FORECASTING MODELS OF ACTIVITY IN
INDUSTRIAL AND COMMERCIAL
BUILDING

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Definitions and Abbreviations

\( \delta \)  Rate of depreciation of capital assets.

\( \Delta K_t \) Change in capital stock, i.e. net investment, in period \( t \).

\( \lambda(t) \) Costate variable in optimal control problems. It represents the marginal valuation of capital.

\( \pi_t \) Profits, i.e. revenue net of labour and capital costs, in period \( t \).

\( \pi'_t \) Expected profits in period \( t \).

\( \theta_t \) Adjustment factor for income earned abroad by UK companies and income from financial assets in period \( t \).

\( \rho \) Internal rate of return or marginal efficiency of capital.

\( \rho_t \) Rate of discount in period \( t \).

\( \tau_t \) Rate of corporate income taxation in period \( t \).

\( \zeta_t \) Technology shock in period \( t \).

\( a_t \) The value of net investment in industrial buildings as a share of the value of net investment in industrial and commercial buildings in period \( t \).

\( A_t \) Present value of future allowances associated with a unit investment in period \( t \).

\( A'_t \) Present value of future allowances associated with a unit investment in commercial buildings in period \( t \).

ACF Sample correlogram.

ACS(a) Augmented construction system (a).

ACS(b) Augmented construction system (b).

ACS(c) Augmented construction system (c).

ACT Advanced corporation tax.

ADF Augmented Dickey-Fuller.

AIC Akaike Information Criterion.

\( A'_t \) Present value of future allowances associated with a unit investment in industrial buildings in period \( t \).

AMS Augmented macro system.

AR Autoregressive.

ARMA Autoregressive moving average.

ARIMA Autoregressive integrated moving average.

\( B_t \) The present value of allowances remaining on past investment in period \( t \).
BCI 
Building cost index in period $t$.

BCIS Building Cost Information Services

BCS Basic construction system.

BGP Breusch-Godfrey-Pagan test.

$B_i^C$, The present value of allowances remaining on past investment in industrial buildings in period $t$.

$B_i^C$, The present value of allowances remaining on past investment in commercial buildings in period $t$.

BMS Basic macro system.

$BO_t$, Business optimism in period $t$.

c, User cost of capital in period $t$.

$CE_t$, Employees in employment in the construction sector in period $t$.

CBI Confederation of British Industry

CES Constant elasticity of substitution.

CF Common factor.

CSO Central Statistical Office (now the Office of National Statistics).

$CU_t$, Capacity utilisation in period $t$.

DfEE Department for Education and Employment.

DH Durbin's $h$ statistic.

$DIP_t$, Firms' annualised debt interest payments in period $t$.

DoE Department of the Environment.

$DPO_t$, Firms' annualised dividend payments on ordinary shares in period $t$.

$DPP_t$, Firms' annualised dividend payments on preference shares in period $t$.

DW Durbin-Watson statistic.

ECM Error correction mechanism.

$E_t$, Total employment in period $t$.

ET Economic Trends.

$ETAS$ Economic Trends Annual Supplement.

$F_t$, Total number of periods, after period $t$, in which an investment made in period $s$ receives a writing down allowance.

FCIs Financial companies and institutions.

GDFCF Gross Domestic Fixed Capital Formation.

GDP Gross Domestic Product.

GMSFE Geometric mean squared forecast error.
GNP  Gross National Product.
$GP_t$  Gross after-tax profits.
$H$  Hamiltonian for an optimal control problem.
$HCS$  Housing and Construction Statistics.
$HEGY$  Hylleburg, Engle, Granger and Yoo (1993).
$I_t$  Gross investment in period $t$.
$ICCs$  Industrial and commercial companies.
$II_t$  Investment intentions in period $t$.
$IRS$  Inland Revenue Statistics.
$i^*_t$  Investment allowances associated with industrial buildings in period $t$.
$i^t$  Initial allowances associated with industrial buildings in period $t$.
$i^{**}_t$  Writing down allowances associated with industrial buildings in period $t$.
$K_t$  Capital stock in period $t$.
$K^*_t$  Desired, or optimal, capital stock in period $t$.
$K^a_t$  Derived measure of the stock of privately owned industrial and commercial buildings in period $t$.
$K^b_t$  Derived measure of the stock of privately owned industrial and commercial buildings in period $t$.
$K^f_t$  Preferred measure of the stock of privately owned industrial and commercial buildings in period $t$.
$L(t)$  Quantity of labour employed by the firm at time $t$.
$LAK_t$  Natural logarithm of net investment in period $t$.
$LCU_t$  Natural logarithm of capacity utilisation in period $t$.
$LE_t$  Natural logarithm of total employment in period $t$.
$LM$  Lagrange multiplier.
$LO_t$  Natural logarithm of the real value of new orders, placed by the private sector, for industrial and commercial buildings in period $t$.
$LOK_t$  Natural logarithm of optimal capital stock in period $t$.
$LLR_t$  Natural logarithm of industrial and commercial companies’ liquidity ratio in period $t$.
$lr$  Likelihood ratio.
$LR_t$  Industrial and commercial companies’ liquidity ratio in period $t$.
$LRC_t$  Natural logarithm of the real user cost of capital in period $t$.
$LTPI_t$  Natural logarithm of the tender price index in period $t$. 

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$LW_t$  Natural logarithm of the index of real wages in the construction sector in period $t$.

$LY_t$  Natural logarithm of real private sector output in period $t$.

$LY_t^A$  Natural logarithm of real aggregate output in period $t$.

$m(t)$  Costate variable in optimal control problems. It represents the current marginal valuation of capital or the shadow price of capital.

MA  Moving average.

MAFE  Mean absolute forecast error.

MAPFE  Mean absolute percentage forecast error.

$MCI_t$  Market conditions index in period $t$.

MdAPFE  Median absolute percentage forecast error.

$MEC$  Marginal efficiency of capital or the internal rate of return.

$MEI$  Marginal efficiency of investment.

MSFE  Mean squared forecast error.

MSPFE  Mean squared percentage forecast error.

$n$  Life span of a capital asset.

$N_t$  Number of years for which an investment made in period $t$ receives a writing down allowance.

$O_t$  Real value of new orders, placed by the private sector, for industrial and commercial buildings in period $t$.

OLS  Ordinary least squares.

ONS  Office of National Statistics.

$p_t$  Price of output in period $t$.

PACF  Sample partial autocorrelation function.

$PDA_t$  Profits due abroad in period $t$.

PP  Phillips-Perron.

PS  Chow test for predictive stability.

$PV_t$  Present discounted value of future marginal products of capital in period $t$.

$q, q_t$  Tobin's marginal $q$ (in period $t$).

$q_t^A$  Tobin's average $q$ in period $t$.

$Q_t$  $\left( q_t^A - 1 \right) \frac{\hat{z}_t}{p_t}$.

$R^2$  Coefficient of determination.

$\hat{R}^2$  Adjusted $R^2$.

$r_t$  Rate of interest in period $t$. 

xix
\( r_t^A \)  
Capitalisation rate for profits due abroad in period \( t \).

\( r_t^D \)  
Capitalisation rate for debt interest in period \( t \).

\( r_t^O \)  
Capitalisation rate for ordinary shares in period \( t \).

\( r_t^P \)  
Capitalisation rate for preference shares in period \( t \).

\( RE_t \)  
Construction employment relative to total employment in period \( t \).

RIBA  
Royal Institution of Building Architects

\( RIR_t \)  
Real interest rate in period \( t \).

RMSFE  
Root mean squared forecast error.

RMSPFE  
Root mean squared percentage forecast error.

SARIMA  
Seasonal ARIMA model.

SC  
Schwarz Criterion.

SE  
Standard error.

SSE  
Sum of squared errors.

\( T \)  
Number of observations or sample size.

\( TPI_t \)  
Tender price index in period \( t \).

\( V_t \)  
Market value of companies and financial institutions in period \( t \).

\( V(t) \)  
Net worth of a firm at time \( t \).

VAR  
Vector autoregression.

VECM  
Vector error correction model.

\( w_t \)  
Price of labour in period \( t \).

\( W_t \)  
An index of real wages in the construction sector in period \( t \).

\( Y_t \)  
Real value of private sector output in period \( t \).

\( Y_t^A \)  
Real value of aggregate output (at market prices) in period \( t \).

\( Y_t^e \)  
Expected output in period \( t \).

\( z_t \)  
Price of investment goods in period \( t \).

