterly time series data, because it gave different results. To see the differences between them in the application, the Rotterdam model of equation (5.6) that used the change of real income as independent variable is estimated, and it gave totally different results than the Rotterdam model when it used the volume index of the change in total consumption as independent variable as given by equation (5.7). The main results of estimation are reported in Table 5.4.

Table 5.4
The main results of applying the Rotterdam model using the change in real income as independent variable as given by Equation (5.6).

<table>
<thead>
<tr>
<th>Commodity group</th>
<th>Marginal Budget share ( (\beta_i) )</th>
<th>Own price coefficient ( (Y_{mi}) )</th>
<th>( R^2 )</th>
<th>( d.w )</th>
<th>Expenditure elasticity ( (\eta_i) )</th>
<th>Compensated own price elasticity ( (\epsilon_{mi}^*) )</th>
<th>Uncompensated own price elasticity ( (\epsilon_{mi}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverage</td>
<td>0.008 (0.0185)</td>
<td>-0.013 (0.0157)</td>
<td>0.020</td>
<td>0.800</td>
<td>0.047 (-0.076)</td>
<td>-0.084 (-0.143)</td>
<td></td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.031 (0.0315)</td>
<td>-0.010 (0.0453)</td>
<td>0.088</td>
<td>2.513</td>
<td>0.443 (-0.143)</td>
<td>-0.174 (-0.193)</td>
<td></td>
</tr>
<tr>
<td>Housing</td>
<td>0.186 (0.1098)</td>
<td>-0.001 (0.0476)</td>
<td>0.114</td>
<td>2.563</td>
<td>1.329 (-0.007)</td>
<td>-0.193 (-0.380)</td>
<td></td>
</tr>
<tr>
<td>Transportation</td>
<td>0.045 (0.0394)</td>
<td>0.060 (0.0305)</td>
<td>0.068</td>
<td>2.278</td>
<td>0.563 (0.750)</td>
<td>0.705 (-0.380)</td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>0.730 (0.5438)</td>
<td>0.189 (0.6612)</td>
<td>0.108</td>
<td>2.177</td>
<td>1.352 (0.350)</td>
<td>-0.380 (-0.350)</td>
<td></td>
</tr>
</tbody>
</table>

* Standard error of the estimates in the parentheses.

As can be seen from Table 5.4 that the results of expenditure elasticities indicate that the demand for Food & beverages, Clothing & footwear, and Transportation commodity group are classified as necessities products, while the demand for Housing and Other commodity group are classified as luxuries products. Second, all of uncompensated own-price elasticities are found negative except that for Transportation commodity group. The own compensated price elasticities for Transportation and Other commodity group are found positive as not expected in
this type of analysis, and hence violated the demand theory law. Third, using the change in real income instead of the volume index of the change in total consumption in Rotterdam model led to producing different own price elasticity for all commodity groups, because, the Marshallian price elasticities are related to income elasticities by the homogeneity condition \( \sum_{j=1}^{n} e_{ij} = -\eta_i \). See Tables 5.2, 5.3 and 5.4. Four, all the coefficients of the marginal budget share and own price are not statistically significant. Five, the coefficient of determination suggest that the overall fit is low. The values of the Durbin-Watson statistics indicate that there is no evidence of first-order serial correction in the residuals for most of equations except that for Food & beverage commodity group equation (See Table 5.4).

According to the results obtained above it seems that the Rotterdam model is not fitting to the Bahrain quarterly time series data for two main reasons: First due to the two versions of the Rotterdam models (5.6 and 5.7) producing different results which supposed to give similar results. Secondly based on the statistical consideration which indicated that the low fitted of equations and most of parameters of equation are not statistically significant and finally, based on the assumption that the volume index of the change in total consumption can be interpreted as a measure of the change in real income 
\[
(\sum \omega_i d \log q_i = (d \log x - \sum \omega_i d \log p_i))
\]
in the Rotterdam model is not true in the case of Bahrain quarterly time series data.

When the homogenous restriction is imposed the Rotterdam Model of Equation 5.7 transfer to the other form is given by
\[
w_i^* Dq_i = \beta_i DQ_i + \sum_{j=1}^{J} \gamma_{ij} (Dp_{ij} - Dp_{ij}) + u_i
\]  
(5.11)

The homogeneity system above can be fitted to Bahrain data either by using restricted estimation \( \sum \gamma_i = 0 \) or by dropping one equation from the system,
and using the price of that commodity group dropped as a normalizer, (as regressor price ratios). The estimation method of OLS is still valid (Judge et. al., 1985, p.468). Madden (1995, p.105) indicated that, if the right hand variables are the same for all equations, this is equivalent to running separate OLS regression for each good. The estimation obtained is identical, except when the cross equation restriction of symmetry is imposed.

The parameters of the deleted equation can be calculated by the adding up conditions, and the standard errors of the parameters of the deleted equation can be calculated as follows:

\[ S.E(\gamma_{sj}) = \sqrt{\sum Var(\gamma_{s}) + 2cov(\gamma_{s}, \gamma_{k})} \quad \text{for} \quad j \neq k \]  

Table 5. 5 shown the marginal budget share, the coefficients of own and cross price, and in addition to the main statistical results. The main findings of this application are as follows: (1) As a whole, fit seemed to be worse for all equations in the system. (2) Only three out of five estimated \( \beta \)'s coefficients are statistically significant at 5% level (t-values of greater than 2 and positive), and these results which are found here are the same as in the free Rotterdam and for the same commodity groups (Food & Beverages, Clothing & Footwear, and Transportation).

(3) The Durbin-Watson statistics results show that there is no first-order serial correlation in the residuals in all equations of the system. But, it has been observed form the empirical work that imposition of homogeneity restriction led to slightly increased \( R^2 \) values for three equations of the system, and there is a marginal increase in the values of D.W statistical results for some equations. Deaton and Mullebauer (1980), Blanciforti and Green (1983), pointed out that the decrease in the D.W statistic is more drastic for commodity groups where homogeneity is strongly rejected. In respect to the present study the evidence indicates that the homogeneity restriction is accepted for all commodity groups,
but the values of the D.W statistic are not changed for most commodity groups except for Food and Beverages. The findings show that the value is marginally increased (See Tables 5.1 and 5.5).

Table 5.5
Estimated parameters of homogenous Rotterdam Model and summary of the statistical results classified according to commodity group equation-Usual model

<table>
<thead>
<tr>
<th>Commodity groups equations</th>
<th>Marginal budget share ( \beta_i )</th>
<th>Food &amp; beverages ( \gamma_{i1} )</th>
<th>Clothing &amp; footwear ( \gamma_{i2} )</th>
<th>Housing ( \gamma_{i3} )</th>
<th>Transportation ( \gamma_{i4} )</th>
<th>Other ( \gamma_{i5} )</th>
<th>( R^2 )</th>
<th>( d.W^b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverages</td>
<td>0.490 (0.045)</td>
<td>-0.023 (.054)</td>
<td>-0.162 (.094)</td>
<td>0.017 (.030)</td>
<td>0.018 (.054)</td>
<td>0.150 (.078)</td>
<td>0.65</td>
<td>2.17</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.178 (.055)</td>
<td>0.091 (.065)</td>
<td>-0.006 (.114)</td>
<td>0.070 (.036)</td>
<td>-0.108 (.066)</td>
<td>-0.047 (.094)</td>
<td>0.40</td>
<td>2.52</td>
</tr>
<tr>
<td>Housing</td>
<td>0.116 (.098)</td>
<td>-0.253 (.116)</td>
<td>0.433 (.203)</td>
<td>-0.119 (.064)</td>
<td>0.033 (.118)</td>
<td>-0.094 (.168)</td>
<td>0.36</td>
<td>2.54</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.166 (.063)</td>
<td>0.031 (.075)</td>
<td>-0.147 (.131)</td>
<td>0.008 (.041)</td>
<td>0.081 (.076)</td>
<td>0.028 (.108)</td>
<td>0.30</td>
<td>2.58</td>
</tr>
<tr>
<td>Other</td>
<td>0.050 (.122)</td>
<td>0.155 (.146)</td>
<td>-0.118 (.255)</td>
<td>0.024 (.081)</td>
<td>-0.024 (.148)</td>
<td>-0.036 (.231)</td>
<td>0.21</td>
<td>2.17</td>
</tr>
</tbody>
</table>

(a)- Standard error of the estimates in parentheses.
(b)- \( d_u \) and \( d_i \) are 1.80 and 1.48 respectively and imply that there is a positive serial correlation in the residual of the equations.

(4) All own price coefficients are negative except for the Transportation commodity group. There only one of cross price coefficient, \( \gamma_y \), is significant at the five- percent significance level (1 out of 25). (5) The own price coefficient estimates are smaller in absolute values in homogeneity restricted model than it was in the unrestricted (free) Rotterdam Model.

The elasticities of homogeneity in the Rotterdam Model are computed based on the results of Table 5.5. Table 5.6 indicates that: (1) Elasticity estimates with
respect to total expenditure are similar to those acquired from the unrestricted model. The demand for two commodity groups is inelastic, and these groups are Housing and the Other group. (2) The classification of commodity groups between luxuries and necessities products are the same as in the case of the free Rotterdam Model. (3) All uncompensated own price elasticities are negative except for the Transportation commodity group which is found positive, and the latter results indicate that the commodity group has violated the principle of the demand theory. Furthermore, the compensated own-price elasticities estimates as presented in Table 5.6, indicate that most of these elasticities are negative except for the Transportation commodity group which is found positive as not expected in this type of analysis. Finally, uncompensated price elasticities are bigger than compensated own price elasticities in the absolute values.

The above findings are consistent with prior knowledge in the sense that the quantity demand for any commodity group is related inverse to its own price. This result was true for all commodity groups except for Transportation commodity group. See Table 5.6.

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>Total expenditure elasticity ($\eta_1$)</th>
<th>Un- compensated own price elasticity ($e_{ii}$)</th>
<th>Compensated own price elasticity ($e_{ii}^*$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverage</td>
<td>2.882</td>
<td>-0.625</td>
<td>-0.135</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>2.543</td>
<td>-0.264</td>
<td>-0.086</td>
</tr>
<tr>
<td>Housing</td>
<td>0.829</td>
<td>-0.966</td>
<td>-0.850</td>
</tr>
<tr>
<td>Transportation</td>
<td>2.075</td>
<td>0.847</td>
<td>1.013</td>
</tr>
<tr>
<td>Other</td>
<td>0.093</td>
<td>-0.117</td>
<td>-0.067</td>
</tr>
</tbody>
</table>

Table 5.6
Total expenditure elasticities, uncompensated-compensated own price elasticities for the homogenous Rotterdam demand system - Usual model.
The Rotterdam Model can be used to test the homogeneity restriction through linear restrictions on fixed parameters. In order to investigate the validity of the homogeneity restriction an F-test comparing the sum of residuals square of Equation 5.8 and 6.9 must be fulfilled. The F-statistics for each equation are calculated as follows:

\[
F = \frac{(RSSH_O - RSSH_A) / M}{RSSH_A / (T - K)}
\]  

(5.13)

where \( RSSH_O \), \( RSSH_A \) are the residual sum of square of restricted and non-restricted equations respectively, \( M \) is the number of restrictions, \( T \) is the number of observations and \( K \) is the number of parameters in each equation. Therefore, the homogeneity restriction tested equation by equation using the critical values of the F-statistic \( F_{1, 74} \) at 5 and 1 percent levels of significance are 3.96 and 6.96 respectively.

Table 5.7 presents the F-statistic calculated for each individual commodity group equation in addition to the sum of the estimated price coefficients. These F-calculated for each equation of commodity group are all less than the critical value of F at 5% and 1% significance levels. This suggests that homogeneity be not rejected for all commodity groups. This implies that an equiproportion increase in prices and total expenditure will not change the quantity demanded for all of commodity groups in the demand system (Food & beverages, Clothing & footwear, Housing, Transportation and Other commodity group). Thus, the homogeneity hypothesis cannot be rejected at the 5% and 1% levels of significance for all equations of the usual Rotterdam Demand System Model.
Table 5.7

Test for the homogeneity restriction for equation by equation basis for unrestricted Rotterdam demand system - Usual model.

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>$\sum_j r_{jU}$</th>
<th>$F - values^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food and beverage</td>
<td>0.220</td>
<td>2.065</td>
</tr>
<tr>
<td>Clothing and footwear</td>
<td>-0.050</td>
<td>0.082</td>
</tr>
<tr>
<td>Housing</td>
<td>-0.308</td>
<td>0.850</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.134</td>
<td>0.386</td>
</tr>
<tr>
<td>Other</td>
<td>0.007</td>
<td>0.000</td>
</tr>
</tbody>
</table>

(*) F-test for the validity of the homogeneity restriction: critical values $F_{0.05} = 3.69, F_{0.01} = 6.96$ for 0.05 and 0.01 level of significance

When symmetry is imposed the system cannot be estimated equation by equation, therefore, the Iterative Seemingly Unrelated Regression (ISUR) estimation method was used. This method produces Maximum Likelihood (ML) estimates for linear equation system and producers parameters estimates invariant to the choice of the deleted equation (David et al., 1996, p.77). Table 5.8 presents the parameters estimated with a summary of statistical results.

The major findings of this application are as follows: (1) There is a reduction in the values of coefficients of determination ($R^2$) and $d.w$ results for some equations of the system. These results are obtained when the comparison is made between the symmetry model and the past two versions (unrestricted and homogenous) of the Rotterdam model. (2) Three of the estimated $\beta$, coefficients for the commodity groups are statistically significant (t-values of greater than 2) and positive. These results held for unrestricted and homogenous models and for the same commodity groups. (3) One of the own price coefficients, $\gamma_u$, is statistically significant at the five-percent level (1 out of 15), and that for the Housing equation. (4) All own price coefficients, $\gamma_u$, are negative except for the Transportation equation. (5) The values of $d.w$ statistics are all above the value
2 for all equations of the symmetry Rotterdam Model. The results indicate that there is no first-order serial correlation in those equations.

Table 5.8
Estimated parameters of the symmetric Rotterdam model and summary of the statistical results classified according to commodity group equation-Usual model

<table>
<thead>
<tr>
<th>Commodity group equations</th>
<th>Marginal budget share</th>
<th>Food &amp; beverage</th>
<th>Clothing &amp; footwear</th>
<th>Housing</th>
<th>Transportation</th>
<th>Other</th>
<th>$R^2$</th>
<th>$d.w$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma_{11}$</td>
<td>$\gamma_{12}$</td>
<td>$\gamma_{13}$</td>
<td>$\gamma_{14}$</td>
<td>$\gamma_{15}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food &amp; beverage</td>
<td>0.509 (0.044)</td>
<td>-0.042 (0.052)</td>
<td>0.064 (0.056)</td>
<td>0.020 (0.029)</td>
<td>0.041 (0.043)</td>
<td>0.185 (0.082)</td>
<td>0.65</td>
<td>2.16</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.172 (0.054)</td>
<td>-0.025 (0.138)</td>
<td>0.071 (0.036)</td>
<td>-0.105 (0.062)</td>
<td>-0.056 (0.101)</td>
<td></td>
<td>0.40</td>
<td>2.53</td>
</tr>
<tr>
<td>Housing</td>
<td>0.130 (0.096)</td>
<td>-0.145 (0.060)</td>
<td>-0.010 (0.039)</td>
<td>-0.090 (0.183)</td>
<td></td>
<td></td>
<td>0.30</td>
<td>2.52</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.174 (0.062)</td>
<td></td>
<td>0.093 (0.074)</td>
<td>0.064 (0.114)</td>
<td></td>
<td></td>
<td>0.31</td>
<td>2.58</td>
</tr>
<tr>
<td>Other</td>
<td>0.015 (0.018)</td>
<td></td>
<td></td>
<td>-0.094 (0.252)</td>
<td></td>
<td></td>
<td>0.21</td>
<td>2.11</td>
</tr>
</tbody>
</table>

- Standard error of the estimates in parentheses.

Based on these estimated parameters, the expenditure and price elasticities are computed and presented in Table 5.9. Table 5.9 indicates that elasticity estimates with respect to total expenditure under the symmetry restriction are smaller than those acquired from the unrestricted model, but are bigger than those obtained from homogeneous systems for all commodity groups.
Table 5.9
Total expenditure elasticities, uncompensated-compensated own price elasticities for the symmetric Rotterdam demand system - Usual model.

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>Total expenditure elasticity ($\eta_i$)</th>
<th>Un-compensated own price elasticity ($e_{ii}$)</th>
<th>Compensated own price elasticity ($e^{*}_{ii}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverage</td>
<td>2.994</td>
<td>-0.756</td>
<td>-0.247</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>2.457</td>
<td>-0.529</td>
<td>-0.357</td>
</tr>
<tr>
<td>Housing</td>
<td>0.929</td>
<td>-1.166</td>
<td>-1.036</td>
</tr>
<tr>
<td>Transportation</td>
<td>2.175</td>
<td>0.626</td>
<td>0.800</td>
</tr>
<tr>
<td>Other</td>
<td>0.028</td>
<td>-0.189</td>
<td>-0.174</td>
</tr>
</tbody>
</table>

All compensated and uncompensated own price elasticities are negative except for the Transportation commodity group. The uncompensated own-price elasticity of Housing is higher than unity within the symmetric system. The own price elasticities (uncompensated and compensated) for all commodity groups are less than those obtained from the unrestricted and homogeneity restriction model. These results are obtained when the comparison is made between symmetric and two previous versions of the usual Rotterdam Model. (see Tables 5.2, 5.3, 5.6, and 5.9).

To clarify the findings regarding the elasticities for the usual Rotterdam Demand System Model. Table 5.10 presents the estimated total expenditure elasticities along with uncompensated and compensated own-price elasticities for the three versions of the usual Rotterdam Demand System Model. With respect to estimated elasticities the results indicate that there are no differences in the results regarding the classification of expenditure elasticities. All types of Rotterdam versions share the classification, and that is the Food & Beverages, Clothing & Footwear, and Transportation categories represent luxury products.
Table 5.10
Elasticity estimates for Rotterdam demand system model according to the type of the model and commodity group- Usual model

<table>
<thead>
<tr>
<th>Type of model</th>
<th>Unrestricted version</th>
<th>Homogenous version</th>
<th>Symmetric version</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commodity groups</td>
<td>$e_{ii}$</td>
<td>$e^*_{ii}$</td>
<td>$\eta_i$</td>
</tr>
<tr>
<td>Food &amp; beverages</td>
<td>-0.484</td>
<td>0.024</td>
<td>2.988</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>-0.630</td>
<td>-0.457</td>
<td>2.471</td>
</tr>
<tr>
<td>Housing</td>
<td>-1.070</td>
<td>-0.979</td>
<td>0.650</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.961</td>
<td>1.138</td>
<td>2.213</td>
</tr>
<tr>
<td>Other</td>
<td>-0.113</td>
<td>-0.063</td>
<td>0.093</td>
</tr>
</tbody>
</table>

$-\eta_i$ is expenditure elasticities. $e_{ii}$ and $e^*_{ii}$ are uncompensated-compensated own price elasticities respectively.

Furthermore, the uncompensated own price elasticities are greater than compensated own price elasticities in the absolute values for all commodity groups except for the Transportation category due to violation of the principle of the demand theory, which is due to the positive sign of he own price coefficient of this category. The findings can be seen clearly from Table 5.10.

5.4 Testing for the Hypotheses of the Demand Theory- Usual Model

Symmetry given homogeneity, unlike homogeneity, cannot be tested on an equation by equation basis and a large sample likelihood ratio test (Thomas, 1996, p.256) can be used for the system as a whole. The likelihood ratio test involves comparison of the logarithmic likelihood values for the different models.

Under the null hypothesis of the restricted model, twice the difference between the maximum logarithmic likelihood value for the unrestricted model and that value for the restricted model is asymptotically distributed as chi-square statistic with the number of degrees of freedom equal to the number of restrictions imposed. The results of this test are presented in Table 5.11 for three versions of
the usual Rotterdam Demand System. According to the log likelihood ratio test it is detected that homogeneity is not rejected over the no-restrictions case. Both symmetry given homogeneity and symmetry over the no restriction also cannot be rejected at the 5% level of significance. (See Table 5.11).

<table>
<thead>
<tr>
<th>Type of model</th>
<th>Logarithmic likelihood value</th>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>-2log λ</th>
<th>d.f</th>
<th>Critical Value (5%)</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>844.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homogeneity</td>
<td>843.47</td>
<td>Homogeneity</td>
<td>No-restrictions</td>
<td>2.85</td>
<td>4</td>
<td>9.49</td>
<td>Accepted</td>
</tr>
<tr>
<td>Symmetry</td>
<td>840.63</td>
<td>Symmetry</td>
<td>No-restrictions</td>
<td>8.54</td>
<td>6</td>
<td>12.59</td>
<td>Accepted</td>
</tr>
<tr>
<td>Homogeneity &amp;</td>
<td>837.56</td>
<td>Symmetry</td>
<td>Homogeneity</td>
<td>14.68</td>
<td>10</td>
<td>18.31</td>
<td>Accepted</td>
</tr>
</tbody>
</table>

Log likelihood ratio is asymptotically distributed as $\chi^2$ with $h$ degrees of freedom, where $h$ is the number of restrictions imposed.

5.5 Overall Performance of the Dynamic Rotterdam Model.

The estimation of the dynamic Rotterdam Demand System was summarized in Table 5.12: It presents the estimated coefficients associated with total expenditure, own-cross price, and lagged consumption.

The dynamic version of the Rotterdam Model is restricted after habit persistence is included into the usual version. The purpose of this application is to examine how past consumption patterns effect Bahrain consumers' consumption behavior, and then to estimate elasticities for the unrestricted dynamic Rotterdam Model, followed by imposing and testing the restrictions implied by demand theory.
The dynamic coefficients reflect persistence in budget allocations when it is positive and stock effects when it is negative. As far as auto-correlation is concerned in these specifications since lagged consumption is on the right hand side of the dynamic Rotterdam Model, all equations were estimated individually and simultaneously to check if any auto-correlation existed.

The major findings of this applications are as follows: First, three marginal budget shares are significantly different from zero, and these are Food & Beverages, Clothing & Footwear, and Transportation commodity groups equation. Second, the most important aspect is the examination of the significance of the habit coefficient. All of the lagged consumption parameters are significantly different from zero at 5% level.

<table>
<thead>
<tr>
<th>Commodity group equations</th>
<th>Translation lag $\alpha_i$</th>
<th>Marginal budget share $\beta_i$</th>
<th>Price parameters $\gamma_{i1}, \gamma_{i2}, \gamma_{i3}, \gamma_{i4}, \gamma_{i5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverages</td>
<td>-0.294</td>
<td>0.412</td>
<td>0.006, -0.071, -0.002, 0.010, 0.191</td>
</tr>
<tr>
<td></td>
<td>(0.072)</td>
<td>(0.048)</td>
<td>(0.054), (0.113), (0.029), (0.050), (0.078)</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>-0.327</td>
<td>0.126</td>
<td>0.148, -0.119, 0.053, -0.084, -0.020</td>
</tr>
<tr>
<td></td>
<td>(0.097)</td>
<td>(0.054)</td>
<td>(0.067), (0.140), (0.036), (0.062), (0.099)</td>
</tr>
<tr>
<td>Housing</td>
<td>-0.335</td>
<td>0.162</td>
<td>-0.134, 0.042, -0.125, 0.019, -0.150</td>
</tr>
<tr>
<td></td>
<td>(0.103)</td>
<td>(0.099)</td>
<td>(0.127), (0.261), (0.064), (0.115), (0.179)</td>
</tr>
<tr>
<td>Transportation</td>
<td>-0.291</td>
<td>0.132</td>
<td>0.091, -0.147, 0.015, 0.084, 0.060</td>
</tr>
<tr>
<td></td>
<td>(0.104)</td>
<td>(0.065)</td>
<td>(0.082), (0.171), (0.042), (0.076), (0.118)</td>
</tr>
<tr>
<td>Other</td>
<td>1.247</td>
<td>0.168</td>
<td>-0.111, 0.295, 0.059, -0.029, -0.081</td>
</tr>
<tr>
<td></td>
<td>(1.90)</td>
<td>(1.39)</td>
<td>(1.174), (1.360), (0.089), (1.159), (2.49)</td>
</tr>
</tbody>
</table>

The coefficients of determination ($R^2$) for Food & Beverage, Clothing & Footwear, Housing, Transportation, and Other are 0.73, 0.45, 0.30, 0.39, and 0.21 respectively. Note that as the four demand equations are estimated together, as system, the $R^2$ have not been maximized. The Durbin Watson (D.W) Statistics for Food and Beverage, Clothing & Footwear, Housing, Transportation, and Other are, (1.79), (2.04), (2.16), (2.14), (1.95), respectively. For demand systems obeying the adding-up property, the D.W statistics for individual equations are not precise measures of auto-correlation (Bewley, 1986)
Most of the dynamic coefficients of lagged consumption (the Translation lag parameters) are negative indicating dominance of inventory effects (stock effects), while the positive sign of lagged consumption coefficient for other commodity groups’ equation suggests dominance of habit persistence. Sexauer (1979) mentioned that the length of the time period of an observation has an important influence on the relative strengths of the inventory and habit effects. The shorter the time interval of an observation, the more likely inventory effects is to dominate habit effects. The results of this study indicate that most of the lagged consumption estimates have negative signs and this result could be related to the short time interval of an observation (using quarterly time series rather than annual time series).

Furthermore, there is no autocorrelation in all equations of the dynamic model, but the $R^2$ results indicate that the equations are low fitted except for the Food & Beverages commodity group equation (73%). Finally, the coefficients of prices are significant at 5% level for only 2 out of 25.

Based on the results, which were presented in Table 5.12 the elasticities of demand with respect to total expenditure and to own and cross price for each commodity group were evaluated at the mean values of the corresponding budget shares and are presented in Table 5.13.

The evaluated demand elasticities with respect to total expenditure indicate that the demand for Food & Beverages, Clothing & Footwear, Housing, and Transportation commodity group are elastic and large, while the demand for Other groups products is inelastic. This suggests that the latter groups be classified as necessities (which implies that these commodities are ranked highest in consumer total expenditure responses) and the demand for the former commodity groups products are classified as luxuries, in the sense of having expenditure elasticities bigger than one. As income increases, Bahrain consumers tend to buy relatively more from four commodity groups products.
namely; Food & Beverages, Clothing & Footwear, Housing, and Transportation commodity group, and less from Other commodity groups products. This also indicates that Bahrain has a potential for these product markets.

Table 5.13
Total expenditure and own-cross price elasticities estimates for Rotterdam model with translation parameters dependent on changes in lagged consumption for Bahrain personal consumption expenditures-Unrestricted dynamic model.

<table>
<thead>
<tr>
<th>Commodity group</th>
<th>Total expenditure elasticity (( \eta_i ))</th>
<th>Elasticity of demand with respect to price of each commodity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Food &amp; beverage</td>
</tr>
<tr>
<td>Food &amp; beverage</td>
<td>2.424</td>
<td>-0.376</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>1.800</td>
<td>1.808</td>
</tr>
<tr>
<td>Housing</td>
<td>1.157</td>
<td>-1.154</td>
</tr>
<tr>
<td>Transportation</td>
<td>1.650</td>
<td>0.857</td>
</tr>
<tr>
<td>Other</td>
<td>0.311</td>
<td>0.152</td>
</tr>
</tbody>
</table>

The uncompensated own-price elasticities indicate that the demand for all groups of commodities, namely; Food & Beverages, Transportation and Other commodity groups are inelastic, except for Clothing & Footwear and Housing commodity group, which seem to be elastic (See Table 5.13).

Further, in terms of the cross price elasticities other commodity group has an inelastic income and own price elasticities which indicate that this market is saturated and their relative budget share is losing importance as part of total expenditure.

Uncompensated own price elasticities are greater than compensated own price elasticities in the absolute values, except for Transportation commodity groups, as a result of obtaining a positive signs for both compensated and uncompensated
own price elasticities for that category. Moreover, three compensated own-price elasticity estimates are found negative and consistent with a prior expectation regarding their sign. See Table 5.14

<table>
<thead>
<tr>
<th>Elasticties</th>
<th>Food &amp; beverage</th>
<th>Clothing &amp; footwear</th>
<th>Housing</th>
<th>Transportation</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverage</td>
<td>0.035</td>
<td>-0.418</td>
<td>-0.012</td>
<td>0.059</td>
<td>1.124</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>2.114</td>
<td>-1.700</td>
<td>0.757</td>
<td>-1.200</td>
<td>-0.286</td>
</tr>
<tr>
<td>Housing</td>
<td>-0.957</td>
<td>0.300</td>
<td>-0.893</td>
<td>0.136</td>
<td>-1.071</td>
</tr>
<tr>
<td>Transportation</td>
<td>1.138</td>
<td>-1.838</td>
<td>0.188</td>
<td>1.050</td>
<td>0.750</td>
</tr>
<tr>
<td>Other</td>
<td>0.206</td>
<td>-0.546</td>
<td>-0.109</td>
<td>0.054</td>
<td>-0.150</td>
</tr>
</tbody>
</table>

However, the sign of own compensated price elasticity for Food & Beverages and Transportation were found to be positive as not expected, and thus violated the demand theory law. The sign of cross price elasticities reflects the type of relationships (positive sign for substitution and negative sign for complement relationship) that exists between commodity groups (see Table 5.14).

5.6 Testing for the Hypotheses of the Demand Theory- Dynamic Model

The hypothesis test for restrictions in the Dynamic Rotterdam Demand System is reported in Table 5.15 Overall, as in the Usual Rotterdam Demand System Model all types of restrictions implied by the demand theory are accepted. Thus, the restrictions of demand theory held for both the usual and dynamic Rotterdam Demand System Models.
<table>
<thead>
<tr>
<th>Type of model</th>
<th>Logarithmic Likelihood value</th>
<th>$-2\log \lambda$</th>
<th>d.f</th>
<th>Critical value (0.05)</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>858.47</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homogeneity</td>
<td>857.33</td>
<td>2.28</td>
<td>4</td>
<td>9.49</td>
<td>Accepted</td>
</tr>
<tr>
<td>Symmetry</td>
<td>855.12</td>
<td>6.70</td>
<td>6</td>
<td>12.59</td>
<td>Accepted</td>
</tr>
<tr>
<td>Homogeneity &amp; Symmetry</td>
<td>851.28</td>
<td>14.38</td>
<td>10</td>
<td>18.31</td>
<td>Accepted</td>
</tr>
</tbody>
</table>

Log likelihood ratio is asymptotically distributed as $\chi^2$ with $h$ degrees of freedom, where $h$ is the number of restrictions imposed.

As shown from this application that all restrictions are accepted as suggested by results. This can be related to either the dynamic specification adopted in this chapter is appropriate or could be also related to other reasons - such as flexible spending patterns in the short run (the time period length of an observation is quarterly data), which led to accepting the demand theory principle. Secondly due to inclusion of both percentages of the profit margin and import duty on the values of imported goods. These two percentages are very high in Bahrain and mainly affect consumer expenditure behavior. Accordingly, the decisions of the consumer's behavior in this context are not difficult to observe even from the quarterly time series data.

5.7 A Test of the Lagged Consumption Effects in the Rotterdam Model.

To test for the presence of lagged consumption effects in the Rotterdam Demand System as group and not as an individual equation the likelihood ratio test procedure was used. It can be shown that minus two times the logarithm of the ratio or the restricted to the unrestricted likelihood function ($-2\log \lambda$) has an asymptotic chi-square distribution with the degree of freedom equal to the number of linearly independent restrictions. The computed value of $-2\ln\lambda$ is
equal to 27.14 when testing the hypothesis: $H_0 : \forall F_i = 0$. This value is greater than the critical value of $\chi^2_{0.05,4} = 9.49$ which suggests the presence of lagged consumption effects.

5.8 Conclusion

The first section of this current chapter focused on applying the Usual Rotterdam Demand System to Bahrain quarterly data for the period 1979Q1 to 1998Q4. The major findings of this application are presented as follows:

- Three out of five marginal budget shares were found to be significantly different from zero at 5% level. There were Food & Beverages, Clothing & Footwear, and Other commodity group. This is true for only 3 out of 25 price coefficients which were obtained from unrestricted mode, 1 out of 25 for homogeneity model, and 1 out of 15 for symmetric version of the Rotterdam Demand System.

- The signs of own price coefficients are found negative for three commodity groups in unrestricted version (Clothing & Footwear, Housing, and Other commodity groups). While for homogeneity and symmetric versions, it was found that signs of own price were negative for most of the categories except for the Transportation category. This implied that the estimates matrix $\{y_i\}$ was not negative semi-definite as required by the second order conditions.

- The results of the $d.w$ statistic indicate that the model has not suffered from first-order serial correlation in the residuals for all equations of the system. In addition, the $R^2$ values suggested that the overall fit is not sensible for most of the equations except for the Food & Beverage equation.

- Compensated and uncompensated own price elasticities often have the same sign and the former is smaller than the latter in absolute values. The
result is true for all commodity groups except for the Food & Beverages category which is found to have a different sign (positive compensated own price and negative uncompensated own price).

- The signs of the cross price elasticities indicated that the relationships between categories seem to be more substitution than complement.
- In terms of expenditure elasticities, Food & Beverages, Clothing & footwear, and Transportation are classified as luxuries, while the demand for Housing and other commodity groups are classified as necessities, and these results are found for all versions of Usual Rotterdam Model.
- The testing for homogeneity restriction is accepted as equation by equation basis at both 5% and 1% levels. This result is for all commodity groups equations.
- Finally, based on the log likelihood ratio test it is detected that homogeneity is not rejected over the no-restrictions case when the test is carried out for the whole system and not on an equation by equation basis. Both symmetry given homogeneity and symmetry over the no restriction are also rejected at the 5% level of significance.

However, the second section of this current chapter focuses on applying the Dynamic Rotterdam Demand System Model to Bahrain quarterly data for the period 1979Q1 to 1998Q4. The major findings of this application are presented as follows:

- Three of the coefficients of budget shares are significant different from zero at the 5% level. (Food & Beverages, Clothing & Footwear, and Transportation category equation). The coefficients of prices are significant at the 5% level for only 2 out of 25.
- All the coefficients of lagged consumption are significantly different from zero at the 5% level.
• There is no auto-correlation in all equations of the dynamic model, but the $R^2$ results indicate that the equation are low fitted except for the Food & Beverages commodity group equation (73%).

• Most lagged consumption estimates have negative signs suggesting dominance of inventory effects, and this result could be related to the short time interval of an observation that is used in this application.

• The evaluated demand elasticities with respect to total expenditure indicates that the demand for Food & Beverages, Clothing & Footwear, Housing, and Transportation commodity group are elastic and large, while the demand for Other groups products is inelastic. This suggests that the latter groups be classified as necessities and the demand for the former commodity groups products be classified as luxuries.

• The results also suggest that as income increases, Bahrain consumers tend to buy relatively more from four commodity groups products namely: Food & Beverages, Clothing & Footwear, Housing and Transportation commodity group, and less from the Other commodity groups products. This also indicates that Bahrain has potential for these product markets.

• The uncompensated own-price elasticities indicated that the demand for all commodity groups (Food & Beverages, Clothing & Footwear, Transportation and Other commodity group) are inelastic except for the Housing commodity group which appears to be elastic.

• In terms of the cross price elasticities, the Other commodity groups has an inelastic income and own price elasticities which indicate that this market is saturated and its relative budget share is losing importance as part of total expenditure.

• Uncompensated own price elasticities are greater than compensated own price elasticities in the absolute values, except for the Transportation commodity groups due to obtaining positive signs for both compensated and uncompensated own price elasticities for that category.
• Moreover, three compensated own-price elasticity estimates are found negative and consistent with prior expectation regarding their sign. However, the sign of own compensated price elasticity for Food & Beverages and Transportation were unexpectedly found to be positive and this violated the demand theory law.

• With respect to the hypothesis tests in the Dynamic Rotterdam Model, this was based on the likelihood ratio test and the findings indicated that all types of restrictions are not rejected.

• A hypothesis test of the lagged consumption effects in the Rotterdam Model suggests the presence of a lagged consumption effect.

• Lastly, the assumption that the volume index of the change in total consumption can be considered as a measure of the change in real income in the Rotterdam demand system model \( (\sum w_i d \log q_i = (d \log x - \sum w_i d \log p_i)) \), is not true in the case of Bahrain quarterly data, because using the change in real income as independent variable in the Rotterdam model demand system model gave totally different results (in the term of expenditure and price elasticities) than those obtained from using the volume index of the change in total consumption as independent variable in Rotterdam demand system model.
References

Journals


**Books**


6.1 Introduction

The Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980) is one of the most widely used flexible demand specifications. While the AIDS possesses many desirable properties, it may be difficult to estimate. To simplify the estimation problem, Deaton and Muellbauer (p.315) suggested using a linear approximation. To linear approximate the Almost Ideal Demand System (LA/AIDS), it uses Stone's price index to normalize AIDS, and has been employed in the vast majority of empirical applications of the AIDS model with a variety of formulas to compute it. The quality of the approximation to true AIDS depends on the parameters and the collinearity among the exogenous price variable.

Recently, Moschini (1995) suggests to use Tornqvist's price index instead of Stone index \( p^* \), because the linear AI with the standard Stone index \( p^* \) performs rather poorly, particularly in producing extremely biased estimates of income which will spill over to price elasticities as well through the homogeneity condition (where the Marshallinan price elasticities are related to income elasticities by the homogeneity condition \( \sum_{j=1}^n e_{ij} = -\eta_j \)). Accordingly, Moschini (1995) suggested using the Tornqvist's index to estimate a linear approximation to the AI demand model and that the standard Stone index should be avoided as the latter is dependent on the nature of the data (units of measurements). Accordingly, in this study, the Tornqvist price index has been utilized instead of Stone's price index.
In this chapter estimation of the AIDS as proposed by Deaton and Muellbauer (1980) and its dynamic specification proposed by Blanciforti and Green (1983) will be discussed. Its applications will be regarding Bahrain personal expenditure quarterly data for the period 1979Q1 to 1998Q4. The outline of this chapter is as follows: Firstly, a brief overview of the model and the estimation method. Secondly, overall performance of the models which will be included in the main results and a discussion of their implications, testing the applicability of restrictions emanating from demand theory, and finally, conclusions drawn from the above applications.

6.2 Model Specification and Estimation Method - Static Model

Theoretically AIDS is superior to any demand system as Deaton and Muellbauer (1980, p.312) recorded the following properties: The AIDS model gives an arbitrary approximation to any demand system; it satisfies the axioms of choice exactly; aggregates perfectly over consumers without invoking parallel linear Engle curves; it has a functional form which is consistent with known household budget data. Moreover, the theoretical restrictions can be imposed through linear restrictions on the parameters and tests of these restrictions. The AIDS model in its general form is nonlinear. In practice, however, by a suitable approximation to the price index, it is made linear.

The AIDS demand system was specified by starting from a specific class of preferences, which by the theorems of Muellbauer (1975, 1976) allows perfect aggregation over consumers. Basically, the perfect or exact aggregation property ensures that aggregate demand curves will be consistent with maximizing behavior by the representative consumer or optimizing behavior in the aggregate: the representation of market demands as if they were the outcome of decisions by a rational representative consumer (see appendix D). These preferences, known as the PIGLOG class, are represented via the cost or expenditure function, which defines the minimum expenditure necessary to attain a specific utility level at given prices. This function, $C(u, p)$, was donated by Deaton and Muellbauer (1980a) and defines
the PIGLOG class as a convex combination of the cost of subsistence and bliss. Deaton and Muellbauer (1980a) have represented this cost function as follows

\[ \log C(u, p) = (1 - u) \log(a(p)) + u \log(b(p)) \]  

(6.1)

where \( u(0 < u < 1) \) represents utility, \( p \) is the price vector, \( a(p) \) and \( b(p) \) are interpreted as the cost of subsistence and bliss respectively. In order to have a flexible functional form, which must possess enough parameters, the following functional forms for \( \log a(p) \) and \( \log b(p) \) have been selected. Thus:

\[ \log a(p) = a_0 + \sum_k a_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_{ij} \log p_k \log p_j \]  

(6.1.1)

\[ \log b(p) = \log a(p) + B_0 \prod_k p_k^\gamma \]  

(6.1.2)

So that the AIDS cost function is written as:

\[ \log c(u, p) = a_0 + \sum_k a_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_{ij} \log p_k \log p_j + uB_0 \prod_k p_k^\gamma \]  

(6.2)

where \( a_0, B_0 \) and \( \gamma_{ij} \) are the parameters, \( p_k \) is price, and \( u \) is a specified utility level.

The demand functions can be derived directly from Equation 6.2. Deaton and Muellbauer (1980a) recorded that: “The flexible functional form of the AI demand system cost function implies that the demand functions derived from it are first order approximations to any set of demand functions derived from utility maximizing behavior”

It is a fundamental property of the cost function that its price derivatives are the quantities demanded.

Thus: \[ \frac{\partial c(u, p)}{\partial p_i} = q_i(u, p) = q_i \]  

(6.2.1)

for \( i = 1, 2, \ldots, n \)

In other words, the derivatives of the cost function with respect to prices will result in the optimal Hicksian quantity demand function.
Since the total cost \(c(u,p)\) is the same as the total expenditure, \(x\) for an optimizing individual and since the cost function is stated in logarithmic terms, and if, differentiated with respect to \(\log p\), obtains:

\[
\frac{\partial \log c(u,p)}{\partial \log p} = \frac{\partial c(u,p)}{\partial p} = \frac{p}{c(u,p)} = \frac{p}{x} = w_i(u,p)
\]

for \(i=1,2,\ldots,n\)

where \(w_i\) is the budget share of commodity \((i)\)

Moreover, Equation 6.2.2 states that cost elasticity of commodity \(i\) equals the budget share of commodity \(i\) and that, the budget shares are functions of prices and utility. Hence, the logarithmic differentiation of Equation 6.2 with respect to \(\log p\), gives the budget shares as a function of prices and utility.

\[
\frac{\partial \log c(u,p)}{\partial \log p} = w_i = a_i + \sum_j \gamma_{ij} \log p_j + B_i u B_o \prod_k p_k^{\beta_i} = w_i(u,p)
\]

where \(\gamma_{ij} = \frac{1}{2}(\gamma_{ij}^* + \gamma_{ji}^*)\).

By inversion of \(\log c(u,p) = \log x\) in Equation 6.2, for the utility maximization, total expenditure \(x\) is equal to \(c(u,p)\). This results in the indirect utility function, which is a function of \(p\) and \(x\).

\[
v(p,x) = u = \frac{\log x - (a_o + \sum_k a_k \log p_k + \frac{1}{2} \sum_j \gamma_{kj}^\ast \log p_k \log p_j)}{B_o \prod_k p_k^{\beta_i}}
\]

By substituting Equation 6.4 into Equation 6.3 the budget share as a function of \(p\) and \(x\) is given

\[
w_i = a_i + \sum_j \gamma_{ij} \log p_j + \gamma_i (\log x - (a_o + \sum_k a_k \log p_k + \frac{1}{2} \sum_j \gamma_{kj}^\ast \log p_k \log p_j))
\]

Deaton and Muellbauer defined the price index as:

\[
\log p = a_o + \sum_k a_k \log p_k + \gamma_i (\log x - (a_o + \sum_k a_k \log p_k + \frac{1}{2} \sum_j \gamma_{kj}^\ast \log p_k \log p_j)
\]

Hence, the Equation 6.5 can be reduced to a more simple form by inverting Equation 6.5 where:
\[ w_i = a_i + \sum \gamma_j \log p_j + B_i \log \left( \frac{x}{p} \right) \]  

Equation 6.7 states that the budget share of commodity \( i \) changes due to changes in relative prices and real expenditure. Hence, Equation 6.7 and Equation 6.5 represent the Almost Ideal Demand System approach (Deaton and Muellbauer, 1980a, p.316). Thus, the AIDS model is derived from an underlying structure of consumer preferences via a cost expenditure function of the form (6.2).

Stone's (1953) price index can be utilized because it is an excellent approximation to the Deaton price index (Deaton, 1980a, p.317). Stone (1953) defined the price index as:

\[ \log p^* = \sum w_k \log p_k \]  

(6.8)

Deaton and Muellbauer show that empirically it makes little difference in using \( p \) instead of \( p^* \) (Assuming that \( p \equiv \xi p^* \)). Therefore, Equation 6.7 can be rewritten as follows:

\[ w_i = a_i^* + \sum \gamma_j \log p_j + B_i \log \left( \frac{x}{p} \right) \]  

(6.9)

where \( (p^*) \) is the price index by this definition, and is equal to \( p \equiv \xi p^* \), and hence \( a_i^* = a_i - B_i \log \xi \)

Equation 6.9 is known as the Linear Approximate Almost Ideal Demand System (LA/AIDS). According to Equation 6.9, the intercept \( a_i \) represents the average budget share when the logarithm of prices and real expenditure is (0). The parameter \( \gamma_j \) measures the change in \( i \)-th budget share corresponding to a percentage change in \( p_j \), holding all other variables, in case \( (\frac{x}{p}) \), constant, whereas, \( B_i \), measures the change in the budget share corresponding to a percentage change in \( (\frac{x}{p}) \) with \( p_j \), held constant, these add zero and can assume either negative or positive values. Hence, the commodity \( i \)-th represents a necessary item if \( B_i \) is negative, which
means that the i-th budget share decreases with the logarithm of real expenditure, while, in the case of \( B_i \) positive, the i-th commodity is a luxury item, as a result of the i-th budget share increasing with the logarithm of real expenditure.

Recently, Moschini (1995) suggested using Tornqvist's price index instead of Stone index \( p^* \). The Tornqvist's price index is defined as:

\[
\ln p_i = \frac{1}{2} \sum_{m=1}^{s} (w_m + w_m^0) \ln \left( \frac{p_{it}}{p_i^0} \right)
\]

where the zero superscript denotes base period values.

Moschini (1995) made an attempt to estimate AIDS by applying different price indices in linear forms and the main result of this study was: Tornqvist's price index was the best in producing the same results as the true nonlinear AI model.

In order to make AIDS consistent with the theory of demand the following restrictions on the parameters of the Equation 6.5 is required:

Adding-up restriction:

This restriction will be met if \( \sum_{i=1}^{s} a_i = 1, \sum_{i=1}^{s} B_i = 0 \) and \( \sum_{i=1}^{s} \gamma_{ij} = 0 \).

Homogeneity restriction:

For the homogeneity restriction to be met, if \( \gamma \) is restricted, so that \( \sum_{j} \gamma_{ij} = 0 \).

Slutsky symmetry restriction:

Eventually, the Slutsky symmetry restriction is satisfied if \( \gamma_{ij} = \gamma_{ji} \) (for all \( i, j = 1,2,\ldots,n \), and \( i \neq j \)).

Negativity restriction:

As it is true for other flexible functional forms, negativity cannot be ensured by any restrictions on the parameters alone. It can however be checked for any given estimates by calculating the eigenvalues of the Slutsky matrix \( s_{ij} \). According to Deaton and Muellbauer (1980b), in practice it is easier not to use \( s_{ij} \) but
where \( \delta_{ij} \) is the Kronecker delta that is unity if \( i \neq j \).

The second-order condition of the equilibrium is fulfilled when the matrix \( k_v \) is negative semi-definite. Under that condition, the consumer is considered to be in stable equilibrium.

The above restrictions not only decrease the dimensionality of the parameter space but also ensure that own-price, cross-price, and expenditure elasticities are consistent with the neoclassical theory.

The uncompensated and compensated own-price, cross-prices, and expenditure or income elasticities are as follows: For uncompensated own price elasticity:

\[
e_a = \frac{1}{w_i} \left( \frac{\partial w_i}{\partial \log p_i} \right) - 1 = \frac{\gamma_a}{w_i} - 1
\]

While, for uncompensated cross-price elasticity:

\[
e_a = \frac{1}{w_i} \left( \frac{\partial w_i}{\partial \log p_j} \right) = \frac{\gamma_a}{w_i}
\]

However, for compensated price elasticities:

\[
e_a^* = \eta_a w_i + e_a
\]

\[
e_a^* = \eta_j w_j + e_a
\]

Finally, for expenditure elasticity:

\[
\eta = 1 + \left( \frac{\eta}{w_i} \right)
\]

When estimating demand systems, one equation must be omitted. This procedure avoids inherent singularity in the variance-covariance matrix of the residuals across equations. In other words, because of adding up \( \sum w_i = 1 \) for all \( t \), it therefore follows that the contemporaneous covariance matrix is singular, with this singularity problem typically handled in estimation by deleting an equation. The commodity
that is arbitrarily omitted in the model is other commodity group's equation. Barten (1969) shows that if full information maximum likelihood (FIML) estimation techniques are used, the estimates obtained are invariant with respect to the equation deleted. Using (ISUR) will provide the similar estimates, which are equal to (FIML) estimates (David et al., 996, p.77). The parameters of the deleted equation can be calculated by the adding up condition, and the standard errors of the parameters of the deleted equation can be calculated as follows:

\[ S.E(\gamma_{ij}) = \sqrt{\sum_i Var(\gamma_{ij}) + 2\text{cov}(\gamma_{ij}, \gamma_{ik})} \quad \text{for} \quad j \neq k \quad (6.17) \]

The unrestricted static LA/AIDS model consisting of the demand system can be estimated using Ordinary Least Square method (OLS)) since the same explanatory variables appear in all share equations (Judge et. al., 1985, p.468). Homogeneity can be imposed using either restricted estimation or the price of one commodity as normalizer using as regressors price ratios, hence OLS estimation is still valid (Mergos and Donatos, 1989, p.985). When symmetry restriction is imposed the system cannot be estimated equation by equation, an Iterative Seemingly Unrelated Regression technique (ISUR) is needed.

Added to the above, the Tomqvist's price index will be replaced by the true price index of Deaton and Muellbauer (1980) in both Equations 6.9 and 6.19, so as to normalize the model. The homogeneity and symmetry restrictions will be imposed and tested for both static (LA/AIDS) and dynamic versions of AIDS.

Depending on the above presentation, the system of LA/AIDS will be applied to Bahrain using the quarterly data for the period 1979Q1 to 1998Q4. A five-item classification of personal consumer expenditure was used: (1) Food & Beverages, (2) Clothing & Footwear, (3) Housing, (4) Transportation, and (5), other commodity group.
6.3 Overall Performance of the Static LA/AIDS Model.

The empirical implementation of the model is accomplished by estimating the system of Equation 6.9. The unrestricted (free) LA/AIDS model was estimated for each of the five commodity groups separately by OLS. Table 6.1 presents the intercepts $a$, marginal budget shares $\beta$, and the $\gamma$ estimate together with the row sum of $\gamma$ estimates, the coefficients determination, $R^2$, and the values of the Durbin-Watson statistics ($d.w$).

The estimates of $\beta$, classify the demand for Food & Beverages products, Clothing & Footwear, Housing, and Transportation as necessities, while the Other commodity group is classified as luxuries. In the AIDS model $\beta$'s negative imply necessities while positive $\beta$'s suggest luxuries. These findings provide an interesting contrast with the findings of the most recent studies such as the study of Mergos and Donatos (1989a, p.987) to the Greece data, Deaton and Muellbauer (1980a) to the British data, Pashardes (1993, pp. 910-11) to the UK expenditure data, and Al-Taub (1995, p.210) to annual Jordanian time series data. In all of these studies the Food and Beverages category are classified as necessities as in Bahrain.

Furthermore, the results of this application indicate that 7 out of 35 parameter estimates are significantly different from zero at the 5% level of significance, as indicated by the t-values (the t-values can be obtained by dividing the parameters on its own standard errors). In addition, 3 out of 25 of the price coefficients have t-values absolutely greater than two, while two of the marginal budget shares are significant at 5% level, and these results are for Food & Beverages, and other commodity group equation.
Table 6.1
Estimated parameters of unrestricted LA/AIDS model and summary of statistical results classified according to commodity group equation - Static model

<table>
<thead>
<tr>
<th>Commodity groups equations</th>
<th>Intercept</th>
<th>Marginal budget share</th>
<th>Food &amp; beverages</th>
<th>Clothing &amp; footwear</th>
<th>Housing</th>
<th>Transportation</th>
<th>Other</th>
<th>( R^2 )</th>
<th>( dw^* )</th>
<th>( \sum \gamma_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverages</td>
<td>0.584</td>
<td>-0.127</td>
<td>0.039</td>
<td>0.016</td>
<td>0.001</td>
<td>-0.009</td>
<td>0.74</td>
<td>0.16</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.272)</td>
<td>(0.043)</td>
<td>(0.046)</td>
<td>(0.047)</td>
<td>(0.012)</td>
<td>(0.019)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.194</td>
<td>-0.033</td>
<td>-0.010</td>
<td>0.047</td>
<td>0.003</td>
<td>-0.049</td>
<td>0.39</td>
<td>1.07</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.245)</td>
<td>(0.038)</td>
<td>(0.046)</td>
<td>(0.042)</td>
<td>(0.011)</td>
<td>(0.017)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing</td>
<td>1.144</td>
<td>-0.090</td>
<td>-0.040</td>
<td>-0.135</td>
<td>0.029</td>
<td>0.048</td>
<td>0.33</td>
<td>1.24</td>
<td>-0.094</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.500)</td>
<td>(0.078)</td>
<td>(0.092)</td>
<td>(0.085)</td>
<td>(0.023)</td>
<td>(0.035)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transportation</td>
<td>0.531</td>
<td>-0.075</td>
<td>0.049</td>
<td>-0.159</td>
<td>-0.026</td>
<td>0.037</td>
<td>0.54</td>
<td>0.89</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.324)</td>
<td>(0.051)</td>
<td>(0.060)</td>
<td>(0.055)</td>
<td>(0.015)</td>
<td>(0.023)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>-1.452</td>
<td>0.331</td>
<td>0.037</td>
<td>0.250</td>
<td>-0.059</td>
<td>0.173</td>
<td>0.61</td>
<td>0.80</td>
<td>-0.003</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.88)</td>
<td>(1.139)</td>
<td>(1.195)</td>
<td>(1.151)</td>
<td>(0.040)</td>
<td>(0.062)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- (Standard error of the estimates in the parentheses).

(*) \( d_u \) and \( d_i \) are 1.83 and 1.45 respectively and imply that there is a positive serial correlation in the residual of all equations.

The findings are not considered to be better than the findings obtained from previous studies. Deaton and Muellbauer (1980a) in their study found that only 22 out of 64 of the estimate price coefficients had t-values bigger than two in the absolute value, Ray (1980, p.598) when adopted AIDS to Indian data acquired 13 out of 81 of the price coefficients were significant, and lastly, Blanciforti (1982, p.132) applied AIDS to USA data and found that 42 out of 121 of the price coefficients were significant. Al-Taub (1995, p.212) applied the AIDS model to Jordanian annual data and found that 11 out of 36 had t-values absolutely bigger than two. Finally, the row sum \( \sum \gamma_i \) for some of the individual equation is very small and very close to zero, which is supposed to be under the homogeneity condition (see Table 6.1).

With respect to the coefficients of determination \( R^2 \), it ranges between 0.33 for the Housing commodity group to 0.74 for the Food & Beverage commodity group. The
results suggest the presence of positive first-order serial correlation in all equations in the system.

Demand elasticities of LA/AIDS for the unrestricted Equation 6.9 are reported in Table 6.2 below. Table 6.2 presents total expenditure elasticity along with uncompensated-compensated own price elasticities, and cross-uncompensated price elasticities. With respect to the evaluated total expenditure elasticities, the demand for Food & Beverages, Clothing & Footwear, Housing along with Transportation commodity group are inelastic, however, the demand for other commodity group is elastic. This means that as income increases, Bahraini consumers tend to spend relatively more on “Other” products and less on the remaining products (Food & Beverage, Clothing Footwear, Housing, Transportation products) as suggest by the results of applying the LA/AIDS model. This finding is consistent with the marginal budget shares estimates.

Table 6.2
Total expenditure elasticities, Uncompensated own-cross price elasticities, and compensated own-price elasticities for unrestricted LA/AIDS: Static model.

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>Total expenditure elasticity ($\eta_i$)</th>
<th>Elasticity of demand with respect to price of</th>
<th>Compensated own price elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Food &amp; beverage</td>
<td>Clothing &amp; footwear</td>
</tr>
<tr>
<td>Food &amp; beverage</td>
<td>0.253</td>
<td>-0.771</td>
<td>-0.906</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.529</td>
<td>-1.167</td>
<td>-0.329</td>
</tr>
<tr>
<td>Housing</td>
<td>0.357</td>
<td>-1.286</td>
<td>-1.957</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.063</td>
<td>-0.388</td>
<td>-2.988</td>
</tr>
<tr>
<td>Other</td>
<td>1.602</td>
<td>-1.067</td>
<td>-0.582</td>
</tr>
</tbody>
</table>

Furthermore, the results indicate that the Food & Beverages commodity group together with Clothing & Footwear, Housing, and Transportation are classified as a "necessity" for the Bahraini consumer in the sense of having expenditure elasticities less than one while the demand for the other commodity groups is classified as "luxuries" with expenditure elasticity estimates consistently in excess of one. See Table 6.2.
The uncompensated own-price elasticities show that the demand for all categories is inelastic. Further, Table 6.2 recorded that all of compensated and uncompensated own price elasticities are negative. The latter findings are consistent with prior expectations regarding their sign. Furthermore, it appears from Table 6.2 that the compensated own price elasticity for all commodity groups is less than the uncompensated own price elasticity in absolute values. Finally, the signs of the cross-price elasticities reflect the substitution or complementary relationship that exists between commodity groups.

The homogeneity restriction can be imposed using either restricted estimation \((\sum y_i = 0)\) or the price of one commodity as a normalizer using as regressors price ratios, hence the OLS estimation is still valid. When the homogeneity restriction was imposed the results were slightly different from the unrestricted LA/AIDS model. Table 6.3 reported the parameter structural along with the main statistics results for the homogenous LA/AIDS model.

The results of Table 6.3 show that the overall fit is the worst for all equations of the LA/AIDS model. This conclusion is reached when a comparison is made between the free LA/AIDS model and the homogenous LA/AIDS model. This can be seen from the values of the, \(R^2\) and the Durbin-Waston statistical results. The aim of estimating the homogeneity version of the model as equation by equation will help in testing the homogeneity restriction as an equation by equation basis and not as a system basis. The system basis test will be presented in section (6.4) of this chapter. However, the results of the equation by equation basis test are reported in Table 6.5 and the discussion and analysis is also presented in the same Table 6.5.
Table 6.3
Estimated parameters of homogenous LA/AIDS model and summary of statistical results classified according to commodity group equation- Static model

<table>
<thead>
<tr>
<th>Commodity groups equation</th>
<th>Intercept</th>
<th>Marginal budget share</th>
<th>Food &amp; beverages</th>
<th>Clothing &amp; footwear</th>
<th>Housing</th>
<th>Transportation</th>
<th>Other</th>
<th>$R^2$</th>
<th>d.w</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverages</td>
<td>0.823</td>
<td>-0.106</td>
<td>-0.020</td>
<td>0.019</td>
<td>0.012</td>
<td>0.084</td>
<td>-0.095</td>
<td>0.72</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>(0.249)</td>
<td>(0.042)</td>
<td>(0.042)</td>
<td>(0.047)</td>
<td>(0.011)</td>
<td>(0.015)</td>
<td>(0.058)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.248</td>
<td>-0.029</td>
<td>-0.024</td>
<td>0.048</td>
<td>0.005</td>
<td>0.033</td>
<td>-0.062</td>
<td>0.39</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>(0.219)</td>
<td>(0.037)</td>
<td>(0.037)</td>
<td>(0.042)</td>
<td>(0.010)</td>
<td>(0.014)</td>
<td>(0.051)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing</td>
<td>0.856</td>
<td>-0.120</td>
<td>-0.139</td>
<td>0.017</td>
<td>0.020</td>
<td>0.072</td>
<td>-0.120</td>
<td>0.30</td>
<td>128</td>
</tr>
<tr>
<td></td>
<td>(0.450)</td>
<td>(0.076)</td>
<td>(0.076)</td>
<td>(0.085)</td>
<td>(0.021)</td>
<td>(0.028)</td>
<td>(0.106)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transportation</td>
<td>0.534</td>
<td>-0.074</td>
<td>0.048</td>
<td>-0.159</td>
<td>0.026</td>
<td>0.037</td>
<td>0.047</td>
<td>0.54</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>(0.289)</td>
<td>(0.049)</td>
<td>(0.048)</td>
<td>(0.056)</td>
<td>(0.013)</td>
<td>(0.018)</td>
<td>(0.068)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>-1.461</td>
<td>0.330</td>
<td>-0.035</td>
<td>0.230</td>
<td>-0.060</td>
<td>-0.174</td>
<td>0.038</td>
<td>0.61</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>(0.790)</td>
<td>(0.134)</td>
<td>(0.132)</td>
<td>(0.150)</td>
<td>(0.036)</td>
<td>(0.049)</td>
<td>(0.186)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*)- (Standard error of the estimates in the parentheses)

Table 6.4 recorded the evaluated elasticities of demand with respect to total expenditure, along with uncompensated and compensated own-price elasticities that is obtained from the homogenous system. As usual the elasticities are estimated at the mean values of the budget shares of each commodity group. Based on the results reported in Table 6.4, one of the commodity groups is elastic with respect to total expenditure (this is being the Other commodity groups) while the remainder of the commodity groups is inelastic with respect to the total expenditure elasticity. Lastly, the classifications results of commodity groups between necessities and luxuries are the same as is obtained in the case of the unrestricted (free) LA/AIDS model.

The uncompensated own price elasticity estimates show that the demand for the Food & Beverages commodity group is elastic only while the demand for the remainder commodity groups is inelastic with respect to price. These findings are not the same as in the case of the LA/AIDS without restrictions. One remarkable
result is that the own price elasticity for Food & Beverages is relatively very big in comparison to estimated results obtained from different studies. For instance, Ray (1980, p.599) when estimating the AIDS using the data from India, the own price elasticity was 0.331, Merges and Donatos (1989a, p.981) obtained about -0.44 of own price elasticity when they estimated the model for Greece. Pashardes (1993, p.911) reported about -0.17 for the UK. The overestimation of Food & Beverages elasticity can be related to the variables that are omitted from the model.

In addition, Table 6.4 recorded that all of compensated uncompensated own cross price elasticities are negative. The latter findings are consistent with a prior expectation regarding their sign. Furthermore, it appears from Table 6.4 that the size of the own compensated price elasticities for all commodity groups is less than the size of own uncompensated price elasticities in absolute values.

The empirical test for homogeneity is summarized in Table 6.5. Column (1) gives the sum of the estimated coefficients and column (2) the F-test. The results indicate rejection of the homogeneity hypothesis at the 0.05 level for only the Food and Beverages commodity group equation. However, the homogeneity is not rejected for four remaining commodity group equations of the static LA/AIDS model. This means that for these commodity groups that are not rejected, the homogeneity restriction, a proportional increase in prices and total expenditure will increase

<table>
<thead>
<tr>
<th>Commodity groups equations</th>
<th>$\eta_i$</th>
<th>$e_i^u$</th>
<th>$e_i^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverages</td>
<td>0.376</td>
<td>-1.118</td>
<td>-1.054</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.586</td>
<td>-0.314</td>
<td>-0.273</td>
</tr>
<tr>
<td>Housing</td>
<td>0.143</td>
<td>-0.857</td>
<td>-0.859</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.075</td>
<td>-0.538</td>
<td>-0.532</td>
</tr>
<tr>
<td>Other</td>
<td>1.609</td>
<td>-0.930</td>
<td>-0.061</td>
</tr>
</tbody>
</table>

(*$\eta_i$ is expenditure elasticities - $e_i^u$, and $e_i^*$, are uncompensated and compensated own price elasticities respectively.)
expenditure on all four commodity groups as implied by the row sum $\sum_j \gamma_j$ of estimated coefficients reported in Table 6.5.

<table>
<thead>
<tr>
<th>Commodity groups equations</th>
<th>$\sum_j \gamma_j$</th>
<th>$F$ - value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food and beverages</td>
<td>0.077</td>
<td>3.956</td>
</tr>
<tr>
<td>Clothing and footwear</td>
<td>0.018</td>
<td>0.249</td>
</tr>
<tr>
<td>Housing</td>
<td>-0.094</td>
<td>1.700</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>Other</td>
<td>-0.003</td>
<td>0.000</td>
</tr>
</tbody>
</table>

(*) : Tabulated $F$-value for (1, 73) is 3.92 for 0.05 level.

Symmetry, unlike homogeneity, can be tested on an equation by equation basis. Imposing the symmetry restrictions on the system of Equation 6.9, the model was estimated by the means of (ISUR), which is equivalent to maximum likelihood estimation when it converges (David et. al., 1996, p. 77). Parameter estimates along with the coefficients of determination, $R^2$, $d.w$ statistics and the standard errors of the estimates are presented in Table 6.6.

The statistic results which are indicated by $R^2$ and $d.w$ suggest that the model is less fitted than both free and homogenous models (see Tables 6.1, 6.3 and 6.6). The t-ratios also show that 3 out of 15 one price coefficients $\gamma_j$ are significant at the 5% significant level. Second, the marginal budget share for the Food & Beverages commodity group equations is statistically significant at a five percent level. In all versions of the static LA/AIDS the results show that the only commodity group that is classified as luxuries with respect to total expenditure elasticity is the other commodity groups. Finally, comparing the elasticities reported in Table 6.7 with those reported in Tables 6.2 and 6.4 shows a lot of variation among the three sets of results regarding total expenditure and own compensated-uncompensated elasticities. Differences between models are better seen in the context of the values of own-price and expenditure elasticities.
Table 6.6
Estimated parameters of symmetric LA/AIDS model and summary of statistical results classified by commodity group equation - Static model.

<table>
<thead>
<tr>
<th>Commodity groups equation</th>
<th>Intercept (a_i)</th>
<th>Marginal budget share (\beta_i)</th>
<th>Food &amp; beverages (\gamma_{11})</th>
<th>Clothing &amp; footwear (\gamma_{12})</th>
<th>Housing (\gamma_{13})</th>
<th>Transportation (\gamma_{14})</th>
<th>Other (\gamma_{15})</th>
<th>(R^2)</th>
<th>d.w</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverages</td>
<td>0.515</td>
<td>-0.145</td>
<td>0.065</td>
<td>0.027</td>
<td>-0.003</td>
<td>0.046</td>
<td>-0.019</td>
<td>0.73</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(0.247)</td>
<td>(0.036)</td>
<td>(0.039)</td>
<td>(0.027)</td>
<td>(0.011)</td>
<td>(0.015)</td>
<td>(0.052)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.334</td>
<td>-0.078</td>
<td>-0.018</td>
<td>-0.005</td>
<td>0.002</td>
<td>0.039</td>
<td></td>
<td>0.37</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>(0.227)</td>
<td>(0.034)</td>
<td>(0.034)</td>
<td>(0.010)</td>
<td>(0.015)</td>
<td>(0.050)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing</td>
<td>0.715</td>
<td>-0.097</td>
<td></td>
<td>0.024</td>
<td>0.031</td>
<td>-0.044</td>
<td></td>
<td>0.28</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>(0.386)</td>
<td>(0.053)</td>
<td></td>
<td>(0.021)</td>
<td>(0.014)</td>
<td>(0.062)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transportation</td>
<td>0.049</td>
<td>-0.027</td>
<td></td>
<td>0.051</td>
<td>-0.085</td>
<td></td>
<td>0.046</td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.268)</td>
<td>(0.043)</td>
<td></td>
<td>(0.019)</td>
<td>(0.059)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td>-0.613</td>
<td>0.347</td>
<td></td>
<td></td>
<td></td>
<td>-0.109</td>
<td></td>
<td>0.53</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>(0.577)</td>
<td>(0.084)</td>
<td></td>
<td></td>
<td></td>
<td>(0.112)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*)- (Standard error of the estimates in the parentheses.

As shown in Tables 6.2, 6.4, and 6.7 differences in the estimated elasticities between estimations without restrictions and imposing homogeneity and symmetry are large. For example, in the static model, the own-price elasticity of the Food & Beverages commodity group ranges from -0.771 without restrictions to -1.118 and -0.618 respectively when homogeneity and symmetry are imposed. Own price of uncompensated elasticities is found negative for all models satisfying the negativity constraint. Price elasticities tend to become larger in moving from the unrestricted to the symmetric version. Expenditure elasticities tend to become bigger for most of the groups except for the Housing commodity group (homogeneity restriction); however, under symmetric restriction all became smaller except Transportation and Other commodity groups.
Table 6.7
Total expenditure elasticities, uncompensated-compensated own price elasticities for the symmetric LA/AIDS model. - Static model

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>$\eta_1$</th>
<th>$e^*_{ii}$</th>
<th>$e^*_{ii}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverages</td>
<td>0.147</td>
<td>-0.618</td>
<td>-0.593</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>-0.113</td>
<td>-1.257</td>
<td>-1.265</td>
</tr>
<tr>
<td>Housing</td>
<td>0.306</td>
<td>-0.832</td>
<td>-0.789</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.663</td>
<td>-0.363</td>
<td>-0.310</td>
</tr>
<tr>
<td>Other</td>
<td>1.643</td>
<td>-0.798</td>
<td>0.089</td>
</tr>
</tbody>
</table>

(*) $\eta_1$ is expenditure elasticities. $e^*_{ii}$ and $e^*_{ii}$ are uncompensated and compensated own price elasticities respectively.

In some other cases, the change in the estimated elasticity's after imposing the restrictions is drastic. For example, there are cases in the static model where there is a change from being price inelastic to being price elastic (e.g. Food & Beverages and for the Clothing & Footwear commodity group). Furthermore, there is only one violation of the negativity property (compensated own-price), and that for the Other commodity group when symmetry is imposed in the static model. Merges and Donatos (1989, p.991) obtained similar results and which were for the Clothing category but for uncompensated own-price. This result was reached when Merges and Donatos imposed the symmetry restriction on the LA/AIDS model using annual Greece data. Merges and Donatos related the reason to the presence of higher-serial correlation in the clothing category equation.

Moreover, it is also found that the expenditure elasticity of Clothing & Footwear is negative. The validity of negative income elasticity for this category is verified by the fact that the consumption per capita has declined over the observation period. Sasaki (1996, p. 43) also obtained negative expenditure elasticity when the LA/AIDS was applied to Japanese data in order to analyze consumer demand for 16 commodity categories.

6.4 Testing for the Hypotheses of Demand Theory- Static Model

In addition to the estimates reported in Tables 6.1, 6.3, and 6.6 homogeneity and symmetry restrictions were imposed and tested, which is implied by the demand theory. The results in Table 6.8 indicate that the homogeneity is rejected over the no
restrictions case. Symmetry given homogeneity is strongly rejected. Symmetry is also clearly rejected over the no-restrictions case (see Table 6.8).

**Table 6.8**

**Hypotheses tests for restrictions in the LA/AIDS model - Static model.**

<table>
<thead>
<tr>
<th>Type of the model</th>
<th>Logarithmic likelihood value</th>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>$-2\ln \lambda$</th>
<th>d.f</th>
<th>Critical value (5%)</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>951.65</td>
<td>Homogeneity</td>
<td>No-restrictions</td>
<td>9.50</td>
<td>4</td>
<td>9.49</td>
<td>Rejected</td>
</tr>
<tr>
<td>Homogeneity</td>
<td>946.90</td>
<td>Homogeneity</td>
<td>No-restrictions</td>
<td>15.08</td>
<td>6</td>
<td>12.59</td>
<td>Rejected</td>
</tr>
<tr>
<td>Symmetry</td>
<td>944.11</td>
<td>Symmetry</td>
<td>No-restrictions</td>
<td>30.56</td>
<td>10</td>
<td>18.31</td>
<td>Rejected</td>
</tr>
<tr>
<td>Homogeneity + Symmetry</td>
<td>936.37</td>
<td>Symmetry</td>
<td>Homogeneity</td>
<td>30.56</td>
<td>10</td>
<td>18.31</td>
<td>Rejected</td>
</tr>
</tbody>
</table>

Log likelihood ratio is asymptotically distributed as $\chi^2$ with $h$ degrees of freedom, where $h$ is the number of restrictions imposed.

In general the static model tends to be rejected by the data; hence, it is difficult to depend on the parameters of these models for testing hypotheses or computing the demand elasticities for the products of the commodity groups for this study. The results have an important role for modeling and prediction of consumer behavior. Therefore, relying on these results will also lead to bias in the estimation of income and price elasticities, which consequently results in the productions of behavior to be quite inaccurate and the latter could be the policy maker of the country in question.

This chapter has made another attempt to insert a dynamic factor which might affect mainly the consumer behavior expenditure and this dynamic factor could be improved by the overall performance of the model and will provide logical evidence about Bahrain's consumer behavior towards the commodities in the local markets.

**6.5 A Dynamic Version of LA/AIDS Model**

Three explanations have been offered as reasons for inconsistencies between the data and theory: (1) use of inappropriate statistics in view of the short time series of data available (Laitinen, 1978); (2) neglect of system dynamics (Blanciforti and Green, 1983; Anderson and Blundell, 1983); and (3) problems of aggregation, i.e, the
expenditure categories are simply too broad for the concept of utility maximization to be meaningful (Boroah, 1985). As a result, of these explanations, there has been continuing attention and great support given in recent years to the role of habit formation in consumer's allocation decisions. This can be seen in the form of including a dynamics factor into direct demand systems. Such a dynamic approach has been established by Pollak and Wales (1969), Pollak (1970), Phlips (1972), Manser (1976), Blanciforti and Green (1983), Ray (1984), Alessic and Kapteyn (1991), Heien and Durham (1991), Kim (1994), and recently by Matthew et. al. (1997) and others. Some of these authors allow parameters associated with particular goods to depend only on their own lagged consumption, while the others approaches allow parameters associated with a particular goods to depend not only on their own lagged consumption but also on lagged consumption of all other goods in the system.

Accordingly, the purpose of this chapter is also to estimate a dynamic specification proposed by Blanciforti and Green (1983). It should be noted that the researcher of this study made an attempt to apply different approaches of the dynamic AIDS model for different authors. All of those approaches gave us bad results for Bahrain data except the dynamic LA/AIDS model of Blanciforti and Green (1983) which has been found to be the best among these approaches.

The remainder of the section is organized as follows: The DLA/AIDS is presented and explanation as to how the estimation method is used. The DLA/AIDS model will be applied to the same sets of data that is used in the static LA/AIDS model. The main results of this application will be presented and discussed. Conclusions and implications are summarized in the final section of this chapter.
6.6 Model Specification and Estimated Method- Dynamic Model

Blanciforti and Green ('1983) established the dynamic version of the AIDS incorporating habit effects in the form of Pollak and Wales (1969). Blanciforti and Green ('1983) redefined the parameters \( a_i \) in the static AIDS model as:

\[
 a_i = a_i^* + a_i^{**} x_{t-1} ,
\]  

(6.18)

where \( x_{t-1} \) denotes lagged purchases (quantity) of item (i). It is worth noting that \( a_i^{**} \)'s allowing dependence of the original AIDS parameters on the past consumption of only their own lagged consumption item (via the lagged purchase \( x_{t-1} \)). The corresponding dynamic system of Blanciforti and Green (1983) include simple linear habit effects given by:

\[
w_i = a_i^* + a_i^{**} x_{t-1} + \sum_j \gamma_j \log p_j + \beta_i \log \left( \frac{x_i}{p_i} \right) + u_i ,
\]  

(6.19)

where \( \ln p^* \) is given by (6.20)

\[
\log p_i^* = a_0 + \sum_i (a_i^* + a_i^{**} x_{t-1}) \log p_i + \frac{1}{2} \sum_j \sum \gamma_j \log p_j \log p_i
\]  

(6.20)

Blanciforti and Green (1983) did not impose adding up on the \( a_i^{**} \) parameters. This is because they considered only lagged own-quantity effects in each individual share equation. Alessic and Kapteyn (1991) argue that it is not possible to impose adding up in such a system without restricting all \( a_i^{**} \)'s to equal zero.

The dynamic model of Equation 6.19 will be estimated using the (ISUR) method and by following the same procedure and price index used for the static model. First, the model is estimated without restrictions, then with homogeneity imposed, and finally with symmetry imposed. The estimation results and discussions are offered in the following sections of this chapter.

6.7 Overall Performance of the Dynamic LA/AIDS Model

Table 6.9 presents parameter estimates along with main statistical results for the dynamic LA/AIDS model (6.9), which consists of five commodity groups. All the
dynamic parameters are significant and point to dynamic AIDS representing a significant generalization of its static version. This was confirmed by a huge drop in likelihood on the imposition of the restriction that all the dynamic adjustment parameters are zero. The significance of all of $a_i^\pi$, establish the importance of introducing linear habit form in modeling consumer expenditure. The coefficient of lagged consumption reflects persistence in budget allocations when it is positive and stock effects when it is negative. The coefficient of lagged consumption indicates that expenditure on other commodity group displays existence of stock effects whereas expenditure on all of the four commodities groups is persistent in budget allocations. There is not much variability in the estimation of $(a_i^\pi)$ as observed from the unrestricted to the symmetric models whether in the magnitude or in sign (see Table 6.10).

Table 6.9
Estimated parameters (standard error) for unrestricted LA\AIDS with main statistical results according to commodity group equation- Dynamic model.

<table>
<thead>
<tr>
<th>Parameters estimates</th>
<th>Food &amp; beverages equation</th>
<th>Clothing &amp; footwear equation</th>
<th>Housing equation</th>
<th>Transportation equation</th>
<th>Other equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_i^\pi$</td>
<td>0.922 (0.118)</td>
<td>0.408 (0.242)</td>
<td>1.187 (0.521)</td>
<td>0.449 (0.295)</td>
<td>-1.966 (0.657)</td>
</tr>
<tr>
<td>$a_i^\pi$</td>
<td>0.002 (0.001)</td>
<td>0.001 (0.002)</td>
<td>0.001 (0.002)</td>
<td>0.001 (0.002)</td>
<td>-0.004 (0.004)</td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>-0.016 (0.022)</td>
<td>-0.023 (0.046)</td>
<td>-0.044 (0.097)</td>
<td>0.008 (0.055)</td>
<td>0.074 (0.123)</td>
</tr>
<tr>
<td>$\gamma_{i2}$</td>
<td>0.035 (0.018)</td>
<td>0.063 (0.039)</td>
<td>-0.067 (0.087)</td>
<td>-0.084 (0.049)</td>
<td>0.052 (0.109)</td>
</tr>
<tr>
<td>$\gamma_{i3}$</td>
<td>-0.005 (0.005)</td>
<td>0.003 (0.010)</td>
<td>0.054 (0.023)</td>
<td>0.005 (0.013)</td>
<td>-0.057 (0.029)</td>
</tr>
<tr>
<td>$\gamma_{i4}$</td>
<td>0.018 (0.008)</td>
<td>0.021 (0.016)</td>
<td>0.044 (0.033)</td>
<td>0.062 (0.019)</td>
<td>-0.146 (0.042)</td>
</tr>
<tr>
<td>$\gamma_{i5}$</td>
<td>-0.064 (0.026)</td>
<td>-0.080 (0.054)</td>
<td>-0.091 (0.114)</td>
<td>0.003 (0.066)</td>
<td>0.221 (0.145)</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>-0.123 (0.018)</td>
<td>-0.050 (0.037)</td>
<td>-0.109 (0.080)</td>
<td>-0.063 (0.045)</td>
<td>0.345 (0.101)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.96</td>
<td>0.53</td>
<td>0.48</td>
<td>0.63</td>
<td>0.75</td>
</tr>
<tr>
<td>d.w</td>
<td>1.71</td>
<td>2.03</td>
<td>1.60</td>
<td>1.67</td>
<td>1.59</td>
</tr>
</tbody>
</table>

(*) - $d_u$ and $d_l$ are 1.68 and 1.28 respectively and imply that there is a rough absence of auto-correlation in the residual of all equations.
The marginal budget share for the Food & Beverages commodity group is the only budget share which is found significant at 5% level, while in the static model it was found that both budget shares for Food & Beverages and other commodity groups were significant at a 5% level. Six out of 25 price coefficients are significant at 5% level whereas in the static model it was discovered that 3 out of 25 price coefficients were significant at 5% level.