\( Z_t \)  
Stock of inventories held by companies and financial institutions in period \( t \).
Abstract

Despite its importance in national income, the level of activity in the construction sector has received little attention in the economics literature. The lack of studies attempting to forecast construction activity is surprising given that its volatility is often regarded as destabilising to the economy. Here, we model an important and growing component of construction, namely private industrial and commercial building. Construction activity is typically measured by output. To the extent that new construction output represents capital formation, output can be modelled as an investment problem. The theoretical investment literature is disparate and confusing but here, the leading models are presented in a unified framework in which the similarities and differences between them can be easily identified. We then go on to estimate a number of the models empirically. Some are econometric models consistent with traditional theories of investment. Others are based on vector autoregression (VAR) analysis which provides a largely statistical representation of a set of variables with minimum use of a priori restrictions but in which long-run relationships are preserved. The data required for model estimation is considerable and complicated by the effects of investment incentives embodied in the tax system. The forecasting performance of all the models is evaluated against forecasts generated by a benchmark model suggested by the data rather than by economic theory. In terms of forecasting performance, some of the investment models considered here are shown to be superior to the benchmark model.
1. Introduction

1.1 Background and Motivation

Workload or activity in the construction sector is typically measured by output. The Department of the Environment defines construction output as the value of work done by contractors in a specified period. The importance of construction in the overall economy can be seen in Figure 1.1. In this figure, we plot total construction output as a share of GDP over the period between 1955q1 and 1996q4. On average, construction has contributed an average of 9.7% to GDP, although the contribution has been as high as 12.7%, in 1967, and as low as 6.4%, in 1977. Since 1977, the share of total output provided by construction appears to have shown a trend increase.

The volatility of construction output is evident from Figure 1.2 which plots the annual percentage change in construction output together with the annual percentage change in GDP. Whilst both series are clearly cyclical, construction output displays much more volatility than the GDP. Indeed, in the year to 1964q1 construction output expanded by over 30% (from a low base associated with the particularly severe winter in 1963q1), whereas in the year to 1979q3 construction output contracted by more than 20%. These statistics compare with extremities in the GDP series of 11.5% in 1973q1 and -4.5% in 1980q2.

In addition to having greater amplitude than general economic cycles, the cycles in construction activity tend to be greater in amplitude than those of most other industries. This is due, in part, to the fact that new work, and to a lesser extent repair and maintenance, can usually be postponed in times of financial difficulty. Businesses are reluctant to invest in new buildings when there is little confidence in the future. Hence, at these times they tend to postpone the development of new buildings and improvements. Private individuals tend to have less money and less confidence when the economy has lost buoyancy and employment prospects are less certain. Thus, the onset of recession in an economy is likely to be accompanied by a fall in private orders to the construction industry. However, the fall in orders is likely to be much greater in amplitude than a fall in an industry that produces consumption goods and services, since many consumption goods are necessary to sustain living.
Figure 1.1: Total construction output as a percentage of GDP

Figure 1.2: Annual percentage changes in total construction output and GDP
The volatility of construction output has also been attributed to heavy government involvement in the industry. The importance of the public sector as a client of the construction industry has wide ranging implications on the industry and the economy more widely. Given that such a large component of construction is commissioned by the public sector, large changes in government’s capital expenditure will result in large fluctuations in construction output. However, the long and variable lags in the construction process can accentuate this effect. Changes in construction output lead to changes in the level of employment in the industry, changes in incomes, demand and ultimately the level of output of other industries. This relationship between construction output and aggregate demand has provided successive governments with the opportunity to influence aggregate demand through their capital expenditure budgets. In other words, this involvement furnishes the government with a means of regulating the economy. The long and variable lags in the construction process present governments with some difficulties in the timing of their actions. Unless governments can see problems far in advance, the effect of its action will often not be felt until after it is required. If the major effect of its action comes much later, it may well prove to have an effect contrary to that desired. In other words, policy intended to be counter-cyclical or stabilising to the economy can in fact be destabilising. If the construction cycle is broadly concurrent with the economic cycle, the amplitude of the former will be reinforced by badly timed policy intended to be counter-cyclical. According to this view, the government’s declining involvement in the industry over recent years should, ceteris paribus, result in less violent swings in building activity.

It is also argued that the long and variable lags in the construction process are themselves a source of inherent instability sufficient to explain the exceptional volatility of the sector. The period between the decision to build and completion of the project can be many years in the case of a large office complex. During an upswing, the long lag between an increase in users’ demand for floor space and the completion of new buildings gives rise to shortages in floor space in the market place. Because short-run supply is so inflexible, this can prompt substantial rises in rents or prices, encouraging developers to increase building activity. When this new space eventually comes on stream, it can bring about sharply falling rents and a long depression in building activity. Thus, the sector can be said to have its own internal dynamics and
these dynamics give rise to the industry’s endogenous supply side cycle. This can be very pronounced (as indicated by Figure 1.2) and may be amplified or dampened by economic or demographic factors affecting the demand for buildings by users and the supply of building by developers.

Although this so-called cobweb model provides us with an intuitively appealing basis for cyclical variations in the construction sector, it assumes that agents are myopic i.e. it assumes that they take no account of the future in decision making. Common sense would suggest that some measure of foresight on the part of agents should serve to stabilise the cycle. In particular, a high level of construction activity must imply a lower real price subsequently, and expectations of this should dampen the cobweb. The role of expectations in damping building cycles is explored in Sampson and Skinner (1995).

Whatever the cause of the exceptional volatility of construction output, these fluctuations are damaging to the overall efficiency of the industry. The total process of construction is long and spans many different occupations and skills. Therefore, a sudden halt to the starts of projects followed by the later commencement in more favourable economic conditions, leads to disruption to the design and planning processes as well as to the work done on site. In addition to creating inefficiencies, the disruption of projects provides a substantial disincentive to invest in plant and skills.

The undesirable effects of fluctuations in construction activity can be alleviated by improved information on future demand conditions. Forecasts of future demand will help construction industry agents to plan such that new buildings come on stream at times when it is required. Forecasts can also help governments, intent on using its involvement in construction as a tool for stabilisation, to gain improved information on future demand and the appropriate timing of necessary action. To the extent that forecasts may be useful in evening out demand for construction services, they may also contribute to improving the efficiency of the sector.

In this thesis we aim to develop forecasting models of an important and growing component of construction, namely private industrial and commercial building. In other words, the component of interest is that construction for industrial and commercial use commissioned by private sector clients. In focusing on this
component of construction we are ignoring a substantial proportion of the sector. First, we rule out the value of work done repairing and maintaining existing buildings. In 1990, work carried out on repair and maintenance accounted for 44% of total construction output. Repair and maintenance is much less cyclical than new work due, in part, to the fact that it can not be as easily delayed and tends to be done in smaller and less discrete quantities making delay less desirable. Second, we do not consider the value of new work in housing. The factors determining new work in housing are likely to be substantially different from those determining new work in private industrial and commercial building. This is especially true since a large, although decreasing, proportion of new housing is commissioned by the public sector (20% in 1996) and this is likely to be rather insensitive to changes in the economic climate. Thirdly, we rule out other public work which historically has included new work on buildings to be used for educational and health services and infrastructure. As such, this reflects not so much the dynamics of market behaviour, but rather a response to society’s needs or a periodic shift in public policy. Our a priori feeling is that public sector activity will not lend itself to the type of dynamic modelling suitable for analysing the behaviour of the private sector market. Finally, it excludes new work on infrastructure commissioned by the private sector. This component was virtually zero prior to the 1980s, but has grown rapidly in the last few years as a result of government policies such as privatisation and the Private Finance Initiative etc. Industrial and commercial building is likely to be much more sensitive to the changes in the economic environment than other construction work.

The real value of new work done in private industrial and commercial building is plotted for the period since 1955 in Figure 1.3. The real value of new work in private industrial and commercial building displays a clear upward trend over the period: growth is particularly strong between 1955 and 1965 and again between 1985 and 1990. In addition, since the early 1970s this component has accounted for an increasing share of all new construction work. This is clear from Figure 1.4 which plots the real value of new work in private industrial and commercial building as a percentage of the real value of all new construction work.

This sector is particularly volatile and sensitive to changes in the economic climate. Therefore, it likely to benefit from forecasting to a greater degree than other
Figure 1.3: Real new private industrial and commercial construction output

Figure 1.4: Real new private industrial and commercial construction output as a percentage of total new construction output
components of new work which are more difficult to model due to government decision making based on non-market criteria.

1.2 Construction Activity as an Investment Problem

To the extent that new construction output represents fixed capital formation, output can be modelled as an investment problem. Fixed capital formation is defined as investment in physical productive assets that yield a continuous service beyond the period of account in which they are purchased. To the extent that a new building represents an addition to a country's productive capacity, yields stream of services over its lifetime, and requires foregoing current consumption to finance its development, it embodies physical investment. In national income accounting, investment in the provision of social products such as roads, hospitals and schools undertaken by the government is counted as government expenditure; thus investment expenditure is normally defined as consisting only of private sector investment spending.

Also, national income accounting typically distinguishes between three components of investment. Firstly, residential construction is investment ultimately for the use of home owners and is known to be very sensitive to even minor changes in the market rate of interest (see Swan (1970) for a study of factors affecting the residential investment). The second major portion of aggregate investment is inventory investment which is by far the most volatile component of investment. Firms tend to use inventory changes as buffers against variations in the sales of goods and services, and, for this reason, inventory investment is known to be very sensitive to the overall level of economic activity and especially short term fluctuations in sales. Inventories may also be held for speculative purposes. The third and largest component of aggregate investment is fixed business investment; it incorporates expenditure on structures and producer durable equipment. The term 'structures' is defined here as buildings and civil engineering works.

Fixed business investment can also be split into gross and net investment: gross investment is the total amount of investment and is the sum of net investment and
replacement investment. The latter is designed merely to replace the amount of capital that has deteriorated or been scrapped.