Table 6.9 shows that there is no evidence of auto-correlation in any equations of the dynamic model as indicated by the values of the $d.w$ statistics. The $R^2$ value ranges from 0.48 for Housing equation to 0.96 for the Food & Beverages equation. The statistical results indicate that all equations for the dynamic version are more fit than the unrestricted static LA/AIDS model. Therefore, the data provides evidence to support a dynamic behavior for the Bahrain data rather than utilization of the static version of the LA/AIDS model.

Furthermore, the imposition of homogeneity and symmetry restriction leads only a to marginal drop in the statistical results but both the homogeneity and symmetry AIDS versions share the following results with the unrestricted version: First, all the coefficients of past consumption are significant at 5% level. Second, the marginal budget shares for Food & Beverages and Other commodity group are only statistically significant at 5% level which is the same as in the free version of the dynamic model. However, in the symmetric version of the dynamic model, most of the marginal budget shares are significantly different from zero at 5% level, however, the marginal budget share for the Transportation commodity group equation is not statistically significant at 5% level.

Moreover; there are only 6 out of 25 prices coefficient which are statistically significant at 5% level in the homogeneity model while there is a presence of serial auto-correlation in the Housing commodity group equation. In the case of the symmetry model there are only 4 out 15 price coefficients that are significant at 5% level, and there is a presence of serial auto-correlation in both Housing and
Transportation commodity group equations. It should be noted here that when there is a drop in the statistical results, the values of $R^2$ increase.

Table 6.10

<table>
<thead>
<tr>
<th>Commodity group</th>
<th>$\gamma_u$</th>
<th>$\beta_i$</th>
<th>$\alpha_i$</th>
<th>$\gamma_u$</th>
<th>$\beta_i$</th>
<th>$\alpha_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverages</td>
<td>0.006</td>
<td>-0.130</td>
<td>0.002</td>
<td>-0.008</td>
<td>-0.132</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.018)</td>
<td>(0.001)</td>
<td>(0.020)</td>
<td>(0.017)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.063</td>
<td>-0.053</td>
<td>0.001</td>
<td>0.020</td>
<td>-0.081</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.037)</td>
<td>(0.002)</td>
<td>(0.032)</td>
<td>(0.033)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Housing</td>
<td>0.041</td>
<td>-0.128</td>
<td>0.001</td>
<td>0.048</td>
<td>-0.151</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.079)</td>
<td>(0.002)</td>
<td>(0.021)</td>
<td>(0.055)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.061</td>
<td>-0.063</td>
<td>0.001</td>
<td>-0.078</td>
<td>-0.040</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.044)</td>
<td>(0.002)</td>
<td>(0.051)</td>
<td>(0.039)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Other</td>
<td>0.114</td>
<td>0.374</td>
<td>-0.004</td>
<td>0.216</td>
<td>0.404</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.130)</td>
<td>(0.100)</td>
<td>(0.004)</td>
<td>(0.094)</td>
<td>(0.077)</td>
<td>(0.003)</td>
</tr>
</tbody>
</table>

(*) $\gamma_u$ is own-price coefficient, $\beta_i$ is the marginal budget share, and $\alpha_i$ is a lagged consumption coefficient.

Demand elasticities represent very important economic indicators for the policy analysis. Based on the parameters estimated from the dynamic unrestricted AIDS, Table 6.11 presents the evaluated elasticities of demand with respect to total expenditure, together with respect to uncompensated and compensated own price elasticities.

The elasticities were evaluated at the mean values of the budget shares of each commodity group. Grounded on the evaluated elasticities, all commodity groups are inelastic with respect to total expenditure. However, it is found that the demand for the other commodity group is elastic. This indicates that all commodity groups are a necessity for Bahraini consumers except the other commodity group. This is indicated by the results of all versions of the dynamic LA/AIDS model. Thus, the other commodity group is classified as luxuries only in these applications of the dynamic AIDS. In all the versions results, as income increases, Bahraini
consumers tend to buy relatively more from other commodity group products and less from the rest of the four products (see Table 6.11)

Table 6.11
Total expenditure elasticities, compensated and uncompensated own price elasticities of the LA/AIDS- Dynamic model

<table>
<thead>
<tr>
<th>Commodity group (l)</th>
<th>Unrestricted version</th>
<th>Homogenous version</th>
<th>Symmetric version</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\eta$, $e_y$, $e_y^*$</td>
<td>$\eta$, $e_y$, $e_y^*$</td>
<td>$\eta$, $e_y$, $e_y^*$</td>
</tr>
<tr>
<td>Food &amp; beverages</td>
<td>0.276, -1.109, -1.047</td>
<td>0.235, -0.965, -0.925</td>
<td>0.224, -1.047, -1.009</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>0.286, -0.100, -0.080</td>
<td>0.243, -0.100, -0.083</td>
<td>-0.157, -0.714, -0.725</td>
</tr>
<tr>
<td>Housing</td>
<td>0.221, -0.614, -0.583</td>
<td>0.086, -0.707, -0.695</td>
<td>-0.386, -0.657, -0.668</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.213, -0.225, -0.208</td>
<td>0.213, -0.238, -0.221</td>
<td>0.500, -0.113, -0.073</td>
</tr>
<tr>
<td>Other</td>
<td>1.639, -0.591, 0.294</td>
<td>1.693, -0.789, 0.125</td>
<td>1.784, -0.600, 0.344</td>
</tr>
</tbody>
</table>

(*) - $\eta$, is expenditure elasticities. - $e_y^*$, and $e_y^*$, are uncompensated and compensated own price elasticities respectively.

The uncompensated own-price elasticities indicate that the demand for Food & Beverages and other commodity groups is elastic with respect to the price whereas the remaining categories appear to be price inelastic. This is true for the symmetric model. However, it is found that the demand for all commodity groups is inelastic in the homogenous model. In addition, all the uncompensated own price elasticities estimates are negative and consistent with prior expectations regarding their sign. On the other hand, the results of compensated own-price elasticities are negative for all commodity groups except the sign of own-price compensated elasticity for the Other commodity group which is found positive (see Table 6.11).

6.8 Testing for the Hypotheses of Demand Theory-Dynamic Model

Having presented the discussion above of the dynamic LA/AIDS model, it is now necessary to test for the consistency of the general restrictions implied by the theory. The testing for general restrictions will be on the basis of the log likelihood ratio test. The likelihood ratio test involves a comparison of the logarithmic likelihood values for the different models. Under the null hypothesis of the restricted model, twice the difference between the maximum logarithmic likelihood value for the unrestricted
model and that value for the restricted model is asymptotically distributed as a chi-square statistic with the number of degrees of freedom equal to the number of restrictions imposed. Table 6.12 shows the logarithmic likelihood values for the different models.

When the unrestricted model is compared to alternative models of dynamic AIDS, the results in Table 6.12 indicate that all types of restrictions should be accepted. The results of this study support Deaton's observations in which symmetry could be accepted when the maintained hypothesis included homogeneity. Deaton reached this conclusion from the application of the model to the British expenditure data.

<table>
<thead>
<tr>
<th>Type of model</th>
<th>Logarithmic Likelihood value</th>
<th>$-2 \ln \lambda$</th>
<th>d.f</th>
<th>Critical value at (0.05)</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconstrained</td>
<td>1026.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homogeneity</td>
<td>1023.7</td>
<td>5.6</td>
<td>4</td>
<td>9.49</td>
<td>Accepted</td>
</tr>
<tr>
<td>Symmetry</td>
<td>1021.5</td>
<td>10.0</td>
<td>6</td>
<td>12.59</td>
<td>Accepted</td>
</tr>
<tr>
<td>Homogeneity &amp; Symmetry</td>
<td>1019.8</td>
<td>13.4</td>
<td>10</td>
<td>18.31</td>
<td>Accepted</td>
</tr>
</tbody>
</table>

Log likelihood ratio is asymptotically distributed as $\chi^2$ with $h$ degrees of freedom, where $h$ is the number of restrictions imposed.

6.9 Negativity Restriction

The final restriction implied by utility maximization which the author wishes test is negativity. Utility maximization implies that the matrix of Slutsky coefficients be symmetric and negative semi-definite. In particular, negativity implies that the consumer's cost function be concave. A well-known result from linear algebra states that a necessary and sufficient condition for a real symmetric matrix to be negative semi-definite is that all the eigenvalues of that matrix should be less than or equal to zero (Johnston, 1984, p.151). This condition can be visually inspected for both static and dynamic demand systems.
Table (6.13) shows the eigenvalues for both systems. Three of the five eigenvalues are negative for the static LA/AIDS model, while two of five eigenvalues are negative for the DLA/AIDS model. In most cases those eigenvalues which are positive are quite small in size, suggesting that the matrices may not be that far from being negative semi-definite.

As proved from the application, all types of restrictions are not rejected as suggested by results which are reported in Table 6.12. The reasons for non rejection could imply that either the dynamic specification adopted in this chapter is appropriate or it is related to other reasons - such as flexible spending patterns in the short run, which led to accepting the demand theory principles. This means that the decisions of the consumer's behavior are not hard to observe in this context.

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>Static LA/AIDS model</th>
<th>Dynamic LA/AIDS model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverages</td>
<td>-0.028</td>
<td>-0.016</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>-0.062</td>
<td>-0.009</td>
</tr>
<tr>
<td>Housing</td>
<td>-0.051</td>
<td>0.047</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.007</td>
<td>0.012</td>
</tr>
<tr>
<td>Other</td>
<td>0.596</td>
<td>0.901</td>
</tr>
</tbody>
</table>

6.10 A Hypothesis Test of the Lagged Consumption Effects in the LA/AIDS

To test for the presence of lagged consumption effects in the LA/AIDS the likelihood ratio test procedure was used. It can be shown that minus two times the logarithm of the ratio or the restricted to the unrestricted likelihood function \((-2 \ln \lambda)\) has asymptotic chi-square distributed with a degree of freedom equal to the number of linearly independent restrictions. The computed value of \(-2 \ln \lambda\) was equal to 149.7 when testing the hypothesis: \(H_0 : \Gamma = 0\) This value is greater than the critical value of \(\chi^2_{0.01,4} = 9.49\) which suggests the presence of lagged consumption (habit) effects.
6.11 Conclusion

Consumer demand analysis is carried out by the Almost Ideal Demand System
developed by Deaton and Muellbauer (1980) and its dynamic version proposed by
Blancifoti and Green (1983) to Bahrain expenditure data. The aim of the application
is to acquire an explanation of the movement in the distribution of consumer's
expenditure in Bahrain. The application will be relate to Bahrain personal
expenditure data for the period 1979Q1-1998Q4. A five-item classification of
consumer expenditure is used, namely, Food & Beverages, Clothing & Footwear,
Housing, Transportation, and Other commodity group.

The principle conclusion for the static model LA/AIDS are presented as follows:

- The estimates of $\beta_i$ has classified the demand for Food & Beverages products,
  Clothing & Footwear, Housing and Transportation commodity group as
  necessities while the demand for Other commodity groups is classified as
  luxuries.

- The results of this application indicate that 7 out of 35 parameter estimates are
  significantly different from zero at the 5% level of significance, as indicated by
  the t- values. Furthermore, 3 out of 25 of the price coefficients have t-values
  absolutely greater than two. The latter is true only for the budget shares of Food
  & Beverages and Other commodity groups.

- With respect to the coefficients of determination, $R^2$ suggest that the overall fit
  is not sensible for all equations in the system. $R^2$ ranges between 33% for the
  Housing commodity group to 74% for Food and Beverages commodity group.

- The $d.w$ results suggest that there is serious auto-correlation in the system and
  are not fitted as whole.

- Homogeneity is accepted for most of the commodity groups except for the
  Food and Beverages commodity group.

- When the restrictions (homogeneity and symmetric) are imposed the results are
  found to be worse than the model within the unrestricted static model and
  particularly for homogenous model.
• Imposition of homogeneity did not lead to significant changes in expenditure elasticities as the symmetry imposition results indicate. However, there is variability in the magnitude of own price and expenditure.

• Also, the imposition of restrictions produced the same results in terms of the classification of the demand for the products between necessities and luxury products.

• Furthermore, there was a drop in the log likelihood values when the comparison was made between the restricted and unrestricted model.

• Finally, imposition of restrictions implied by the demand theory is rejected as suggested by the hypothesis tests.

However, the results of the dynamic LA\AIDS indicate the following results:

• Overall performance of these applications indicated that the unrestricted dynamics of AIDS is preferred among the alternative versions of the dynamic LA\AIDS. This is due primarily to most estimated parameters being more significant within the unrestricted model. Secondly, as a result of acquiring sensible statistical results for unrestricted dynamic model.

• The dynamic parameters are all significant and point to dynamic LA\AIDS representing a significant generalization of its static version. This is confirmed by a huge drop in likelihood on the imposition of the restriction that all the dynamic adjustment parameters are zero.

• Statistically, the dynamic model is clearly supported by the data. There are rough absences of auto-correlation in all equations of dynamic model as indicated by the values of the D.W statistics. The $R^2$ values are improved in the DLA\AIDS model, which indicates that all equations are better, fitted than the static LA\AIDS model.

• Based on the evaluated elasticities most commodities groups are inelastic with respect to total expenditure. This indicates that all commodities appear to be a necessity for the Bahrain consumer except the other commodity group.
The uncompensated own-price elasticities indicate that the demand for the Food & Beverages commodity group is elastic with respect to the price in free and symmetric versions, whereas all other categories appear to be price inelastic especially under the homogeneity restriction.

All the uncompensated and compensated own price elasticities estimates are negative and consistent with a prior expectation regarding their signs. On the other hand, the results of compensated own-price elasticities are negative for most commodity groups except the sign of own-price compensated elasticity for other commodity group which is found positive.

Testing for the hypothesis for the consistency of the general restrictions implied by the theory the test has shown that both homogeneity and homogeneity and symmetry restrictions should not be rejected at the 5 percent level of significance. The test has also indicated that the symmetry over no restriction is not rejected at 5% level of significant. The testing for general restrictions was based on the likelihood ratio test.

The final restriction implied by utility maximization is negativity restriction. The results of the study suggested that the matrix of Slutsky might not be that far from being negative semi-definite.

Finally, a hypothesis test of the habit effects in the LA/AIDS model suggest the presence of lagged consumption effects on consumer behavior expenditure, the result of which is based on likelihood ratio test procedure.

Thus it seems that the acceptance of the demand theory in the aggregate data in LA/AIDS model which is caused by either the dynamic specification adopted in this chapter is appropriate or could also be also related to other reasons such as flexible spending. Therefore, the study suggests further work is needed on other issues such as the properties of time series. The analysis and the discussions of these issues will be presented in Chapter 8 utilizing the same sets of the data and the AIDS model.
References

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Chapter 7

Systems of Demand Equations: An Empirical Comparison of Alternative Specifications

7.1 Introduction.

A variety of complete systems of demand equations have been introduced for econometric utilization. Most of these specifications rest on the neo-classical demand theory, and most of them introduce restrictions so as to simplify the model and reduce the number of parameters. Theoretically none of the models dominate its competitors. Therefore, the choice of form for empirical work must be made on empirical grounds. The question is which model performs best and reflects the data of the economy? Which model should the demand analyst select as a basis for her or his policy recommendations?

In applied demand analysis one of the important decisions to be made is the selection of the model. The appropriate choice of model depends upon the purpose of the model. There are many models which can be used to examine theoretical demand restrictions, such as the AIDS model and a likelihood ratio technique can be used to test for the validity of demand restrictions. However, using the non-flexible form such as LES does not help in testing the demand restrictions because the homogeneity and Slutsky symmetry conditions hold automatically for LES model. The selection between the dynamic and static model depends on the type of commodity. For example, to investigate the demand for durable goods the dynamic model rather than the static model need to be used. The latter choice depends on the type of data used if the length of the run is very close, i.e. weekly or monthly rather than annually. It is likely that
habit or inventory effects will be present even for non-durables. In this case, the dynamic model would be more appropriate than the static model (Green et al. (1995, p.486)). However, in this study, which model performs best and reflects the Bahrain data well among available alternative models has been investigated. Different criteria have been introduced to enable a choice to be made among the various models, such as the Akaike Criterion (AIC), the Schwartz' Criterion (SC) and recently, Pollak and Wales (1991) who introduced the Likelihood Dominance Criterion (LDC) as a new method of model selection. This chapter will use all of these procedures in model selection.

This chapter considers the six popular demand system equations which have been discussed in the previous chapters, namely: The linear expenditure system (LES), the dynamic version of the linear expenditure system (DLES), the almost ideal demand system (AIDS), the dynamic version of the almost ideal demand system (DAIDS), the usual Rotterdam model (RM) and the dynamic version of the usual Rotterdam model (DRM). A comparison of the models is provided in terms of their empirical usefulness, basing the latter comparison on the same set of data which is used in this study. This chapter starts with the various procedures used for model selection, followed by the demand systems. The empirical results are then reported which contain elasticity estimates for various demand system models as well as the results from the model selection procedures, and conclusion are drawn.

7.2 Model Selection Criteria

There are many approaches to model selection. Recently, Pollak and Wales (1991) developed a new approach for model selection which is called the Likelihood Dominance Criterion (LDC). It is based on the differences of likelihood. It is designed to compare two models according to some criterion without the need to construct the composite model. If is difficult to estimate the composite model because of the large number of parameters which have to be
estimated. This is usually associated with a high degree of multicollinearity. Pollak and Wales (1991) provide a new approach to model selection, which is consistent with the classical statistical approach to hypothesis testing. Pollak and Wales (1991, p.236) reported that the LDC would be decisive for model selection. The LDC is presented below:

**Likelihood Dominance Criterion**

1. The LDC prefers $H_1$ to $H_2$ if $L_2 - L_1 < \frac{[C(n_2 + 1) - C(n_1 + 1)]}{2}$
2. The LDC prefers $H_2$ to $H_1$ if $L_2 - L_1 > \frac{[C(n_1 - n_2 + 1) - C(1)]}{2}$
3. The LDC is indecisive between $H_1$ and $H_2$ if $\frac{[C(n_2 - n_1 + 1) - C(1)]}{2} < L_2 - L_1 > \frac{[C(n_1 + 1) - C(n_2 + 1)]}{2}$

where $n_1$ and $n_2$ are the number of parameters in $H_1$ and $H_2$ respectively, and the $C$ (chi-square distribution) is evaluated at the 5% level of significance.

There are several other criteria has which to select models. Akaike’s (1973) information criterion and Schwarz’s (1978) criterion are simple model selection rules depend only on the log-likelihood values ($L_i$), the number of parameters and, in one case, the number of observations ($T$). Akaike’s (1973) information criterion selects the model with the largest value of $L_i - n_i$. Schwarz’s (1978) criterion which selects the model with the largest value of $L_i - n_i (\frac{1}{2} \log T)$

Pollak and Wales (1991) rewrote the Akaike rule, by choosing $H_2$ over $H_1$, if and only if $L_2 - L_1 > n_2 - n_1$ (Pollak and Wales, 1991, p.239). Pollak and Wales obtained the same conclusion when the LDC was applied. The only information that is needed for model selection is the value of the log likelihood and the number of parameters associated with each model. Pairwise comparisons using the LDC require reference to the chi-square Table.
In Pollak and Wales (1991) rewriting Schwarz criterion gives that: $H_2$ is chosen over $H_1$ if and only if $L_2 - L_1 > n_2 - n_1 \left( \frac{1}{2} \log T \right)$ (Pollak and Wales, 1991, p.239). However, all the model selection procedures presented above require that the models in question have the same dependent variable. It is difficult to apply these procedures since the three main models, which have been estimated in this study (LES, LA\AIDS, and RM), have different dependent variables. It could be applied only in cases where the comparison is made between the static and dynamic version (i.e. - LES and DLES) because both models have the same dependent variable. Accordingly, the applications of Akaike and Schwarz model selection criteria, along with LDC, will be used only in the case where the dependent variables are the same.

Barten (1993, p.143) indicated that the average information inaccuracy has been widely used where the dependent variables are not the same for different demand system model. The concept of the average information inaccuracy was introduced by Theil (1967). Parks (1969) made the first attempt at an empirical comparison of various demand systems. Later it was used by Klevmarken (1979) and by Barten (1989). Basically it compares the budget shares generated by the estimated system with the actual ones. Taking the (weighted) average over the equations and over the sample, a relatively high average information inaccuracy is taken to be an indicator of less satisfactory behavior. An average information inaccuracy can be defined as:

$$I = \frac{1}{T} \sum_{i} w_i \log \left( w_i / \hat{w}_i \right)$$

(7.1)

where $w_i$, $\hat{w}_i$, and $T$ are observed budget share, estimated budget share, and the number of observations respectively. Accordingly, in the first step, this procedure will be applied to select one of the three main static demand systems.

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1 The model which has fewer parameters is illustrated by $H_1$, while the model which has more parameters is illustrated by $H_2$.\n
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that best reflects the Bahrain data. These models are: the Linear Expenditure System, the Linear Almost Ideal Demand System and the Rotterdam model. In the second step, the application of the three procedures (Akaike, Schwarz, along with LDC) will take place in order to select the static or the dynamic system.

7.3 Demand System Models

In this chapter, six demand systems are considered. The six models considered are the linear expenditure system (LES), the dynamic version linear expenditure system (DLES), the almost ideal demand system (AIDS), the dynamic version of the almost ideal demand system (DAIDS), Rotterdam model (RM), and the dynamic version of the Rotterdam model (DRM). A brief description of each demand system can be found in chapters five, six, and seven of this study.

The LES is an additive system and is given by:

$$ p_iq_i = p_iy_i + B_i(x - \sum_{k=1}^{n} p_ky_k) \quad i = 1, 2, \ldots, n $$

(7.2)

where $$ q_i > y_i > 0, \quad 0 < B_i < 1, \quad \text{and} \quad \sum_{i} B_i = 1 $$ normalization restriction.

Equation 7.2 describes the expenditure the i-th commodity, as a linear function of income ($x$) and the $n$ prices ($p$). An interpretation of Equation 7.2 as suggested by Samuelson (1947-48) is as follows: At the beginning of the period, the consumer purchases $y_i$ units of i-th commodity, this is the minimum purchased required of the commodity at each period without regard to prices. For all $n$ commodities this amount to $\sum p_kq_k$. The remaining amount of income after that expenditure is $x - \sum_k p_kq_k$ which is called supernumerary expenditure or income and is allocated among the $n$ commodities in fixed proportions $B_i(B_1, B_2, \ldots, B_n)$.

The system is non-flexible and homogenous and Slutsky symmetry conditions hold automatically. Thus, these theoretical restrictions cannot be tested.
However, the LES does exactly aggregate over consumers and may be a viable model for a highly aggregated group of commodities.

In recent years there has been continuing attention given to the role of habit formation in consumer’s allocation decisions. This can be seen by the inclusion of dynamics factors in direct demand systems, for example studies by Poland Wales (1969), Pollak (1970), Philips (1972), Manser (1976), Blanciforti and Green (1983), Heien and Durham (1991), Alessic and Kapteyn (1991), and recently by Matthew et al. (1997).

Pollak (1970) developed the static LES demand system model of consumer behavior to study the impact of habit formation. The main assumption is that the necessary quantity of a commodity, $\gamma_i$, depends upon previous consumption of that commodity. Accordingly, it is implied that the necessary quantity of each commodity is a linear function of the past period consumption level of that commodity.

The formulation for that necessary quantity can be defined as:

$$\gamma_u = \gamma^*_u + \Gamma_i q_{n-1}, \quad \text{(for } i=1, 2, \ldots, n)$$

(7.3)

where, $\gamma^*_u$ was defined as the "physiologically necessary" component and $\Gamma_i q_{n-1}$ was defined as the "psychologically necessary" component of $\gamma_u$. Accordingly, by substituting Equation 7.3 into Equation 7.2 and then maximization subject to the budget constraint yields the dynamic version of the LES.

The Dynamic version of the LES is

$$p_u q_u = p_u (\gamma^*_u + \Gamma_i q_{n-1}) + \beta_i \{ (x_i - \sum_k p_u (\gamma^*_k + \Gamma_k q_{n-1})) + u_i \}$$

(7.4)

Thus, Pollak's hypothesis implies that past consumption influenced current preferences and current demand. In Equation 7.4 $p_u (\gamma^*_u + \Gamma_i q_{n-1})$ is now committed expenditure on commodity (i), and $p_u \Gamma_i q_{n-1}$ is the part of the
minimum required expenditure consequent from the purchase in the current time period at the current market price of that similar amount of the commodity that had been purchased in the past period, \( R_i q_{u-1} \). Following Samuelson's (1947-48) argument that the amount that involves, for all \( n \) commodities together is
\[
\sum_k P_k (\gamma_k^* + R_k q_{u-1})
\]
while the remaining amount of income after that expenditure is
\[
(x_i - \sum_k P_k (\gamma_k^* + R_k q_{u-1}))
\]
which is called supernumerary expenditure or income, and is allocated among the \( n \) commodities in fixed proportions, \( B_i \).

The linear approximate almost ideal demand system (LA/AIDS) is given by
\[
w_i = a_i^* + \sum_j \gamma_j^* \log p_j + B_i \log \left( \frac{x_i}{p} \right)
\]
(7.5)
where \( \log p^* \) is Tornqvist's price index and will be used to replace the true priced index of Deaton and Muellbauer (1980). The Tornqvist's price index is given by
\[
\ln p_t = \frac{1}{2} \sum_{i=1}^{n} (w_i + w_i^0) \ln \left( \frac{p_{it}}{p_{it}^0} \right)
\]
(7.6)
The primary reason as to why this system is frequently used is related to the ease of estimation. However, the usual homogeneity conditions, \( \sum_j \gamma_j = 0 \), and the Slutsky restriction, \( \gamma_j = \gamma_j^* \), associated with the AIDS, are often employed in empirical applications.

However, the AIDS of Deaton and Muellbauer (1980) is defined as
\[
w_i = a_i + \sum_j \gamma_j \log p_j + B_i \log \left( \frac{x_i}{p} \right)
\]
(7.7)
where \( \log p = a_0 + \sum_k a_k \log p_k + \frac{1}{2} \sum_k \sum_j \gamma_j^* \log p_k \log p_j \).

The AIDS is theoretically plausible and can be used to test the theoretical restrictions of homogeneity and symmetry, and possesses other desirable
properties. One drawback with the AIDS is the difficulty of estimating the large number of parameters needed if more than a few commodities are considered. In order to allow for short run dynamic effects, the Blanciforti and Green (1983) model's of the dynamic version of the AIDS will be used.

Blanciforti and Green (1983) established the dynamic version of the AIDS incorporating habit effects of the form of Pollak and Wales (1969). Blanciforti and Green (1983) redefine the parameters \( \alpha_i \) in the static AIDS model as:

\[
\alpha_i = \alpha_i^* + \alpha_i^{**} x_{i,t-1}
\]

(7.8)

where \( x_{i,t-1} \) denotes lagged purchases (quantity) of item \( i \). It is worth noting that \( \alpha_i^{**} \)'s allowed dependence of the original AIDS parameters on the past consumption of only own lagged consumption of item \( i \), via the lagged purchase \( x_{i,t-1} \) of commodity \( i \)-th. The corresponding dynamic system of Blanciforti and Green (1983) that include simple linear habit effects is then given by:

\[
w_i = \alpha_i^* + \alpha_i^{**} x_{i,t-1} + \sum_j \gamma_j \log p_{j\mu} + \beta \log\left(\frac{x_i}{p_i}\right) + u_i,
\]

(7.9)

where

\[
\log p_i = a_0 + \sum_i (\alpha_i^* + \alpha_i^{**} x_{i,t-1}) \log p_{i\mu} + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \log p_{i\mu} \log p_{j\mu}
\]

Blanciforti and Green (1983) did not impose adding up on the \( \alpha_i^{**} \) parameters. This is because they considered only lagged own-quantity effects in each individual share equation. Alessic and Kapteyn (1991) argue that it is not possible to impose adding up in such a system without restricting all \( \alpha_i^{**} \)'s to equal zero. In addition, the dynamic model of Equation 7.9 was estimated using the (ISUR) method.

The Rotterdam Differential Demand Model, developed by Barten (1964) and Theil (1965), is based not on a particular utility function but more generally, on a
first-order approximation to the demand functions themselves. Barten (1967) defined the Rotterdam Model by the following form:

\[ w_t d \log q_t = B_t (d \log x - \sum_k w_k d \log p_k) + \sum_j k_j d \log p_j \]  

(7.10)

Equation 7.10 is formulated in terms of infinitesimal changes, the Rotterdam model, however is a finite change version of (7.10) which can be written for empirical purposes as (Theil et al. 1987).

\[ w_t^* Dq_t = B_t DQ_t + \sum_j y_j Dp_j + u_t \]  

(7.11)

where:
- \( w_t^* = (w_t + w_{t-1}) / 2 \), \( D \) denotes the log difference operator,
- \( Dq_t = \log q_t - \log q_{t-1} \)
- \( DQ_t = \sum_i w_i D \log q_i \approx (d \log x - \sum_i w_i d \log p_i) \) is a finite change version of the Divisia volume index, (Theil and Clements 1987, p.25).
- \( B_t \) are the marginal budget shares, while, \( y_j \) are the parameters of the Slutsky matrix.
- \( u_t \) is a disturbance term with the following assumptions: \( E[u_{it}] = 0 \), \( E[u_{it}, u_{it'}] = 0 \), for \( t \neq t' \) and \( E[u_{it}, u_{jt}] = \omega_{ij} \) for \( t=t' \) (i,j=1,...,n).

It is assumed that the random disturbances are un-correlated across observations, but are correlated across equations for the same observations. The contemporaneous covariance matrix \( U = [\omega_{ij}] \) is singular. In the light of the equality

\[ \sum_i w_t^* Dq_i = DQ_t \]  

(7.12)

is a value weighted average of the logarithmic differences of the quantities demanded. It is a volume index of the change in total consumption and can be interpreted for our purpose as a measure of the change in real income.

\[2\] For the derivation of the Rotterdam model see Page 151-152.
Amongst the attractive features of the Rotterdam model is the relative ease of imposition and testing of such restrictions as aggregation, homogeneity and symmetry. The adding up restriction implies $\sum \beta_i = 1$, $\sum \gamma_i = 0$.

Homogeneity implies $\sum \gamma_i = 0$, while symmetry implies $\gamma_i = \gamma_j$ $(i \neq j)$.

Unlike the AIDS models, negativity can be imposed by the condition that the matrix $[\gamma_y]$ be negative ($\gamma_y < 0$) semi-definite of rank $n - 1$, where $n$ is the number of commodity.

Brown and Lee (1992) developed the usual Rotterdam demand system by extending the Rotterdam model to include lagged consumption throughout translation parameters, allowing for habit and inventory effects. They stated that the demand impact of lagged consumption could be specified through the translation parameters. The effect of the past consumption is introduced into the model by letting the translation term $z_i$ depend on lagged consumption. They hypothesized that:

$$z_i = w_{i-1} d \log q_{i-1}$$

Then the Rotterdam model with translation is:

$$w_i d \log q_i = z_i - d_i \sum_j z_j + \beta_i d \log Q_i + \sum_j \gamma_y d \log p_j,$$

where $z_i = \frac{p_i \gamma_i}{\chi} d \log \gamma$, the log change in the translation parameters $(\gamma_i)$ weighted by the share of total expenditure committed to the good, $p_i \gamma_i / \chi$. The left-hand side variables in Equation 7.14 can be viewed as the percentage change in demand weighted by the budget share. The difference between the usual Rotterdam model and Brown and Lee (1992) specification is the first two terms on the right hand side of Equation 7.14, which involve changes in the translation parameters. The first term $(z_i)$ is a direct effect due to a change in the translation parameter for the good in question, while the second term $d_i \sum_j z_j$ is an indirect
income effect due to a change in supernumerary income caused by the over all charge in the translation parameters. Changes in the translation parameters can be viewed as preference changes, and the resultant direct \((z_i)\) and indirect \((\sum_j z_j)\) effects, which lead to a re-allocation of income.

The effects of past consumption can be introduced into the model by letting the translation term \(z_t\) depend on the lagged consumption, as indicated by Equation 7.13. Equation 7.13 states that the weighted log change in the translation parameters equals a constant times the weighted log change in lagged consumption. However, Equation 7.14 is not based on separability assumptions with attention focused on the conditional demand equations for separable groups of goods (Brown and Lee, p.2). In Equation 7.13 the parameter \(a\) is normally expected to be negative (positive) when inventory (habit) effects dominate. The shorter the time interval of an observation, the more likely inventory effects are to dominate habit effects.

Equation 7.14 is applied to the Bahrain quarterly data in this study using the dynamic Rotterdam model.

7.4 Empirical Results
Bahrain quarterly data from 1979-1998 are employed, which has a total of 80 observations, for Food & Beverages, Clothing & Footwear, Housing, Transportation, and Other. The data are used for the estimation and model selection procedures. The sources of the data and method of collecting have been discussed in Chapter 3. This section reports the empirical results which contain both the elasticity estimates for various models and the results from model selection procedures.
7.4.1 Elasticities.

Own price and total expenditure elasticities are presented in Table 7.1 for each of the six models. In every case, except for the Transportation commodity group in the Usual Dynamic Rotterdam model, the own price elasticities are negative. The largest own price elasticities in absolute value are for Clothing and Footwear with values ranging from -0.088 for the DLES model to -1.826 for the dynamic Rotterdam Model. For the remaining commodities (with the exception of Food & Beverages in the dynamic LA/AIDS model, Housing in the Usual Dynamic Rotterdam Model, and Clothing & Footwear in the Dynamic Rotterdam Model), all the own price elasticity estimates are less than one in absolute value implying own-price inelastic goods.

Table 7.1
Estimated own-price elasticities according to five commodity groups and type of the demand system model

<table>
<thead>
<tr>
<th>Demand System Models</th>
<th>Food &amp; beverage</th>
<th>Clothing &amp; footwear</th>
<th>Housing</th>
<th>Transportation</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Linear Expenditure System</td>
<td>-0.402</td>
<td>-0.139</td>
<td>-0.851</td>
<td>-0.865</td>
<td>-0.665</td>
</tr>
<tr>
<td>Dynamic Linear Expenditure System</td>
<td>-0.216</td>
<td>-0.088</td>
<td>-0.419</td>
<td>-0.024</td>
<td>-0.428</td>
</tr>
<tr>
<td>Static Linear Almost Ideal Demand System</td>
<td>-0.771</td>
<td>-0.329</td>
<td>-0.793</td>
<td>-0.538</td>
<td>-0.933</td>
</tr>
<tr>
<td>Dynamic Linear Almost Ideal Demand System</td>
<td>-1.109</td>
<td>-0.100</td>
<td>-0.614</td>
<td>-0.225</td>
<td>-0.591</td>
</tr>
<tr>
<td>Usual Rotterdam Model</td>
<td>-0.484</td>
<td>-0.630</td>
<td>-1.070</td>
<td>0.961</td>
<td>-0.113</td>
</tr>
<tr>
<td>Dynamic Rotterdam Model</td>
<td>-0.376</td>
<td>-1.826</td>
<td>-1.055</td>
<td>0.918</td>
<td>-0.319</td>
</tr>
</tbody>
</table>

The values of the own price elasticities vary substantially across models. For Food & Beverages the range is from -0.216 to -1.109, for Housing it is -0.419 to -1.070, for Transportation the values range from -0.024 to -0.865 and for Other commodity groups the values range from -0.113 to -0.933.

With respect to the total expenditure elasticity estimates, for all commodities and across all models, the estimated total expenditure elasticities are positive (see Table 7.2). The estimated Food & Beverage total expenditure elasticity has the largest value for the usual Rotterdam Model. 16 out of 30 of the total
expenditure elasticity estimates for the remaining goods are less than one. As with the own price elasticity estimates, the values of the total expenditure estimates vary widely across models. For Food & Beverages, the range is from 0.253 to 2.988. For Clothing & Footwear, the range is from 0.114 for the LES model to 2.471 for the Usual Rotterdam Model. For Housing, the values range from 0.221 to 2.198. For the Transportation commodity group, the values range from 0.063 to 2.213. Finally for Other commodity group, the values range from 0.093 for Usual Rotterdam Model to 1.639 for Dynamic LA/AIDS model. Results of elasticities in Table 7.2 reveal that large differences exist in magnitudes of elasticity estimates due to different model specifications.

From Table 7.2 the results of static linear expenditure system suggest that Housing, Transportation and Other commodity groups are luxury products, while the Food & beverage, and Clothing & footwear are necessities (as income increases, Bahrain consumers tend to spend relatively more on Housing, Transportation and Other commodity groups products and less on the Food & beverage and Clothing products). According to this model, Bahrain is a possible for the Housing, Transportation and Other commodity groups markets. However, the results of the dynamic linear expenditure system gave different results. This model suggests that as income increases, Bahrain consumers tend to spend relatively more on Food & beverage and Housing, commodity groups products and less on Clothing & footwear, Transportation and Other commodity groups products. The linear expenditure system used the total expenditure (income) level and prices level as independent variables, while the expenditure on commodity (i) as dependent variable.
Table 7.2
Estimated total expenditure elasticities according to five commodity groups and type of the demand system model.

<table>
<thead>
<tr>
<th>Demand system models</th>
<th>Food &amp; beverages</th>
<th>Clothing &amp; footwear</th>
<th>Housing</th>
<th>Transportation</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static linear expenditure system</td>
<td>0.465</td>
<td>0.114</td>
<td>1.486</td>
<td>1.563</td>
<td>1.074</td>
</tr>
<tr>
<td>Dynamic Linear expenditure system</td>
<td>1.059</td>
<td>0.886</td>
<td>2.193</td>
<td>0.438</td>
<td>0.770</td>
</tr>
<tr>
<td>Static Linear Almost Ideal Demand System</td>
<td>0.253</td>
<td>0.529</td>
<td>0.357</td>
<td>0.063</td>
<td>1.602</td>
</tr>
<tr>
<td>Dynamic Linear Almost Ideal demand system</td>
<td>0.276</td>
<td>0.286</td>
<td>0.221</td>
<td>0.213</td>
<td>1.639</td>
</tr>
<tr>
<td>Usual Rotterdam Model *</td>
<td>2.988</td>
<td>2.471</td>
<td>0.650</td>
<td>2.213</td>
<td>0.093</td>
</tr>
<tr>
<td>Dynamic Rotterdam Model *</td>
<td>2.424</td>
<td>1.800</td>
<td>1.157</td>
<td>1.650</td>
<td>0.311</td>
</tr>
<tr>
<td>Usual Rotterdam Model **</td>
<td>0.047</td>
<td>0.443</td>
<td>1.329</td>
<td>0.563</td>
<td>1.352</td>
</tr>
<tr>
<td>Dynamic Rotterdam Model **</td>
<td>0.094</td>
<td>0.243</td>
<td>1.207</td>
<td>0.525</td>
<td>1.400</td>
</tr>
</tbody>
</table>

- * The results of the Rotterdam model using the volume index of the change in total consumption \(DQ = \sum w_i D \log q_i\) as independent variable.
- ** The results of the Rotterdam model using the change in the log of the real income \(d \log x - \sum w_i d \log p_i\) as independent variable.