Net investment in buildings (as distinct from structures) corresponds to that part of construction output attributable to the value of new work in private industrial and commercial building. It should be stressed that the part of construction output contributed by the value of repair and maintenance works is not part of capital formation or investment since this activity does not produce fixed assets but merely preserves existing buildings.

Despite the correspondence between investment expenditure and construction output, there are important differences in the two concepts. For example, geographical coverage of the two series differs; the expenditure data is collected for the UK, whereas the output data covers Great Britain. In addition, the expenditure data includes expenditure on infrastructure (which has increased rapidly since the commencement of the privatisation programme), whereas the private industrial and commercial construction output data does not. Moreover, there are differences in the timing of data. Recall, the output data measures the value of work done during a
specified period. Actual expenditure may not be coincident in timing with the actual execution of work. These, and other differences which receive more detailed consideration in Section 4.2.1.3, are not thought to be of first order importance and, in some respects, the output series may be a superior measure of investment in private industrial and commercial buildings. The close relationship between net fixed business investment in non-residential structures and new private industrial and commercial construction output is shown in Figure 1.5 which plots annual real value data for the two series.

1.3 The Modelling Strategy

In this thesis, the value of new work in private industrial and commercial building is modelled as an investment problem. The theoretical literature on non-residential fixed investment suggests four dominant theories of investment. Strictly speaking, these theories focus almost exclusively on the demand side of the industry and little consideration is given to the implications for investment of capital goods producing industry. The resulting relationships between investment and its determinants are of the single equation variety. We estimate equations for private industrial and commercial building that are consistent with these dominant theories and typical of the equations estimated in the empirical literature. In developing these equations we make use of modern time series econometric techniques. These techniques have rarely been applied to the investment field. This is due, in part, to the fact that much of the literature was developed prior to the introduction of these techniques. Moreover, to our knowledge, the fields of construction activity and investment behaviour have not previously been synthesised in this way.

As with all structural equations, interpretation of investment equation parameters may be blurred and identification compromised if, for example, an exogenous variable has multiple interpretations. In response to potential problems of this kind Sims (1980) has argued for a relatively non-structural approach to modelling. Sims believed that the restrictions needed to identify the econometric structure are ‘incredible’ and this belief provided the basis for the new methodology of vector autoregressive (VAR) modelling. VAR analysis provides a largely statistical representation of a set of
variables with minimum use of a priori restrictions. This procedure involves regressing contemporaneous values of each variable on lagged values of all the variables in the model. No variables are treated as exogenous and no variable is excluded from any of the equations in the model. This implies that everything causes everything else and there is no room for assuming anything more than very general economic principles as a starting point. Sims' methodology is often referred to by its critics as atheoretical macroeconometrics. However, the standard defence is that there is literally no variable that may not enter the consumer's demand or labour supply function and, therefore, there can be no variable that can be excluded from the maximisation of lifetime utility problem. In such a formulation, one need not worry about the dynamics of the relationship between investment and its determinants which have been the source of much empirical difficulty. One of the criticisms of this approach is that variables entering into the system must be stationary in order that the spurious regression problem can be avoided. Given that many economic variables are non-stationary and must be first differenced to become stationary, many VARs contain first differenced data. As such, there is a loss of long-run information. Johansen (1988) suggested a reparameterisation of a pure VAR which allows long-run information to be retained. The resulting system is sometimes referred to as a vector error correction model (VECM). VECM and VAR models are particularly useful for forecasting exercises, since the forecaster does not need to worry about the economic theory underlying the model (this allows empirical specifications to differ from those suggested by theory) and, moreover, does not need to make assumptions about the values of exogenous variables in the forecast period. This is in contrast with standard econometric forecasting where forecasts have to be conditioned on knowledge of the exogenous variables.

The poor forecasting performance of many macroeconomic models in the late 1980s and early 1990s led many to consider whether outside information may be used to improve forecasts. One of the areas in which discussion has focused is in the use of leading indicators. The 'cyclical analysis' approach to forecasting was developed by Burns and Mitchell (1946) in the US. It was introduced to the UK by O'Dea (1975) and CSO (1975). A later appraisal of the technique is available in Auerbach (1982). Broadly speaking, these techniques involve studying a large number of series of
economic data for systematic timing relationships between their corresponding turning points. Those variables that turn earlier than the general economic cycle are known as leading indicators and it is this group of variables that carries most information of use to the forecaster.

Of course, there are limitations to the use of leading indicators. Given that turning points in economic activity generally occur once every two or three years, there are long periods in which the leading indicators do nothing but suggest that a turning point has not yet occurred. Further, since the timing relationships are determined by data analysis rather than a complete structural econometric model containing theoretical priors, a turn in a leading indicator can not in itself be used as a prediction of a coming cyclical turn and, moreover, it can not be used as a tool for policy analysis. At most it gives an indication of something to look for as new observations arrive and, as such, provides only a useful complement to mainstream macroeconometric models. Although the inclusion of leading indicators in structural models may be not be justifiable on theoretical grounds, no such justification is needed for including these variables in atheoretical models. Marsland and Weale (1992) synthesise the leading indicator and VAR approaches to forecasting. In their study, leading indicators are used as extra inputs into a VAR model of macroeconomic aggregates and the forecasting performance of the resulting system is compared with the published forecasts from standard UK macroeconometric models.

In this work, we develop a number of VAR models of construction output. Some of the VAR models are consistent with the dominant theories of investment. However, we also make use of the flexibility of the VAR approach to modelling by developing systems that are based rather more on intuition than a strict interpretation of investment theory. Some of these systems are augmented with variables thought to be leading indicators of construction output. Given that there are often substantial empirical difficulties associated with the development of investment equations (which will be discussed in Chapter 2), it is odd that researchers in the field of investment have not made use of VAR analysis. In using VARs many of the empirical difficulties are bypassed allowing the researcher to focus on assessing the validity of the determinants of investment posited by alternative theories. Not only is the use of VAR
models new to the investment field, it is has not previously been applied in analysis of
the construction sector.

The forecasting performance of the VAR models and the single equation models
corresponding to the dominant theories of investment behaviour are compared with
each other and with the performance of a benchmark ARIMA model. Until recently,
the task of comparing the accuracy of sets of forecasts from alternative models was
extremely complex. However, recent developments in the fields of forecast evaluation
have made a scientific approach to such tasks more manageable. In this work we make
use of these new developments in evaluating the accuracy of forecasts of construction
output generated using our models.

1.4 Outline of the Thesis

The post-war investment literature has been largely dominated by the empirical testing
of four different theories of investment behaviour. A large part of Chapter 2 is spent
outlining these four theories with a view to arriving at, for each theory, an investment
equation of the kind frequently estimated in the empirical literature. The first model
considered is the accelerator model. This posits a simple relationship in which
investment is a function of current and lagged changes in output. Although lacking
theoretical underpinnings, the accelerator dominated the literature prior to the 1960s
neoclassical research programme on investment.

A number of more sophisticated models have been developed subsequently. The first
of these models is the neoclassical model originated by Jorgenson (1963). Jorgenson’s
model attempts to remedy the defects of the accelerator model by developing a model
based on the neoclassical principle of optimisation behaviour. The model relates
desired capital stock to interest rates, output, capital prices and tax policies and is
based on maximising net worth, defined as the sum of the stream of discounted future
net profits. However, although the theoretical model is more realistic than the
accelerator model and, moreover, consistent with microeconomic theory, it has been
heavily criticised. From these criticisms, two other models have evolved. The
Jorgenson model assumes that the range of factor substitution possibilities \textit{ex post}, i.e.
after capital has been purchased and installed, are identical to those available \textit{ex ante}.
i.e. prior to the purchasing of capital. In practice, the substitutability between factors of production is likely to be more limited \textit{ex post} than \textit{ex ante}. The third model we outline, the putty-clay model, embodies this idea. A second major criticism of the neoclassical model has lead to the development of the cost of adjustment model. In Jorgenson’s theoretical model the firm is able to adjust its actual capital stock instantaneously to the desired level. This implies an infinite rate of investment. To implement the model empirically, therefore, the neoclassical model requires an \textit{ad hoc} assumption of delivery lags which allows a distributed lag relationship between investment and its determinants to be estimated. The cost of adjustment model simply incorporates the assumption of delivery lags (or equivalently, other factors which imply additional costs of adjusting capital stock) into the firm’s optimisation problem. By building the costs of adjustment into the firm’s optimisation problem, that is, by building into the theory, the dichotomy between Jorgenson’s theory of investment and its empirical implementation is resolved.

The fourth theory to dominate the literature is that developed by Tobin and Brainard in the late 1970s. On the surface, this appeared to be unrelated to previous work on the theory of investment behaviour. Put simply, the theory states that a firm will engage in investment if its value on the stock market increases as a result of an investment by more than it costs the firm to undertake the investment. Although the theory appears to be unrelated to previous theories of investment, Abel (1979) and Hayashi (1982), among others, show that it is in fact equivalent to the basic neoclassical theory of investment modified by adjustment costs. Put another way, Tobin’s theory is a particular type of adjustment cost model.

This literature is disparate and confusing, but here the four dominant theories are presented in a unified framework in which the similarities and differences between the models can be easily identified. The relationship between these models and a number of peripheral models is also explored.