The static and the dynamic linear almost ideal demand system provided similar results. Both models suggest that Food & beverage, Clothing & footwear, Housing and Transportation are necessities, while the “Other” commodity group is luxury products. As income increases, Bahrain consumer tend to spend less on Food & beverage, Clothing & footwear, Housing and Transportation commodity group and relatively more on “Other” products. The linear almost ideal demand system used the log real income level and the log prices level as independent variable, while the budget share of commodity (i) as dependent variable.

Table 7.2 also indicates that size of the expenditure elasticities for Food & beverage and Clothing and footwear are very big for the Rotterdam model than those obtained from the linear expenditure system and from the linear almost ideal demand system. The main reason for obtaining this results is due to the using the volume index of the change in total consumption as independent variable in the Rotterdam model rather than using the change in the log real income (due to the assumption in the Rotterdam model that the volume index of the change in total consumption can be interpreted as a measure of the change in the log real income \(\sum w_i D \log q_i \approx (d \log x - \sum w_i d \log p_i)\).
To see the differences, the Rotterdam model re-estimated by using the change in the log real income as independent variable in the Rotterdam model. The results of total expenditure ($\eta_i$) elasticity of these estimation for both the usual and dynamic Rotterdam model indicate that Food & beverage, Clothing & footwear, and Transportation commodity group are necessities product, while the Housing and Other commodity groups are luxury products. According to the results of these estimations of the Rotterdam model, this demand system model is not fitting to the Bahrain data quarterly times series data for two main reasons: First due to the two versions of the Rotterdam models (Equation 5.6 and 5.7 of Chapter 5) producing different results in the term of total expenditure and price elasticities which supposed that the two versions of this model give the same results. Secondly based on the statistical consideration which indicates that the low fitted of equations and most of parameters of equation are not statistically significant (see Table 5.4).

Finally, the assumption that the volume index of the change in total consumption can be considered as a measure of the change in log real income in the Rotterdam ($\sum w_u d \log q_u \approx (d \log x_i - \sum w_i d \log p_i)$), is not true in the case of Bahrain quarterly time series data, because using the change in the log real income as independent variable gave totally different results (in the term of expenditure and price elasticities) than those obtained from using the volume index of the change in total consumption as independent variable in Rotterdam demand system model.

7.4.2 Model Selection.

Based on the average information inaccuracy, the static LA/AIDS model was preferred over both the static Linear Expenditure system and Usual Rotterdam Model. The average information inaccuracy for the LA/AIDS is 0.0022, while for the Linear Expenditure System and Rotterdam Model is 0.1295 and 0.3181 respectively. A relatively low average information inaccuracy is taken to be an
indicator of highly satisfactory behavior. The selection of LA/AIDS model is due to its having the smallest information inaccuracy for the entire sample period. Thus, the LA/AIDS model dominates all of the other models on the basis of the average information inaccuracy.

The log Likelihood values, and with Difference in the log Likelihood values for the dynamic and static versions, for various demand system models, and that needed for the application of Akaike, Schwarz's and LDC criterions are given in Table 7.3.

<table>
<thead>
<tr>
<th>Demand system models</th>
<th>log Likelihood values</th>
<th>Difference in log Likelihood's values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic Linear Almost Ideal Demand System</td>
<td>1026.5</td>
<td>-</td>
</tr>
<tr>
<td>Static Linear Almost Ideal Demand System</td>
<td>960.44</td>
<td>72.4</td>
</tr>
<tr>
<td>Dynamic Rotterdam Model versus:</td>
<td>858.5</td>
<td>-</td>
</tr>
<tr>
<td>Usual Rotterdam Model</td>
<td>849.9</td>
<td>8.6</td>
</tr>
<tr>
<td>Dynamic Linear Expenditure System Model versus:</td>
<td>-961.4</td>
<td>-</td>
</tr>
<tr>
<td>Static Linear Expenditure System Model</td>
<td>-1092.0</td>
<td>130.6</td>
</tr>
</tbody>
</table>

With respect to model selection, the Akaike criterion selects the Dynamic Linear Almost Ideal Demand System in preference to the Linear Almost Ideal Demand System. As the difference in the log Likelihood value is greater bigger than the difference in the number of the parameters of the models in question, the Dynamic Rotterdam Model is preferred over the Usual Rotterdam Model. Finally, the test indicates that the dynamic linear expenditure system is preferred to the static linear expenditure system.

Schwarz's criterion, the same results were provided as obtained with Akaike's criterion. However, in this case it needs the Difference in the Likelihood's to be
The Likelihood Dominance Criterion (LDC) suggests that the dynamic model is preferable to its static version for each of the three main demand systems. -i.e. It prefers the Dynamic Almost Ideal Demand System to its static Linear Almost Ideal Demand System. The Difference in the Likelihood is greater than the critical 5% i.e value of 3.61. Thus the Likelihood Dominance (LDC) and the other two criteria which are Akaike's and Schwarz's, same results are provide. It could be said here that these criteria prefer a model with more parameters rather a model with few parameters.

Green et al. (1995, p. 497) applied the two procedures for model selection using Canadian quarterly data from 1947 to 1987, for durables, semi-durables, services and non-durables. The main conclusion for model selection procedures is that the Likelihood Dominance Criteria (LDC) yields the same rankings of the demand systems as the other two criteria (Akaike's and Schwarz's).

Again all the dynamic demand systems are tested against their static versions, but in this case, the test is based on the log likelihood ratio statistic \((-2\ln L)\). Under the null hypothesis, the LR statistic is distributed asymptotically as chi-square, with the number of degrees of freedom equal to the number of restrictions to be tested. As can be seen from Table 7.4 both the dynamic forms of the LA/AIDS and LES models performed significantly better than their static versions (LES and LA/AIDS models). However, the dynamic Rotterdam Model did not perform significantly better than its static counterpart (Usual Rotterdam Model).

Based on the model selection procedures and on the statistically consideration, and due to the acceptance all the restrictions implied by demand theory (homogeneity, symmetry, symmetry given homogeneity), the dynamic linear

---

3 where \( T \) is the number of observations. In our case it is 80 observations. \( n \) is the number of parameters for the model in question. It is 28, 24, and 24 for static LA/AIDS, static LES, and Usual Rotterdam model respectively, while for the Dynamic models of these previously static model are 32, 28, and 28 respectively.
The expenditure demand system model (LA\AIDS) is selected as best among other demand system models.

Table 7.4

<table>
<thead>
<tr>
<th>Demand System Models</th>
<th>(-2\ln \lambda)</th>
<th>d.f</th>
<th>(\chi^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Dynamic Linear Expenditure System versus:</td>
<td>144.8</td>
<td>12</td>
<td>21.03</td>
</tr>
<tr>
<td>-Static Linear Expenditure System</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-Dynamic Linear Almost Ideal Demand System</td>
<td>149.7</td>
<td>32</td>
<td>55.76</td>
</tr>
<tr>
<td>versus: Static Linear Almost Ideal Demand System</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-Dynamic Rotterdam Model versus:</td>
<td>17.14</td>
<td>28</td>
<td>41.34</td>
</tr>
<tr>
<td>-Usual Rotterdam Model</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7.5 Conclusion

The conclusions in this chapter are based on a highly aggregated commodity classification consisting of Food & Beverages, Clothing & Footwear, Housing, Transportation, and Other commodity group; using a quarterly time series data set for the period (1979Q1-1998Q4). The main conclusions of this chapter are as follows: First, the static Linear Almost Ideal Demand System (LA\AIDS) is selected over other static demand system models, namely, the Linear Expenditure System (LES) and the Rotterdam Model (RM). The selection was based on the average information criterion. Second, based on Akaike's, Schwarz's and LDC criteria, it is found that all the dynamic models performed better statistically than did their static counterpart.

Based on the Likelihood ratio statistic, both the dynamic LES and the LA\AIDS model performed significantly better than their static counterparts. However, the Dynamic Rotterdam model's did not perform better than its static version (Usual Rotterdam model). The dynamic LA\AIDS performed better and was therefore selected over its static counterpart. However, the static LA\AIDS is preferred to
the other static demand systems, because the selection on the average information criterion. Moreover, the dynamic LA/AIDS model presented and discussed in Chapter 6 also conforms to the restrictions and principles implied by demand theory. This model therefore is recommended for future applications to Bahrain data.

Economic results from the estimation of the dynamic LA/AIDS suggest that “Other” is luxury products, while the rest of commodities are necessities (as income increases, Bahrain consumers tend to spend relatively more on “Other” products and less on the remaining products. Therefore, Bahrain is a possible for the “Other” commodity group market. Moreover, Food and beverages own price elasticity shows the highest values (bigger than one). Food and beverages commodity group responds more to own-price changes than the rest of the products do. Retail price strategies will become more important in food market. Clothing and footwear, Housing, and Transportation have inelastic income and own price elasticities that three markets are saturated and their relative budget share are losing importance as part of total expenditure. Producer and retailers must diversify products in order to find a place in the markets. Clothing and footwear consumption is less responsive to change in price and in total expenditure. On the other hand, the “Other” commodity group response to prices changes is inelastic, but income elasticity is above the unity. It means that the relative importance of these products on total expenditure is not going to change significantly and thus price policies to encourage consumption will not have any effectiveness. In this case, it would be more appropriate to apply market strategies focusing on the “Other” group products such education and medical care, and personal care.
References

Journals


**Books**


Chapter 8

Testing for Unit Roots and Order of Cointegration in Bahrain Commodity Demand Data

8.1. Introduction.

"The analysis of previous studies of demand systems indicates that the homogeneity restriction of the demand theory is often found to be rejected by the data. Such studies indicate that times series issues are partly responsible for rejection found in previous studies" (Serena, 1995, p.147).

In other studies researchers often fail to take account of the time series properties of the data, which provide the link between the consumer theory and short run consumer action. This means that researchers do not benefit from the relevant data to improve their modeling. To demonstrate it is helpful to assume the single equation specification.

\[ Y_t = \beta X_t + \varepsilon_t \]  

This specification is called the "Average Economic Regression" (AER) (Gilbert, 1986). The main target of the researcher is to obtain an efficient and unbiased estimate of \( \beta \) and to explain the findings. On the other hand, there are many problems which might exist in the estimation of AER, such as: serial correlation, multicollinearity, heteroscedasticity, simultaneity, wrong signs, or insignificant coefficients (Definitions, testing for these problems, and how to overcome them are in Gujarati (1995)). The task of the econometrician in response to these problems, is to re-specify the model in some way, to add or subtract variables, change the definition of variables and so forth, until eventually, he/she gets an equation which
has all the correct signs, statistically significant coefficients, a Durbin-Waston statistic of around 2, a relatively high $R^2$ etc. Usually, researchers use data transformations to ensure that Gauss-Markov or other appropriate assumptions are maintained. Frequently, the result of such transformations is several models, from several econometricians using similar data, providing evidence for opposing theories. Looked at in this way the "science" of empirical economic analysis does not appear to be very scientific.

The London School of Economics (LSE) approach, which is outlined by Gilbert (1986), focuses on model specification and validation in a time series context. In the LSE approach there are four basic steps in the model specification process: First to marginalizes those factors which are thought not to matter a great deal in the Data Generating Processes (DGP). Secondly, to condition the endogenous variables on the exogenous variables. The third step is to represent a simple equation which best illustrates the relationship between the endogenous and exogenous variables. Finally, to estimate the unknown parameters in the representation. The LSE approach gives attention to the time series properties of the variables, which are included in the basic model. Therefore, testing for unit roots (non-stationarity) and cointegration (existence of a long-run relationship) is the major target of the LSE approach. The AER approach depends totally on theory for the design of the model. It assumes that the designation is accurate, and that the estimation method represents the main solution for any problem which might arise from the test statistic findings. Hendry and Richard (1983) provide six criteria for model selection. These criteria are as follows: First, the model must be data admissible: This means that the data are generated from the specific model. For example the budget share of AIDS model which represents the dependent variable lies in the interval (0, 1). If the dependent

---

1 Hendry's (1982) approach to econometrics is grounded in the concept of the DGP. This is the joint probability of the sample data (on both endogenous and exogenous variables). Thus, $X_t$ is the vector of observations on all these variables in period $t$, and $X_{t, r} = (x_{1,t}, ..., x_{r,t})$. The joint probability of the sample $X_t$ may be written as $\prod_{t=1}^T D(x_t / X_{t-1}, \theta)$. where $\theta$ is a vector of parameters of the joint density function $D$. 

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variable lies outside this range, it will violate the data admissibility criterion. In addition, the expenditure, budget share, prices and quantity demand data must be positive in a demand system specification. Second, the model must be consistent with theory. In this thesis the demand system e.g. (Almost Ideal Demand System (AIDS)) is derived from consumer theory, which provides a good description of consumer behavior for empirical models. Third, satisfactory models should have regressors that are (at least) weakly\(^2\) exogenous. Weak exogeneity implies that knowing the data generating process of the regressors will not add any useful information in the estimation of the parameters of the model. The Neo-classical Consumer Theory implies that consumers' expenditure is conditional upon prices and income. Therefore, prices and income must either be weakly exogenous or be treated as endogenous and estimated within a simultaneous system of equations. If the regressors are not weakly exogenous, then they are endogenous by default and must be jointly modeled in a simultaneous system. Fourth, satisfactory models must exhibit parameter consistency. This is essential if the model is to be of any use in forecasting or in policy simulation, both of which require that the same parameter values, apply inside and outside the sample period. The primary aim of the LSE approach, which is to test a given specification which has been derived from theory. The neo-classical theory of the consumer provides a well-defined theoretical basis for empirical models describing consumer behavior. Fifth, a satisfactory model must be data coherent. It is a requirement that the differences between the fitted values generated by the model and the actual values should be random, in the sense that they should not be predictable from their own past history models. Sixth, a satisfactory model should encompass a wide range of rival models. A model encompasses other models when it can explain the results obtained in these models.

\(^2\) Weak exogeneity is one of four types introduced by Engel, Hendry and Richard (1983). These are: weak, strong, super, and strict exogeneity. The utilization of these types depends on the aim of the model. Weak exogeneity is required if the model is used for conditional inference only. Strong exogeneity is needed for prediction. Super exogeneity is needed when the aim is policy analysis based on the estimated model. Strict exogeneity is needed to make reliable forecasts.
A model that satisfies all six criteria is said to be congruent with the evidence. It is notable that the major criterion in the AER methodology is absent from this list. This criterion of AER is to maximize $R^2$ subject to Durbin-Watson near 2. The econometrician can obtain a high $R^2$ by the addition of further explanatory variables, but in general this will result in apparent structural non-constancy as additional data points are added and so parameter constancy will be violated. This present study focuses attention on the variables that are included in the demand system and explain consumer behavior consistent with the consumer theory. Variables such as price, expenditure, and quantity demanded must be connected in the long-run.

In econometric terms the variables for prices, income, and budget share in the equations of the demand system should form a cointegrated system if the theory is to be consistent with the data. If the variables in the demand system are not cointegrated, tests for homogeneity are not very important because of the inherent conflict between the model and the data even in the absence of constraints. Large empirical studies of demand systems have proceeded to test for homogeneity without testing whether the demand system is cointegrated.

Accordingly, the structure of this chapter is as follows: In the first section the definition and the concept of stationarity is presented. I discuss the statistical implications of finding a series to be non-stationary (containing unit roots). An application of the Dickey-Fuller approach for testing for unit roots in each time series used in the LA/AIDS model is provided.

The second section of this chapter describes cointegration techniques. It seeks to determine whether or not an equilibrium relationship exists among the variables that make-up the LA/AIDS model. If a strong relationship exists then this relationship may be indicates support for the underlying theory of consumer behavior. Therefore, in this section the static LA/AIDS model will considered as a test of the Granger Representation Theorem.
The last section of this chapter explores the assumption that the inclusion of short-run dynamics can establish a long run relationship. The reconciliation of the long-run theory with short-run observation needs a dynamic framework which has not formerly received consideration in the evaluation of a system of consumer demand. The Error Correction Mechanism (ECM) model is dynamic in nature and uses both the long-run and the short-run information embedded in trended variables such as expenditure, income and prices which are used in consumer demand analysis. The representation by ECM will reflect the theory and yield a sufficient statistical description of dynamic consumer behavior. Accordingly, a dynamic version of the LA/AIDS model is estimated using a Full Information Maximum Likelihood (FIML) approach to cointegration. Within this framework implications drawn from the theory of consumer are tested.

The discussion and analysis of econometrics issues utilizes the AIDS developed by Deaton and Muellbauer, 1980a) as the framework for analysis. The attraction of AIDS is that it gives an arbitrary first-order approximation to any demand system satisfying the axioms of choice (almost) exactly. It aggregates perfectly without invoking the assumption of parallel linear Engel curves. Also it has a function in form which is consistent with known household budget data. It is well-known that many of the desirable properties of AIDS are possessed by one or other of the Rotterdam Translog models and neither possesses all of them simultaneously (Clements, Selvanahan, and Selvanahan, 1996, p.71). The AIDS model in its general form is nonlinear. In practice, however, by a suitable approximation to the price index, it is made linear. AIDS can also be used to test the restrictions of homogeneity and symmetry (Barnett 1984, P.285)). Furthermore, the particular (nonlinear) Engel curve of the AI model implies that if a commodity is income inelastic (a necessity), an increase in income not only decrease the share of income allocated to that commodity, but also reduces its income elasticity. This type of nonlinear Engel curve has worked well in empirical studies, particularly those modeling food demand (Moschini,1998, p350). Finally, the AIDS model performed better and was selected among other demand system models it reflected
the Bahrain data well. AIDS is best viewed as a description of long-run equilibrium behavior.

Section I

8.2 Stationarity

A stochastic process \((X_t)\) is said to be stationary if the joint and conditional probability distribution of the process is unchanged when displaced in time. Thus, a stochastic process \((X_t)\) is said to be stationary if

\[
\begin{align*}
(1) & \quad E(X_t) = \mu = \text{constant for all } t; \\
(2) & \quad Var(X_t) = \sigma^2 = \text{constant for all } t; \text{ and} \\
(3) & \quad Cov(X_t, X_{t+n}) = \sigma_j \text{ constant for all } t.
\end{align*}
\]

Therefore, the mean and the variance of the process are constant over time while the value of the autocovariance between any two periods depends only on the gap between the periods, and not on the actual time at which this covariance is considered. If one or more of the conditions above are not fulfilled, the process is said to be non-stationary.

Under non-stationary processes, there are two main effects of using the non-stationary time series. The first effect is on policy analysis. In order to explain this, it useful to assume the case where \(d = 1\): Hence, the series \(X_t\) is found to be integrated of order one, and \(X_t\) can be defined as:

\[
X_t = X_{t-1} + \varepsilon_t, \quad \text{or} \quad \Delta X_t = \varepsilon_t
\]

(8.2)

Analogously, any autoregressive process that has a coefficient \(\rho\) equal to one in the equation below:

\[
X_t = \rho X_{t-1} + \varepsilon_t
\]

(8.3)

\(d\) is the number of times a non-stationary stochastic series requires differencing (d) to provide a stationary series and is called the order of integration of that series. This is denoted as \(I(d)\).
is described as having a unit root. Assuming $\varepsilon_t$ is generated by a white noise\textsuperscript{4} process with variance, $\sigma^2$, then (8.3) is also known as a simple random walk. Repeated substitution of:

$$X_{t-1} = \rho X_{t-2} + \varepsilon_{t-1}; \quad X_{t-2} = \rho X_{t-3} + \varepsilon_{t-2}$$

(8.4)

into Equation 8.2 provides

$$X_t = \sum_{j=0}^{\tau} \varepsilon_{t-j}$$

(8.5)

If $\varepsilon_t$ is determined to be the result of a shock or shocks to these prices then it is clear that these disturbances will have a permanent impact on the series as they accumulate and their effects do not die down. This is the reason that the term “integrated” is used to describe $X_t$ which amounts to an accumulation of persistent shocks when a unit root exists. However, if $X_t$ was generated by a stationary process, i.e. $|\rho| < 1$ in Equation 8.3, then similar repeated substitution produces:

$$X_t = \sum_{j=0}^{\tau} \rho^j \varepsilon_{t-j}$$

(8.6)

Equation 8.6 above shows that the effect of the shocks to the system decreases over time. Hence, a significant degree of emphasis must be placed on tests for unit roots in any policy analysis that uses economic time series.

As mentioned above, in the case of non-stationary process, the effect of a shock to the system will continue over time. An example of such a shock is an import tariff on clothing or any imported category. Such a tax changes the series representing the price of clothing permanently and the demand for clothing is permanently altered, even if the tax itself is removed. Thus, the consumption of clothing before and after the tariff is no longer directly comparable. The data generating process has been permanently changed and as a result the manner in which consumers make clothing decisions has been permanently changed. Accordingly, the decision makers must use great caution when making decisions that will affect consumer expenditure in any of expenditure categories considered in any analysis.

\textsuperscript{4} White noise process is utilized to explain error terms that are independent and have the same means and variances. Furthermore, a white noise process cannot be predicted from its previous values.
The second implication of non-stationarity in time series is in the statistical analysis of consumer issues. If the non-stationary series used the resulting test statistics will not have the standard distributions. In other words, the classical regression methods of statistical inference break down when non-stationary series are used. The statistical implications of analyzing variables that contain unit roots are as profound and significant as those for policy. Although the mean of a non-stationary process, $X_t$, may be zero, $E(X_t) = 0$, its variance is not constant over time. Using the same repeated substitution used to describe a non-stationary process as an accumulation of errors, the variance of the non-stationary process can be expressed as follows:

$$Var(X_t) = Var(\varepsilon_t) + \cdots + Var(\varepsilon_{t-1}) = \sigma^2 \tau$$ \hspace{1cm} (8.7)

The equation above shows that the variance is growing over time, and it will be a function of time when $\rho = 1$.

For $|\rho| < 1$ the variance is constant, it is not a function of time as in the case for a non-stationary process, as can be seen from equation 8.8. Consequently for a stationary process the impact of a shock to the system will not continue indefinitely over time.

$$Var(X_t) = \rho^0 Var(\varepsilon_t) + \rho^1 Var(\varepsilon_{t-1}) + \cdots + \rho^{T-1} Var(\varepsilon_{t-T})$$ \hspace{1cm} (8.8)

$$= \sigma^2 (1 + \rho + \rho^2 + \cdots + \rho^{T-1})$$

$$= \sigma^2 / (1 - \rho^T) \quad \text{as } T \to \infty$$

For more technical details, suppose that a variable $x_t$ is generated by the following process:

$$x_t = \rho x_{t-1} + \varepsilon_t$$ \hspace{1cm} (8.9)

In equation 8.9, if $\rho = 1$ then $x_t$ will be non-stationary and it is possible to rearrange and accumulate $x_t$ for different periods, starting with an initial value of $x_{t-n}$, to obtain:
\[ x_t = x_{t-n} + \sum_{j=0}^{n-1} u_{t-j} \]  \hspace{1cm} (8.9.a)

That is, the current value of \( x_t \) depends on its initial value and all disturbances accruing between \( t - n+1 \) and \( t \), while the variance of \( x_t \) is \( t \sigma^2 \) and this increases to become infinitely large as \( t \to \infty \). In fact \( x_t \) does not converge to a mean value in any normal sense since if at some point \( x_t = c \) then the expected time until \( x_t \) again returns to \( c \) is infinite. Thus, a non-stationary series has a different mean at different points in time and its variance increases with the sample size. See Harris (1995, pp. 15-16).

However, if \( |\rho| < 1 \) then \( x_t \) will be stationary and it is possible rearrange and accumulate \( x_t \) for different periods, starting with an initial value of \( x_{t-n} \), to obtain:

\[ x_t = \rho^* x_{t-n} + \sum_{j=0}^{n-1} \rho^j u_{t-j} \]  \hspace{1cm} (8.9.b)

Since \( |\rho| < 1 \), as \( n \to \infty \) (8.9.b) reduces to \( x_t \) being determined solely by a finite moving average (MA) process of order \( n \) with most weight being placed on the first elements of the disturbance term (i.e., \( u_t, \rho u_{t-1}, \rho^2 u_{t-2}, \ldots \)). Thus, when \( x_t \) is stationary, it has a constant mean and variance which are independent of time. In this simple example where \( x_t \) is determined by (8.9), \( x_t \) has a mean of 0 and a variance of \( (\sigma^2 (1 - \rho^2)) \). Hence, a stationary series tends to return to its mean value and fluctuate around it within a more-or less constant range (i.e., it has a finite variance). See Harris (1995, pp. 15-16).

According to the above discussion, all aspects of statistical inference depend heavily on the variance of a process. This means that all findings of past studies of demand systems which do not give particular attention to the properties of time series issues must be reviewed and re-investigated to see whether the data on consumer behavior are consistent with the underlying theory of demand.

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Granger and Newbold (1974) carried out a Monte Carlo experiment in which the non-stationary series were generated as independent random walks such as in regression Equation 8.3, (when $\rho = 1$) which has frequently a high $R^2$ value, and at the same time a low Durbin-Watson statistic. In such a case, the usual t-tests of statistical significance can be very misleading because they reject the null hypothesis of no relationship frequently. They accept as significant relationships that are too often spurious. Accordingly, Granger and Newbold suggest that it is better to estimate the relationship in first differences of the variables rather than in the levels of variables.

Granger and Newbold (1974) and Philips (1986) demonstrated that such a result can be a consequence of a "spurious regression", which will possess the following statistical properties:

1. The regression coefficients will not converge in probability to constants and will have non-degenerating limiting distributions as the sample size increases. Additionally, the distribution of the intercept diverges as the sample size increases.
2. The coefficient of correlation will have a non-degenerate limiting distribution as the sample size increases. The Durbin-Watson statistic converges to zero.
3. More seriously, the distribution of the $t$ and $F$ statistics diverge as the sample size increases, so that there are no asymptotically correct critical values. Granger and Newbold (1974) contended that these tests are seriously biased towards the rejection of the null hypothesis of no relationship even when the series are independent random walks.

A non-stationary series which can be transformed to a stationary series by differencing $d$ times is said to be integrated to the order of $d$, and denoted as $x_t \sim I(d)$. If the series is stationary then no differencing is necessary, that is: $x_t \sim I(0)$. If two series or more are integrated of different orders then they must be moving apart from each other over the long run. Therefore, before analyzing the
long run relationship between a set of variables it is important to test their order of integration.

In a system of two or more variables it is essential to test the presence of a unit root in all variables before the cointegrating regression is performed. If this hypothesis is not rejected then the process can proceed to test for cointegration. Empirical studies reveal that many macroeconomic series have been found to be integrated of order one, $I(1)$ (Nelson and Plosser, (1982), Granger, (1986), Engle and Grange, (1987)). This means that if two series are $I(1)$, then a linear combination of the series, $z_t = y_t - \alpha - x_t$ may be of $I(0)$.

8.3 Testing for Unit Roots

There are a total of 80 quarterly observations from 1979: 1 to 1998: 4 available for estimation of the LA/AIDS model. The purpose of the analysis in this chapter is to test for unit roots in the variables to be used in the demand system. The relevant variables are the budget shares for each commodity group ($w_i$), the logarithms of the prices ($p_i$), and the logarithm of real total expenditure ($x/p$). Before carrying out the test for unit roots, time series (variables) on the level and first differences of each time series is plotted. Simply by looking at the time path of these figures (8.1.a - 8.11.b) will provide a first indication of the stationarity, or non-stationarity of the process.

The figures below indicate that the levels of the most of the variables of this study do not satisfy the formal requirements of stationarity as most series do not have a variance that is independent of time. Most variables in the levels contain the effects of the event or shocks as a permanent element of their time series behavior. Thus, all the historical events or shocks can be removed or lost if only the data is interpreted of order one. The graphical findings can be confirmed with formal tests for unit roots.
Fig. 8.1.a: Budget share of food & beverages (79Q1-98Q4)

Fig. 8.1.b: One period changes in the budget share of food & beverages
Fig. 8.2. a: Budget share of clothing & footwear (79Q1-98Q4)

Fig. 8.2.b: One period changes in the budget share of clothing & footwear
Fig. 8.3.a: Budget share of housing (79Q1-98Q4)

Fig. 8.3.b: One period changes in the budget share of housing
Fig. 8.4.a: Budget share of transportation (79Q1-98Q4)

Fig. (8.4.b): One period changes in the budget share of transportation
Fig. 8.5.a: Budget share of other (79Q1-98Q4)

Fig. 8.5.b: One period changes in the budget share of other
Fig. 8.6.a: Natural log price index of food & beverages (79Q1-98Q4)

Fig. 8.6.b: One period changes in the natural log price index of food & beverages
Fig. 8.7.a: Natural log price index of clothing & footwear (79Q1-98Q4)

Fig. 8.7.b: One period changes in the natural log price index of clothing & footwear
Fig. 8.8.a: Natural log price index of housing (79Q1-98Q4)

Fig. 8.8.b: One period changes in the natural log price of housing
Fig. 8.9.a: Natural log price index of transportation (79Q1-98Q4)

Fig. 8.9.b: One period changes in the natural log price of transportation
Fig. 8.10.a: Natural log price index of other (79Q1-98Q4)

Fig. 8.10.b: One period changes in the natural log price index of other
Fig. 8.11.a: Natural log of total real expenditure (79Q1-98Q4)

Fig 8.11.b: One period changes in the natural log of total real expenditure
Several tests for unit roots have been developed to test for the unit roots in the level of a series. Engle and Granger (1987) provided a detailed review and comparison of some of these tests, but the authors recommended the use of the Dickey-Fuller (1979, 1981) or the Augmented Dickey Fuller (ADF) test. The latter is the most widely used in testing for unit roots.

The basic form of the Dickey Fuller (DF) test consists of running one of the following OLS regressions:

\[
\Delta X_t = \beta X_{t-1} + \varepsilon_t \quad \text{(8.10a)}
\]

\[
\Delta X_t = \alpha_0 + \beta X_{t-1} + \varepsilon_t \quad \text{(8.10b)}
\]

\[
\Delta X_t = \alpha_0 + \beta X_{t-1} + \alpha_1 t + \varepsilon_t \quad \text{(8.10c)}
\]

where \( \Delta X_t = X_t - X_{t-1} \)

That is, by regressing the first difference of \( X_t \) on its value lagged by one time period.

If \( \beta \) is significantly less than zero, the null hypothesis that the variable contains a unit root would be rejected. The ratio of \( \hat{\beta} \) to its standard error provides the essential test statistic of the Dickey-Fuller test. Granger and Newbold (1974), Philips (1986), and Davidson and Mackinno (1993) reported that the distribution of the test statistic does not follow a standard distribution when integrated processes are included in the regression. The \( t \)-ratio for \( \beta \) does not have a limiting normal distribution. The distribution is so negatively skewed that most of its mass falls below zero. Hence, the critical values in the left-hand tail should be less than those that would be observed in the conventional student \( t \) distribution.

---

5 In conducting the test using model (8.9), it is assumed that the mean value of \( X_t \) is zero. If this is not the case, an intercept should be included.

6 \( X_t \) is a series whether it is stationary or not is being determined.
Accordingly, the used $t$ tests are not appropriate for testing the null hypothesis of a unit root ($\beta = 0$). Similarly, the case would be for $F$ tests used to test some joint specification where a unit root is suspected. Dicky and Fuller and others have used Monte Carlo simulations to calculate tables of critical values based on the distribution of the statistics gained from such tests for non-stationarity. These resulting tables of critical values are in absolute value higher than the normal critical values.

If a lagged dependent variable is included in Equation 8.9 then the test becomes the Augmented Dickey Fuller test (ADF) as in Equation 8.10 below.

$$AX_i = \alpha + \beta X_{t-1} + \sum_{i=1}^{p} \theta_i AX_{t-i} + \epsilon_t$$  (8.11)

The lagged first difference terms are included in ADF regression equation in order to minimize the problem of serial correlation. The null hypothesis of a unit root should be rejected if $\beta$ is significantly less than zero.

In addition it may be necessary to add a constant and/or a time trend to make sure that the residuals are white noise. The same asymptotic critical values calculated for DF-tests can be used to test the null of nonstationarity (Davidson and Mackinnon, 1993, p. 711). For negative values of $\hat{\beta}$ if the calculated value of the ratio $\hat{\beta}$ to its standard error is larger in absolute value than the critical value from the table, then the null of a unit root is rejected.

For every series used in this study, an ADF regression of the following form is applied:

$$AX_i = \alpha_0 + \beta X_{t-1} + \sum_{i=1}^{p} \theta_i AX_{t-i} + \alpha_l t + \epsilon_t$$  (8.12)

where $X$ represents each variable covered in the analysis of consumer demand.
The null hypothesis is that a unit root exists in all of the series frequently used in demand analysis. If $\beta$ in Equation 8.11 is not significantly different than zero, than the null hypothesis of non-stationary ($H_0 : nonstationary$) is accepted, or there is indication that the series is not stationary.

The second row of Table 8.1 shows the statistical t-values for $\beta$ in Equation 8.11. It is observed seems from the second row of the Table 8.1 that all series exhibit an evidence of non-stationarity in the levels.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$w_{f_{ab}}$</th>
<th>$w_{c_{af}}$</th>
<th>$w_h$</th>
<th>$w_f$</th>
<th>$l_p_{f_{ab}}$</th>
<th>$l_p_{c_{af}}$</th>
<th>$l_p_h$</th>
<th>$l_p_f$</th>
<th>$l_p_o$</th>
<th>$l(x/p)$</th>
</tr>
</thead>
</table>

The Augmented -Dicky-Fuller (ADF) statistic (ADF) is computed based on the following test equation:

$$\Delta x_t = \alpha_0 + \beta x_{t-1} + \sum_{i=1}^n \theta_i \Delta x_{t-i} + \alpha_1 t + \varepsilon_t$$

Where $\Delta$ denotes the first-difference operator and $x_t$ stands for each variable which make-up the LA/AIDS of this study.

The critical values for 1% and 5% are (-4.09) and (-3.47). These are generated using the Pc-Give Econometric Software Package and are based on the surface response estimates give in MacKinnon (1991).

where $w_{f_{ab}}, w_{c_{af}}, w_h, w_f$ and $w_o$ represent the budget shares of Food & Beverage, Clothing & Footwear, Housing, Transportation, and Other commodity group respectively, where, $l_p_{f_{ab}}, l_p_{c_{af}}, l_p_h, l_p_f, l_p_o$ represent the logarithmic price indices for Food & Beverage, Clothing & Footwear, Housing, Transportation, and Other, respectively. Where $l(x/p)$ represents real total expenditure.

This confirms the results obtained from the graphical analysis of all series. Thus, at 1% level of significance, the null of unit roots cannot be rejected for all variables, however, at 5% level of significance, the null of unit roots can be rejected for three of the variables and these are the price of Housing category ($l_p_h$), the price of Other category ($l_p_o$) and real expenditure ($l(x/p)$) respectively.
The ADF test can also be used to test for non-stationarity in the first and second difference series by running the following regressions.

\[ \Delta^2 X_t = \alpha_0 + \beta X_{t-1} + \sum_{i=1}^{p-1} \theta_i \Delta^2 X_{t-i} + \alpha_t + \epsilon_t \]  \hfill (8.13)

\[ \Delta^1 X_t = \alpha_0 + \beta X_{t-1} + \sum_{i=1}^{p-1} \theta_i \Delta^1 X_{t-i} + \alpha_t + \epsilon_t \]  \hfill (8.14)

Equation 8.13 is the same as the Equation 8.12 but the difference is that one period changes of each process \((\Delta X_t)\) has replaced the level \((X_t)\). The results of the ADF test for non-stationarity in the first difference terms are presented in Table 8.2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>(W_{f&amp;b})</th>
<th>(W_{c&amp;f})</th>
<th>(W_h)</th>
<th>(W_t)</th>
<th>(W_o)</th>
<th>(l p_{f&amp;b})</th>
<th>(l p_{c&amp;f})</th>
<th>(l p_h)</th>
<th>(l p_t)</th>
<th>(l p_o)</th>
<th>(l(x/p))</th>
</tr>
</thead>
</table>

The Augmented-Dicky-Fuller (ADF) statistic is computed based on the following test equation:

\[ \Delta X_t = \alpha_0 + \beta X_{t-1} + \sum_{i=1}^{p-1} \theta_i \Delta X_{t-i} + \alpha_t + \epsilon_t \]  

where \(\Delta\) denotes the first-difference operator and \(X_t\) stands for each variable which make-up the LA/AIDS of this study.

The critical values for 1% and 5% are (-4.09) and (-3.47). These are generated using the Pc-Give Econometric Software Package and are based on the surface response estimates given in MacKinnon (1991).

The findings are included in the second row of Table 8.2. The ADF test shows that all of the series can be made stationary by first differencing at 1% and 5% levels of significance except the budget share of Food and Beverages, which can be made stationary by second differencing at 1% and 5% levels of significance. An ADF-test results for budget share of Food & Beverage at the second difference is -8.510. In short, there is an indication that all of the series are integrated of order one I(1) at 1% level of significance. Accordingly, the Equation 8.13 was run for the budget share of Food & Beverages, since the latter variable is not stationary in the first order.
difference terms. In general, the previous results amount to powerful evidence of non-stationarity in the majority of the time series generally utilized in a consumer demand system.

Thus, it appears from the above discussions that studying the properties of time series that are commonly used in consumer demand is important for both statistical analysis and the policy of consumer issues at every level of inference. The disadvantages implicated in the statistical analysis of integrated processes can be avoided when estimation of the AIDS model rely on using data covered in the unit roots test which performed above. The application of this approach seems that most of the processes included in this study are integrated of order one, I(1). This result has significant implications particularly for decision-makers in the economy when new policies are being applied which may result in a change or shock to this non-stationary series. In short, accurately studying the effect of the policy requires information such as prices, and expenditures must be represented by stationary series. Since the non-stationary series will have a permanent rather than a transitory effects in AIDS model over time, the policy which will be adopted does not cause desirable results and will not die over time. Thus, the consumer decision making process with respect to any commodity will be permanently changed.

Section II:

8.4 The Concept of Cointegration.

To define what is meant by cointegration requires considering a simple relationship of the form

\[ Y_t = \beta X_t + \varepsilon_t \]  \hspace{1cm} (8.15)

where \( \varepsilon_t \) is the disequilibrium error.

Engle and Granger (1987) maintain that if an equilibrium relationship exists between certain economic variables, the values of errors should be small. Thus, the variables
must not drift too far apart when scaled. In our example, the scaled transformations placed on $Y$ and $X$, are $1$ and $-\beta$ respectively. The latter is called the scaled cointegrating vector $[1,-\beta]$. Under the equilibrium relationship which is basically two or more series that are cointegrated, Granger (1981) supports the utilization of an Error Correction Mechanism (ECM) to interpret the short run adjustment process toward equilibrium. This argument is known as the Granger Representation Theorem.

The formal definition of cointegration of two variables developed by Engle and Granger (1987) is as follows: Time series $x_t$ and $y_t$ are said to be cointegrated of order $d,b$ where $d \geq b \geq 0$ written as $x_t,y_t \sim CI(d,b)$ if
- Both series are integrated of order $d$
- There exists a linear combination of these variables. If $\alpha_1 x_t + \alpha_2 y_t$, which is integrated of order $d - b$, The vector $[\alpha_1, \alpha_2]$ is called a cointegrating vector.

Suppose that $x_t$, $y_t$ are both $I(1)$ and that the long run relationship between them is given by:

$$y_t^* = \beta x_t$$  \hspace{1cm} (8.16)

An important case occurs if the variables $x_t$, $y_t$ are $CI(1,1)$, and have the cointegrating vector $[1,-\beta]$, so that the deviations of $y_t$ from its long run path $y_t^*$ are $I(0)$. If this is the case, a model in first differences incorporating an error correction mechanism can be sensibly developed. Specifically for the Equation 8.17:

$$\Delta y_t = \beta_1 x_t + \beta_2 (y_{t-1} - \beta x_{t-1}) + \varepsilon_t$$  \hspace{1cm} (8.17)

Both the dependent variables $\Delta y_t$ and the regressors $\Delta x_t$ and $(y_{t-1} - \beta x_{t-1})$ are $I(0)$ Hence, there is no danger of estimating a spurious regression due to the presence of stochastic deterministic trends in the data. The model includes both a
long run solution and has an error correction mechanism (ECM) when $\beta_2$ is negative.