After presenting the underlying theory for each model of investment we outline the major empirical results associated with that model. Like the theoretical literature, the empirical literature on nonresidential fixed investment is vast. Of greatest interest are UK studies in which models of construction investment are estimated with a view to forecasting. Unfortunately, studies of this kind are too few in number to draw
inference on the relative performance of investment models. We therefore consider
the literature more broadly and review studies of equipment expenditure and
aggregate expenditure more generally. Moreover, since there are relatively few UK
studies of investment, most of the studies surveyed are based on US data. In addition,
in many of the studies considered, models are estimated with a view to policy analysis
rather than forecasting. In an attempt to make the survey more manageable we focus
on time series studies. The number of cross-section and panel data studies has
increased dramatically in recent years and have much to offer in terms of improving
our understanding of firms' investment behaviour. However, to the extent that we aim
to develop models with aggregate time series data, these studies receive limited
attention: important results are referred to in passing. There are a handful of mainly
US studies whose primary objective is to compare the forecasting performance of
alternative econometric models of investment. Given that the objective of these
studies overlaps our objective, we consider these in some detail.

It is noteworthy that the principles of economic analysis are being applied to problems
in the field of surveying and building with increasing regularity. There are a number
of recent studies in this field that use statistical techniques to develop forecasting
models of construction variables. Although, these studies tend not to be based on
secure theoretical foundations, they are included in the literature survey on the
grounds that the objective of these studies is very much related to ours.

We consider issues related to the estimation of models and forecasting of private
industrial and commercial construction output in Chapter 3. The survey of the
literature on investment models contained within Chapter 2 is largely devoid of
discussion of econometric issues and, to a large extent, this reflects an absence of
discussion in the literature. For example, issues such as the examination and treatment
of non-stationarities in the data, have not received attention in the investment
literature. These are important issues since the distribution of conventional test
statistics in econometrics is calculated under the assumption that the residual series is
stationary. Moreover, the regression of one non-stationary time series on another may
lead statistical analysis to suggest a relationship between series even where one does
not exist. Given this, one might be surprised to find a conspicuous absence of such
testing in the empirical investment literature. However, this absence is due, in part, to
the fact that much of the literature was developed prior to realisation of the importance of this issue.

The fact that many economic time series are known to be non-stationary implies that testing for non-stationarity is essential to the development of valid econometric equations. Also, it is important that the procedures for dealing with non-stationarity are followed. In Chapter 3 we describe a number of popular tests for non-stationarity and describe the procedures for dealing with it. To the extent that (later in the thesis) we examine data for non-stationarities and estimate equations consistent with the findings from such tests this work represents an advance on the existing investment literature.

Also in Chapter 3, we outline the VAR approach to modelling. In particular, we focus on a description of the Johansen procedure. Although this procedure (and for that matter, the various tests for non-stationarity and the means of dealing with it) are well known in the econometrics literature, they are outlined in this work since they have not previously been applied or considered in the context of investment behaviour or in analysis of the construction sector.

In the final part of Chapter 3 we consider issues surrounding the evaluation of forecast performance. There are a number of studies in the empirical investment literature which aim to systematically compare the forecast performance of alternative investment models. On the whole the methods of comparison in these studies are crude, being based on statistics summarising the sample evidence (e.g. the mean square forecast error) and visual inspection of plots of forecasts and forecasts errors. In this thesis we adopt more rigorous methods of comparing forecasts. For example we use recently developed procedures to test whether one set of forecasts is significantly better than another in a statistical sense. In Chapter 3 we discuss the underlying objectives of an evaluation exercise, some frequently used measures of forecast accuracy and some of the problems associated with these measures. We describe the procedures to be used in this thesis and outline attempts to create conditions under which forecasts derived from different generating mechanisms can be fairly compared. We also discuss the parameters of the forecasting contest to be conducted in this work..
Chapter 4 relates to the data. In the first part of the chapter we provide the definitions and sources of data, details of how variables are constructed and limitations of the resulting measures. This part of the chapter is organised into two components. First, we consider the data suggested by the four dominant theories of investment behaviour. We examine the relationship between construction output data and investment expenditure data with a view to determining the suitability of using theories of the latter to model the former. The data requirements are considerable and complicated by the effects of investment incentives in the tax system. Among the determinants suggested by theory are the user cost of capital and Tobin's $q$. Considerable effort is spent constructing these measures and adjusting them for the effects of taxation. Second, we consider other variables thought to have an impact on this kind of construction output. These are used in the VAR models. Some of these variables are suggested by other theories of investment, some have been found to be significant in the empirical literature, and some are thought to be leading indicators of investment. In the second part of the chapter, we examine the statistical properties of all series used in subsequent analysis. In particular, we examine the data for evidence of nonstationarity using a number of unit root tests. In cases where these tests fail unambiguously to determine the presence of unit roots, we employ spectral techniques to provide more evidence. It should be noted that the power of these tests is generally low and, as such, care must be exercised in interpreting results.

Chapter 5 is concerned with model estimation. This chapter divides naturally into three parts. In the first part we consider the issues surrounding the estimation of the four dominant theories and analyse the results of estimation and diagnostic testing. The models are estimated such that they are consistent with the findings of unit root tests presented in Chapter 4. In addition, we provide a comparison of the within sample properties of each of these models. In the second part, we develop a number of ARIMA models of construction output. Several models are found to provide an adequate description of the data and four models, each corresponding to a different transformation of the data, satisfy the diagnostics. Further discrimination between models is reserved until Chapter 6 when the forecasting abilities of these models is assessed. The ‘winning’ model provides the benchmark model against which the forecasting performance of the econometric and VAR models is compared. In the
third part of the chapter we estimate a number of VAR models. Some of these VAR models are loosely consistent with theory, others make use of leading indicators, data from the construction sector and macro variables.

In Chapter 6 we use the models of private industrial and commercial construction output estimated in Chapter 5 to generate one-step-ahead, four-step-ahead and twelve-step-ahead *ex post* forecasts for the period from 1993q4 to 1996q4. In the first part of the evaluation exercise we determine the benchmark model from four ARIMA models found to describe the private industrial and commercial construction output data adequately. We then evaluate the forecasting performance of the econometric and VAR models. The forecast performance of each econometric investment equation is first compared with the performance of the other investment equations and then with the performance of the benchmark ARIMA model. The accuracy of forecasts generated using a particular VAR model is similarly compared with the accuracy of forecasts from other VARs prior to making comparisons with the benchmark model. In the final part of the evaluation exercise we examine the relative forecast performance of the econometric investment equations and the VAR models.

Finally, in Chapter 7, we summarise the major findings and achievements of this work, outline some of its weaknesses and suggest some avenues of future research. However, we begin with a review of the investment literature.
Since we are to model construction output as an investment problem it is appropriate to begin by reviewing existing theories of investment. The investment literature has been largely dominated by the empirical testing of four different theories of investment behaviour. In the first part of this chapter we outline these theories and review the associated empirical evidence. We consider first the accelerator model of investment, the most prominent model prior to the neoclassical revolution of the 1960’s. The principal attraction of the accelerator model is its simplicity: it posits a relationship between the aggregate investment and the change in aggregate output. However, its simplicity, and the fact that the relationship between investment and output is fundamentally *ad hoc* (in the sense that is not consistent with economic theory), resulted in criticism which led to the development of a number of other, more sophisticated, models of investment. The first of these models is the neoclassical model originated by Jorgenson (1963). Jorgenson attempts to remedy the defects of the accelerator model by developing a model based on the neoclassical principle of optimisation behaviour. The model relates desired capital stock to interest rates, output, capital prices and tax policies and is based on maximising net worth, defined as the sum of the stream of discounted future net profits. Jorgenson maximises net worth over an infinite time horizon subject to a standard neoclassical production function and the constraint that the rate of growth of capital stock is equal to net investment. However, although the theoretical model is more realistic than the accelerator model and, moreover, consistent with microeconomic theory, efforts to implement the model empirically have resulted in a number of substantial criticisms.

The Jorgenson model assumes that the range of factor substitution possibilities *ex post*, i.e. after capital has been purchased and installed, are identical to those available *ex ante*, i.e. prior to the purchasing of capital. In practice, the substitutability between factors of production is likely to be limited *ex post*, in contrast to substitution possibilities *ex ante*. The putty-clay model of investment was born from this criticism of the neoclassical model and is the third of the models that we consider below. A
second major criticism of the neoclassical model has lead to the development of the cost of adjustment model. The empirical implementation of the neoclassical model requires an \textit{ad hoc} assumption of delivery lags which allows a distributed lag relationship between investment and its determinants to be estimated. Without such an assumption, investment takes place at an infinite rate. The cost of adjustment model simply incorporates the assumption of delivery lags (or equivalently, other factors which imply additional costs of adjusting capital stock) into the firm's optimisation problem. By building the costs of adjustment into the firm's optimisation problem, that is, by building into the theory, the dichotomy between Jorgenson's theory of investment and its empirical implementation is resolved.

The neoclassical theory of investment, and variants thereof, dominated in the literature from its inception in the early sixties to the late seventies. In the late seventies, Tobin and Brainard posited a new theory of investment which, on the surface, appeared to be unrelated to work that was being undertaken in the ongoing neoclassical research programme. Simply stated, Tobin and Brainard suggest that a firm will engage in investment if its value on the stock market increases as a result of the investment by more than it costs the firm to undertake the investment. Although the theory appears to be unrelated to previous theories of investment, Abel (1979) and Hayashi (1982), among others, show that it is in fact equivalent to the basic neoclassical theory of investment modified by adjustment costs. Put another way, Tobin's theory is a particular type of adjustment cost model. This interpretation of the cost of adjustment model is the fourth theory of investment considered in this chapter.

We aim to provide a comprehensive outline of the development of each of these theories. The links between these theories have often been difficult to grasp, in part, because they lack a common framework. To this end, we present the theories in a unified framework in which the similarities and differences between theories can be easily identified. To simplify analysis, we do not explicitly include taxation and other policy instruments into the models at this stage. The fiscal factors affecting investment are considered more fully in Chapter 4. Such a treatment allows a clearer exposition of the relationship between theories, a matter which is considered explicitly in Section 2.9.
There are a number of points that ought to be made about our discussion of the empirical studies associated with each of the theories of investment. First, the empirical literature associated with each of these theories of investment is vast. Given our interest in new private industrial and commercial construction output, we would ideally like to focus attention on empirical studies of net investment in buildings. However, very few studies focus entirely on investment in buildings. A small number consider investment in structures (which includes an element of civil engineering) but such investment tends to be analysed alongside investment in plant and machinery. More usually, the aim of studies is to model aggregate investment or investment in plant and machinery alone. This survey reflects this bent.

Second, the vast majority of studies in investment are based on US data. However, given our interest in UK construction, we pay particular attention to UK studies wherever possible. This is not to over-emphasise the contribution of British economists to the empirical investment literature, but merely because these are the studies more relevant to our own work.

Third, since the introduction of the neoclassical principle of optimisation, investment theory has developed in the context of the individual firm. This is reflected by the recent increase in popularity of empirical studies using disaggregated data. Also increasing in popularity of late has been the use of panel data to estimate investment equations. Whilst cross-sectional and panel data estimation have much to offer in terms of improving our understanding of firm’s investment behaviour, this thesis is primarily concerned with developing aggregate time series models and, as such, this survey of the empirical investment literature focuses primarily on aggregate studies. However, important results originating from studies using disaggregated data are noted. We consider the issue of aggregation at in Section 2.9.

The fourth point to note about our coverage of empirical studies is an absence of studies assessing forecasting performance. Unfortunately, the forecasting performance of investment models is rarely considered in the mainstream literature. Studies have been primarily concerned with policy analysis. However, there are a handful of published studies in which the empirical performance of alternative models is compared. One of the popular tools of model comparison in this group of papers is forecast performance. We consider these studies in some detail in Section 2.7.
Finally, in order not to detract from the thread of the underlying survey, we minimise discussion of econometric issues in what follows. The estimation of empirical investment equations raises many econometric issues. A discussion of the econometric issues relevant to our modelling approach is contained in Chapter 3. In delaying discussion of such issues we are able to focus on the economic debate which is dominated by arguments over the relative importance of price variables (such as tax and interest rates) versus quantity variables (such as output, sales, or liquidity) as determinants of investment.

We are fortunate to have a number of excellent surveys of nonresidential fixed investment at our disposal. Meyer and Kuh (1957) and Eisner and Strotz (1963) survey the early literature. These surveys are dominated by empirical analysis of the accelerator model. Jorgenson (1971) and Nickell (1978) survey the empirical results associated with the neoclassical model and its many variants. A first rate account of the modern literature has been provided by Chirinko (1993). In addition to discussion of some of the early studies, Chirinko presents a very detailed account of the main issues raised by researchers’ attempts to estimate cost of adjustment models. We draw upon these surveys in this chapter.

In addition to outlining the development of these four dominant theories of investment and providing a survey of the empirical work associated with each of these theories, we briefly introduce a number of other theories that have contributed to the development of investment theory. These models, which include the Keynesian model, the cash-flow, profit and liquidity models, and the irreversibility theory of investment, receive much less attention in modern empirical literature. A short account of each of these theories is provided in this chapter, since all of them are in some way related to the four main theories considered and have in some way contributed to the development of investment theory. These are considered in Section 2.8.

The literature associated with models of nonresidential fixed investment, however, forms only part of our area of interest in this chapter. Since in this thesis we are modelling new private industrial and commercial construction output, we are also interested in studies that aim to model and forecast new construction output. The distinct lack of such studies in the economics literature has already been noted.
However, the tools of economic analysis are being applied to problems of surveying and construction with increasing regularity. This has been particularly evident in the analysis of the construction cycle, as more and more studies in the field of building and surveying adopt econometric techniques to develop models for forecasting construction variables. Many of these studies are motivated by the desire to obtain more accurate information about future conditions in the market that may aid agents’ planning and decision making and thereby help to dampen the volatility of the construction cycle. Although this work tends to lack sound theoretical underpinnings, we include a brief review of the empirical work from this field on the grounds that the objective of this body of research is more clearly related to the objective of this work.

The review is organised as follows. In Sections 2.2 to 2.6 we develop each of the main theories of investment behaviour with a view to arriving at an estimable equation of the kind typically estimated in the literature. We also present empirical results associated with each of these theories. In Section 2.7 we consider a number of papers in which the empirical performance of various competing models of investment are tested and compared. In addition to providing a summary of issues raised in previous sections, it is hoped that this section may cast light on some of the empirical problems likely to be encountered in the estimation of models in this thesis. In Section 2.8 we consider other theories of investment and in Section 2.9 we provide an overview of the literature in which we make explicit the links between the theoretical models and summarise empirical findings associated with them. Finally, we review empirical work from the fields of building and surveying in Section 2.10.

2.2 The Accelerator Model

The accelerator model of aggregate investment behaviour, as posited by Clark (1917), was one of the earliest attempts to rationalise the volatility of aggregate investment expenditures. The implicit assumption of a fixed capital-output ratio in this model implies that prices, wages, taxes and interest rates have no direct effect on capital spending. We begin by outlining the theory and in Section 2.2.4 we examine the associated empirical literature.
### 2.2.1 The Simple and Flexible Accelerators

Denoting real output $Y_t$, and the optimal capital stock $K_t^*$, the naive accelerator is given by

$$K_t^* = y Y_t \quad (2.1)$$

where $y$ is the fixed capital-output ratio. In addition to the fact that the optimal capital stock bears a fixed relationship to output, the capital stock is always optimally adjusted implying that $K_t = K_t^*$. Here, net investment is given by

$$\Delta K_t = K_t - K_{t-1} = y (Y_t - Y_{t-1}) \quad (2.2)$$

As a result of the assumption of instantaneous adjustment, the naive accelerator model of investment has not fared well in empirical analysis. Koyck (1954) generalised this original accelerator model so that the adjustment of capital stock to its desired, or optimal, level was no longer instantaneous. Instead, the adjustment was assumed to be a constant proportion, $v$, of the difference between the actual and desired levels of capital stock, that is,

$$\Delta K_t = K_t - K_{t-1} = v (K_t^* - K_{t-1}) \quad (2.3)$$

The so called flexible accelerator model may be derived by substituting equation 2.1 into equation 2.3. This yields

$$\Delta K_t = K_t - K_{t-1} = yv Y_t - v K_{t-1} \quad (2.4)$$

or

$$K_t = yv Y_t + (1 - v)K_{t-1} \quad (2.5)$$

By writing equation 2.5 for different time periods, $t-1$, $t-2$, $t-3$, etc. and repeatedly substituting each such equation back into equation 2.5 we get a distributed lag relationship with geometrically declining weights. This is given by

$$K_t = y [v Y_t + v(1 - v) Y_{t-1} + v(1 - v)^2 Y_{t-2} + \ldots] \quad (2.6)$$

or
There are two notable aspects to equation 2.7. First, the change in capital stock, i.e. net investment, depends on current and lagged changes in output, hence the term ‘accelerator’ model. Second, changes in output in period $t$ affect investment not only in the current period but in future periods also. Conversely, investment in period $t$ is a result of current and past changes in output. More distant changes carry successively smaller weights, hence the geometric distributed lag formulation.

The investment function defined in equation 2.4 or 2.7 is in terms of net investment. Assuming replacement investment is a constant proportion of last period’s capital stock, a gross investment function may be obtained by adding $SK_{t-1}$ to both sides of equation 2.4 or 2.7 to give

$$K_t - K_{t-1} = \gamma \left[ v(Y_t - Y_{t-1}) + v(1 - v)(Y_{t-1} - Y_{t-2}) + v(1 - v)^2 (Y_{t-2} - Y_{t-3}) + \ldots \right]$$

(2.7)

$$\Delta K_t = \gamma v \sum_{i=0}^{\infty} (1 - v)^i \Delta Y_{t-i}$$

Equations of this form are frequently estimated in the empirical literature. We consider specific examples in the following chapter.