The various possibilities of integration and cointegration in (8.17) are as follows:
1) If $y_t \sim I(1)$ and $x_t \sim I(0)$ then $u_t \sim I(1)$ and the variables $x_t$, $y_t$ are not cointegrated.
2) If $y_t \sim I(1)$ and $x_t \sim I(1)$ then it may be that $u_t \sim I(0)$ and the variables $x_t$, $y_t$ are cointegrated, only if $[\beta, -I]$ constitutes a cointegrating vector.
3) If $y_t \sim I(0)$ and $x_t \sim I(0)$ then $u_t \sim I(0)$ and the cointegration inquiry does not make sense.
4) If $y_t \sim I(0)$ and $x_t \sim I(1)$ then $u_t \sim I(1)$ and the variables $x_t$, $y_t$ are not cointegrated.

8.5 Estimation and Testing for the Cointegration in LA/AIDS.

This section will describe how to estimate the cointegrating vector and testing for cointegration. The commonly used method to obtain cointegrating vector is known as the Engle Granger two-step procedure. The procedure begins with the estimation of a given specification that is implied by the theory. For example, consider the basic regression, which is meant to model some long-run relationships:

$$Y_t = \beta X_t + \varepsilon_t$$  \hspace{1cm} (8.18)

or

$$\varepsilon_t = Y_t - \beta X_t$$  \hspace{1cm} (8.19)

The residual $\varepsilon_t$ can be used as a first step to test for the existence of the long relationship or cointegration. Formally, the null hypothesis states that the processes are not cointegrated. Thus, $H_0 : \rho = 1$ in the regression below

$$\varepsilon_t = \rho \varepsilon_{t-1} + \nu_t$$  \hspace{1cm} (8.20)

where $\nu_t$ is white noise. Supposing that $Y_t$ and $X_t$ are $I(1)$, the relationship which provides the residual $\varepsilon_t$ is one of cointegration if $\varepsilon_t$ is $I(0)$. Engle and Granger
(1987) recommended applying the Augmented Dickey-Fuller (ADF) test. The ADF test is known as the augmented Engle Granger (AEG) test due to the utilization of the residual based test for cointegration. In this test it needs to first run the regression of this form:

\[ \Delta \epsilon_t = \phi \epsilon_{t-1} + \sum \delta_i \Delta \epsilon_{t-i} + \nu_t \]  
(8.21)

Testing the null hypothesis of a unit root is \( \phi = 0 \). The ratio of \( \hat{\phi} \) to its standard error can be compared in absolute value to the critical values listed in tables of Davidson and Mackinnon (1993, p. 722). The critical values for the AEG test vary with the number of elements in the cointegrating vector need to be estimated. Table 20.2 in Davidson and Mackinnon (1993, p. 722) accommodates cointegration tests when the cointegrating vector has 7 or fewer elements. If the number of elements in the cointegrating vector exceeds 7, then critical values must be calculated through Monte Carlo simulations. If the calculated value exceeds the critical value, then the null of no cointegration is rejected. Thus, the coefficients estimated in equation (8.18) yield a cointegrating vector that does not allow \( Y_t \) and \( X_t \) to drift too far apart.

The second step of the Engle and Granger approach is conditional on finding a long run relationship in the first step. Given the existence of cointegration, the estimated long run coefficients can be included in the error correction model by substituting for the error correction term with the lagged values of the estimated residuals from the cointegrating regression. Thus, the error correction representation can now be written in the following form:

\[ \Delta Y_t = \alpha \Delta X_t + \lambda \epsilon_{t-1} + \nu_t \]  
(8.22)

Formally, the null hypothesis states that there is a long run relationship, or the existence of cointegration between \( Y_t \) and \( X_t \), Thus, \( H_0 : \lambda = 0 \)

An ECM can be obtained from the equilibrium model in Equation 8.18. When estimated in first differences is
\[
\Delta Y_t = \alpha \Delta X_t + \Delta \epsilon_{t-1} + \alpha \Delta X_t + \epsilon_t - \epsilon_{t-1}
\]  \hspace{1cm} (8.23)

substituting of \( Y_{t-1} = \beta X_{t-1} + \epsilon_{t-1} \) or \( \epsilon_{t-1} = Y_{t-1} - \beta X_{t-1} \) from the Equation 8.18 into Equation 8.23 provides an ECM with full adjustment:

\[
\Delta Y_t = \alpha \Delta X_t + -\lambda(Y_{t-1} - \beta X_{t-1}) + \epsilon_t
\]  \hspace{1cm} (8.24)

Full adjustment to past error is evident in the fact that the coefficient of the error correction term \((Y_{t-1} - \beta X_{t-1})\) is negative. If the adjustment to past error occurs more slowly, then the coefficient of the error correction term is less than one (in absolute value). However, in the case where there is no adjustment to past error, the coefficient of the error correction term becomes zero (see Thomas, 1996, p. 432).

Following the Engle-Granger two-step procedure, the static AIDS model can be considered as a potential cointegrating regression. Thus, the test for cointegration using AIDS model amounts to a general empirical test of the microeconomic theory of the consumer. If the elements that build up the AIDS specification are found to be cointegrated then the AIDS specification can be considered an adequate representation of long-run consumer behavior. Moreover, the dynamics of consumer behavior could then be incorporated through an ECM.

Therefore, the static AIDS model is estimated as a test of the Granger Representation Theorem. The five static regressions that make up the AIDS model are those in Equation 6.9 of Chapter 6. This means that the AIDS model describes long-run consumer demand in this system of five expenditure commodity groups. The residuals \((\epsilon_i)\) from the five OLS regressions represented in (6.9) are tested for nonstationarity and therefore, cointegration, using a residual-based cointegration test with critical values which can be found in Davidson and Mackinnon (1993, p. 722). The test involves running the five OLS regressions of the form

\[
\Delta \epsilon_t = \phi \epsilon_{t-1} + \sum \delta_i \Delta \epsilon_{t-i} + \nu_t
\]  \hspace{1cm} (8.25)
and testing the null hypothesis of a unit root. That is, testing the hypothesis that \( \phi = 0 \).

Table 8.3 indicates the results of the residual-based cointegration tests. In the static LA/AIDS model, one of the five equations appears to represent a cointegrating regression. The null hypothesis of no cointegration is not rejected at the conventional 1% level of significance for any of the equations in the static LA/AIDS model, however, at the 5% level of significance; the null hypothesis of no cointegration is rejected for only the Housing equation. In general, the static LA/AIDS model does not appear to describe a long run equilibrium relationship. This result is not surprising given the Granger Representation Theorem. The variables are integrated of the same order, \( I(d) \). Therefore, this test amounts to a test of the Granger Representation Theorem. The theoretical framework implied by the AIDS model is not supported by the data. These series do not appear to be related within the framework of the LA/AIDS model in the long run and this result hold for most equations in the system as seen from the results in Table 8.3. Thus, the Engle-Granger tests are not precise and in particular, lack power (Thomas, 1996, p.431). The procedure fails to detect a long run relationship even when one exists. With the author's data set, the tests fail to detect cointegration.

According to the application and presentation above, all past results of empirical studies of demand system that pointed to testing postulated economic models of consumer behavior must be approached with extreme wariness, especially inferences in the context of static models of demand systems which seem to be at best ambiguous.
Table 8.3:

<table>
<thead>
<tr>
<th>Dependent variable ($w_i$)</th>
<th>Static LA/AIDS model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget share of Food &amp; beverages ($w_{f&amp;b}$)</td>
<td>-3.6717</td>
</tr>
<tr>
<td>Budget share of Clothing &amp; footwear ($w_{c&amp;f}$)</td>
<td>-3.3499</td>
</tr>
<tr>
<td>Budget share of Housing ($w_h$)</td>
<td>-5.2347</td>
</tr>
<tr>
<td>Budget share of Transportation ($w_t$)</td>
<td>-4.3170</td>
</tr>
<tr>
<td>Budget share of Other ($w_o$)</td>
<td>-4.0401</td>
</tr>
</tbody>
</table>

The critical values are reported by Davidson and Mackinnon (1993, p.722). For six right-hand side variables are: -4.42 ($p = .10$), -4.71 ($p = .05$) and -5.25 ($p = .01$).

It should be noted here that the Engle-Granger two-step method is valid only if there is only one cointegrating vector, and if I (1) variables are linked by more than one cointegrating vector, the method is no longer applicable. OLS estimation of the cointegrating regression no longer provides consistent estimates of any of the cointegrating vectors. The inapplicability of the Engel-Granger method in the multivariate case implies that other methods of estimation have to be employed. Johansen (1990) suggests a maximum likelihood approach. The Johansen procedure, therefore, provides estimates of all cointegrating vectors in the multivariate case, and the application can be seen in the next section of this chapter. Therefore, the next section provides a maximum likelihood approach which includes the dynamic behavior of the data while testing the neoclassical model of consumer behavior.
Section III

8.6 The Johansen VAR Maximum Likelihood Estimator.

Johansen (1988) and Johansen and Juselius (1990) developed a method that gives a much more general application of cointegration techniques. Johansen's procedure is based on the estimation of a vector autoregression (VAR). Given \( p \) dimensional \( z \) vector (all variables in the system), a vector autoregression of order \( k \) could be written as

\[
z_t = \Pi_1 z_{t-1} + \ldots + \Pi_k z_{t-k} + \mu + \varepsilon_t
\]  

(8.26)

where the \( \varepsilon_t \) are iid \( N_p(0, \Psi) \) and \( \mu \) is a constant term. This model can be written in difference terms as follows:

\[
\Delta z_t = \Gamma \Delta z_{t-1} + \ldots + \Pi_{k-1} \Delta z_{t-k-1} + \Pi_k \varepsilon_{t-k} + \mu + \varepsilon_t
\]  

(8.27)

where

\[
\Gamma = -I + \Pi_1 + \ldots + \Pi_k, \quad I \text{ is a unit matrix, } i=1, (k-1),
\]

and

\[
\Pi = -(I + \Pi_1 + \ldots + \Pi_k)
\]

Hence, \( \Pi \) is the multivariate distributed lag function of the first equation in this section with the lag operator \( L \), set equal to 1, which is the usual way of determining long term multipliers.

Three situations now arise related to the rank of \( \Pi \):

1) If \( \Pi \) has full rank equal to \( p \), then it can be shown that \( z \) must be stationary, thus estimation of the system in levels presents no immediate problem.
2) If the rank of \( \Pi \) is zero, then \( \Pi \) is a null matrix, and there is no cointegration. Thus, the model can be estimated in first differences only and in this case, there is no information gained from the error correction term.
3) If the rank of $\Pi$ is equal to $0 < r < p$ then $\Pi$ (there are $r$ cointegrating vectors) can be written as the product of two matrices, $\alpha$ and $\beta$, i.e. $\Pi = \alpha \beta$

The cointegrating space is defined by $\beta$ and the adjustment factors are defined by $\alpha$, and both $\beta$ and $\alpha$, are $p \times r$ matrices. The $\Pi$ is known as the long run impact matrix. In the long run impact matrix, $\alpha$ represents the feedback, loading or adjustment matrix, which measures the speed of adjustment of a particular variable with respect to periods of disequilibrium, while $\beta$ represents the cointegrating matrix where $\beta' z_t \sim I(0)$ when $z_t \sim I(1)$.

The differenced model can be written as:

$$z_{0t} = \Pi z_{lt} + \Pi z_{kt} + \epsilon_t$$

where

$$z_{0t}, z_{lt} = [\Delta z_{t-1}, \ldots, \Delta z_{t-k+1}]$$

and

$$z_{kt} = z_{t-k}$$

If the effect of $z_{lt}$ on both $z_{0t}$ and $z_{kt}$ is partial led out then $\Pi$ can be estimated by regressing the adjusted $z_{0t}$ on $z_{kt}$. These adjusted values are simply the residuals of two multivariate regressions on $z_{lt}$ i.e.

$$R_o = z_0 - z_1(z_1'z_1)^{-1}z_1'z_0$$

and

$$R_k = z_k - z_1(z_1'z_1)^{-1}z_1'z_k$$

Four $p \times p$ matrices are then computed and denoted as $S_{ok} = T^{-1}R_oR_k$, $S_{oo} = T^{-1}R_oR_o$, $S_{kk} = T^{-1}R_kR_k$, and $S_{ko} = S_{ko}'$

where $T$ is the sample size. Using these matrices, the roots or eigenvalues of the polynomial equation in $\lambda$ are the solutions to the following determinant or eigenvalue problem
The values of $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$, along with associated eigenvectors, $v$, can be arranged into the matrix $V = \{v_1, v_2, v_3, \ldots, v_n\}$. The eigenvectors can be then normalized utilizing $V' S_\beta V = I$. If the relationship between the variables in $z_t$ is one of cointegration, then the first $r$ eigenvectors are the cointegrating vectors. Therefore, $v_1, v_2, \ldots, v_r$ are the columns that form the cointegrating matrix $\beta$.

Johansen (1988) indicates that these eigenvectors are the maximum likelihood estimates of $\beta$ when the cointegration exists.

There are two formal tests for the number of cointegrating vectors $r$. These tests are described in Johansen and Juselius (1990) and involve the computation of two likelihood ratio test statistics. The first is a test of the null hypothesis of $r$ cointegrating vectors against the general alternative of $n$ cointegrating vectors and are referred to as $LR_{\text{trace}}$. The likelihood ratio test statistic is calculated as

$$LR_{\text{trace}} = -T \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_i) \quad r = 0, 1, 2, \ldots, n-2, n-1.$$  \hfill (8.30)

The $LR_{\text{trace}}$ statistic tests the null hypothesis that the number of distinct cointegration vectors is less than or equal to $r$ against a general alternative. The second likelihood ratio statistic is used in the test of the null hypothesis of $r$ cointegrating vector against the alternative hypothesis of $r + 1$ cointegrating vectors. The statistic is calculated as

$$LR_{\text{max}} = -T \ln(1 - \hat{\lambda}_r) \quad r = 0, 1, 2, \ldots, n-2, n-1.$$  \hfill (8.31)

Once the value of $r$ is determined, and the $\beta$ matrix is set equal to the values of the first $r$ cointegrating vectors, the elements of $\alpha$ in $Pt = \alpha \beta'$ can be obtained from the first $r$ columns in the matrix $S_\alpha v$. Therefore, using the method summarized above, the matrix of long run parameters, $\beta$ can be obtained which can be normalized to a given dependent variable. Moreover, the adjustment matrix $\alpha$,
which describes the speed in which particular variables move back to equilibrium once they have been out of equilibrium, can be estimated. The advantage of Johansen's procedure is that, the short-run dynamics of the variables is also taken into consideration.

In this chapter's, the Johansen procedure is applied to test linear combinations of expenditure shares, prices and total real expenditure that are hypothesized to describe the co-movement between these series. This technique discussed in Johansen (1988), along with critical values reported in Johansen and Juselius (1990), can be used to identify the number of cointegrating vectors, to estimate the cointegrating matrix itself and to derive inference based on these estimates.

8.7 Empirical Results of VAR Maximum Likelihood Estimator

In this section an attempt has been made to estimate a dynamic version of the AIDS. The data set ranges from 1979/Q1 to 1998/Q4 and represents Bahrain consumption in five major expenditure categories. Tests for unit roots indicate that the series used in the system are again time heterogeneous. The aim is to include the time heterogeneity of the data into the model. Therefore, a dynamic version of the AIDS will be estimated using Johansen's approach as described in the past section. Relying on this structure, the consumer theory is tested.

8.7.1 FIML Estimation of the Dynamic LAVIDS- as individual equation

The Maximum Likelihood approach can be used to tests for cointegration in five commodity groups of AIDS system, namely: Food & Beverage, Clothing & Footwear, Housing, Transportation, and finally, Other commodity groups. A VAR (3) system is estimated in the equation by equation analysis. VAR (3), k=3, means that there are three lag of each of the variables included in the system. The choice of the lag-length of the VAR (3) was based on Schwarz Bayesian Criterion (SBC).
where: \( z_{ii}, z_{2i}, z_{3i}, z_{4i}, \) and \( z_{5i} \) are determined by all the variables in the Food & beverage equation, Clothing & footwear equation, Housing equation, Transportation equation, and Other commodity group equation respectively.

- \( w_i \) is the budget share of commodity groups (i), \( \ell_{p, \text{F&B}} \) is the price of Food & beverages, \( \ell_{p, \text{C&F}} \) is the price of Clothing & footwear, \( \ell_{p, \text{H}} \) is the price of Housing, \( \ell_{p, \text{T}} \) is the price of the Transportation, \( \ell_{p, \text{O}} \) is the price of Other commodity group, and finally, \( l(x/p) \) is real total expenditure.

The \( LR_{\max} \) and \( LR_{\text{trace}} \) are computed and presented in Tables 8.4, 8.5, 8.6, 8.7, and 8.8 to test for cointegration among the variables for each of commodity groups. Tables 8.4, 8.5, 8.6, 8.7 and 8.8 show \( LR_{\text{trace}} \) with the critical values at 5% level of significance. The statistics are computed for potential value of \( r \) in each of the five vector autoregressions. The major results are calculated as follows: under the \( LR_{\max} \) test statistic, the null hypothesis of no \((r = 0)\) cointegration is not accepted in favor of alternative cointegration \((r = 1)\) in all of five commodity groups. The computed values \( LR_{\max} \) statistics are greater than the critical values of 45.3 at five percent level of significance for all commodity groups. The subsequent hypothesis that the number of cointegrating vectors is at most one is also rejected in favor of the alternative of at least two cointegrating vectors \((r = 2)\).

The computed values of \( LR_{\max} \) statistics are greater than the critical values of 39.4 at five percent level of significance for all commodity groups. The subsequent hypothesis that the number of cointegrating vectors at \((r \leq 2)\) is also rejected in
favor of the alternative of at least three cointegrating vectors ($r = 3$). The computed values of $LR_{\text{max}}$ statistics are greater than the critical values of 33.50 at five percent level of significance for all commodity, as shown clearly by Tables 8.4, 8.5, 8.6, 8.7, and 8.8. However, the calculated value of the $LR_{\text{max}}$ statistics is less than the critical value 27.1 at 5% level under the null hypothesis of $(r < 3)$ which is tested relative to the alternative of $(r = 4)$.

Thus, the results of the maximum eigenvalues test imply that there are three cointegrating vectors in every two equations, namely, food & beverages and clothing & footwear commodity groups of AIDS model. This can be seen clearly by looking at Tables 8.4 and 8.5, while the test also indicates that there are four cointegrating vectors in equations of housing, transportation, and other commodity groups.

With respect to $LR_{\text{trace}}$ test statistic the results suggests that there are four cointegrating relationships for four commodity groups equations, namely: Food & Beverages, Clothing & Footwear, Housing, and Other commodity group equation. However, the calculated values of the $LR_{\text{max}}$ and $LR_{\text{trace}}$ statistics are less than the critical values of $LR_{\text{max}}$ (31.9) and $LR_{\text{trace}}$ (53.8) at 1% level under the null hypothesis of $(r < 3)$ tested relative to the alternative of $(r = 4)$. Thus, the results of the maximum eigenvalues test imply that there are three cointegrating vectors in every two equations, namely, Food & Beverages and Clothing & Footwear commodity groups of LA/ AIDS model.

For the Transportation commodity group, the calculated value of the $LR_{\text{trace}}$ test statistic is less than the critical value (15.4) at 5% of significance when the null hypothesis of $(r \leq 5)$ is tested relative to the alternative of $(r = 6)$. Hence, the results of the maximum eigenvalue test imply that there are five cointegrating vectors in the Transportation commodity group.
However, the calculated values of the $LR_{\max}$ (16.94) and $LR_{\text{trace}}$ (30.81) statistics are less than the critical values of $LR_{\max}$ (25.5) and $LR_{\text{trace}}$ (35.4) at 1% level under the null hypothesis of $(r \leq 4)$ which is tested relative to the alternative of $(r = 5)$. Thus, the results of the maximum eigenvalues test imply that there are four cointegrating vectors in Transportation commodity group of LA/AIDS model. Enders (1995, p.393), reported that the results of the $LR_{\max}$ and $LR_{\text{trace}}$ tests can conflict. The $LR_{\max}$ test has the sharper alternative hypothesis. It is usually preferred for trying to pin down the number of cointegrating vectors.

According to the above findings, there is a possibility of three cointegrating vectors in two commodity groups (Food & Beverages and Clothing & Footwear), equations, whereas there is a possibility of four cointegrating vectors in Housing, Transportation and other commodity group equations.

![Table 8.4](image)

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\lambda_i$</th>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>$LR_{\max}$</th>
<th>95%</th>
<th>$LR_{\text{trace}}$</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.563</td>
<td>$r = 0$</td>
<td>$r = 1$</td>
<td>63.8</td>
<td>45.3</td>
<td>207.8</td>
<td>124.62</td>
</tr>
<tr>
<td>2</td>
<td>0.529</td>
<td>$r \leq 1$</td>
<td>$r = 2$</td>
<td>57.92</td>
<td>39.4</td>
<td>144.0</td>
<td>94.2</td>
</tr>
<tr>
<td>3</td>
<td>0.362</td>
<td>$r \leq 2$</td>
<td>$r = 3$</td>
<td>33.59</td>
<td>33.5</td>
<td>861.1</td>
<td>68.5</td>
</tr>
<tr>
<td>4</td>
<td>0.256</td>
<td>$r \leq 3$</td>
<td>$r = 4$</td>
<td>22.8</td>
<td>27.1</td>
<td>51.5</td>
<td>47.2</td>
</tr>
<tr>
<td>5</td>
<td>0.205</td>
<td>$r \leq 4$</td>
<td>$r = 5$</td>
<td>17.63</td>
<td>21.0</td>
<td>28.7</td>
<td>29.7</td>
</tr>
<tr>
<td>6</td>
<td>0.098</td>
<td>$r \leq 5$</td>
<td>$r = 6$</td>
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<td>14.1</td>
<td>11.1</td>
<td>15.4</td>
</tr>
<tr>
<td>7</td>
<td>0.040</td>
<td>$r \leq 6$</td>
<td>$r = 7$</td>
<td>3.16</td>
<td>3.8</td>
<td>3.2</td>
<td>3.8</td>
</tr>
</tbody>
</table>
Table 8.5
Test for cointegration rank for Clothing & Footwear commodity group equation of LA/AIDS model

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \lambda_i )</th>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>( LR_{\max} )</th>
<th>95%</th>
<th>( LR_{trace} )</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.581</td>
<td>( r = 0 )</td>
<td>( r = 1 )</td>
<td>66.89</td>
<td>45.3</td>
<td>209.2</td>
<td>124.62</td>
</tr>
<tr>
<td>2</td>
<td>0.508</td>
<td>( r \leq 1 )</td>
<td>( r = 2 )</td>
<td>54.58</td>
<td>39.4</td>
<td>142.30</td>
<td>94.2</td>
</tr>
<tr>
<td>3</td>
<td>0.394</td>
<td>( r \leq 2 )</td>
<td>( r = 3 )</td>
<td>38.53</td>
<td>33.5</td>
<td>87.76</td>
<td>68.5</td>
</tr>
<tr>
<td>4</td>
<td>0.242</td>
<td>( r \leq 3 )</td>
<td>( r = 4 )</td>
<td>21.30</td>
<td>27.1</td>
<td>49.23</td>
<td>47.2</td>
</tr>
<tr>
<td>5</td>
<td>0.146</td>
<td>( r \leq 4 )</td>
<td>( r = 5 )</td>
<td>12.19</td>
<td>21.0</td>
<td>27.94</td>
<td>29.7</td>
</tr>
<tr>
<td>6</td>
<td>0.135</td>
<td>( r \leq 5 )</td>
<td>( r = 6 )</td>
<td>11.19</td>
<td>14.1</td>
<td>15.74</td>
<td>15.4</td>
</tr>
<tr>
<td>7</td>
<td>0.057</td>
<td>( r \leq 6 )</td>
<td>( r = 7 )</td>
<td>4.55</td>
<td>3.8</td>
<td>4.55</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Table 8.6
Test for cointegration rank for Housing commodity group equation of LA/AIDS model

<table>
<thead>
<tr>
<th>( i )</th>
<th>( \lambda_i )</th>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>( LR_{\max} )</th>
<th>95%</th>
<th>( LR_{trace} )</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.651</td>
<td>( r = 0 )</td>
<td>( r = 1 )</td>
<td>80.95</td>
<td>45.3</td>
<td>235.00</td>
<td>124.62</td>
</tr>
<tr>
<td>2</td>
<td>0.543</td>
<td>( r \leq 1 )</td>
<td>( r = 2 )</td>
<td>60.27</td>
<td>39.4</td>
<td>154.00</td>
<td>94.2</td>
</tr>
<tr>
<td>3</td>
<td>0.394</td>
<td>( r \leq 2 )</td>
<td>( r = 3 )</td>
<td>38.53</td>
<td>33.5</td>
<td>93.74</td>
<td>68.5</td>
</tr>
<tr>
<td>4</td>
<td>0.303</td>
<td>( r \leq 3 )</td>
<td>( r = 4 )</td>
<td>27.81</td>
<td>27.1</td>
<td>55.21</td>
<td>47.2</td>
</tr>
<tr>
<td>5</td>
<td>0.186</td>
<td>( r \leq 4 )</td>
<td>( r = 5 )</td>
<td>15.82</td>
<td>21.0</td>
<td>27.4</td>
<td>29.7</td>
</tr>
<tr>
<td>6</td>
<td>0.110</td>
<td>( r \leq 5 )</td>
<td>( r = 6 )</td>
<td>9.01</td>
<td>14.1</td>
<td>11.58</td>
<td>15.4</td>
</tr>
<tr>
<td>7</td>
<td>0.033</td>
<td>( r \leq 6 )</td>
<td>( r = 7 )</td>
<td>2.57</td>
<td>3.8</td>
<td>2.57</td>
<td>3.8</td>
</tr>
</tbody>
</table>

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### Table 8.7
**Test for cointegration rank for Transportation commodity group equation of LA/AIDS model**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\lambda_i$</th>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>$LR_{\text{max}}$</th>
<th>95%</th>
<th>$LR_{\text{trace}}$</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.571</td>
<td>$r = 0$</td>
<td>$r = 1$</td>
<td>65.2</td>
<td>45.3</td>
<td>217.4</td>
<td>124.62</td>
</tr>
<tr>
<td>2</td>
<td>0.500</td>
<td>$r \leq 1$</td>
<td>$r = 2$</td>
<td>53.43</td>
<td>39.4</td>
<td>152.2</td>
<td>94.2</td>
</tr>
<tr>
<td>3</td>
<td>0.402</td>
<td>$r \leq 2$</td>
<td>$r = 3$</td>
<td>39.58</td>
<td>33.5</td>
<td>98.74</td>
<td>68.5</td>
</tr>
<tr>
<td>4</td>
<td>0.308</td>
<td>$r \leq 3$</td>
<td>$r = 4$</td>
<td>28.35</td>
<td>27.1</td>
<td>59.15</td>
<td>47.2</td>
</tr>
<tr>
<td>5</td>
<td>0.197</td>
<td>$r \leq 4$</td>
<td>$r = 5$</td>
<td>16.94</td>
<td>21.0</td>
<td>30.81</td>
<td>29.7</td>
</tr>
<tr>
<td>6</td>
<td>0.130</td>
<td>$r \leq 5$</td>
<td>$r = 6$</td>
<td>10.73</td>
<td>14.1</td>
<td>13.87</td>
<td>15.4</td>
</tr>
<tr>
<td>7</td>
<td>0.040</td>
<td>$r \leq 6$</td>
<td>$r = 7$</td>
<td>3.15</td>
<td>3.8</td>
<td>3.15</td>
<td>3.8</td>
</tr>
</tbody>
</table>

### Table 8.8
**Test for cointegration rank for Other commodity group equation of LA/AIDS model**

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\lambda_i$</th>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>$LR_{\text{max}}$</th>
<th>95%</th>
<th>$LR_{\text{trace}}$</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.604</td>
<td>$r = 0$</td>
<td>$r = 1$</td>
<td>71.28</td>
<td>45.3</td>
<td>228.1</td>
<td>124.62</td>
</tr>
<tr>
<td>2</td>
<td>0.522</td>
<td>$r \leq 1$</td>
<td>$r = 2$</td>
<td>56.78</td>
<td>39.4</td>
<td>156.9</td>
<td>94.2</td>
</tr>
<tr>
<td>3</td>
<td>0.418</td>
<td>$r \leq 2$</td>
<td>$r = 3$</td>
<td>41.66</td>
<td>33.5</td>
<td>100.1</td>
<td>68.5</td>
</tr>
<tr>
<td>4</td>
<td>0.320</td>
<td>$r \leq 3$</td>
<td>$r = 4$</td>
<td>29.70</td>
<td>27.1</td>
<td>58.43</td>
<td>47.2</td>
</tr>
<tr>
<td>5</td>
<td>0.184</td>
<td>$r \leq 4$</td>
<td>$r = 5$</td>
<td>15.68</td>
<td>21.0</td>
<td>28.73</td>
<td>29.7</td>
</tr>
<tr>
<td>6</td>
<td>0.116</td>
<td>$r \leq 5$</td>
<td>$r = 6$</td>
<td>9.49</td>
<td>14.1</td>
<td>13.05</td>
<td>15.4</td>
</tr>
<tr>
<td>7</td>
<td>0.045</td>
<td>$r \leq 6$</td>
<td>$r = 7$</td>
<td>3.56</td>
<td>3.8</td>
<td>3.56</td>
<td>3.8</td>
</tr>
</tbody>
</table>
The eigenvectors, which are also called cointegrating vectors, form the cointegrating matrix $\beta$. Accordingly, each matrix has been determined to be of rank 4 for all commodity groups except Food & Beverages. The Clothing & Footwear commodity group will have only three cointegrating vectors. The following Table 8.9 presents the eigenvectors ($\beta'$'s) computed in the VAR (3). All the estimated parameters for all categories are normalized by the corresponding budget shares.

Upon the initial investigation, it appears that the parameters of the third vector are closer to the demand theory principle for four main reasons: First; most own price coefficients for commodity groups are negative for the third equilibrium relationships (vector $\beta_3$). Second, the parameters of the real total expenditure are less than one. Third, the sum of the real total expenditure coefficients is very close to zero for the third equilibrium relationship of the LA/AIDS model. Fourth, the classification of the demand for products is more logical under the third equilibrium relationship and this result was obtained under the dynamic LA/AIDS model in chapter 6.

Table 8.9 presents the three equilibrium relationships of Bahrain data of $z_{it}$ in rows. The estimated parameters are normalized by $w_{f&b}$. The budget share devoted to Food & Beverage implies that a stationary relationship exists between the variables in $z_{it}$. In other words, the components of $\beta''$ are the parameters that preserve a long run relationship between the variables in the Food & Beverages equation in this demand system model. The adjustment coefficients ($\alpha_r$) acquired in the vector autoregressions of $z_{it}$, which match to the cointegration coefficients of Table 8.9 are presented in Table 8.10.

Among the variables in the third vector of Table 8.9, it appears that the price coefficient of the Other commodity group and own price of Food & Beverages commodity group have the greater influence on the budget share of the Food & Beverages category. The negative coefficient for real total expenditure is in
agreement with the dynamic LA/AIDS model in chapter 6, the latter results were obtained without taking into account the time series issues. From the results of this chapter, goods in the Food & Beverages commodity group show a cointegrating relationship in which the demand for Food & Beverages products is treated as necessities by Bahrain's consumers.

Table 8.9
Estimated cointegrating vectors ($\beta_i$) for the VAR (3) for Food and beverage commodity group equation ($z_{it}$).

<table>
<thead>
<tr>
<th>$\beta$'s</th>
<th>$w_{f&amp;b}$</th>
<th>$l_{p_{f&amp;b}}$</th>
<th>$l_{p_{c&amp;f}}$</th>
<th>$l_{p_h}$</th>
<th>$l_{p_i}$</th>
<th>$l_{p_o}$</th>
<th>$l(x/p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>1.000</td>
<td>-0.055</td>
<td>0.818</td>
<td>0.118</td>
<td>-0.432</td>
<td>-1.016</td>
<td>-3.140</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.000</td>
<td>0.173</td>
<td>-0.122</td>
<td>-0.032</td>
<td>-0.011</td>
<td>-0.035</td>
<td>-0.011</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>1.000</td>
<td>-0.477</td>
<td>-0.056</td>
<td>-0.126</td>
<td>-0.251</td>
<td>0.940</td>
<td>-0.158</td>
</tr>
</tbody>
</table>

where $w_{f&b}$ is the budget share of Food and beverages, and other variables as above are defined.

By looking at the average speed of adjustment towards the estimated equilibrium state a low coefficient indicates slow adjustment and a high coefficient indicates rapid adjustment. In general the adjustment coefficients were slow for all categories. The third equilibrium relationships for all categories (which measure the change in budget share) the average speed adjustments which match these cointegrating vectors for the third relationship are presented in Table 8.10. However, the adjustment coefficients for the rest of commodity groups are presented in Tables 8.12, 8.14, 8.16, and 8.18.

With respect to the rate of adjustment, which seems very low, and in the third equilibrium relationship, changes in the price of Transportation ($l_{p_i}$) account for most feedback when the consumers find that they are not spending the expected amount on the Food & Beverages commodity group.
Table 8.10

Estimated adjustment coefficients ($\alpha_i$) for the VAR (3) of for Food and beverage commodity group equation ($z_{lt}$).

<table>
<thead>
<tr>
<th>$\alpha_i$</th>
<th>$w_{f&amp;b}$</th>
<th>$lp_{f&amp;b}$</th>
<th>$lp_{c&amp;f}$</th>
<th>$lp_h$</th>
<th>$lp_t$</th>
<th>$lp_o$</th>
<th>$l(x/p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.002</td>
<td>0.019</td>
<td>0.005</td>
<td>0.186</td>
<td>0.025</td>
<td>0.000</td>
<td>-0.144</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.001</td>
<td>-0.289</td>
<td>-0.106</td>
<td>0.775</td>
<td>-0.048</td>
<td>-0.143</td>
<td>0.185</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.001</td>
<td>0.002</td>
<td>-0.010</td>
<td>0.006</td>
<td>0.025</td>
<td>-0.009</td>
<td>-0.003</td>
</tr>
</tbody>
</table>

The cointegrating matrix for the VAR (3) of $z_{lt}$ is computed and presented in Table 8.11. All the cointegrating vectors in Table 8.11 are normalized with respect to the budget share of Clothing & Footwear commodity group. These vectors provide a stationary relationship among the variables in $z_{lt}$ such that $\beta'z_{lt} \sim I(0)$.

The greatest influence on the budget share of Clothing & Footwear stem from the prices of Food & Beverages and Other categories (see Table 8.11). The positive sign of the coefficient for real total expenditure implies that the portion of the budget going toward Clothing & Footwear can be explained by a cointegrating relationship in which Clothing & Footwear are considered luxuries by Bahrain's consumers.

Table 8.11

Estimated cointegrating vectors ($\beta_i$) for the VAR (3) for Clothing and footwear commodity group equation ($z_{lt}$).

<table>
<thead>
<tr>
<th>$\beta$'s</th>
<th>$w_{c&amp;f}$</th>
<th>$lp_{f&amp;b}$</th>
<th>$lp_{c&amp;f}$</th>
<th>$lp_h$</th>
<th>$lp_t$</th>
<th>$lp_o$</th>
<th>$l(x/p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>1.000</td>
<td>-0.546</td>
<td>0.528</td>
<td>0.209</td>
<td>-0.093</td>
<td>-0.236</td>
<td>1.157</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.000</td>
<td>3.239</td>
<td>-0.624</td>
<td>-0.433</td>
<td>-0.099</td>
<td>-2.418</td>
<td>-1.360</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>1.000</td>
<td>0.339</td>
<td>-0.009</td>
<td>0.114</td>
<td>0.239</td>
<td>-0.691</td>
<td>0.187</td>
</tr>
</tbody>
</table>

where, $w_{c&f}$ is budget share of Clothing and footwear commodity group
The greatest influence on the budget share of Clothing & footwear stem from the prices of Food & Beverages and Other categories (see Table 8.11). The positive sign of the coefficient for real total expenditure implies that the portion of the budget going toward Clothing & Footwear can be explained by a cointegrating relationship in which Clothing & Footwear are considered luxuries by Bahrain's consumers.

The adjustment coefficients \( (\alpha_i) \) acquired in the vector autoregressions of \( z_{2t} \) which match the cointegration coefficients of Table 8.11 are presented in Table 8.12. In the third cointegrating vector, the bigger weights or adjustment coefficients apply to the price of Transportation, \( (l_{p_t}) \), but was very slow due to the great shock in the system and will take long time to stabilize down. These coefficients indicate the relative importance of every element in \( z_{2t} \) when the processes are out of equilibrium and moving back in the direction of equilibrium.

**Table 8.12**

<table>
<thead>
<tr>
<th>( \alpha_i )</th>
<th>( w_{c&amp;f} )</th>
<th>( l_{p_{f&amp;b}} )</th>
<th>( l_{p_{c&amp;f}} )</th>
<th>( l_{p_h} )</th>
<th>( l_{p_t} )</th>
<th>( l_{p_c} )</th>
<th>( l(x/p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>-0.034</td>
<td>-0.068</td>
<td>-0.054</td>
<td>0.792</td>
<td>0.041</td>
<td>0.059</td>
<td>-0.242</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>-0.006</td>
<td>-0.325</td>
<td>-0.063</td>
<td>0.302</td>
<td>-0.155</td>
<td>0.185</td>
<td>0.364</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>-0.000</td>
<td>-0.001</td>
<td>-0.002</td>
<td>0.001</td>
<td>0.005</td>
<td>-0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

The cointegrating matrix for the VAR (3) of \( z_{2t} \) is computed and presented in Table 8.13. All the cointegrating vectors in Table 8.11 are normalized with respect to budget share of Housing commodity group. The budget share devoted to the Housing implies a stationary relationship among the variables in \( z_{3t} \), such
that $\beta' z_{3t} \sim I(0)$  The greatest influence on the budget share of Housing stem from the price of Other commodity group and real total expenditure (see Table 8.13).

Table 8.13
Estimated cointegrating vectors ($\beta_i'$) for the VAR (3) for Housing commodity group equation ($z_{3t}$).

<table>
<thead>
<tr>
<th>$\beta_i$'s</th>
<th>$w_h$</th>
<th>$lp_{f&amp;b}$</th>
<th>$lp_{e&amp;f}$</th>
<th>$lp_h$</th>
<th>$lp_t$</th>
<th>$lp_o$</th>
<th>$l(x/p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>1.000</td>
<td>-0.293</td>
<td>-0.769</td>
<td>-0.184</td>
<td>0.164</td>
<td>-0.909</td>
<td>1.597</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.000</td>
<td>5.255</td>
<td>-2.259</td>
<td>-1.010</td>
<td>-0.492</td>
<td>-1.676</td>
<td>-1.162</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>1.000</td>
<td>-0.164</td>
<td>-0.177</td>
<td>0.092</td>
<td>-0.248</td>
<td>0.877</td>
<td>-0.657</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>1.000</td>
<td>0.430</td>
<td>-0.092</td>
<td>0.024</td>
<td>0.222</td>
<td>-0.606</td>
<td>-0.130</td>
</tr>
</tbody>
</table>

where $w_h$ is the budget share of Housing commodity group.

The negative sign of the coefficient for real total expenditure implies that the portion of the budget going towards Housing can be explained by a cointegrating relationship in which housing products are considered necessities by Bahrain consumers.

The adjustment coefficients ($\alpha_i$) acquired in the vector autoregressions of $z_{3t}$ which match the cointegration coefficients of Table 8.13 are presented in Table 8.14. In the third cointegrating vector, the biggest weights or adjustment coefficients among the price coefficients are those applied to the price of the Housing commodity groups ($lp_h$) and then to the price of Transportation commodity groups ($lp_t$). These coefficients indicate the relative importance of every element in $z_{3t}$ when the processes are out of equilibrium and moving back in the direction of equilibrium.
Table 8.14
Estimated adjustment coefficients ($\alpha_i$) for the VAR (3) for Housing commodity group equation ($z_{3t}$).

<table>
<thead>
<tr>
<th>$\alpha_i$</th>
<th>$w$</th>
<th>$l_p f_{ab}$</th>
<th>$l_p c_{ef}$</th>
<th>$l_p h$</th>
<th>$l_p r$</th>
<th>$l_p o$</th>
<th>$l(x/p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>-0.151</td>
<td>0.003</td>
<td>0.026</td>
<td>-0.363</td>
<td>-0.075</td>
<td>0.009</td>
<td>0.185</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.060</td>
<td>-0.359</td>
<td>-0.110</td>
<td>0.636</td>
<td>-0.034</td>
<td>0.178</td>
<td>0.265</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>-0.048</td>
<td>-0.003</td>
<td>0.034</td>
<td>0.066</td>
<td>-0.040</td>
<td>0.038</td>
<td>-0.030</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.005</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.004</td>
<td>-0.012</td>
<td>0.000</td>
<td>0.005</td>
</tr>
</tbody>
</table>

The cointegrating matrix for the VAR (3) of $z_{4t}$ is computed and presented in Table 8.15. All the cointegrating vectors in Table 8.15 are normalized with respect to the budget share of the Transportation commodity group. The budget share devoted to the Transportation commodity group implies that a stationary relationship among the variables is $z_{4t} \beta' z_{4t} \sim I(0)$. The greatest influence on the budget share of Transportation stem from the price of Other commodity group ($l_p o$) and real total expenditure $l(x/p)$ See Table 8.15.

Table 8.15
Estimated cointegrating vectors ($\beta_i$) for the VAR (3) for Transportation commodity group equation ($z_{4t}$).

<table>
<thead>
<tr>
<th>$\beta_i$</th>
<th>$w$</th>
<th>$l_p f_{ab}$</th>
<th>$l_p c_{ef}$</th>
<th>$l_p h$</th>
<th>$l_p r$</th>
<th>$l_p o$</th>
<th>$l(x/p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>1.000</td>
<td>0.278</td>
<td>-0.562</td>
<td>-0.116</td>
<td>0.049</td>
<td>0.612</td>
<td>0.676</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>1.000</td>
<td>-5.967</td>
<td>1.669</td>
<td>1.062</td>
<td>0.250</td>
<td>3.528</td>
<td>1.001</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>1.000</td>
<td>-0.116</td>
<td>-0.170</td>
<td>-0.083</td>
<td>-0.227</td>
<td>0.783</td>
<td>-0.539</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>1.000</td>
<td>0.292</td>
<td>-0.247</td>
<td>0.041</td>
<td>0.092</td>
<td>-0.111</td>
<td>-0.160</td>
</tr>
</tbody>
</table>

where $w_t$ is the budget share of Transportation commodity group.
The negative sign coefficient for real total expenditure implies that the portion of the budget going toward transportation can be explained by a cointegrating relationship in which transportation products are considered necessities by Bahraini consumers.

The adjustment coefficients \( (\alpha_i) \) acquired in the vector autoregressions of \( z_{4t} \) which match to the cointegration coefficients of Table 8.15 are presented in Table 8.16. In the third cointegrating vector, the biggest weights or adjustment coefficients apply to the price of Transportation \( (l_p) \) and secondly to the price of the Housing commodity group \( (l_{ph}) \). These coefficients indicate the relative importance of every element in \( z_{4t} \) when the processes are out of equilibrium and moving back in the direction of equilibrium.

<table>
<thead>
<tr>
<th>( \alpha's )</th>
<th>( w_t )</th>
<th>( l_p _ak )</th>
<th>( l_p _cf )</th>
<th>( l_p _h )</th>
<th>( l_p _l )</th>
<th>( l_p _o )</th>
<th>( l(x/p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>-0.175</td>
<td>0.047</td>
<td>-0.066</td>
<td>-0.742</td>
<td>-0.222</td>
<td>-0.012</td>
<td>-0.393</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>0.004</td>
<td>-0.335</td>
<td>-0.086</td>
<td>0.589</td>
<td>-0.122</td>
<td>0.199</td>
<td>0.281</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>-0.024</td>
<td>0.006</td>
<td>0.036</td>
<td>0.055</td>
<td>-0.075</td>
<td>0.026</td>
<td>-0.032</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>0.014</td>
<td>-0.003</td>
<td>0.000</td>
<td>-0.018</td>
<td>-0.016</td>
<td>0.000</td>
<td>0.007</td>
</tr>
</tbody>
</table>

The cointegrating matrix for the VAR (3) of \( z_{5t} \) is computed and presented in Table 8.17. All the cointegrating vectors in Table 8.17 are normalized with respect to the budget share of Other commodity groups. The budget share devoted to the Other category implies that a stationary relationship among the variables in \( z_{5t} \) is that \( \beta' z_{5t} \sim I(0) \).
Table 8.17
Estimated cointegrating vectors (\( \beta' \)) for the VAR (3) for the Other commodity group equation (\( z_{st} \))

<table>
<thead>
<tr>
<th>( \beta' )</th>
<th>( w_o )</th>
<th>( lP_{ft} )</th>
<th>( lP_{c+f} )</th>
<th>( lP_h )</th>
<th>( lP_t )</th>
<th>( lP_o )</th>
<th>( l(x/p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>1.000</td>
<td>-0.324</td>
<td>4.006</td>
<td>0.827</td>
<td>-2.121</td>
<td>-5.207</td>
<td>-14.075</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>1.000</td>
<td>-16.546</td>
<td>7.628</td>
<td>3.574</td>
<td>1.324</td>
<td>4.282</td>
<td>1.320</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>1.000</td>
<td>0.095</td>
<td>0.304</td>
<td>0.183</td>
<td>0.257</td>
<td>-1.473</td>
<td>0.965</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>1.000</td>
<td>-1.137</td>
<td>0.135</td>
<td>-0.049</td>
<td>-0.589</td>
<td>1.424</td>
<td>0.208</td>
</tr>
</tbody>
</table>

where \( w_o \) is the budget share of Other commodity group.

The greatest influence on the budget share of Other commodity group stem from its own price and real total expenditure (see Table 8.17). The positive sign coefficient for total real expenditure implies that the portion of the budget going towards other commodity groups can be explained by a cointegrating relationship in which other category products are considered luxuries by Bahraini consumers.

It appears from Tables 8.9, 8.11, 8.13, 8.15, and 8.17 of the estimated cointegrating vectors, that the price of the Other commodity group has the most effect on the budget shares of most of the commodity groups. The adjustment coefficients (\( \alpha_j \)) acquired in the vector autoregressions of \( z_{st} \), which match the cointegration coefficients of Table 8.17 are presented in Table 8.18. In the third cointegrating vector the biggest weights or adjustment coefficients is applied to the price of the Housing commodity group (\( lP_h \)) and then to the price Transportation commodity group (\( lP_t \)). These coefficients indicate the relative importance of every element in \( z_{st} \) when the processes are out of equilibrium and moving back in the direction of equilibrium.
Table 8.18
Estimated adjustment coefficients \( (\alpha_i) \) for the VAR (3) for the Other commodity group equation \( (z_{it}) \).

<table>
<thead>
<tr>
<th>( \alpha )'s</th>
<th>( w_o )</th>
<th>( lp_{f&amp;b} )</th>
<th>( lp_{z&amp;f} )</th>
<th>( lp_h )</th>
<th>( lp_i )</th>
<th>( lp_o )</th>
<th>( l(x/p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>-0.018</td>
<td>0.003</td>
<td>-0.001</td>
<td>0.048</td>
<td>0.008</td>
<td>-0.001</td>
<td>-0.030</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>-0.093</td>
<td>-0.331</td>
<td>-0.109</td>
<td>0.704</td>
<td>0.001</td>
<td>0.173</td>
<td>0.186</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>0.079</td>
<td>0.008</td>
<td>0.039</td>
<td>0.048</td>
<td>-0.040</td>
<td>0.034</td>
<td>-0.017</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>-0.006</td>
<td>-0.002</td>
<td>-0.001</td>
<td>-0.003</td>
<td>-0.019</td>
<td>0.001</td>
<td>0.004</td>
</tr>
</tbody>
</table>

It appears from the adjustment coefficients that most of the weights were applied to the prices of the Transportation \((lp_f)\) and then to the price of Housing \((lp_h)\) commodity groups. (See Tables 8.10, 8.12, 8.14, 8.16 and 8.18).

8.7.1.1 Estimated Elasticities for the Dynamic LA/AIDS

Most of the previous studies of consumer demand and the previous chapters of this study computed several of the relative expenditure price elasticities using estimated parameters at mean budget share values. Accordingly, the analysis in this chapter indicates that all budget shares follow a non-stationarity process, the mean and variance of budget shares are functions of time, and hence, the computed elasticities will also be functions of time. Therefore, using these results will lead the decision maker of the country to adopt the wrong policies, regardless of the statistical properties of the estimated coefficients.

Thus, the elasticities will be computed after reaching the stationary budget shares series to avoid the problems indicated above. The expenditure and own price elasticities are computed using the estimated cointegrating vectors and the same formulas as in chapter 6. Total expenditure elasticities present in Table 8.19 and these results indicate that two commodity groups namely Clothing & Footwear, and
Other classify luxuries products, while the remaining commodity groups seem to be necessities (See Table 8.19).

Table 8.19
Total expenditure elasticities own price elasticities for the dynamic LA/AIDS model (Johansen FIML approach) - equation by equation.

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>Total expenditure elasticities $\eta_i$</th>
<th>Uncompensated own-price elasticities $e_{ii}$</th>
<th>Compensated own-price elasticities $e^*_{ii}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food &amp; beverage</td>
<td>0.071</td>
<td>-3.806</td>
<td>-3.794</td>
</tr>
<tr>
<td>Clothing &amp; footwear</td>
<td>3.671</td>
<td>-1.129</td>
<td>-0.872</td>
</tr>
<tr>
<td>Housing</td>
<td>-3.693</td>
<td>-0.343</td>
<td>-0.860</td>
</tr>
<tr>
<td>Transportation</td>
<td>-5.738</td>
<td>-3.838</td>
<td>-4.297</td>
</tr>
<tr>
<td>Other</td>
<td>2.787</td>
<td>-3.728</td>
<td>-2.223</td>
</tr>
</tbody>
</table>

(*$\eta_i$ is expenditure elasticities. $e_{ii}$ and $e^*_{ii}$ are uncompensated own and cross price elasticities respectively. $e^*_{ii}$ is compensated own price elasticity.

However, in the application of the dynamic model of LA/AIDS in chapter 6, without taking account of the time series properties, the results indicated that the only other products are classified as luxuries, while the rest of the commodity groups are classified as necessities. The validity of negative expenditure elasticities for both Housing and Transportation in Table 8.19 can be verified by the fact that consumption per capita has declined over the observation period for these commodity groups.

Further, all the own price compensated and uncompensated elasticities are observed to be negative, as expected, given the law of demand. Finally, the relative higher price elasticities of most commodity groups also indicate that price intervention for these products of commodity groups can have a significant impact on consumption. So it is important for the policy-makers to consider the effects of price and subsidy policies on these goods. This analysis also has important implications for...
government production strategies relating to price and subsidy policies and production of reasonable standards of social welfare.

8.7.2 FIML Estimation of the Dynamic LA/AIDS - as full system.
The Maximum Likelihood approach can also be used also to tests for cointegration for the complete demand system of LA/AIDS model, which includes all budget, shares, prices and total expenditure variables. The selection of VAR (3) was based on the Schwarz Bayesian Criterion (SBC). A VAR (3) system is estimated for $z_t$.

where $z_t$ is

$$z_t = (w_{fb}, w_{csf}, w_t, w_o; l_p_{fb}, l_p_{csf}, l_p_h, l_p_t, l_p_o, I(x/p))$$ (8.37)

and it is determined by the all variables in the full demand system model which contains of five commodity groups equations.

In order to estimate the system one, the equation must be dropped to avoid perfect collinearity within the system. In this application, the equation of clothing & footwear commodity group was dropped.

The $LR_{max}$ and $LR_{trace}$ are computed to test for cointegration over the variables in full LA/AIDS model. With respect to the consumer theory there should be four stationary relationships within the LA/AIDS model. Table 8.20 presents the values of statistics of both $LR_{max}$ and $LR_{trace}$ and critical values computed for every potential value of $r$ in the vector autoregression.

From the first investigation, it appears from the results of the maximum eigenvalue test $LR_{max}$; the null of no cointegration ($r = 0$) is rejected in favor of the alternative of cointegration ($r = 1$). The computed value of the $LR_{max}$ statistic exceeds the critical value of 66.2 at the 5% level of significance. The subsequent hypothesis that the number of cointegrating vectors is at most one ($r = 1$) is also rejected in favor of the alternative of at most two cointegrating ($r = 2$). The computed $LR_{max}$
The statistic is greater than the critical value of 61.3 that signs the 95% confidence interval. The null hypothesis of two cointegrating vectors \((r \leq 2)\) tested against that of three cointegrating vectors \((r = 3)\) is also rejected. The computed \(L_{\text{max}}\) statistic is greater than the critical value of 55.5 that signs the 95% confidence interval. Furthermore, the null hypothesis of three cointegrating vectors \((r \leq 3)\) is tested against that of four cointegrating vectors \((r = 4)\) is also rejected. The computed \(L_{\text{max}}\) statistic is greater than the critical value of 49.4 that signs the 95% confidence interval.

### Table 8.20

**Test for cointegration rank for full dynamic system of LA/AIDS model**

<table>
<thead>
<tr>
<th>(i)</th>
<th>(\lambda_i)</th>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>(L_{\text{max}})</th>
<th>95%</th>
<th>(L_{\text{max}})</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.760</td>
<td>(r = 0)</td>
<td>(r = 1)</td>
<td>109.9</td>
<td>66.2</td>
<td>453.5</td>
<td>263.4</td>
</tr>
<tr>
<td>2</td>
<td>0.671</td>
<td>(r \leq 1)</td>
<td>(r = 2)</td>
<td>85.58</td>
<td>61.3</td>
<td>343.6</td>
<td>222.2</td>
</tr>
<tr>
<td>3</td>
<td>0.551</td>
<td>(r \leq 2)</td>
<td>(r = 3)</td>
<td>62.09</td>
<td>55.5</td>
<td>258.0</td>
<td>182.8</td>
</tr>
<tr>
<td>4</td>
<td>0.522</td>
<td>(r \leq 3)</td>
<td>(r = 4)</td>
<td>56.76</td>
<td>49.4</td>
<td>195.9</td>
<td>146.8</td>
</tr>
<tr>
<td>5</td>
<td>0.423</td>
<td>(r \leq 4)</td>
<td>(r = 5)</td>
<td>42.33</td>
<td>44.0</td>
<td>139.2</td>
<td>114.9</td>
</tr>
<tr>
<td>6</td>
<td>0.386</td>
<td>(r \leq 5)</td>
<td>(r = 6)</td>
<td>37.55</td>
<td>37.5</td>
<td>96.86</td>
<td>87.3</td>
</tr>
<tr>
<td>7</td>
<td>0.234</td>
<td>(r \leq 6)</td>
<td>(r = 7)</td>
<td>20.56</td>
<td>31.5</td>
<td>59.31</td>
<td>63.0</td>
</tr>
<tr>
<td>8</td>
<td>0.202</td>
<td>(r \leq 7)</td>
<td>(r = 8)</td>
<td>17.39</td>
<td>25.5</td>
<td>38.75</td>
<td>42.4</td>
</tr>
<tr>
<td>9</td>
<td>0.176</td>
<td>(r \leq 8)</td>
<td>(r = 9)</td>
<td>14.88</td>
<td>19.0</td>
<td>21.35</td>
<td>25.3</td>
</tr>
<tr>
<td>10</td>
<td>0.081</td>
<td>(r \leq 9)</td>
<td>(r = 10)</td>
<td>6.47</td>
<td>12.3</td>
<td>6.47</td>
<td>12.3</td>
</tr>
</tbody>
</table>

However, the null of four cointegrating vectors \((r \leq 4)\) against that of five cointegrating \((r = 5)\) vectors is not rejected at the 5% level of significance as the critical value is 44.0. The findings suggest that there are four cointegrating vectors in this system, which is equal to the number implied by the consumer theory.
With respect to the $LR_{\text{trace}}$ trace test, the results indicate that the first test is of the null of no cointegration ($r = 0$) against the general alternative of cointegration ($r \leq n$). The computed value of the $LR_{\text{trace}}$ statistic exceeds the critical value of 263.4 suggesting cointegration among the variables in the full system. With further investigation, the null hypotheses ($r \leq 1$), ($r \leq 2$), ($r \leq 3$), ($r \leq 4$) and ($r \leq 5$) are also rejected.

Finally, at the 5% level of significance, the value of ($r \leq 6$) is not rejected due to the statistical value (59.3) less than critical value (63.0) at 5% level of significance. The findings suggest that there are six cointegrating vectors in this system. These findings can be clearly seen in Table 8.20.

However, the calculated values of the $LR_{\text{max}}$ (42.33) and $LR_{\text{trace}}$ (139.2) statistics are less than the critical values of $LR_{\text{max}}$ (51.31) and $LR_{\text{trace}}$ (143.1) at 1% level, when the null hypothesis of ($r \leq 4$) is tested relative to the alternative of ($r = 5$). Thus, the results of the maximum eigenvalues test imply that there are four cointegrating vectors in the full dynamic system of LA/AIDS model.

Tables 8.21 and 8.22 present the four estimated cointegrating vectors ($\beta'$ matrix) and the parallel adjustment coefficient ($\alpha'$ matrix) estimated for the full system. The main results of a test of homogeneity and test for the significance of every variable in the cointegrating matrix are also presented in Table 8.21.

According to Table 8.21, it appears from the positive eigenvalue estimated for the real expenditure variables in the third and four eigenvectors that two commodity groups namely, Transportation and Other products represent luxury products for Bahraini consumers, while Food & Beverages and Housing commodity groups expenditures are relative necessities. By the adding-up condition, it also appears that
the demand for the products of the Clothing & Footwear commodity groups seem to be necessities products with respect to the result.

In chapter 6, the static and the dynamic models of LA/AIDS, without taking account of the time series properties, classified only the other commodity groups as luxuries while the rest of the commodity groups are classified as necessities.

Table 8.21
Estimated cointegrating vectors (β') for the VAR (3) of \( z_t \), and the test statistics for the full system of LA/AIDS model

<table>
<thead>
<tr>
<th>( \beta' )</th>
<th>( w_{tfb} )</th>
<th>( w_h )</th>
<th>( w_i )</th>
<th>( w_o )</th>
<th>( lp_{tfb} )</th>
<th>( lp_{c,f} )</th>
<th>( lp_h )</th>
<th>( lp_i )</th>
<th>( lp_o )</th>
<th>( l(x/p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_1 )</td>
<td>1.000</td>
<td>0.086</td>
<td>0.294</td>
<td>0.104</td>
<td>0.122</td>
<td>0.035</td>
<td>0.001</td>
<td>0.013</td>
<td>0.022</td>
<td>-0.185</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>-18.45</td>
<td>1.000</td>
<td>3.110</td>
<td>0.179</td>
<td>2.769</td>
<td>1.821</td>
<td>-0.329</td>
<td>0.184</td>
<td>0.018</td>
<td>-0.202</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>0.684</td>
<td>1.027</td>
<td>1.000</td>
<td>0.090</td>
<td>-0.444</td>
<td>0.265</td>
<td>0.040</td>
<td>-0.100</td>
<td>-0.229</td>
<td>0.232</td>
</tr>
<tr>
<td>( \beta_4 )</td>
<td>13.581</td>
<td>-3.276</td>
<td>-0.612</td>
<td>1.000</td>
<td>-1.341</td>
<td>1.060</td>
<td>0.017</td>
<td>0.372</td>
<td>-1.083</td>
<td>0.417</td>
</tr>
<tr>
<td>( LR_{\beta=0} )</td>
<td>31.21</td>
<td>19.85</td>
<td>38.52</td>
<td>12.48</td>
<td>34.56</td>
<td>28.16</td>
<td>23.38</td>
<td>17.57</td>
<td>54.33</td>
<td>49.62</td>
</tr>
</tbody>
</table>

where \( z_t \) is determined by the all variables in the full demand system model which contains of five commodity groups equations.

Furthermore, it appears from Table 8.21 that all of the price coefficients for the own-price variables are negative except that for the Food & Beverage commodity group which is found to be positive. Table 8.22 shows the estimated adjustment coefficients for the VAR (3) of \( z_t - \alpha' \), for the full dynamic LA/AIDS model.

The adjustment matrix \( \alpha' \), suggests that in general the influence on the consumer adjustment process back towards equilibrium is not very slow as in the case of individual equation for all price variables in the system as indicated from the Table 8.22, but in regard to the strongest one, it seems from 8.22 that for the first, second, and third cointegrating vectors are from the price of Housing commodity group,
while for the four cointegrating vectors are from real total expenditure and then from the price of the Food & Beverages commodity group.

### Table 8.22

<table>
<thead>
<tr>
<th>$\alpha$'s</th>
<th>$w_{f&amp;c}$</th>
<th>$w_h$</th>
<th>$w_t$</th>
<th>$w_o$</th>
<th>$lp_{f&amp;h}$</th>
<th>$lp_{e&amp;f}$</th>
<th>$lp_h$</th>
<th>$lp_t$</th>
<th>$lp_o$</th>
<th>$l(x/p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>-0.020</td>
<td>1.218</td>
<td>0.838</td>
<td>-1.974</td>
<td>1.363</td>
<td>0.376</td>
<td>3.476</td>
<td>2.011</td>
<td>0.316</td>
<td>-2.959</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.009</td>
<td>-0.061</td>
<td>0.010</td>
<td>0.030</td>
<td>0.205</td>
<td>0.064</td>
<td>-0.462</td>
<td>0.074</td>
<td>-0.049</td>
<td>-0.115</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>-0.028</td>
<td>0.820</td>
<td>-0.027</td>
<td>-0.844</td>
<td>-0.258</td>
<td>-0.211</td>
<td>-0.855</td>
<td>-0.358</td>
<td>-0.523</td>
<td>0.242</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>0.025</td>
<td>-0.221</td>
<td>-0.004</td>
<td>0.272</td>
<td>-0.086</td>
<td>-0.039</td>
<td>-0.048</td>
<td>0.078</td>
<td>-0.084</td>
<td>-0.150</td>
</tr>
</tbody>
</table>

where $z_r$ is determined by the all variables in the full demand system model which contains of five commodity groups equations.

Johansen and Juselius (1990) introduced a hypothesis testing within the VAR approach, which utilized the identical procedure of reduced rank regression to acquire eigenvalues without the restriction imposed, $\lambda$, and with a restriction imposed, $\lambda_{R_r}$. Hence, the test of a linear restriction depends on the likelihood ratio idea. The likelihood ratio test can be defined as:

$$LR_r = -T\sum_i \ln\{(1-\lambda_{R_r})/(1-\lambda_i)\}$$

which is asymptotically $\chi^2$ distributed with $r$ degrees of freedom. The null hypothesis indicates that the restriction holds while the alternative suggests a rejection of a given restriction (the values of the $LR_r$ statistic is bigger).

The imposition of the homogeneity restriction can be easily applied by following the same procedures as used in chapter 6. The homogeneity restriction means that the sum (cointegrating vectors) of the price coefficients is equal to zero. That is, $\sum \beta_p = 0$. In other words, $\beta_p = -\beta_{p_1} - \beta_{p_2} - \beta_{p_3} - \beta_{p_4}$. With respect to the LR test
results, it is suggested that the homogeneity restriction of degree zero among all price coefficients for full cointegrating vectors of the dynamic system of LA/AIDS model is not accepted. The critical value for $\chi^2_{(4)}$ is 9.49 at the 5% level of significance is less than the LR computed values (37.11) of the all cointegrating equilibrium relationships of the full dynamic LA/AIDS model.

However, the homogeneity restriction is accepted only for dynamic versions of LA/AIDS, while, for the static LA/AIDS model it is rejected (see chapter 6). The rejection of the homogeneity restriction according to the consumer theory implies that the consumer suffers from money illusion. From this current study, the sum of the price coefficients does not equal zero, as the results of price variables data represent only the portion of price that consumers pay to obtain the goods and services, but the actual prices which is implied by the theory of the consumer demand cannot be observed.

Furthermore, Table 8.21 recorded the likelihood ratio (LR) computed for a test of the significance of all variables that were included in the cointegrating vectors. The restriction that is imposed is defined as $\beta_s = 0$. The main findings of these impositions suggest that all the variables are significant and must be included in the dynamic LA/AIDS. These results are for full system analysis. Furthermore, most variables of budget share are observed to be significant in the cointegrating range and this supports the hypothesis of simultaneity among the consumer decision making processes.

8.8 Summary and Conclusion.
This chapter is divided into three main sections. In first section I a definition of the word stationary and its significant implications for both policy and for statistical analysis was given. In this section, a graphical analysis is utilized. Formal tests for unit roots are applied and the results mostly conformed with the graphical analysis. The results indicate that most of the time series that will be used in estimating the
linear almost ideal demand system model (LA/AIDS) are shown to be integrated of order one, $I(1)$. In section II the concept of cointegration was presented and description of the procedures that are usually applied to estimate the cointegrating vector and testing for cointegration was given. In this section, the LA/AIDS model is estimated to determine any equilibrium relationships that might exist between the variables of the model. Therefore, the cointegrating techniques are used to provide a framework for a general test of the consumer theory. The results of the residual-based cointegration tests indicate that the null of no cointegration is not rejected at 1% level of significance, as the series are determined not to move together in the long-run within the structure defined by the LA/AIDS model.

In the third section III another attempt was made to analyze the co-movement among budget shares, prices and total expenditure time series. In this section, the Johansen (1988) and Johansen and Juselius (1990) approach is employed. The major aims of doing this analysis is firstly to specify any equilibrium relationships that might exist between the LA/AIDS variables and secondly to test the principle of demand theory of consumer behavior. The main results of applied Johansen (1988) and the Johansen and Juselius (1990) approach are as follows: (1) In the equation by equation analysis the results indicate that there are three equilibrium relationships in Food & Beverages and Clothing & Footwear commodity groups. While in Housing, Transportation, and Other commodity groups, the findings suggest that there are four cointegrating relationships. (2) It seems from the analysis of cointegrating vector that the price of the Other commodity groups has the most effect on the budget shares of most of the commodity groups. However, the analyses of the adjustment coefficients indicate that most of the weights apply to the price of the Transportation commodity group and then to the price of Housing commodity group. (3) The classification of the demand for Clothing & Footwear and Other commodity groups are as luxury products while the remaining commodity groups of the LA/AIDS are as necessity products. (4) All the own price elasticities for compensated and
uncompensated are negative but not all of the former are bigger than the latter as expected.

However, the analyses of a full dynamic complete system which includes all budget shares, prices, and expenditure series suggest four equilibrium relationships which are implied by the demand theory. Moreover, the estimated eigenvectors produce sensible parameters in the context of the consumer theory, that most of the parameters are significant. However, in the full system analysis the classification of the commodity groups between luxuries and necessities is different to that obtained in the individual analysis. In the full system analyses it classifies the Transportation and Other commodity groups as luxuries products, while the rest of commodity groups (namely, Food & Beverages, Clothing & Footwear, and Housing) are as necessities products.

The homogeneity restriction is imposed and the results indicate that this restriction is rejected by the data. Accordingly, the argument that the time series issues are partly responsible for this rejection is not always true.

In short, the equilibrium relationship within the variables is obtained according to the theory of consumer behavior that underlies the LA\AIDS model but the information used does not completely reflect the theory of consumer demand by using Johansen (1988) and the Johansen and Juselius (1990) approach.
References

Journals


**Books**


9.1 Introduction

Statistical demand analyses are of two types: (1) those based on aggregative time series data which depend on quantities and prices of commodities and on aggregate income, and (2) those based on incomes and expenditures of a cross-section of individual families in a given period of time. Earlier work in this study was based on the first type. This chapter focuses on the second type, which is often called a "family or household budget study." This type of study has a relatively long history and has been carried out, in one way or another, in practically every country, which has modern economic statistics. The data are usually collected from a sample of families, which record their expenditures on various commodities for a given period of time (usually not more than one month). The survey can be used as source of data for econometric analysis.

A household budget study is based on inter-household comparison in the same period of time, and therefore it follows that the commodity prices are approximately constant for all households. As a result, the effect of prices on consumption cannot be studied effectively from cross-section data. The latter relationship must be left to time series studies. Income elasticities computed from budget studies predict the effect of specified changes in income on the consumption of individual commodities. However, the transition from cross-section elasticities to elasticities for the economy as a whole is by no means a simple matter. The advantages of doing a cross-section
study is that the cross section income elasticities are useful in predicting changes in aggregate time series data if the effects of price changes are first eliminated from the time series analyses. This type of study can give a first indicator of consumption patterns. Time series data require time to measure the effects of changes in economic variables.

The background to the presents Engel Curve analysis, using data from household income and an expenditure survey, is the fact that the government of Bahrain since December 2000 has eliminated all tax imposed on basic food and has made a reduction in the tax imposed on clothing and footwear imports, plus most imported luxuries. The reduction in tax is from 10% to 7.5%. For motor car imports the tax decreased from 20% to 15%. In addition, the government of Bahrain has reduced electricity and water charges by 50%-25% for the household-sectors. This new tax regime has resulted in substantial income and price changes.

This study examines consumer demand so as to provide the policy maker with some knowledge about consumers' demand and the preference structure.

An Engel Curve is a well-known tool used to determine the pattern of consumption expenditure of various consumer classes in a society. The estimation of Engel curves has a long tradition in empirical economics literature since the formulation of Engel's Law (1857) which states that the income elasticity of food is always less than unity. An Engel Curve describes the relationship between a household's expenditure on a specific commodity and total household expenditure or income. The Engel elasticity of a commodity measures the percentage change in household total expenditure on that commodity relative to a percentage change in household total expenditure or income.

Empirical investigation reveals that different functions may be suitable for different commodities. The choice of a particular function should generally be governed by properties such as goodness of fit, computational simplicity, additivity and
theoretical feasibility (Iyengar, 1967, p.86). Estimating coefficients from functions lacking in these desirable properties may lead to erroneous results.

The Engel Curve has been widely adopted for studying the effects on consumer at changes in the tax structure of the economy. The Engel Curve can also be utilized for anticipating demand led changes in economic structure. Added to that, it is useful for market researchers in forecasting the demand for groups of goods and services.

This chapter analyzes the patterns of consumption in Bahrain using the data from the Family Budget Survey (1994-1995), which provides a picture of the broad spectrum of household expenditure in addition to demand relations and the elasticities of the major consumption items. There is no econometric evidence concerning expenditure behavior in Bahrain. The present research is the first in a planned series of econometric studies to analyze the pattern of consumption in Bahrain using data from the survey.

The structure of this section is as follows: First, functional forms of Engel Curves and the estimation method are discussed in section 9.2. The discussions of previous studies relating to the estimation of Engel Curves are all provided in this section. Secondly, data for household's expenditure on commodity groups are presented in section 9.3. Thirdly, analysis of data and overall results are offered in section 9.4. Fourthly, the hypothesis relating to the household's size examined in section 9.5. Finally, concluding remarks are provided in section 9.6.

9.2 Engel Curve Forms and Estimation Method

Numerous functional forms have been suggested and utilized as the basis for estimating Engel Curves because different functions may be appropriate for different commodities. The choice of a specific function should usually be governed by the properties such as goodness of fit, computational simplicity, additivity and theoretical feasibility (Iyengar, 1967, p.86). Thus, estimating coefficients from functions lacking in these desirable properties may obviously lead to invalid results. 292
In this study seven functional forms for Engel Curves are considered. These specifications are:

(1) Linear
$$X_t = \alpha_i + \beta_i Y + u_i$$  \hspace{1cm} (9.1)

(2) Semi-log
$$X_t = \alpha_i + \beta_i \log Y + u_i$$  \hspace{1cm} (9.2)

(3) Double-log
$$\log X_t = \alpha_i + \beta_i \log Y + u_i$$  \hspace{1cm} (9.3)

(4) Working-Leser (W-L)
$$w_i = \alpha_i + \beta_i \log Y + u_i$$  \hspace{1cm} (9.4)

(5) Inverse
$$X_t = \alpha_i - \frac{\beta_i}{Y} + u_i$$  \hspace{1cm} (9.5)

(6) Log-inverse
$$\log X_t = \alpha_i - \frac{\beta_i}{Y} + u_i$$  \hspace{1cm} (9.6)

(7) Log-log inverse
$$\log X_t = \alpha_i + \frac{\beta_i}{Y} + \delta_i \log Y + u_i$$  \hspace{1cm} (9.7)

where $X_t$ is the expenditure in the $i$-th commodity, $Y$ is total expenditure, $w_i$ is the budget share of commodity ($i$) and $u_i$ is the disturbance term, $\alpha_i$, $\beta_i$, and $\delta_i$ are parameters. All these functions (1) to (7) have been estimated for all the items by using the cross-section data from the household expenditure and income survey (1994-1995).

Allen and Bowley (1935) estimated the linear functional form (9.1) and it represents the simplest form of estimation and analysis. It has the characteristic of a constant marginal propensity to consume with respect to income given by the regression coefficient, but the income elasticity varies at each level of income. The major findings of the application of this form suggest that the linear relationship is not suitable for a wide range of incomes (Podder, 1971, p.384).

This linear functional form satisfies the adding-up restriction of general demand restrictions. Therefore, means, when a set of linear Engel Curve's is fitted to an additive data set, the OLS regression estimates will satisfy the following restrictions:

$$1- \sum \alpha_i = 0, \quad 2- \sum \beta_i = 1 \quad \text{and} \quad 3- \sum u_i = 0$$
The adding-up restriction is a property of the budget constraint, which is independent of whether the consumer is a utility maximiser. It states that the total of the budget shares has to be equal to one at all expenditure levels. It also implies that the marginal budget shares have to be equal to unity at all expenditure levels (i.e. \( \sum \frac{\partial q_i p_i}{\partial Y} = 1 \)). It should be mentioned here, that the adding-up restriction is the only restriction which can be satisfied under these functional forms since the remaining restrictions of general demand function would not be satisfied. As result, all restrictions are in terms of price derivatives and the latter (all prices) is constant under the cross-section data of household demand.

Kalkwani (1977b, p.13) studied different functional forms of Engel Curves and the main conclusion reached was that the linear form was most preferred. Kalkwani’s result was based on distance function criterion.

The disadvantage of utilizing the linear form is that to the fact that demand elasticities for necessities increase with income (Leser, 1963, p.695), and this can be explained if the income elasticity of this form \( \eta_i = \beta_i / \omega_i \) is considered. According to the definition of elasticity, if increased income while prices are held constant this leads to budget shares of necessities to decrease, but for luxuries it increases. This shows that the demand elasticities for necessities increase as income rises while demand elasticities for luxuries decrease as income increases. Leser (1963) also reported that the linear form is valid for high levels of income and not for low levels of income. These conclusions suggest the linear form should not always fit. Goods with a negative \( \alpha_i \), are luxury goods, i.e. have an income elasticity which declines with increasing income. Goods with a positive \( \alpha_i \), are necessities i.e. have an income elasticity which increases with increasing income. For such inferences Prais and Houthakker (1955) formed different functional forms, such as the semi-log hyperbolic form and log reciprocal form, so as to obtain a better description of observed facts.
The semi-log form (9.2) was applied by Prais and Houthakker (1955) in an early classic study of U.K cross sectional data and the authors pointed out that this curve provided a better fit than the double log curve for food items which tend to become necessities at a relatively low level of total expenditure. However, Kakwani (1977) recorded that the semi-log form, which was found to be the best by Prais and Houthakker (1955) on the multiple correlation criterion, was not selected at all. Kakwani (1977) result was based on the distance function criterion. The elasticity of this curve is usually fitted as \( \eta_i = \beta_i / X \) which therefore declines as total expenditure increases. This function has a more serious problem in that it fails to satisfy the adding-up restrictions. That is, if all Engel Curves are of this form, then the sum of expenditures on all commodities would not equal total expenditure, i.e. \( \sum q_i p_i \neq Y \) (Thomas, 1987, p.21). However, the semi-log relationship has the advantage of a variable marginal propensity as well as variable income elasticity at different levels of income. In other words, as mentioned above, under this functional form the distinction between luxury and necessity can be obtained regardless of the levels of income.

The double log form (9.3) is based on the assumption that the income elasticity is constant over the entire range of income, and is given by regression coefficient \( (\eta_i = \beta_i) \), but the marginal propensity is not constant. The advantage of utilizing this functional form is that provides a better fit for non-food items and durable commodities compared with other functional forms (Prias and Houthakker, 1955). However, this functional form also fails as does the semi-log form to satisfy the adding-up restriction. Samad and Hossain (1993,p.104) demonstrated that the double log function leads to erroneous results, since the expenditure pattern of only a few items can be adequately assessed in the framework of this function. Finally, Samad and Hossain indicated that the double log function has frequently been used in the past to investigate the expenditure pattern in Bangladesh, mainly for reasons of simplicity.
Working (1943) and Leser (1963) introduced W-L form (9.4). This form relates budget shares linearly to the logarithm of total income. Furthermore, this model has attracted a good deal of attention in economic literature, and its popularity is assured since Deaton and Muellbauer (1980) recently proposed the AIDS which collapses to the W-L model in cross-section data. Equation (4) satisfies the theory of demand in the sense that the adding-up criterion is satisfied. Therefore, OLS regression estimates will automatically satisfy the following restrictions:

\[ 1-\sum a_i = l, \quad 2-\sum \beta_i = 0, \quad \text{and} \quad 3-\sum u_i = 0 \]

This model gives an elasticity of the form \( \eta_i = \frac{1 + \beta_i}{w_i} \). According to the elasticity definition of the W-L model, the budget share of necessities declines as income increases, leading the income elasticity of necessity goods to decrease and for the luxury goods to drop toward unity. Moreover, when the consumer becomes wealthy, all goods change to be less luxury (Clements, 1987, p. 15). Added to the above, the elasticity form states that a commodity will be a necessity when \( \beta_i \) is negative and a luxury when \( \beta_i \) is positive. Leser (1963) examined five functional forms of Engel Curves, all of which obey the adding-up criterion. Leser made a comparison between these different functional forms using Irish data and the major conclusion was that the W-L model was the best in comparison to other models. The budget shares specification satisfied the adding-up property and is easy to estimate and interpret. Therefore, it has been selected to study the effects of demographic characteristics of the household on the consumption pattern by introducing the factors in terms of a dummy variable on the right hand side of the budget share equation (4).

Prais and Houthakkar (1955) utilize the inverse (9.5) and log inverse (9.6) forms beside the other functional forms mentioned above except the Working-Leser Model. Prais and Houthakker (1955, p. 82) mentioned that these forms have two important properties. First, there is an initial income, below which a commodity is not purchased, and second, there is a satiety level, that is, a maximum to the quantity
of the commodity consumed which is not exceeded however high income may rise. However, these properties may not be applicable to all commodities. Kakwani (1977b, p.13) utilized different forms of the Engel Curve and pointed out that the double-log inverse model is preferable to the double-log and log inverse. The linear form was preferred. To both, Samad and Hossain (1993) estimated different functional forms of Engel Curve for Bangladesh and the main results were that the log-log inverse functional form gave the best fit for most of the items, followed by the log inverse and semi-log functions, while the inverse function was the least useful.