2.2.2 Two Rationalisations of the Flexible Accelerator

Unlike the simple accelerator model in which net investment is a function only of current changes in output, the flexible accelerator posits that net investment also depends on lagged changes in output. There are two rationalisations as to why lagged changes in output are important. The first is that, for some reason, the firm can adjust only gradually, rather than instantaneously, to changes in demand. Thus, the firm will increase part of its capital stock today in response to a change in output and will go on reacting to that change of output over time. Hence, the distinction between actual and optimal capital stock, $K_t$ and $K^*_t$ respectively. Assuming that the desired capital stock depends solely on output as defined in equation 2.1, so that $\Delta K^*_t = \gamma \Delta Y_t$, then desired
investment is related to current changes in output. Thus, the firm would like to react instantaneously to changes in output (as it implicitly does in the naive version of the model); however, it may be prevented from doing so by delays owing to, say, lags in the delivery of the investment goods, which means that in practice it can only do a fraction of the desired investment each period. Since investment intentions must pass through various stages such as planning, contracting and ordering before becoming expenditures and then may still be subject to delivery lags and construction lags, the lagged output terms reflect the gradual response of investment to changes in final demand.

The second rationalisation of the flexible accelerator model focuses on expectations. Suppose firms form their expectations of future output or demand, on which they base their investment decisions, by looking at past levels of demand. Then, current investment will be a weighted average of current and past changes in output. The following example illustrates this point. Assume that the firm remains optimally adjusted so that actual and desired investment are equal, but that desired capital stock depends upon expected output at time $t$. That is

$$K_t = K^*_t = \gamma Y^*_t$$

(2.10)

This implies that

$$\Delta K_t = \Delta K^*_t = \gamma \Delta Y^*_t = \gamma (Y^*_t - Y^*_{t-1})$$

(2.11)

If we assume adaptive expectations, that is, if firms adjust their expectations by a fraction of the error between actual and expected output last period, then

$$Y^*_t - Y^*_{t-1} = \nu (Y_{t-1} - Y^*_{t-1})$$

(2.12)

By manipulating this equation we can show that

$$Y^*_t = \nu Y_{t-1} + \nu (1 - \nu) Y_{t-2} + \nu (1 - \nu)^2 Y_{t-3} + \ldots = \nu \sum_{i=0}^{\infty} (1 - \nu)^i Y_{t-i-1}$$

(2.13)

That is, the expectation of output at time $t$ is a weighted average of lagged output. Substituting this into equation 2.11 we get
\[ \Delta K_t = \gamma Y_t = \gamma \nu \sum_{i=0}^{\nu} (1 - \nu)^i \Delta Y_{t-i-1} \] (2.14)

which is the flexible accelerator relation. Thus, the lagged output term can also be seen as encompassing output projections, if such projections are derived from past output.

The flexible accelerator, although simple, has often performed better than some of its more sophisticated counterparts when tested empirically but the underlying reason for this good performance remains unclear. We have shown that we can rationalise the flexible accelerator model in one of two ways: the non-instantaneous adjustment of capital stock to its desired level follows from the assumption of delivery lags or from the assumption of adaptive expectations. Does the model explain the data well because firms form their expectations adaptively or because there are delivery lags etc. in acquiring new capital? Could it be a mixture of both? In fact, at an empirical level, there exists considerable difficulty and controversy in establishing the precise role of different factors of the investment process, and this is not confined just to the accelerator theory of investment behaviour.

2.2.3 Underlying Technology and Relative Prices

A similar ambiguity relates to underlying technology of the firm that is assumed in the accelerator theory of investment. The distinguishing feature of the accelerator model is the emphasis on demand or output as the determining factor of investment. Why is there no explicit role for relative prices (interest rates and input prices) and hence profitability? If a firm is in some way demand constrained, such that it can not sell all of its output, then it will choose its inputs, and hence investment decisions in such a way as to minimise the cost of producing this given level of output. Although output is an important determinant of investment, the price of inputs should still affect both the quantity and type of investment that the firm undertakes. So why do these variables not enter the investment equation?

Again there are two possible explanations. The first is that the firm's production possibilities are defined by a fixed coefficient production function such that the firm's isoquants in \((L,K)\) space are right-angled, so that for a cost minimising firm, the
optimal capital will be uniquely determined by output. Here, adding more labour, $L$, whilst maintaining the level of capital stock will not increase output. Similarly, output will be unaffected by increasing capital, $K$, for a given labour input. Thus, there is a unique optimum level of capital (and labour) needed to produce a given output, independent of the relative prices.

The second possible explanation allows capital to be substituted for labour in the production of a given level of output, so that the firm’s isoquants are smoothly downward sloping. In this case, relative prices are crucial in determining the optimal level of capital. The firm, as above, chooses the cheapest combination of capital and labour to produce a given level of output, but the combination changes as relative prices change. Thus, in general, there is no unique relationship between desired capital and output. However, if relative prices remain constant, then output will be the sole determinant of desired capital stock as implied in the accelerator.

Thus, although it is clear that relative prices do not matter in the accelerator model of investment, the reasons why they do not matter are unclear. It may be because firms face fixed coefficient production functions. However, the assumption of constant relative prices in a world where capital can be freely substituted for labour, will also result in an absence of a term for relative prices in an accelerator equation.

Although the accelerator performs well empirically, its failure to assert precisely why relative prices do not matter, together with its failure to identify the source of the non-instantaneous adjustment of capital stock to its desired level, has weakened its position as a credible theory of investment behaviour. However, as mentioned above, the difficulty in establishing the precise role of different factors is not confined to the accelerator theory of investment; as we shall see, it is a general problem common to other theories. We now examine the some of the attempts to apply this model to real data.

2.2.4 Empirical Results with Accelerator Models

The accelerator model dominated the literature prior to the 1960s. This is reflected by the fact that some of the references in this section are rather dated. We begin with some early results and work forward. Many authors substituted profit variables into
the accelerator framework. It is noteworthy that if profits are a function of output, then profit models are empirically indistinguishable from accelerator models. (We consider the relationship between profit theories and accelerator models in more detail in Section 2.8.2). As such, this section also presents empirical results from some early profit models.

The work carried out by Tinbergen (1938, 1939) represents the earliest example of work adopting statistical techniques to investigate accelerator type relationships. Using ordinary least squares, Tinbergen (1938) runs a number of regressions for various industries to explain iron and steel consumption (a proxy for investment) in the US, UK, France and Germany. Regressions adopting the current change in output as the single explanatory variable yielded mixed results. Although the coefficient on the accelerator term was significant in a number of regressions, its size was much lower than predicted by theory. When Tinbergen substitutes current changes in output for a variable measuring profits, significance improves and the size of the coefficient increases but it is still lower than predicted by theory. Tinbergen (1939) comments on these earlier results and concludes that 'there is fairly good evidence that the fluctuations in investment activity are in the main determined by profits earned in industry some months earlier' (p. 49). In a separate analysis of railroad investment in the same countries, Tinbergen (1939) concludes that 'the acceleration principle gives a somewhat better explanation than the profit principle but the regression coefficients are below the theoretical values' (p.130).

Manne (1945) also estimates an equation for railroad investment in the US with pre-war data. Manne’s work differs from that of Tinbergen in that he includes a regressor capturing spare capacity in addition to the standard accelerator variable. The spare capacity variable is given by the ratio of idle to total freight cars. When estimated without this variable, Manne’s results are very similar to Tinbergen’s in that the coefficient on the accelerator term was much lower than predicted by theory, 0.299 rather than unity, and thus provide little support for the simple accelerator. On re-estimating the relationship with the spare capacity variable, the coefficient becomes 0.578 which is much closer to its theoretical value.

Kisselgoff (1951) and Kisselgoff and Modigliani (1957) model gross investment by US electrical utilities as an accelerator relation with positive results. The Kisselgoff
and Modigliani (1957) equations are estimated using least squares over the sample period 1925-1941. They conclude that the acceleration principle is the prime explanation of investment in the electric power industry. They point out that this result may be industry specific, since there existed strong legal and institutional pressures to meet demand which tended to make the level of capital stock in this industry more sensitive to changes in output. Indeed, this possibility is supported by some of the empirical work of Kisselgoff (1951) in which the accelerator variable is found to be insignificant in regression equations of aggregate gross investment. In both studies profits are also found to have some influence, although as Kisselgoff and Modigliani note their role is not very pronounced.

Chenery (1952) compares the performance of the simple and flexible accelerator. Using ordinary least squares to estimate simple accelerator relations for a number of US industries between 1920 and 1940, Chenery finds mixed support for the accelerator model. Measuring investment as the proportionate change in each industry's capital stock, the accelerator variable is found to be significant for some industries (namely electric power, cement, petroleum refinery and paper) but insignificant in equations for other industries (such as steel and zinc). When estimating capacity models, compatible with the flexible accelerator, over similar sample periods, the results improve with equations for all industries containing significant capacity variables.

Eisner (1957) estimates cross sectional relationships to test the acceleration principle. Taking gross investment as a proportion of capital stock as his dependent variable, Eisner uses a data set containing responses from more than 250 firms to estimate investment equations by sector and firm size. The change in output as a proportion of the previous period's output is used as the explanatory variable. Support for the accelerator hypothesis is found in most industrial sectors. When the data is organised according to firm size, Eisner uncovers support for the accelerator relation only among large firms. Moreover, the evidence suggests that these firms were slow to adjust their capital expenditures to the boom at the end of 1950, but were quick to adjust to the decline in 1949. Eisner also finds support for a positive relationship between the level of profits and capital expenditure. In a subsequent paper, Eisner (1960) reported estimates of a distributed lag investment function based on accounting
data for large corporations. Lagged changes in sales and profits were used as alternative explanatory variables. Strong evidence in support of the acceleration component in capital expenditures is reported.