All these forms have the common property that they can be estimated by a simple regression method after an appropriate transformation. Prais and Aitchison (1954) have demonstrated that, whatever the method of grouping, the generalized least-squares estimates are unbiased but have a larger variance than estimates based on individual observations. According to Prais and Aitchison's conclusions ordinary least squares which can be utilized only after an appropriate transformation, i.e. multiplying each class mean by the square root of the corresponding relative frequency in the class. The relative frequency can be defined as $f_i = n_i / N$ where $n_i$ is the number of families earning income in the interval $x_{i-1}$ and $x_i$; $N$ is the total number of families which have been grouped into $(T+1)$ income classes (Kakwani and Podder, 1976, p.142). Thus, the ordinary least squares procedure when applied to the newly transformed variables will result in equivalent estimates as obtained by the generalized least squares.

Prais and Aitchison (1954) have also shown that the variance of the disturbance term is inversely proportional to the number of households within each group and this is a kind of heteroscedasticity. The goodness of fits of the functions have basically been evaluated by (1) the $t-ratios$, (2) adjusted coefficients of determination ($\bar{R}^2$) and (3) standard errors of estimates ($SEE$). Because the functional forms are non-identical, $\bar{R}^2$ and $t-ratios$ alone cannot theoretically be utilized as criteria of good fits. The selection is adopted in which all parameters are statistically significant. Those
functions were selected which showed a lowest value of SEE and as high a value of \( \bar{R}^2 \) as feasible (Rahman, 1980, pp.18-19). These measures were used by Samad and Hossain (1993) when they studied income and expenditure elasticities for the major consumption items in Bangladesh.

Kakwani (1977) made a choice between alternative forms of Engel functions on the grounds of goodness of fit by using distance function rather than multiple correlation coefficient -i.e. \( R^2 \) Jain and Tendulkar (1973) earlier used the distance function as a selection criterion:

\[
D_i^2 = \frac{1}{(n-k)} \sum_{j-i} f_i(X_{ij} - \hat{X}_{ij})^2
\]

(9.8)

where \( X_{ij} \) is the observed level of consumer expenditure in the i-th commodity in the j-th expenditure class, \( \hat{X}_{ij} \) is the expenditure in the i-th commodity in the j-th expenditure class as estimated from a given form of the Engel Curve. \( n \) is the number of expenditure classes and \( k \) is the number of explanatory variables in the given Engel Curve.

There are two main reasons for utilizing the distance function instead of \( R^2 \). First, the comparison of the correlation coefficients of the (9.1), (9.2), (9.4), and (9.5) functional forms with those of the (9.3), (9.6) and (9.7) forms. For the former functional forms, the multiple correlation coefficient measures the degree of explanation of the variance of \( X_i \), while it measures the degree of explanation of the variance of \( \log X_i \) for the latter functional forms, which are, of course, not comparable. The second reason is the problem concerns the grouping of observations which was considered by Prais and Aitchison (1954) and by Cramer (1964). These authors have demonstrated that the correlation coefficient based on grouped data is quite an unsatisfactory estimate of the correlation in the population and consequently, is of little statistical interest.
Furthermore, the estimation of demand relationships with cross-section data treats income as the only independent variable. This means that the number of persons in the family, their age and sex, occupation, religion and similar factors are assumed to be the same for all families. These factors may change over time. Economic applications of estimated demand functions might attach little significance to these factors, which are known as "nuisance" variables. They can, however, be quite important in the underlying relation between expenditure and income, i.e. demographic and other personal characteristics which vary widely over individuals in a single cross-section sample.

Prais and Houthakker (1955) emphasized that households' total expenditure and size are positively correlated and if the latter is not treated explicitly in the formulation of Engel Curve, this may bias the results. Furthermore, variations in household-size have comparatively larger effects on the consumption of certain commodities than variations in total expenditure. The coefficient of household-size captures the effect of economies of scale in consumption among larger households. Houthakker (1957) pointed out that the coefficient of household-size represents a combination of two effects: the first being "specific effect" and the second the "income effect". The "specific effect" refers to the effect resulting from an increase in the need for various commodities when household-size increases. The increase in need is, however, usually less than proportional to the increase in household-size because of "economies of scale" in the large households. The "income effect" refers to the effect when an increase in the household-size makes the family relatively poorer in per capita income terms. If the "specific effect" dominates the "income effect" the coefficient of household-size is positive, and negative otherwise.

Currie (1972, p.27) argued that the omission of family composition from a household budget study and the assignment of its effects to the disturbance term will result in biased estimates of the total expenditure coefficients if there is any

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1 The objective of this chapter is to examine consumption patterns rather than behavior. The study focuses on total expenditure and household-size. Crockett (1967) found that total expenditure and household-size are the most important factors in determining the consumption pattern.
correlation between family composition and family total expenditure. Pollak and Wales (1981) emphasized family composition in the demand function of a household due to its effect on the relative importance of different requirements of the household.

Kakwani (1977b), Ketkar and Ketkar (1987), Binh and Whiteford (1990) and others believe that in the cross sectional studies, households vary not only in their total expenditure but also in age and sex composition, which varies from household to household. In this chapter, the size of the family has been given the status of an independent (explanatory) variable in the demand relationships.

Burner and Al-Mutairi (1993) estimated three functional forms of the Engel Curve (linear, log linear, working-leser) for Kuwait using the family budget survey of 1986-87. The major results are summarized as follows: In general, the findings are in conformity with Engel’s Law, i.e. the share of necessities in total expenditure declines as income increases. The expenditure elasticities of different commodities with respect to household size confirm the existence of economies of scale in the consumption of some of the commodities, particularly food and drink. Finally, the results indicate that the consumption patterns of Kuwaiti and foreign households differ significantly from each other, and these differences in the consumption patterns are found to represent structural as well as behavioral differences.

Accordingly, family composition in the terms of the number of persons will be introduced in this study only and into the double-log form for every commodity group due to data limitations, which do not permit us to do more than this.

9.3 Data and Household's Expenditure on Commodity Groups.

This study is based on the cross-section data reported in the Family Budget Survey (FBS) for the year (1994-1995) which was conducted by the Central Statistical Organization (CSO) of Bahrain. The survey is based on a national sample, covered 3003 households and a comprehensive list of commodities. The commodities were
grouped into (8) major categories, namely: (1) Food Expenses, (2) Clothing Expenses, (3) Housing Related Expenses, (4) Transportation Expenses, (5) Educational Expenses, (6) Medical Care Expenses, (7) Personal Care Expenses, (8) and Miscellaneous Expenses. The total expenditure of the households is classified according to these 8 categories, and for ten income categories as shown in Table 9.1. The data in Table 9.1 will be used to estimate the parameters of each of the seven Engel’s Curve, which are discussed in the section 9.2 and the results of estimating these seven forms will be presented in section 9.4.

However, the most important issue which requires discussion is the independent variable which to be included in the Engel Curve functional forms. Usually, empirical work on family budget studies includes total expenditure as the independent variable instead of total family income, for estimating the Engel elasticity of demand for a commodity. There are two main objections to using total income. The reliability of such data is questioned on two counts. (1): The first problem relates to net family income, which is relevant for estimating the demand relation and not gross income. In the calculation of net income people often forget the exact figures of refunds on income tax. In addition, they often deliberately avoid mentioning subsidiary incomes from property and other sources. (2): The second objection is that even if the correct figures of net family income for the year of the survey are collected, this income can be considered as being made up of two components— the permanent and transitory. It is not possible to separate the permanent family income from the measured family income (Podder, 1971, p.381).

Currie (1972, pp.25-26) argued that the level of income registered in a specific time period may well be distorted by transitory components. The preferred explanatory variable in household budget studies is therefore total expenditure. Friedman (1957) argued that consumption is determined more by permanent than by actual measured income. Houthakker and Taylor (1970) utilized total expenditure rather than income as the explanatory variable in their analysis of the 1960-1 Consumer Expenditure Survey (CES) data and suggested that the first factor loading be much more closely
correlated with total expenditure than with income. For this reason, the total expenditure as the indicator of purchasing power has been considered, again for all the items. That is the per capita income variable in the equations (1) to (7) has been substituted by the corresponding per capita expenditure variable. Furthermore, the budget shares are also calculated from total expenditure data to ensure that the OLS estimates meet the adding-up restriction. Added to the above, may tend to underestimate their income. In the present study therefore total expenditure will be utilized as the independent variable in the Engel functional forms of empirical work.

In order to estimate the demand parameters, a set of grouped data (Table 9.1), which has been published by the Central Statistical Organization of Bahrain, has been utilized. The set of grouped data was from the survey of households and was based on a national sample, covered 3003 households and a comprehensive list of commodities. The sample size was considered by the Department of Statistics to be representative, since it was selected from various social classes and from different locations. The sample survey aimed at collecting data relating to a household’s income, households expenditure on goods and services, composition of the family, age of the head of household, job status of each family member, marital status and higher education of the head of household. In this study the grouped data will be used, ungrouped data could not be used because this was not made available by the Central Statistical Organization.

Thus, expenditure is grouped by the Central Statistical Organization into eight major commodity groups as shown in Table 9.1. Data has been classified according to per capita total expenditures on each commodity group classified according to the family's income per annum. The data are used to estimate an Engel Curve for each commodity group.

Table 9.1 indicates that the highest proportion of consumption expenditure of Bahraini households is spent in the housing commodity group. Expenditure in the housing commodity group accounts for 32.2% of total expenditure. Expenditure on
the food commodity group represents the second highest proportion, and accounts for 22.9% of total expenditure. Expenditures on 'transportation' and 'miscellaneous' are 14.8% and 13.2% of total expenditure respectively. These results indicate that 83.1% of household's total expenditure is devoted to housing, food, transportation and miscellaneous commodity groups.

In addition, it can be observed from Table 9.1 that there are signalized differences in the distribution of total expenditure according to the class of annual household's income. For the class of annual household income, 2400 to 9600, people spend a higher proportion on the food commodity group than on the housing commodity group. For the classes of annual household income, '9601 to 30000' and 'less than 2400', people tend to spend a higher proportion on the housing categories.

However, the expenditure on transportation and miscellaneous, represents the third and fourth relatively important need for the people within all classes of annual household income. Furthermore, it is observed from Table 9.1 that people tend to spend more than their income and that for the first four classes of annual household income. This could be related to the underestimation of their income during the survey periods (the reporters may not provide a full account of all income from all sources). Second, some consumer units, classified in lower income classes could have expenditure levels that are more typical of upper income consumer units. Their expenditures raise the average expenditure levels of the income class in which they are classified. Additional reasons include: Consumer units whose members are spell of unemployed, may draw on their savings to maintain their expenditure. Self employed consumers may experience business losses that result in low or even negative incomes but they are able to maintain their expenditures by borrowing or relying on savings. Retirees may rely on savings and investments.
| Class of annual household income | Food | Clothing | Housing | Transportation | Education | Medical care | Personal care | Miscellaneous | Total Expenditure | Per capita income |
|---------------------------------|------|----------|---------|---------------|-----------|-------------|--------------|---------------|-----------------|-----------------|-----------------|
| Less than 2400                  | 304.8| 48.4     | 313.6   | 48.1          | 10.0      | 7.8         | 25.9         | 47.7          | 803.3           | 528.2           |
| 2400-4800                       | 310.5| 62.4     | 266.8   | 80.9          | 25.6      | 8.0         | 31.5         | 64.8          | 850.4           | 586.7           |
| 4801-7200                       | 342.5| 83.6     | 308.4   | 123.2         | 27.1      | 10.6        | 38.0         | 96.7          | 1030.3          | 948.2           |
| 7201-9600                       | 382.5| 109.3    | 376.0   | 155.3         | 30.9      | 18.1        | 47.8         | 128.9         | 1248.9          | 1250.0          |
| 9601-12000                      | 402.6| 130.4    | 471.6   | 206.1         | 47.1      | 22.0        | 60.9         | 187.5         | 1528.2          | 1529.3          |
| 12001-15000                     | 434.8| 149.8    | 570.0   | 252.8         | 69.1      | 31.9        | 65.7         | 254.8         | 1828.9          | 2010.5          |
| 15001-18000                     | 499.1| 172.4    | 783.8   | 350.2         | 104.7     | 35.7        | 80.5         | 348.2         | 2387.3          | 2416.2          |
| 18001-24000                     | 512.3| 175.1    | 1016.1  | 488.4         | 131.7     | 48.5        | 92.1         | 413.9         | 2866.5          | 2933.7          |
| 24001-30000                     | 571.0| 212.0    | 1294.6  | 728.1         | 264.7     | 139.0       | 137.9        | 714.0         | 4061.3          | 4506.1          |
| +300000                         | 637.5| 334.3    | 2298.9  | 1274.8        | 528.1     | 260.1       | 166.1        | 1118.9        | 6618.5          | 8137.7          |
| Average per capita expenditure  | 406  | 126.2    | 572.0   | 262.0         | 78.6      | 35.8        | 60.6         | 234.1         | 1775.3          | 9444.3          |
| %                               | 22.9 | 7.1      | 32.2    | 14.8          | 4.4       | 2.0         | 3.4          | 13.2          | 100             | 100             |

Sources: Central Statistical Organization, "Household expenditure and income survey (1994-1995)", Tables (3.85) and (4.48) pages 409 and 536 respectively, Ministry of Cabinet, State of Bahrain.
9.4 Overall Results and Analysis of Data.

The analysis has been carried out with the Household Expenditure and Income Survey (1994-1995) data of Bahrain, which is published by the Central Statistics Organization. It is grouped data based on income deciles. The analysis and the results for each functional forms of Engel's Curve are presented in Table 9.2.

The linear form results have been estimated for each of eight commodity groups. Table 9.2 contains estimates of the marginal budget shares, \( \beta \), the \( \alpha \), estimates and the coefficients of determination, \( R^2 \) for Bahrain budget expenditure. The estimated results seem to be plausible from a statistical point of view, since all the coefficient parameters and intercept terms are significantly different from zero at the five percent level of significance. Overall fit is relatively high, since the values of the coefficients of determination, \( R^2 \) are relatively high for most of the equation estimates within the system, i.e, the value of \( R^2 \) exceeds 0.78 in seven equations out of the eight equations. The values of \( R^2 \) seem to be very low for only the food equation (0.66). However, the standard error of the estimates (SEE) for all intercept terms is very high.

The estimated marginal budget shares, \( \beta \), satisfy a prior knowledge, since each \( \beta \) estimate is greater than zero and less than unity for all commodity groups. Furthermore, the sum of the \( \beta \) estimates is equal to unity and the sum of \( \alpha \) is equal to zero. These results, however, satisfied the additivity condition or Engel aggregation, which is implied by the utility theory. Thus, the estimated results seem to be plausible from statistical and economical points of view. The estimated results for the semi-log fit the data well for most of the commodity group except for the food equation where the \( R^2 \) found is very low (0.21). The goodness of fit ranges between 0.62 for medical care equation and 0.97 for personal care commodity group.
Table 9.2
Linear, Semi-log, and Double log forms results of household expenditure in Bahrain (1994-1995)

| Commodity Groups          | Linear form | | | Semi-log form | | | Double-log form | | |
|---------------------------|-------------|---|---|----------------|---|---|----------------|---|
|                           | $\alpha_i$  | $\beta_i$ | $R^2$ | $\alpha_i$  | $\beta_i$ | $R^2$ | $\alpha_i$  | $\beta_i$ | $R^2$ |
| Food Expenses             | 113.07      | 0.024 | 0.66 | -35.07       | 25.89 | 0.21 | 3.03        | 0.28 | 0.32 |
|                           | (21.44)     | (0.02) |      | (103.81)     | (16.47) |      | (0.91)      | (0.14) |      |
| Clothing Expenses         | 15.09       | 0.04  | 0.79 | -122.82      | 26.01 | 0.90 | -1.90       | 0.88 | 0.86 |
|                           | (5.16)      | (0.02) |      | (19.44)      | (3.09) |      | (0.80)      | (0.15) |      |
| Housing & Related Expenses| -16.38      | 0.36  | 0.99 | -529.1       | 185.23 | 0.80 | -0.89       | 0.96 | 0.98 |
|                           | (9.00)      | (0.01) |      | (235.50)     | (32.08) |      | (0.34)      | (0.05) |      |
| Transportation Expenses   | 41.94       | 0.22  | 0.99 | -638.34      | 16.27 | 0.81 | -5.91       | 1.62 | 0.99 |
|                           | (5.74)      | (0.01) |      | (14.83)      | (19.83) |      | (0.37)      | (0.06) |      |
| Educational Expenses      | -27.14      | 0.09  | 0.92 | -259.69      | 45.97 | 0.68 | -9.06       | 1.91 | 0.96 |
|                           | (6.25)      | (0.01) |      | (71.05)      | (11.27) |      | (0.82)      | (0.13) |      |
| Medical Care Expenses     | -14.04      | 0.05  | 0.88 | -124.92      | 22.01 | 0.62 | -8.72       | 1.73 | 0.91 |
|                           | (14.06)     | (0.05) |      | (38.75)      | (6.15) |      | (1.20)      | (0.19) |      |
| Personal Care Expenses    | 6.77        | 0.02  | 0.88 | -63.16       | 13.23 | 0.97 | -2.34       | 0.86 | 0.93 |
|                           | (1.89)      | (0.01) |      | (5.07)       | (0.80) |      | (0.55)      | (0.09) |      |
| Miscellaneous Expenses    | 35.17       | 0.20  | 0.96 | -563.64      | 103.23 | 0.81 | -5.85       | 1.59 | 0.97 |
|                           | (8.61)      | (0.01) |      | (17.87)      | (112.62) |      | (0.66)      | (0.12) |      |

* Standard errors are given in brackets
Moreover, the t-rations indicate that all the $\beta_i$'s, are significantly different from zero at the 5 percent level, and this is true for all intercept terms except for the food equation. Finally, the SEE is very high for all commodity groups and for both the marginal budget shares estimates and intercept terms.

The estimation of the double log form results are presented in Table 9.2. The coefficients of determination, $R^2$, for the total estimates suggest that the overall fit is good for all commodity groups except for the food commodity group equation. The coefficients of determination $R^2$ range from 0.32 for the food equation to 0.99 for transportation equation. The reason behind the low level of $R^2$ for the food commodity group equation could be attributed to miss specification of the model by omitting relevant variables such as family size or other variables which will be examined in section 9.5 of this chapter. The t-ratio indicates that all the coefficient estimated are significantly different from zero at the 5 percent significance level. Second, the SEE is low for both intercept terms and the marginal budget share estimates. Also, the estimated results of $\beta_i$'s for double log form indicate that the demand for four commodity groups equations are inelastic and these equations are food, clothing, housing, and personal care. These results imply that these commodity groups are necessities while for the remaining commodity groups, the demand for it are luxuries.

The results of other functional forms are presented in Table 9.3 and Table 9.4. The estimated results for the Working-Leser functional form has been presented in Table 9.3. Overall fit for the total estimates seem to be unsatisfactory for clothing, housing, and personal care commodity groups as indicated by $R^2$. On the other hand, $R^2$ for food, transportation, education, and miscellaneous commodity groups are high. The t-ratios indicate that five out of eight of the coefficient estimates $\beta_i$ are significantly different from zero at five- percent significance level. It is observed that the estimates of $\beta_i$, which are not significant, are only for those equations that have low. (See Table 9.3). All the intercept terms are significantly different from zero, $\alpha_i$.  

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<table>
<thead>
<tr>
<th>Commodity Groups</th>
<th>Working-Leser form</th>
<th>Inverse form</th>
<th>Log inverse form</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_i$</td>
<td>$\beta_i$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>Food Expenses</td>
<td>1.24</td>
<td>-0.16</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>Clothing Expenses</td>
<td>0.12</td>
<td>-0.10</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Housing &amp; Related Expenses</td>
<td>0.41</td>
<td>-0.01</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>Transportation Expenses</td>
<td>-0.31</td>
<td>0.07</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Educational Expenses</td>
<td>0.16</td>
<td>0.32</td>
<td>0.76</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Medical Care Expenses</td>
<td>-0.07</td>
<td>0.01</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.00)</td>
<td></td>
</tr>
<tr>
<td>Personal Care Expenses</td>
<td>0.06</td>
<td>-0.10</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>(1.02)</td>
<td>(0.00)</td>
<td></td>
</tr>
<tr>
<td>Miscellaneous Expenses</td>
<td>-0.27</td>
<td>0.06</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.01)</td>
<td></td>
</tr>
</tbody>
</table>

* Standard errors are given in brackets
Furthermore, the SEE is low for all estimates of marginal budget shares. As expected, the sum of $\alpha$, estimates is unity and the sum of $\beta$, estimates is zero and in this application they are very close to these conditions but not exactly.

Although the (W-L) functional form is found unsatisfactory for most of the commodity groups due to very low values of $R^2$, it is selected as a best specification between other functional forms, and this conclusion is based on the distance function as a measure to discriminate among different functional forms of a set of Engel Curves. A small value of this measure favors the model in question.

In this application, the smallest value of distance function is found equal to (0.01) and it belongs to the W-L form, and the second is for log-log inverse (0.23), the third is for double log form (0.33), and the fourth is for log inverse (0.73). However, for linear and semi-log and inverse functional forms, the values of distance functions are very high. The results for the inverse functional form indicate that the goodness of fit is very low for all equations (six equations) except for the clothing commodity group equation, and personal care equations. Second, all the marginal budget shares are significantly different from zero for all equations except the medical care equation. Third, all the intercept terms are significantly different from zero. Finally, all the estimates of inverse functional forms have very high standard errors.

On the other hand, the results for the log inverse functional form indicate that overall the equations fitted very highly, except for the food commodity group equation, which is very low. Second, all the marginal shares estimates and intercept terms are significantly different from zero at 5% level. In addition, the standard error for both the budget shares and constant estimates is very high.

Finally, the results for log-log-inverse are the same as the log inverse, but the difference is that the food equation fitted very well (0.72) under this functional form, and that marginal budget share estimates are significant different from zero at 5% level for only four equations, namely, food, clothing, housing, and personal care.
equations. Finally, the estimate of \( \delta_i \) is not significant for only the clothing equation (See Table 9.4).

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>( \alpha )</th>
<th>( \beta_i )</th>
<th>( \delta_i )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food Expenses</td>
<td>9.24</td>
<td>-348.44</td>
<td>-0.56</td>
<td>0.72</td>
</tr>
<tr>
<td></td>
<td>(2.06)</td>
<td>(109.90)</td>
<td>(0.29)</td>
<td></td>
</tr>
<tr>
<td>Clothing Expenses</td>
<td>3.77</td>
<td>-317.60</td>
<td>0.084</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>(1.64)</td>
<td>(87.61)</td>
<td>(0.23)</td>
<td></td>
</tr>
<tr>
<td>Housing &amp; Related Expenses</td>
<td>-3.58</td>
<td>150.33</td>
<td>1.34</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(0.54)</td>
<td>(28.62)</td>
<td>(0.08)</td>
<td></td>
</tr>
<tr>
<td>Transportation Expenses</td>
<td>-4.10</td>
<td>-101.48</td>
<td>1.36</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(1.08)</td>
<td>(57.87)</td>
<td>(0.15)</td>
<td></td>
</tr>
<tr>
<td>Educational Expenses</td>
<td>-9.43</td>
<td>20.81</td>
<td>1.96</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>(2.85)</td>
<td>(152.19)</td>
<td>(0.40)</td>
<td></td>
</tr>
<tr>
<td>Medical Care Expenses</td>
<td>-14.81</td>
<td>341.55</td>
<td>2.58</td>
<td>0.94</td>
</tr>
<tr>
<td></td>
<td>(3.45)</td>
<td>(184.32)</td>
<td>(0.49)</td>
<td></td>
</tr>
<tr>
<td>Personal Care Expenses</td>
<td>1.88</td>
<td>-247.56</td>
<td>0.25</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(0.77)</td>
<td>(41.32)</td>
<td>(0.11)</td>
<td></td>
</tr>
<tr>
<td>Miscellaneous Expenses</td>
<td>-4.96</td>
<td>-49.99</td>
<td>1.46</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>(2.30)</td>
<td>(122.48)</td>
<td>(0.32)</td>
<td></td>
</tr>
</tbody>
</table>

- Standard errors are given in brackets

9.4.1 Elasticities Estimated for Engel Curve Forms.

It is standard practice to interpret the consumption pattern on the basis of elasticities. To estimate Engel elasticities for the eight commodity groups (food, clothing, housing, transportation, education, medical care, personal care and miscellaneous) the seven functional forms which are presented in section (9.2) must be estimated using the Bahrain household grouped data.

Based on the estimated results illustrated above, Engel elasticities of demand with respect to total expenditure for each commodity group are evaluated at the mean values and are presented in Table 9.5 below. These indicate that the demand for the food and clothing commodity groups is clearly inelastic with respect to total expenditure, indicating that these commodity groups are necessities. The seven functional forms of this study shared these results. The demand for transportation, education, medical care, and miscellaneous commodity groups are estimated by five
functional forms namely, linear, double log, Working-Leser, log inverse, and Log-log inverse forms which are considered to be elastic, indicating that these commodity groups are luxuries. However, in the linear and log-log inverse forms the demand for housing is also considered to be luxury products. While, in the log inverse form, the demand for education is only considered to be luxury products.

The semi log form gives the lowest elasticity estimates whereas the linear and the double log form give the highest elasticity estimates if Working-Leser estimates are excluded. These observations were different than those made by Prais and Houthakker (1955) and Kakwani (1977a). The results of these authors were that the hyperbolic gave the lowest elasticity, while the double log gave the highest elasticity estimates. This conclusion originated from British data, while Kakwani’s (1977a) results were based on Indonesian data.

It is worth mentioning that the Working-Leser form gives the highest elasticity estimates for one commodity groups (food) while the linear functional form gives the highest estimates for housing, education, transportation, medical care, and miscellaneous commodity groups.

Table 9.5
Engel elasticities estimates based on Bahrain household expenditure and income survey for the period (1994-1995)

<table>
<thead>
<tr>
<th>Commodity Groups</th>
<th>Linear form</th>
<th>Double log form</th>
<th>Working Leser (W-L) form</th>
<th>Semi-log form</th>
<th>Inverse form</th>
<th>Log-inverse form</th>
<th>Log-log inverse form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food Expenses</td>
<td>0.08</td>
<td>0.28</td>
<td>0.36</td>
<td>0.04</td>
<td>0.12</td>
<td>0.23</td>
<td>0.02</td>
</tr>
<tr>
<td>Clothing Expenses</td>
<td>0.57</td>
<td>0.88</td>
<td>0.86</td>
<td>0.04</td>
<td>0.20</td>
<td>0.58</td>
<td>0.61</td>
</tr>
<tr>
<td>Housing &amp; Related Expenses</td>
<td>1.09</td>
<td>0.96</td>
<td>0.97</td>
<td>0.31</td>
<td>0.22</td>
<td>0.54</td>
<td>1.09</td>
</tr>
<tr>
<td>Transportation Expenses</td>
<td>1.57</td>
<td>1.62</td>
<td>1.50</td>
<td>0.03</td>
<td>0.25</td>
<td>0.97</td>
<td>1.53</td>
</tr>
<tr>
<td>Educational Expenses</td>
<td>2.25</td>
<td>1.91</td>
<td>1.75</td>
<td>0.08</td>
<td>0.27</td>
<td>1.12</td>
<td>1.93</td>
</tr>
<tr>
<td>Medical Care Expenses</td>
<td>2.50</td>
<td>1.73</td>
<td>1.50</td>
<td>0.04</td>
<td>0.27</td>
<td>0.95</td>
<td>2.01</td>
</tr>
<tr>
<td>Personal Care Expenses</td>
<td>0.67</td>
<td>0.86</td>
<td>0.67</td>
<td>0.02</td>
<td>0.20</td>
<td>0.56</td>
<td>0.66</td>
</tr>
<tr>
<td>Miscellaneous Expenses</td>
<td>1.67</td>
<td>1.59</td>
<td>1.50</td>
<td>0.17</td>
<td>0.25</td>
<td>0.94</td>
<td>1.54</td>
</tr>
</tbody>
</table>

The expenditure elasticities for the three functional forms: Inverse, Log inverse and Log-log inverse are given by \( \eta_i = \beta_i/(\alpha_i Y - \beta_i) \), \( \eta_i = \beta_i(1/Y) \) and \( \eta_i = \delta_i - \beta_i/Y \) respectively (see Poadder, 1971, p. 384).

Moreover, the demand for products for all commodity groups with respect to total expenditure is inelastic by two functional forms, namely semi log, and inverse (see
Finally, the double log form gives the highest estimates elasticity for clothing, transportation, and personal care commodity groups equations.

9.5 Hypothesis Examination.

In reality some other expenditure variables such as the size of the household, age, geographical location, etc. may play a role in the determination of expenditure pattern of the consumers. Therefore, family composition as measured by the number of persons will be introduced into the double log model for each commodity group in order to test the hypothesis that family composition has an affect on Engel Curves. Houthakker (1957) applied the double-log model in the following form for several countries:

\[ \log X_i = \alpha_t + \beta \log Y_i + \gamma_t \log n + u_i \]  

(9.9)

where \( n \) is the number of persons in the household and the other notation is as defined above. Houthakker reported that the coefficient of family composition is significant for most commodity groups and for several countries. Equation 9.9 has been fitted to the Bahrain data and the estimated results are presented in Table 9.6.

<table>
<thead>
<tr>
<th>Commodity groups</th>
<th>( \alpha_t )</th>
<th>( \beta_t )</th>
<th>( \gamma_t )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food Expenses</td>
<td>-4.40 (0.98)</td>
<td>-0.13 (0.22)</td>
<td>0.72 (0.32)</td>
<td>0.60</td>
</tr>
<tr>
<td>Clothing Expenses</td>
<td>-0.27 (0.53)</td>
<td>0.38 (0.12)</td>
<td>0.85 (0.17)</td>
<td>0.96</td>
</tr>
<tr>
<td>Housing &amp; Related Expenses</td>
<td>-1.51 (0.30)</td>
<td>1.15 (0.69)</td>
<td>-0.32 (0.10)</td>
<td>0.99</td>
</tr>
<tr>
<td>Transportation Expenses</td>
<td>-6.32 (0.38)</td>
<td>1.44 (0.09)</td>
<td>0.30 (0.13)</td>
<td>0.99</td>
</tr>
<tr>
<td>Educational Expenses</td>
<td>-9.44 (1.11)</td>
<td>2.03 (0.25)</td>
<td>-0.20 (0.37)</td>
<td>0.97</td>
</tr>
<tr>
<td>Medical Care Expenses</td>
<td>-9.94 (1.50)</td>
<td>2.10 (0.34)</td>
<td>-0.64 (0.50)</td>
<td>0.93</td>
</tr>
<tr>
<td>Personal Care Expenses</td>
<td>-1.40 (0.34)</td>
<td>0.52 (0.08)</td>
<td>-0.59 (0.11)</td>
<td>0.98</td>
</tr>
<tr>
<td>Miscellaneous Expenses</td>
<td>-5.25 (0.85)</td>
<td>1.41 (0.19)</td>
<td>0.31 (0.28)</td>
<td>0.97</td>
</tr>
</tbody>
</table>

- Standard errors are given in brackets

Table 9.6

Elasticities estimates of demand with respect to total expenditure and with respect to family size.
The estimated elasticities with respect to total expenditure for food, clothing, and personal care commodity groups are less than one and therefore, confirm Engel's law for the food commodity group. That is, the demand for food commodity groups is inelastic, which implies that the food commodity group, clothing, and personal care commodity groups are necessity products in the view of Bahrain's consumers. On the other hand, the elasticity estimates for housing, transportation, education, medical care and miscellaneous commodity groups are all significantly greater than one. This suggests that the components of these commodity groups are luxuries.

The estimated elasticities with respect to family size are significantly different from zero for food, clothing, housing, transportation, and personal care commodity groups. The t-ratio indicates that the elasticity estimates with respect to family size for education, medical care and miscellaneous are not significantly different from zero. This implies that family size does not affect the demand for these commodity groups.

From Table 9.6 it appears that the elasticity estimated for commodity groups with respect to family size is negative and less than one for housing, education, and personal care commodity groups, and not significant only for education, housing and medical care commodity groups. The negative elasticity with respect to family size implies that an increase in family size hold per capita total expenditure is constant, leads to a reduction in expenditure on those commodity groups, (housing, education, and personal care) as relatively more resources are needed for relatively necessary items. On the other hand, it is found that elasticities are positive and significantly different from zero for the remaining commodity groups equations except for miscellaneous which are not significant different from zero. Houthakker (1957) found that this elasticity estimate was positive and ranged between 0.2 and 0.35.
9.6 Conclusion

This chapter attempted to estimate seven functional forms of Engel's Curve. The estimation depended on cross-section data taken from the household expenditure and income survey for the year 1994-1995. The data was classified into eight commodity groups according to the total income per annum of each household. Ten income categories were considered. The data indicated that per capita expenditure exceeded per capita income for 4 out of 10 income groups. It was also observed that the highest proportion of expenditure was spent on the housing commodity group, which accounts for about 32.2% of total expenditure.

The results for functional forms suggest that the linear functional form satisfied the adding-up restriction which is implied by the economic theory. Second, it fitted the data well in term of the high coefficient of determination for all equation of commodity groups. Third, all the parameters are significantly different from zero at 5% level. Furthermore, the intercept terms have a high SEE only. The semi-log also fitted the data well for all commodity groups except for the food commodity group but this functional form has a much higher SEE for all estimates of all equations.

The double log functional form fitted the data well in terms of high coefficient of determination except for the food commodity group and all the estimates are significant at 5% level, however, the SEE is very low in comparison to the linear and semi-log functional form. The double log form is selected based on statistical consideration that is, most of the parameters in this model are statistically significant. In addition, this functional form implies a constant elasticity.

On the other hand, the Working- Leser (W-L) functional form results seem to be unsatisfactory for most commodity groups as suggested by the coefficients of determination ($R^2$). But it is selected as the best specification among the other functional forms. The selection is based on the distance function as measurement of selection.
The results for inverse functional form indicate that the goodness of fit is very low for all equations except for the clothing commodity group equation and personal care equations. Second, most of the marginal budget shares are significantly different from zero for all equations, except for the medical care equation. Third, all the intercept terms are significantly different from zero. Finally, all the estimates of inverse functional forms have a very high standard error.

The results for log inverse functional form indicate that overall the equations fitted highly except for the food commodity group equation which is very low. Second, all the marginal shares estimates and intercept terms are significantly different from zero at 5% level. Furthermore, the standard error for both the budget shares and constant estimates is very high. Finally, the results for log log-inverse are the same as the log inverse, but the difference is that the food equation fitted very well (0.72) under this functional form, and that marginal budget share estimates are significant different from zero at 5% level for only four equations (food, clothing, housing, and personal care).

Based on the estimated results, Engel's elasticity of demand with respect to total expenditure for each commodity group is evaluated at the mean values. Whole functional forms suggest that the demand for food and clothing commodity groups is inelastic with respect to total expenditure, indicating that these commodity groups are classified as necessities. Second, the semi-log gave the lowest elasticities estimates for commodity groups. The linear form provided the highest elasticities estimates. Lastly, the demand for the products for all commodity groups with respect to total expenditure is inelastic by two functional forms (semi-log and inverse).

One hypothesis, family size, is examined in this application. Family size is found to be significant for food, clothing, housing, transportation and personal care but is not significant for education, medical care, and miscellaneous commodity groups.
Furthermore, the expenditure elasticities with respect to household-size are negative for housing, education, and personal care commodity groups, and not significant only for education, housing and medical care commodity groups. This implies that an increase in family size, hold per capita total expenditure is constant, leads to a reduction in expenditure on these commodity groups as relatively more resources are needed for relatively necessary items.

When planning for future development to raise the supply of different commodities, it is essential that future levels of demand are accurately predicted, since priorities and investment targets have to be based on demand forecasts, among other things. Reliable estimates of income-expenditure elasticities of different commodities are a pre-requisite. The estimates presented in this study are likely to depict the true consumption patterns accurately. In particular, the commodity-specific estimates for Bahrain households are likely to provide a better understanding of the current consumption patterns. These results might be helpful to the policy makers to observe the changes that might be raised in consumption patterns since the government of Bahrain changed the tax system and electricity and water charges in December, 2000.
References

Journals


**Books**


**Official**

Chapter 10

Income Distribution and Poverty in Bahrain

10.1 Introduction

The results of the household expenditure and income survey for the period 1994-95 indicate that there are marked differences in the distribution of expenditure for most commodity groups, between households in different income categories in Bahrain (Table 9.1 of Chapter 9). Household consumption expenditure behavior is hypothesized to differ by income status. There are significant income disparities between households. Therefore, income distribution in the Bahrain economy represents another related and important issue.

Policymakers in Bahrain need to study income distribution if they are to improve living standards. Bahrain is one of the developing countries for which, as the UN reports, there are still no data on the distribution of income or consumption, or the proportion of the population below the poverty line. A number of other countries with high human development, like Bahrain, also fall into this category. See Table 10.1 and 10.2.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<td>8</td>
<td>-</td>
<td>&lt;2.0</td>
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</tr>
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<td>2</td>
<td>5</td>
<td>&lt;2.0</td>
<td>-</td>
<td>0</td>
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<td>4.4</td>
<td>6</td>
<td>1</td>
<td>&lt;2.0</td>
<td>20.5</td>
<td>2</td>
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<td>9</td>
<td>-</td>
<td>-</td>
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<td>4.5</td>
<td>2</td>
<td>5</td>
<td>6.9</td>
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<tr>
<td>42 Bahamas</td>
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<td>11.8</td>
<td>4.3</td>
<td>4</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
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<td>24.9</td>
<td>-</td>
<td>14</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>48 Qatar</td>
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<td>4.8</td>
<td>19.2</td>
<td>-</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>


Note: A positive figure indicates that the country performs better in income poverty than in human poverty, a negative the opposite.
### Table 10.2

#### Human poverty profile and index for high human development countries (2000)

<table>
<thead>
<tr>
<th>Countries</th>
<th>Human Poverty Index Value (HPI-1) 1998 (%)</th>
<th>Probability of Child under age five not surviving to age 40 (% of cohort) 1995-2000</th>
<th>Adult Illiteracy Rate (age 15 and above) 1999 (%)</th>
<th>Share of income or consumption Richest 20% to Poorest 20% 1987-1998 (°'o)</th>
<th>Share of income or consumption Richest 20% to Poorest 20% 1998-2000 (°'o)</th>
<th>Population below income Poverty line $1 a day (1993 PPP US$) 1989-1998 (%)</th>
<th>National Poverty line 2000 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 Cyprus</td>
<td>4.7</td>
<td>3.2</td>
<td>3.4</td>
<td>7.5</td>
<td>3.5</td>
<td>4.2</td>
<td>25.5</td>
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<tr>
<td>24 Singapore</td>
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<td>2.2</td>
<td>8.2</td>
<td>7.1</td>
<td>3.6</td>
<td>6.6</td>
<td>17.4</td>
</tr>
<tr>
<td>26 Hong Kong, China (SAR)</td>
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<td>2.5</td>
<td>2.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6.8</td>
</tr>
<tr>
<td>30 Barbados</td>
<td>13.7</td>
<td>3.3</td>
<td>19.1</td>
<td>6</td>
<td>18</td>
<td>5</td>
<td>5.4</td>
</tr>
<tr>
<td>31 Korea, Rep. Of</td>
<td>17.9</td>
<td>4.6</td>
<td>9.3</td>
<td>6</td>
<td>4</td>
<td>10</td>
<td>6.1</td>
</tr>
<tr>
<td>32 Kuwait</td>
<td>29.0</td>
<td>3.1</td>
<td>4.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>33 Argentina</td>
<td>5.5</td>
<td>3.1</td>
<td>3.3</td>
<td>6</td>
<td>18</td>
<td>32</td>
<td>17.4</td>
</tr>
<tr>
<td>35 Uruguay</td>
<td>19.6</td>
<td>2.8</td>
<td>19.1</td>
<td>6</td>
<td>18</td>
<td>32</td>
<td>17.4</td>
</tr>
<tr>
<td>39 United Arab Emirates</td>
<td>3.0</td>
<td>4.7</td>
<td>4.6</td>
<td>9</td>
<td>5</td>
<td>10</td>
<td>6.1</td>
</tr>
<tr>
<td>41 Bahrain</td>
<td>5.0</td>
<td>3.9</td>
<td>5.0</td>
<td>6</td>
<td>3</td>
<td>9</td>
<td>6.8</td>
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<tr>
<td>42 Qatar</td>
<td>25.5</td>
<td>4.7</td>
<td>4.6</td>
<td>9</td>
<td>5</td>
<td>10</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Bahrain lacks data on the distribution of income and on income poverty line indicators. The official bodies represented by the Central Statistical Organization and Ministry of Finance and National Economy, do not provide regular annual data on these indicators, and hence they have no ranking in the UN human poverty index. Accordingly, there is a need for studies by economic researchers in Bahrain and research institutes to shed light on these issues and encourage the official bodies in Bahrain to provide the required raw data regarding these indicators on a regular basis. I hope this study can draw attention to the importance of these issues.