Using a data set containing information on 700 US firms in 12 industries, Meyer and Kuh (1957) perform a comprehensive statistical analysis of investment behaviour. Most of their work is with cross sections for a particular year's capital expenditures or with cross sections of the averages for all firm variables within an industry. Much of the emphasis of this work is on the role of liquidity in investment decisions, although they run a number of regressions which include a capacity variable. In their conclusion, Meyer and Kuh declare that the accelerator model worked well for 1946 and 1947 when almost all firms were confronted with favourable market conditions, no excess capacity and no liquidity constraints. In the three subsequent years, liquidity variables were found to be important, particularly so in 1949. Thus, Meyer and Kuh suggest that there is evidence of liquidity and accelerator reactions. Their finding that the accelerator variable works best when in periods when there is little excess capacity, is consistent with the findings of other authors.

The work of Koyck (1954) provides one such example. Using US data for the inter-war years for the freight car, steel, electric light and power, cement, and petroleum refining industries, Koyck estimated a number of distributed lag models of investment behaviour. Throughout Koyck's analysis the reaction of capacity (i.e. capital stock) to changes in output is clearly present. Moreover, the effect is particularly strong in growing industries and in periods of rapid expansion. Evidence in favour of a simple accelerator model is weak, although his analysis of the freight car industry before the first world war yields some evidence that might support this hypothesis. Koyck notes that it was this series that Clark (1917) first used to demonstrate the accelerator hypothesis.

Grunfeld (1960) introduced lagged profits into a flexible accelerator model. He finds that the partial correlation of profits and investment, given capital stock, is insignificant. On the basis of this finding, Grunfeld concludes that current profits are not a good measure of expected profits. As an alternative he suggests that discounted future earnings less the cost of future additions to capital stock might provide a superior measure of expected profits. It is this aspect of Grunfeld's analysis that has
much in common with Tobin’s $q$ theory. We consider this aspect of Grunfeld’s work in Section 2.6.7.

Smyth (1964) has noted that the acceleration principle has been the subject of a large number of statistical investigations which have yielded widely differing results. As well as considering some theoretical aspects of the accelerator model, Smyth surveys and evaluates the empirical evidence and finds that on balance, the statistical evidence is unfavourable to the accelerator model. However, he notes that, generally speaking, statistical studies unfavourable to the accelerator have typically adopted ‘crude formulations and unsatisfactory methods of analysis’ (p. 193). However, he finds in favour of his hypothesis that studies adopting sophisticated statistical methods of analysis ‘invariably produced results favourable to the acceleration principle’ (p. 193). He also notes that those studies adopting more sophisticated statistical methods have ‘invariably produced results favourable to the accelerator principle’ (p. 193).

A number of studies have modelled investment using the accelerator model and assessed its performance against the performance of other models. Among this group of papers is Bischoff (1971b), Clark (1979), Bernanke et al (1988), Jorgenson and Seibert (1968a), Jorgenson et al (1970a, 1970b), Elliot (1973) and Kopcke (1985). We consider the treatment of the accelerator model in these studies and its empirical performance *viz a viz* the performance of other models in Section 2.7. We first turn our attention to other models of investment behaviour.

2.3 The Neoclassical Model

In the accelerator model of investment behaviour there is no role for substitution between capital, labour and other inputs, since the capital-output ratio is assumed constant. In addition, investment does not depend on the price of capital. Jorgenson (1963) and a number of colleagues have attempted to remedy these defects by developing a model based on the neoclassical principle of optimisation behaviour. The model relates desired capital stock to interest rates, output, relative prices and tax policies. However, whilst Jorgenson’s theory provides a clear framework for understanding the factors affecting firms’ capital stock, it does not rationalise either investments or movements to the optimal capital stock. Thus, despite a sound
theoretical foundation, empirical implementation requires a rather ad hoc assumption about the adjustment process.

2.3.1 Some Preliminaries

Jorgenson’s approach is based on the maximisation of function for net worth, $V$, where net worth is defined as the sum of discounted future net profits. Jorgenson maximises net worth over an infinite time horizon subject to a standard neoclassical production function and the constraint that the rate of growth of capital stock is equal to net investment. Since capital goods are durable, firms purchasing long lived plant and equipment, could potentially lock themselves into a situation in which they might not be able to dispose of unwanted capital goods. This implies a complex optimisation problem, one involving uncertainties about the lifetime of capital goods, future input prices and future output demands. In order to make the problem workable, Jorgenson made a number of simplifying assumptions.

Perfect competition is assumed in all markets, including the capital market (so that the firm can borrow or lend any amount at a given rate of interest) and the second-hand capital market. Facing a perfect market for second-hand capital goods, firms need not worry about locking themselves in by purchasing long lived investment goods, since such goods could always be sold on the second-hand market at prices equal to the present value of expected services over their remaining lifetimes. This assumption also allows us to view firms as renting capital to themselves during each time period, charging themselves an implicit rental for capital. Jorgenson also assumes that there are no costs in either the sale or the purchase of investment goods, or the installation of capital. Furthermore, he assumes that capital depreciates at a constant geometric rate. As a result of these assumptions a firm can achieve an optimal capital stock, $K^*_t$, instantaneously. Although the model can be formulated to show the effects of taxation on the optimal capital stock, we exclude the effects of taxation at this stage in the interests of simplicity.
Jorgenson’s simplifying assumptions, reduce a difficult present value optimisation problem to a sequence of one period profit maximisation problems for which the firm chooses capital, labour and output so as to maximise each period’s profits subject to a production function constraint and the constraint that the rate of growth of capital stock is equal to net investment. However, since the firm’s problem is that of maximising profits, or more accurately net worth, over an infinite number of time periods, rather than in a single period, we can not use the tools of static optimisation to solve this problem. Instead, since we are maximising through time subject to constraints that hold at each point in time, we can set up the problem, and solve it, within the framework of dynamic optimisation.

Stated formally, the firm’s problem is written as

\[
\max_{I,J} \quad V(t) = \int_0^\infty \left[ p(t)Y(t) - w(t)L(t) - z(t)I(t) \right] e^{-rt} \, dt
\]

subject to

\[
Y(t) = F[L(t), K(t)]
\]

and

\[
\dot{K}(t) = I(t) - \delta K(t)
\]

where \( V(t) \) is net worth in period \( t \), \( r \) is the rate of interest, \( Y(t) \), \( L(t) \) and \( I(t) \) respectively define the firm’s output, labour and investment in period \( t \), and \( p(t) \), \( w(t) \) and \( z(t) \) denote the firm’s output and input prices in period \( t \). Notice that there are two control variables here, \( I \) and \( L \), that is, firms choose \( I \) and \( L \) so as to maximise net worth. The state variable is capital stock, \( K \). Thus, our problem is to find the optimal controls which maximise the objective function subject to the constraints imposed by the production function and the condition for the growth of capital stock.

If we substitute the production function into the maximand we can solve this optimal control problem by writing the Hamiltonian function as

\[
H = e^{-rt} \left[ p(t)F[L(t), K(t)] - w(t)L(t) - z(t)I(t) \right] + \lambda(t) \left[ I(t) - \delta K(t) \right]
\]

where \( \lambda(t) \) is analogous to the Lagrange multiplier in static constrained optimisation.
problems and is known, in the terminology of optimal control theory, as the costate variable. The Hamiltonian is maximised with respect to the control variables. Dropping the timescripts, the objective function is maximised under the following conditions.

\[
\frac{\partial H}{\partial L} = e^{-rt}[pF_L(L, K) - w] = 0 \tag{2.19}
\]

\[
\frac{\partial H}{\partial I} = -e^{-rt}z + \lambda = 0 \tag{2.20}
\]

\[
\frac{\partial H}{\partial K} = e^{-rt}pF_K(L, K) - \lambda\delta = -\dot{\lambda} \tag{2.21}
\]

\[
\frac{\partial H}{\partial \lambda} = I - \delta K = \dot{K} \tag{2.22}
\]

Equation 2.19 can be written as

\[
\frac{\partial Y}{\partial L} = F_L = \frac{w}{p} \tag{2.23}
\]

Rewriting equation 2.20 as

\[
\lambda = ze^{-rt}, \tag{2.24}
\]

differentiating both sides with respect to \( t \), substituting the expressions for \( \lambda \) and \( \dot{\lambda} \) into equation 2.21 and rearranging, we get

\[
\frac{\partial Y}{\partial K} = F_K = \frac{z(r + \delta) - \dot{z}}{p} \tag{2.25}
\]

where \( \dot{z} \) denotes the rate of change in the price of capital goods. Notice that these conditions are similar to those profit maximisation conditions derived in static problems. Equation 2.23 says that the firm will produce output and adjust its labour force until the marginal product of labour equals the real wage. In other words, the firm alters its employment of labour until the marginal cost of another unit of labour equals the marginal benefit. Since there are no costs to adjusting capital, the same argument can be applied to capital stock. Equation 2.25 says that the firm will continue to adjust its capital stock until the marginal cost of another unit of capital is equal to the marginal benefit. Note that, just as the numerator of the right-hand side of equation 2.23, namely \( w \), is the cost of hiring a unit of labour for one unit of time, the numerator of the right-hand side of equation 2.25 is the cost of hiring a unit of capital.
for one unit of time. It is known as the user cost of capital and for convenience it is rewritten below as

\[ c = z(r + \delta) - \dot{z} \]  

(2.26)

It is determined by three factors: the interest cost of the capital valued at \( r \); the depreciation cost of capital, \( \delta \), and the capital gain or loss on the capital during the period. The user cost of capital is unaffected by whether the firm faces an implicit cost (through owning its capital) or an explicit cost (from renting), since under the assumption of perfect capital markets firms are indifferent between owning and renting.