Ideally we are searching for packages of policies which will increase the productivity of the economy and at the same time reduce inequality and alleviate poverty. Improving the distribution of income and extending social services is the key to human development. The emphasis should be on intervention to meet basic needs in commodities and services. I argue in this thesis that income redistribution and poverty alleviation help to improve the social conditions and the welfare of households, but it is essential that the data are collected as a first step to formulating policy.

This chapter discusses measures of inequality of income and poverty in Bahrain. The poverty line approach has been adopted to measure poverty in Bahrain. I also suggest some policies to reduce inequality of income and poverty alleviation.

10.2. Income Inequality

Income inequality can be measured by any one of three indicators. The first measure is the income share of the of the highest 20 percent, middle 40 percent and lowest 40 percent of households. The second measure is given by the Gini coefficient. The third is the Lorenz Curve. Of the three, the most common statistical indicator of inequality of income is the Gini coefficient.
The finding of the household expenditure and income survey for the period 1994-95 was that there were marked differences in the distribution of expenditure for most commodity groups between different income groups (Table 9.1 of Chapter 9). To supplement these results I have calculated income distribution and measured income inequality. Based on household expenditure and income survey for the period 1994-1995, the results indicate that the poorest 40% of the Bahrain population received 5.67% of income, while the richest 20% received 76.48% of the income (Table 10.3).

<table>
<thead>
<tr>
<th>Indicators of Households Income Distribution</th>
<th>Survey data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1984-85</td>
</tr>
<tr>
<td>-Income Highest 20%</td>
<td>56.98</td>
</tr>
<tr>
<td>-Income Middle 40%</td>
<td>34.16</td>
</tr>
<tr>
<td>-Income Lowest 40%</td>
<td>8.86</td>
</tr>
<tr>
<td>-Ratio of highest 20% to lowest 40%</td>
<td>6.43</td>
</tr>
<tr>
<td>Gini coefficient</td>
<td>0.427</td>
</tr>
<tr>
<td>Mean household income (B.D)</td>
<td>8987.5</td>
</tr>
<tr>
<td>Median household income (B.D)</td>
<td>6300.0</td>
</tr>
</tbody>
</table>

Sources: Computed by the author from official survey

A comparison of the figures for the mid 1990's compared with the household expenditure and income survey for the period 1984-1985 indicates that in the mid 1980's the poorest 40% received 8.86% of the income, while the richest 20% received 56.98%. Further, the analysis indicates that 6.32% of highest income households received 46.24% of income for the period 1994-1995. In 1984-85 the richest 8.87% of households received 33.58% of income. In other words there was no significant improvement in the distribution of income over the period 1984-85 and 1994-95. Analysis of the data also indicates that the middle income household's position deteriorated. They received only 17.85% of income in 1994-95, as compared with 34.16% in the 1984-85 period.

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The mean and median household income in 1984-1985 was 8987.5 B.D and 6300 B.D respectively. By 1994-1995, the mean household income had risen to 9996.1 B.D and the median income to 6697.8 BD.

Bahrain has been unable to achieve a significant improvement in income distribution. The incomes of the rich have increased by a higher percentage than the incomes of the poor. Another indicator of income inequality is the income share of the richest 20% and the poorest 40% of households. The ratio represents the extent of inequality. It can be seen from Table 10.1 that the share of the richest households increased from 6.43% in 1984-1985 to 13.4% in 1994-95.

Inequality of income distribution can be measured by the Gini coefficient, which ranges from zero, for perfect equality, to unity for perfect inequality. The Gini coefficient for inequality was estimated by the author for Bahrain using data from Household Expenditure and Income Survey for the period 1984-1985 and 1994-1995. The Gini coefficient is the ratio of the area between the 45° line and the Lorenz Curve to the entire area below the 45° line (See Figures 1 and 2).

Based on the 1994-95 survey data for Bahrain it was 0.53. It was 0.43 for the 1984-85 period. This means that inequality increased during the period under study. Increased inequality was associated with a small increase in the rate of economic growth, from 1.43% in 1984-85 to 1.84% in 1994-95. The increase in inequality in Bahrain was associated with a high rate of unemployment by Bahrain standards (7%) during the period 1994-95.

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1 The Gini coefficient can be expressed as $G = \left( \frac{2}{n^2 \bar{Y}} \right) \sum (y_i - \bar{Y})$, where $n$ refers to household, while $Y$ refers to income. See Hope (1999, p. 471)
On the basis of international comparison the Gini coefficient for Bahrain was (0.532) below that of middle income countries as a whole (0.569) as reported by Todaro (1994). A Gini ratio of 0.53 indicates a significant level of inequality in the country. Comparable estimates of the Gini coefficient for other countries are reported in Table 10.4.

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>Gini Coefficient (Robinson)</th>
<th>Country</th>
<th>Year</th>
<th>Gini Coefficient (Todaro)</th>
</tr>
</thead>
<tbody>
<tr>
<td>United State</td>
<td>1991</td>
<td>0.379</td>
<td>Jordan</td>
<td>1986</td>
<td>0.35</td>
</tr>
<tr>
<td>Canada</td>
<td>1991</td>
<td>0.276</td>
<td>Peru</td>
<td>1986</td>
<td>0.31</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>1991</td>
<td>0.324</td>
<td>Sir Lanka</td>
<td>1986</td>
<td>0.43</td>
</tr>
<tr>
<td>Japan</td>
<td>1991</td>
<td>0.350</td>
<td>Pakistan</td>
<td>1986</td>
<td>0.37</td>
</tr>
<tr>
<td>Taiwan</td>
<td>1991</td>
<td>0.301</td>
<td>India</td>
<td>1983</td>
<td>0.46</td>
</tr>
<tr>
<td>Colombia</td>
<td>1991</td>
<td>0.503</td>
<td>Bahrain</td>
<td>1994-95</td>
<td>0.53</td>
</tr>
</tbody>
</table>


The third measure of income distribution, the Lorenz Curve, is based on a logarithmic concept. The Lorenz Curve plots the cumulative share of the income received by the cumulative share of households. Starting from the poorest income receiving units. The 45° line which is drawn from the lower left corner to the upper right corners is the line of perfect equality. The further the distance of the Lorenz Curve from the 45° line the greater the inequality. It is apparent from the studies reported in this chapter that the distribution of income worsened in Bahrain in the 1994-1995. This is further demonstrated through the Lorenz Curve in Figure 10.1 and 10.2. Comparing Figure 10.1 and Figure 10.2 the distance of the Lorenz Curve from the 45° line is greater in Figure 10.1 than in Figure 10.1.
Fig. 10.1: Lorenz Curve
(1994-95)

Fig. 10.2: Lorenz Curve
(1984-85)
10.3. Measurement of Poverty

Poverty means spending less on food, on heating, and on clothing than someone an average income (Alcock, 1997, p.3). The magnitude of poverty differs substantially depending on the definition used. Poverty can also be measured by the basic needs approach. In this approach, a basket of basic needs consisting of food, clothing, housing, education, transportation, basic health, personal care, etc. are taken into consideration. In Bahrain a poverty line has been derived on the basis of a monetary equivalent of expenditure sufficient to meet a standard of living which will provide basic needs of shelter, food, clothing, fuel, energy, transportation, communication, education, health and medical care and recreation (Alcock, 1997). I have developed a poverty line of income (PLI) measure. In Bahrain there are no data at present on poverty. The measure of PLI which I have developed is similar to the used by Yusoff et al. (2000) to measure the PLI for the Malaysia. Hence, PLI is average monthly household income per capita (Yusoff et al. 2000, P.42).

The measure I have developed for Bahrain is reported in Table 10.5. Households having incomes below the poverty line are considered poor. Those having incomes less than half of the poverty line is considered to be hardcore poor.

Based on the household survey of 1984-1985, 26.89% of families were living below the poverty line and 34.20% of total Bahraini population was living under the poverty line. Using 1994-1995 survey data 23.62% of total Bahraini families were living below the poverty line and 36.64% of Bahraini population were living below the poverty line.

Table 10.5 indicates that percent of poor families decreased significantly between 1984-85 and 1994-95. But the number of poor families increased from 15799 families in period 1984-85 to 20450 families in the period 1994-1995. Number
and percent of poor in the population also increased from 155211 (34.20%) in 1984-85 to 36.64% (186224) in 1994-95. The classification of hardcore poor, those having incomes less than half of the poverty line income is not however relevant to the Bahrain economy.

<table>
<thead>
<tr>
<th>The Major indicators</th>
<th>Survey data</th>
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<tr>
<td></td>
<td>1984-85</td>
</tr>
<tr>
<td>Economic growth rate</td>
<td>1.43</td>
</tr>
<tr>
<td>Percent of families living below the poverty line</td>
<td>26.89</td>
</tr>
<tr>
<td>Percent of Bahrain population living below the poverty line</td>
<td>34.20</td>
</tr>
<tr>
<td>-Poverty line/ in (B.D)</td>
<td>120.0</td>
</tr>
<tr>
<td>-50% of poverty line</td>
<td>60.00</td>
</tr>
<tr>
<td>Number of families below the poverty line</td>
<td>15799</td>
</tr>
<tr>
<td>Number of people below the poverty line</td>
<td>155211</td>
</tr>
</tbody>
</table>

Sources: Computed by the author from official survey

10.4 Who are the Poor in Bahrain?

If policies are to benefit the poor, it is necessary to know who the poor are. In Bahrain there are no significant numbers of people who are what Lipton (1991) calls the "Ultra poor". These are people in extreme poverty. For example about thirty per cent of the population in sub-Saharan Africa is in extreme poverty, destitute individuals who are heavily concentrated among the old, the frail, women and children. This concept is not relevant to Bahrain. However, poverty in multi-faceted, and although there may be few or none who could be defined as destitute in Bahrain there are different subgroups or levels of poverty from which insights about poverty-relieving policies may be derived.

An analysis of poverty by socio-economic group based on the household income and expenditure survey reveals the following characteristics of the poor in Bahrain.
• The incidence of poverty increases with the size of the household. Large households are more likely to be poor in Bahrain. Families of 8 and above suffer from poverty in Bahrain.

• Households with a high dependency ratio are more likely to be poor in Bahrain.

• The incidence of poverty is higher in households with a single earner.

• Households whose heads are illiterate or with low level of educational attainment are most likely to be poor in Bahrain.

• If the head of the household is a salary/wage earner, living in rented accommodation, and receiving no income transfers, the household is likely to be poor.

• Households whose heads are unemployed or underemployed are most likely to be poor.

The increase in the number of people living under the poverty line between 1984-85 and 1994-95 in part relates to changes household size. I have found that households of 9 members and above mainly suffer from poverty in Bahrain. There has also been increasing wage inequality and unemployment in Bahrain among working households.

Economic growth that increases the demand for labor, particular unskilled labor, tends to reduce poverty. Poverty can be reduced by adopting growth strategies that focus on expanding labor-intensive manufacturing. Improving the human capital of the poor also yields poverty reduction benefits. The success of certain Asian economies in achieving rapid economic growth while reducing inequalities has received considerable attention. In Bahrain political issues such as the Iran-Iraq War (pasted ten years from 1978), the Iraq invasion of Kuwait (1990) and the Howar Island dispute between Bahrain and Qatar (ended 2001) have delayed development and have failed to deliver the foreign investment which could have
created jobs for local labor. A better income tax system and social benefits could also be helping the country reduce poverty and sustain economic growth.

10.5 Conclusion

This chapter has studies income distribution and poverty in Bahrain using survey data for the periods 1984-1985 and 1994-1995. The study indicates that 23.61% of Bahraini families (36.64% of the Bahraini population) are below the poverty line. The poorest 40% of Bahraini people received 5.67% of income. The richest 20% receive 76.48% of income. Middle income households now receive a small proportion of income (1994-95) than they did in 1984-85. There has no significant improvement in the distribution of income over this period. Inequality of income distribution is high in Bahrain, as indicated by the Gini coefficient.

Some policies which may be relevant to improve income distribution and reduce poverty are listed below.

- Development policy should include a human resource development program.
- Higher target growth rates should be accompanied by targets relating to social indicators of development.
- Investment in physical and human infrastructure should be stressed, giving better access to health, education and social amenities.
- There should be an expansion of education and training facilities to increase productivity and promote more rapid economic growth.
- Participation of women in the labor force must be encouraged.
- Unemployment and inflation need to be kept at low levels, with scope for employment creation through export-oriented manufacturing and services sector.
- Reducing wage inequality among working households will help in reducing the numbers in poverty. The income-tax system and social benefits should aim to reduce poverty as well as to promote economic growth.
Additionally, statisticians in Bahrain should encourage carrying out more detailed micro-level surveys and in-depth interviewing to identify the poor, and derive appropriate policies for poverty alleviation.
References

Books


Official


Chapter 11

Conclusion and Wider Implications of the Thesis.

This concluding Chapter is divided into 2 parts. In the first part I summarize the main findings of the econometric tests carried out in Chapters 4, 5, 6, 7, 8, 9, and 10. In the second part I discuss the wider implications of the thesis.

PART I
Conclusion from the Econometric Tests.


Static and dynamic models for three main demand systems have been estimated for Bahrain personal consumption expenditure for the period 1979q1-1998q4. The three demand systems are the linear expenditure systems (LES), the Rotterdam system (RS), and the Almost Ideal Demand System model (AIDS). The static LES and the dynamic LES models give estimates of subsistence levels of expenditure for each of the commodity groups, whereas the Rotterdam system model and the Almost Ideal Demand System model allow for testing the validity of the general restrictions implied by the utility maximization hypotheses. Consumer demand analysis was carried out by using three demand models. The aim was to explain changes in the distribution of consumer's expenditure in Bahrain. A five-item classification of consumer expenditure was used, namely, Food & Beverages, Clothing & Footwear, Housing, Transportation and "Other" Commodity group. Based on the estimation results obtained from the applications of the three models, the following conclusions were drawn.
The results of applying the "Linear Expenditure Demand System"
First, the statistical results indicate that the dynamic LES model of Pollak and Wales (1969) was a better fit to the Bahrain data than its static counterpart. Secondly, as suggested by the dynamic LES model, the highest value of the subsistence expenditure level was for the "Other" commodity group, while the lowest was for the Clothing and Footwear commodity group. The largest part of incremental expenditure goes toward the "Other" commodity group. Thirdly, the results as indicated by the dynamic LES are that 90.79% % of total expenditure is devoted to basic needs in Bahrain. A substantial increase in income is needed to raise living standards above basic needs. Fourthly, expenditure elasticities indicate that there are three commodity groups which are "necessities", namely, Clothing & Footwear, Transportation, and the "Other" commodity group. Food and Beverages and Housing are classified as "luxury" products. In the DLES model the Food and Beverages commodity group is represented a "luxury" product. According to DLES model, it shows that in spite of the rise of the cost of standard of living in Bahrain, the average per capita income has not increased during the period under study. However, the DLES model was not selected as the best model for the Bahrain data. The Dynamic Linear Almost Ideal Demand System model (LA/AIDS) was selected as the best model for Bahrain data and the selection was based on the Likelihood Ratio statistics test.

The results of applying the "Rotterdam System"
First the statistical results indicate that the dynamic Rotterdam system of Brown and Lee (1992) fits better than its static counterpart. Secondly, using total expenditure elasticities, Food & Beverages, Clothing & Footwear, Housing, and Transportation commodity group are classified as "luxuries" The "Other" group is classified as a "necessities" Thirdly, the Food & Beverages commodity group has a positive compensated own price elasticity. The Transportation commodity group has a positive sign for both compensated and uncompensated own price elasticities. These results are not as expected in demand theory. Fourthly, the hypothesis tests indicate that the restrictions implied by demand theory are not rejected in the dynamic
Rotterdam system or in its static counterpart. The test is based on the likelihood ratio procedure. The testing for homogeneity restriction is accepted on an equation by equation basis in the static model at both 5% and 1% levels. This result is for all commodity groups. Further, a hypothesis test of the lagged consumption effects in the Rotterdam system as a whole system model suggests the presence of a lagged consumption effect.

Lastly, the assumption that the volume index of the change in total consumption can be considered as a measure of the change in real income in the Rotterdam \( \sum_w w_i \text{Dlog} q_i \approx (\text{dlog} x_i - \sum_k w_k \text{dlog} p_k) \) is not true in the case of Bahrain data, because using the change in real income as independent variable gave totally different results than those obtained from using the volume index of the change in total consumption as independent variable in Rotterdam demand system model.

- The results of applying the "Linear Almost Ideal Demand System"

First, applications indicate that the unrestricted LA\AIDS dynamic model of Blunetifoti and Green (1983) is preferred to its static counterpart, due to more significant and more sensible statistical results. Secondly, based on computed elasticities, the results indicate that all commodities are "necessities" for the Bahrain consumer except the "Other" commodity group. Thirdly, all of the uncompensated own price elasticities estimates are negative and consistent with a priori expectations regarding their signs. On other hand, the results of compensated own-price elasticities are negative for most commodity groups except the sign of own-price compensated elasticity for the "Other" commodity group which is found to be positive. Fourthly, testing for the consistency of the general restrictions implied by demand theory, the test shows that homogeneity and symmetry restrictions should be not rejected at the 5 percent level of significance. The test also indicates that the symmetry over no restriction is not rejected at 5% level of sign. The testing for general restrictions is based on the likelihood ratio test. Fifthly, a test of the habit
effects in the LA/AIDS model suggests the presence of lagged effects on consumer behavior expenditure. The result is based on the likelihood ratio test procedure. Sixthly, based on the average information criterion, the static Linear Almost Ideal Demand system model (LA/AIDS) is preferred over the other static demand systems, namely the Linear Expenditure system (LES) and the Rotterdam system model (RM). Finally, based on the model selection procedures (on the likelihood ratio statistic) and on the statistically consideration, and due to the acceptance all the restrictions implied by demand theory (homogeneity, symmetry, symmetry given homogeneity), the dynamic linear expenditure demand system model (LA/AIDS) is selected as best among other demand system models.

Economic results from the estimation of the dynamic LA/AIDS suggest that: (1) as income increases, Bahrain consumers tend to spend relatively more on “Other” products and less on the remaining products. Therefore, Bahrain is a possible for the “Other” commodity group market. (2) Food and beverages own price elasticity shows the highest values (bigger than one). A food and beverages commodity group responds more to own-price changes than the rest of the products do. Retail price strategies will become more important in food market. (3) Clothing and footwear, Housing, and Transportation have inelastic income and own price elasticities that three markets are saturated and their relative budget share are losing importance as part of total expenditure. Producer and retailers must diversify products in order to find a place in the markets. (4) Clothing and footwear consumption is less responsive to change in price and in total expenditure. (5) On the other hand, the “Other” commodity group response to prices changes is inelastic, but income elasticity is above the unity. It means that the relative importance of these products on total expenditure is not going to change significantly and thus price policies to encourage consumption will not have any effectiveness. In this case, it would be more appropriate to apply market strategies focusing on the “Other” group products such education and medical care, and personal care.
• Applying "Unit Roots" test and "Cointegration Analyses" to the Bahrain commodity groups data.

Test for unit roots and cointegration for the Bahrain demand data indicate that the series frequently used in demand system analysis are integrated of order one, I(1). Secondly, in the application of the Johansen and Juselius (1990) approach for equation by equation analyses, the results indicate that there are three equilibrium relationships in Food & Beverages and Clothing & Footwear commodity groups. In Housing, Transportation, and "Other" commodity groups there are four cointegrating relationships. Thirdly, the demand for Clothing & Footwear and "Other" commodity group is classified as luxury while the remaining commodity groups are necessity products. Fourthly, all the own price elasticities, compensated and uncompensated are negative. Not all compensated price elasticities are smaller than those of the corresponding uncompensated price elasticities.

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In the Johansen and Juselius (1990) full dynamic complete system analysis, which includes all budget shares, prices, and expenditure series, the results indicate that there are four equilibrium relationships implied by demand theory. The estimated eigenvectors produce sensible parameters in the context of consumer theory. Most of the parameters were significant. Secondly, the classification of commodity groups between "luxuries" and "necessities" is different from those obtained in individual analysis. In full system analysis Transportation and the "Other" commodity group are luxuries. Food & Beverages, Clothing & Footwear, and Housing are necessities. Thirdly, the homogeneity restriction is imposed on the system as a whole. Results indicate that this restriction is rejected by the data. The argument in previous studies which says that time series issues is partly responsible for rejection is not always true.

• The results of estimating "Engel Curves" using data from the Bahrain household expenditure and income survey (1994-1995).

The data were classified into eight commodity groups, according to income per annum of each household. Ten income categories are considered. Statistical results
indicated the double log functional form fitted the data better than the other forms. The double log form is selected based on statistical consideration that is, most of the parameters in this model are statistically significant. In addition, this functional form implies a constant elasticity. Secondly, the Working-Leser (W-L) functional form was unsatisfactory for most of commodity groups as suggested by the coefficients of determination ($R^2$). It was selected as the best specification among the other functional forms based on the distance function as measurement of selection. Thirdly, whole functional forms suggest that the demand for food and clothing is inelastic. With respect to total expenditure, the results indicate that these commodity groups are "necessities". Fourthly, the semi-log gave the lowest elasticities while the linear form provides the highest elasticities. Finally, a hypothesis is examined relating to family size. The result indicates that the size of family affects the demand for food, clothing, housing, transportation and personal care commodity group, but not the demand for education, medical care, and "miscellaneous" commodity groups.

- **Results of measuring "Income Distribution" and "Poverty" in Bahrain.**

The study of income distribution and poverty in Bahrain uses survey data for the period 1984-1985 and 1994-1995. Based on the poverty line approach, Gini coefficient, Lorenz Curve, and other measures and using household expenditure and income data for the period, the results are as follows: (1) 24% of Bahraini families and 37% of the Bahraini population are below the poverty line. Based on household expenditure and income for the period (1984-85), 27% of Bahraini families and 34% of the Bahraini population were at that time living below the poverty line. (2) Further the study confirms that the poorest 40% of Bahraini people received 6% of income, while the richest 20% receive 76% of income, accordingly to survey data of 1994-95. In the period 1984-85, the poorest 40% received 9% of income; the richest 20% received 57% of income. (3) Middle income households became relatively worse off in 1994-95 compared with the 34% of income they had in 1984-85. No significant improvement in the distribution of income occurred within this ten years period of time. (4) The Gini coefficient was 0.53 in 1994-95. It was 0.43 in 1984-85. Inequality of income distribution in Bahrain remains relatively high.
PART II

Wider Implications of the Thesis.

This study highlights important issues regarding consumer demand behavior in Bahrain. The thesis not only provides information on demand structure, but also can be used in areas of economic policy, structural analysis, and forecasting. For instance, a change in the price of a particular commodity will set into motion substitution among other commodities. The dynamic LAVAIDS model which is selected as the best demand system functional form in this study, defines responses in demand to small and large changes in prices and expenditure. For example the dynamic LAVAIDS model indicates that as income increases consumers tend to spend proportionately less on Food & Beverages, Clothing & Footwear, Housing, Transportation commodity groups and more on the "Other" commodity group products.

The demand for Clothing & Footwear, Housing, Transportation and Other commodity group is price inelastic. Price increases do not affect the demand for these commodities. These commodities are important in consumer budgets. Expenditure allocated to food products represents 34.3% of total personal consumption expenditure for low-income groups. For medium and high-income groups this percentage is 23.3% and 12.7% respectively. The government of Bahrain has subsidized basic food products since 1980. However, the subsidy represents only 1.3% of total personal consumption expenditure on food products and only 0.24% of total personal consumption expenditure in Bahrain. I would argue that the subsides should remain because it is important for poorer consumers, though many economists have argued against such subsidies in recent years, because they are behaved to distort markets.

The government also obtains 8% of total personal consumption expenditure as indirect tax. This represents a high burden for the low-income group. The
government of Bahrain imposed an import tax of 20% on all types of cars. The registration fee is the same regardless of the type of cars. This policy affects mainly the low-income group. I would argue that the basic subsidy policy for foods should be retained and the indirect tax policy should be reviewed because of the need to improve income distribution and reduce poverty in Bahrain.

In Chapter 10 a number of suggestions were made for the policy maker to overcome the problem of poverty and inequality of income distribution. These policies focused on the need to include a human resource program in development policy. Higher target growth rates should incorporate an improvement in social indicators. Investment in physical and human infrastructure will increase employment opportunities and these also need to be better access to health, education and social amenities. Employment creation should focus on export-oriented manufacturing and service sectors. Finally, reform of the tax system and improved social benefits can help reduce poverty.

The functional form of the demand system has been selected in this study as a part of a complete macroeconomic model (Delmon) which shows the effects of monetary and fiscal policies. It can shed some light on how personal consumption expenditure is affected by discretionary fiscal and monetary policies. The inclusion of a demand system in the macroeconomic model can used to analyze the effects of possible shocks to the system such as changes in the price index of imports, the exchange rate (dollar -dinar), price of oil, and changes in government expenditure and revenues from both oil and non-oil sectors.

Important opinion-makers during the 1990’s indicated that bodies such as the World Bank and the IMF, have developing countries need to give priority to appropriate macro-economic management, if problems associated with economic instability, rising debt and declining foreign exchange earning are to be avoided. Although Bahrain has not suffered from macroeconomic instability on the scale of African and some Asian and Latin American economies, nevertheless, sound macro-economic management based on appropriate macro-models and good governance is critical for
sustainable and equitable development. The objective of this study, which has been met, is to contribute to a further understanding of the demand scale of the macro models.

Further improvement can, however, be made. These should include

- Collection by research bodies and government agencies of more detailed quarterly data on the consumption groups, i.e. education, health care, personal care, durable and non-durable goods.

- Collection of more detailed data on food expenditure i.e. in a disaggregated form rather than in group commodity form.

- More frequent surveys of household expenditure and incomes. i.e. every five years rather than every ten years.

- Household income and expenditure need to be surveyed not only as average household income, but also as a subset of data for each income group classified by family composition. The availability of this type of data will help in estimating Engel Curves as a full system of equations, which correspond to a complete demand model.

- Improvements in data would be an inducement to further research and could help policy makers in their search for optimal policy packages.

Finally, I have noted that studies of aggregate consumption behavior have begun to feature again in development studies as part of the new emphasis on human development. The UN Human Development Report of 1998 asks how consumption contributes to human development, and concludes that "consumption must be shared, strengthening, socially responsible and sustainable". "Shared" means ensuring basic needs for all. "Strengthening" means building on human capabilities. "Socially responsible" means making sure that the consumption of some does not compromise the well being of others. "Sustainable" means talking into account the choice of future generations. As living standard as rise in Bahrain and the proportion
in poverty falls, the focus of policy will shift towards promoting those patterns of
cconsumption which will promote human development. It is hoped that this thesis
will provide information on which to build future polices.
Appendices

Appendix: A

The Derivation of LES From Three Functional Forms of Consumer Theory
The linear expenditure system can be derived from three different functions mentioned in chapter (2) and subsequent derivation of the LES from these functions is as follows:

(1): The derivation of the demand equations from direct utility function
In order to yield demand equations from utility function (1), there are six steps:

\[ v(q) = \prod_i (q_i - \gamma_i)^{\beta_i} \]  

\[ v(q) = \prod_i (q_i - \gamma_i)^{\beta_i} \quad (1) \]

Step (1): By apply Lagrangean multiplier technique, and maximization of the utility function (1) subject to the budget constraint, yields the following equation:

\[ \text{Max} L = L(q, \lambda) = \sum_i B_i \log(q_i - \gamma_i) + \lambda(x - \sum_i p_i q_i) \quad (2) \]

Step (2): By obtaining the first order conditions and these are as follows

\[ \frac{\partial L}{\partial q_i} = \frac{B_i}{q_i - \gamma_i} - \lambda p_i \quad (2.1) \]

\[ \frac{\partial L}{\partial \lambda} = x - \sum_i p_i q_i \quad (2.2) \]

Step (3): By solving for \( B_i \) and \( \lambda \) and sitting the first-order conditions equal to zero obtains:

\[ B_i = \lambda p_i (q_i - \gamma_i) \quad \text{for } i = 1, 2, \ldots, n \quad (2.3) \]

\[ \lambda = \frac{B_i}{p_i (q_i - \gamma_i)} \quad (2.4) \]

Step (4): Summing \( B_i \) over all \( i \) and including the normalization restriction \( \sum_i B_i = 1 \), yields:

\[ \sum_i B_i = \sum_i \lambda p_i (q_i - \gamma_i) = 1 \quad (2.5) \]
and
\[ \lambda = \frac{l}{(x - \sum_k p_k \gamma_k)} \]  
(2.6)

Step (5): Equating Equation 2.6 and 2.4 yielded the LES equation (expenditure equations).
\[ p_i q_i = p_i \gamma_i + B_i (x - \sum_k p_k \gamma_k) \quad i=1,2,\ldots,n \]  
(3)

Step (6): Dividing Equation 3 by \( p_i \) results in demand equation.
\[ q_i = \gamma_i + \frac{B_i}{p_i} (x - \sum_k p_k \gamma_k) \]  
(4)

Thus, Equation 4 is known as the Marshallian demand function and is a function of prices and income. This equation can be substituted into the direct utility function to obtain the indirect utility function. This can be shown as follows:
\[ \log u(q) = \sum_i B_i \log (q_i - \gamma_i) = \sum_i B_i \left( \log (\gamma_i + \frac{B_i}{p_i} (x - \sum_k p_k \gamma_k)) - \gamma_i \right) \]  
(5)
\[ \log u(q) = \sum_i B_i (\log B_i + \log (x - \sum_k p_k \gamma_k) - \log p_i) = \log \phi(p,x) \]  
(6)

Equation 6 thus obtained is equivalent to the indirect utility function (Equation 7) below.

(2): The derivation of demand equations from indirect utility function.

In order to obtain the derivation of demand equations from indirect utility function (7), we need to utilize Roy's identity.
\[ \psi(x, p) = \frac{(x - \sum_i p_i \gamma_i)}{\prod_i p_i^R} \]  
(7)

The utilization of Roy's identity in the indirect utility function (7) yields the following.
\[ \frac{\partial \psi}{\partial x} = \prod_i p_i^{-R} = \frac{l}{\prod_i p_i^R} \]  
(7.1)
and
\[ \frac{\partial \psi}{\partial p_i} = -\frac{q_i}{\prod_i p_i^R} \quad \text{for } i=1,2,\ldots,n \]  
(7.2)
Thus, the demand function can be obtained by dividing Equation 7.2 by 7.1 which yields

$$\frac{\partial v}{\partial p_i} = -q_i$$

(8)

(3): The derivation of demand equations from the cost function

In order to obtain demand equations from the cost function below (9) needs the following steps,

$$c(u, p) = \sum_i p_i y_i - u \prod_i p_i^b_i$$

(9)

Step (1): Apply Retelling's theorem. The application of Hotellng's theorem yields

$$\frac{\partial c(u, p)}{\partial p_i} = q_i(u, p)$$

(10)

Equation 10 is known as Hicksian- compensated demand equations.

Step (2): By substituting $x$ into the cost function (9), due to total expenditure ($x$), equals $\sum p_i q_i$, which yields

$$x - \sum p_i y_i = u \prod p_i^b_i$$

(11)

Step (3): Substituting Equation 11 into Equation 10 yields

$$y_i + \frac{B_i}{p_i} (x - \sum p_i y_i) = q_i(p, x)$$

(12)

The derivation leads to the Marshallian demand function which is the same as Equation 4 above.
The Derivation of Pigou’s Law From The Linear Expenditure System (LES)

This needs us to utilize the definition of marginal utility of expenditure from the utility maximization problem of the linear expenditure system (LES), i.e.

\[ \lambda = \frac{l}{(x - \sum_k p_k y_k)} = (x - \sum_k p_k y_k)^{-1} \]  \hspace{1cm} (1)

where the income elasticity of \( \lambda \) is defined as,

\[ \frac{\partial \lambda}{\partial x} \lambda = -\frac{1}{x} \lambda \left( \frac{\sum_k p_k y_k}{\left( x - \sum_k p_k y_k \right)^2} \right) \]

\[ = -\frac{x}{x - \sum_k p_k y_k} \]  \hspace{1cm} (2)

where \( x \), refers to expenditure (income), and \( x - \sum_k p_k y_k \) to supernumerary income, whereas, \( \partial \) is defined as the inverse of the income elasticity of \( \lambda \)

\[ \{ -\frac{1}{\lambda} \}. \]

In order to express the relationship between own-price elasticity and income elasticity, we needs to utilize the definition of own-price elasticity. That is shown below. The equation of the linear expenditure system is

\[ q_i = \gamma_i + \frac{B_i}{p_i} (x - \sum_k p_k y_k) \]  \hspace{1cm} (3)

The own-price elasticity can be defined as,

\[ e_{ii} = \frac{\partial q_i}{\partial p_i} \frac{p_i}{q_i} \]  \hspace{1cm} (4)

and hence,

\[ \frac{\partial q_i}{\partial p_i} = -\frac{B_i}{p_i} (x - \sum_k p_k y_k) = \frac{B_i}{p_i} (x - \sum_k p_k y_k) - \frac{B_i}{p_i} \gamma_i \]  \hspace{1cm} (5)
Substituting Equation 5 into Equation 4 yields,

\[ e_u = \frac{B_i}{w_x} (x - \sum_{k} p_k \gamma_k) - \frac{B_i}{q_i} \gamma_i \quad (6) \]

and substituting the definition of \( \delta \) and \( \eta_i = \frac{B_i}{w_i} \) in Equation 6 yields,

\[ e_u = \delta \eta_i - \frac{B_i}{q_i} \gamma_i \quad (7) \]

Due to the two restrictions of LES which are \( 0 < B_i < 1 \), \( q_i \gamma_i > 0 \), when \( q_i \) is positive and \( B_i \) is a small positive number, and supposing \( \gamma_i \) is positive then \( (\frac{B_i \gamma_i}{q_i}) \), the second part in Equation 7, is a small positive number, and since the first part in Equation 7 is big relative to the second part, therefore, \( \delta \eta_i \) dominates the relationship.

The exclusion of the second term of Equation 7 results in

\[ e_u = \delta \eta_i \quad (8) \]

That is Pigou's Law, an approximate result from the assumption of additivity embodied in the LES.
Appendix C.

Aggregation over Households

Muellbeauer (1975) and (1976) shows that if preferences belong to a price independent generalized linear (PICL) class, consistent aggregation across households and the existence of a representative household is possible. According to Muellbeauer, a representative household exists if each average budget share, \( w_i \), can be written as a function of prices, \( p \), and a representative budget level, labeled by \( x_0 \). For \( x_0 \) to exist, the individual budget share equations must have the generalized linear (GL) form

\[
\frac{v_{ih}(x_h, p)}{k_h} = v_h(x_h, p)A_i(p) + B_i(p) + C_{ih}(p)
\]  

(1)

where \( h \) represents h-th family, \( p \) denotes a price vector and \( v_h, A_i, B_i \) and \( C_{ih} \) are functions satisfying \( \sum_i A_i = \sum_h C_{ih} = \sum_i B_i = 1 \). The form of the budget share is then transformed from GL to PIGL form by restricting the function \( v_h \) in (1) to

\[
v_h(x_h, p) = (1 - (x_h/k_h)^{-\alpha})^{\alpha/\gamma}
\]  

(2)

where \( \alpha \) is a constant and \( k_h \), although not a function of \( x_h \), and \( p \) are free to change from household to household. In this case, the budget shares are said to have the price independent generalized linear form (PIGL). In order to be consistent with PIGL class of preferences, the cost function takes the form

\[
\frac{c(u_h, p)/k_h}{k_h} = (1 - u_h)(\alpha(p))^{\alpha} + u_h(b(p))^{\alpha}
\]  

(3)

where \( c \) represents the cost function, \( u \) denotes the utility level of the \( h \)-th family, \( k_h \) represent family composition effects, and \( \alpha(p) \) and \( b(p) \) are
functions of the price \((p)\) vector. When \(\alpha\) tends zero, we obtain the price independent generalized logarithmic, (PICLOG) form

\[
\log\left(\frac{c(u_s,p)}{k_s}\right) = (1-u_s)\log(a(p)) + u_s\log(b(p))
\]

(4)

where \(a(p)\) and \(b(p)\) are linear homogeneous concave functions. In the PIGLOG case, representative expenditure level is independent of prices, and depends only on the distribution of income. The Practical application of the PIGL class requires selection of specific functional forms for the functions \(a(p)\) and \(b(p)\); those leading to the AIDS have been discussed in chapter (2) and its application in chapter (6).
Goldstein and Khan Technique

Since quarterly series on private consumption expenditure do not exist for the entire period 1979-1998, it was necessary to interpolate the annual series to a quarterly basis for some years. The method of interpolation as follows. If $x_{t-1}$, $x_t$, and $x_{t+1}$ are three successive annual observations of a flow variable $x(t)$, the quadratic function passing through the three points in such that:

$$
\int_0^1 (as^2 + bs + c) \, ds = x_{t-1}
$$

$$
\int_0^1 (as^2 + bs + c) \, ds = x_t
$$

$$
\int_0^1 (as^2 + bs + c) \, ds = x_{t+1}
$$

Integrating and solving for $a$, $b$ and $c$ gives

$$
a = 0.5x_{t-1} + 1.0x_t + 0.5x_{t+1}
$$

$$
b = -2.0x_{t-1} + 3.0x_t - 1.0x_{t+1}
$$

$$
c = 1.8333x_{t-1} - 1.1666x_t + 0.333x_{t+1}
$$

Four quarterly figures within any year can be interpolated by

$$
\int_{0.25}^{0.5} (as^2 + bs + cs) \, ds = 0.05469x_{t-1} + 0.23438x_t - 0.03906x_{t+1}
$$

$$
\int_{0.25}^{0.5} (as^2 + bs + cs) \, ds = 0.0078x_{t-1} + 0.26563x_t - 0.02344x_{t+1}
$$

$$
\int_{0.25}^{0.5} (as^2 + bs + cs) \, ds = -0.0234x_{t-1} + 0.26563x_t + 0.00781x_{t+1}
$$

$$
\int_{0.25}^{0.5} (as^2 + bs + cs) \, ds = -0.0391x_{t-1} + 0.23438x_t + 0.05469x_{t+1}
$$

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**Officials**


Gender, group identity and variation in usage of the Berlin Urban Vernacular

Sally Ann Johnson

Thesis for the degree of Doctor of Philosophy

University of Salford
Department of Modern Languages

1991
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