2.3.3 Jorgenson’s Derivation of an Investment Function

How does Jorgenson develop an empirically testable investment equation given this theoretical framework? First, he assumes the specific functional form for the production function, namely a Cobb-Douglas production of the form \( Y = AK^aL^\beta \), with decreasing returns to scale (such that \( a + \beta < 1 \)). The marginal product of capital is therefore

\[ \frac{\partial Y}{\partial K} = \frac{aY}{K} \]  

(2.27)

and so (using the equation 2.26) the first order condition given in equation 2.25 becomes

\[ \frac{\partial Y}{\partial K} = F_k(L, K) = \frac{c}{p} \Rightarrow \frac{aY}{K_t} = \frac{c_t}{p_t} \Rightarrow K_t = \frac{ap_tY_t}{c_t} \]  

(2.28)

which gives a specific form for the desired capital stock. From this, investment orders in each period can be derived as

\[ \Delta K_t^* = a\Delta \left( \frac{pY}{c} \right)_t \]  

(2.29)

Jorgenson now adds the assumption that there are delivery lags for capital goods, such that only a fixed fraction, \( \phi_0 \), of the goods ordered this period are actually delivered, a fraction, \( \phi_1 \), of the orders this period are delivered next period, and so on. So actual investment in any period \( t \) is made up of a fraction of goods ordered and delivered in
period $t$ plus deliveries of investment goods that were ordered in previous periods. That is,

$$\Delta K_t = \phi_0 \Delta K^*_t + \phi_1 \Delta K^*_{t-1} + \phi_2 \Delta K^*_{t-2} + \ldots.$$  \hspace{1cm} (2.30)

Given this assumption, we can derive an investment equation by substituting equation 2.29 into 2.30. This gives

$$\Delta K_t = \phi_0 a \Delta \left( \frac{pY}{c} \right)_t + \phi_1 a \Delta \left( \frac{pY}{c} \right)_{t-1} + \phi_2 a \Delta \left( \frac{pY}{c} \right)_{t-2} + \ldots.$$  \hspace{1cm} (2.31)

Adding $\delta K_{t-1}$ to both sides of equation 2.31 and a constant and error term to the right hand side, we get the basic equation for gross investment estimated by Jorgenson. This is

$$I_t = \kappa + \sum_{i=0}^{\infty} \phi_i \Delta \left( \frac{pY}{c} \right)_{t-i} + \delta K_{t-1} + u_t.$$  \hspace{1cm} (2.32)

Note that if relative prices are constant, then equation 2.32 is equivalent to the accelerator model given by equation 2.9. Investment functions of the form of equation 2.32 have been estimated innumerable times in the empirical literature. We reserve discussion of the empirical performance of such equations until Section 2.3.5.

Jorgenson’s work has been the subject of much criticism and it is to this that we now turn.

2.3.4 Criticisms of the Jorgenson Approach

There are two focal points to this criticism. Firstly, despite the fact that Jorgenson sets up a fully intertemporal optimisation problem with firms maximising profits through time and looking ahead at future variables, the first order condition given in equation 2.25 contains only current variables. In this sense, a number of authors have argued that Jorgenson’s model is not genuinely intertemporal, since the firm does not require information about the future time paths of input and output prices. As such, firms’ investment behaviour is described as ‘myopic’. The firm is simply maximising statically in each time period. From the above exposition of the Jorgenson model it is
obvious that this is simply a result of the assumptions he made about technology and
the assumption that there are no costs of adjustment. The firm need not worry about
the future price of inputs and outputs, since capital goods bought today are the same
(net of depreciation) as those that could have been bought yesterday and those that can
be bought tomorrow. Thus, in perfect capital markets with costless adjustment, the
firm can buy and sell capital costlessly at a price equal to the present discounted value
of the services that the capital yields over its remaining life-span. If the firm finds that
it has too much capital, it can off load it costlessly at a known price. Similarly, if the
firm has too little capital then it can buy (or rent) additional capital at a known price.
This is an unrealistic feature of the original Jorgenson model.

The second focal point for criticism is Jorgenson’s empirical implementation of the
model. Many assumptions underlying the empirical model are inconsistent with the
theoretical model. Objection focuses on two contentious areas. Firstly, there is a major
inconsistency between Jorgenson’s theoretical and empirical treatment of the
adjustment process. Recall, in his theoretical work Jorgenson assumed perfect
competition, perfect foresight, and no costs of adjustment. Under these conditions, the
firm adjusts instantly to a change in the economic conditions and is therefore always
optimally adjusted. Since investment is the rate of change of capital stock, a firm that
responds instantly to a change in the rate of interest for example, has an infinite rate of
investment. However, this results in an investment function that is undefined.
Jorgenson avoids this by introducing an *ad hoc* delivery lag into the empirical model.
The delivery lag is arbitrary and inconsistent with the assumption of costless
adjustment.

Secondly, there are econometric objections arising from the fact that Jorgenson treats
output as exogenous in his final specification. A perfectly competitive firm chooses
the output it wishes to supply so as to maximise profits and as such output is actually
endogenous. The use of ordinary least squares to estimate an equation in which one of
the explanatory variables is endogenous results in biased and inconsistent parameter
estimates. As such, estimation of equation 2.32 by OLS is inconsistent with the
assumption of perfect competition. It is possible to avoid this problem. For example,
by solving for desired output, we can gain an expression for desired capital stock in
terms of exogenous variables (the prices). However, this approach is not adopted by Jorgenson.

Given these inconsistencies, even if one were able to accept Jorgenson’s empirical work, this does not imply acceptance of the theoretical model. Moreover, Jorgenson’s use of a Cobb-Douglas production function and the subsequent derivation of the desired capital stock results in a composite term for output and relative prices in the final estimating equation 2.32. This means that it is impossible to separate out the influence of output and relative prices on investment. In other words, it does not allow a direct test of Jorgenson’s main hypothesis, namely, that relative prices matter.

It is important to emphasise the importance of Jorgenson’s work. It was the first investment function to be rigorously derived from an optimisation model of the firm where all assumptions were clearly set out. In contrast to the accelerator theory, the neoclassical theory emphasises relative prices, interest rates and tax variables as important determinants of investment. (Interest rates and taxation affect investment through the user cost term). It was a pioneering piece of work and, as we shall see in the review of the empirical literature presented in the next section, it dominated the investment literature for more than a decade.

2.3.5 Empirical Results with Neoclassical Models

We have discussed at some length the substantial criticisms that have confronted Jorgenson’s pure neoclassical model of investment. Although there have been a very large number of studies utilising variants of the neoclassical model, there are only a few examples of empirical studies adopting the pure neoclassical model as their theoretical underpinning and this to some extent reflects these criticisms. The most well known of the empirical studies adopting a pure neoclassical framework are those by Jorgenson and his collaborators. It is with these studies, carried out at industry and firm level, that we begin. We also review a number of other important studies in which alternative assumptions inherent in the pure version of the model, and the source of much of its criticism, are relaxed.
2.3.5.1 An Example of Jorgenson’s Approach

A number of Jorgenson’s studies, e.g. Jorgenson (1963, 1965), Hall and Jorgenson (1967, 1969 and 1971), have concerned themselves with the investment behaviour of the US manufacturing sector in aggregate. Others have considered the investment behaviour of individual firms within the manufacturing sector, e.g. Jorgenson and Seibert (1968a and 1968b). Yet other studies have investigated the investment behaviour of industry groups within the manufacturing sector, e.g. Jorgenson and Stephenson (1967a, 1967b and 1969). We focus our attention on the study by Jorgenson and Stephenson (1967a) at the manufacturing subgroup level. We consider this study to be representative of Jorgenson’s wider efforts in this area. Since the policy implications of the alternative theories of investment is not of first order importance here, we pay them only limited attention.

Jorgenson and Stephenson (1967a) adopt what Jorgenson (1963 p. 248) calls ‘a theory of investment behaviour based on the neoclassical theory of optimal capital accumulation’ to explain investment within separate manufacturing industries during the period 1947-60. They also use this theory to examine the shape of the lag distribution. Using quarterly data, investment functions are fitted to 15 sub-industries for manufacturing and to groupings of these industries (total durables, total non-durables and total manufacturing). The fitted investment functions can be viewed as having two components. The first provides an explanation of net investment and this is estimated using the distributed lag function of changes in desired capital stock. An expression for desired capital stock is derived from a Cobb-Douglas production function. The second component is replacement investment and this is assumed to be a constant proportion of capital stock. The resulting specifications, of the form given by equation 2.32 in the previous chapter, are found to provide a very close fit to historical experience for both levels of aggregation. They find no evidence of autocorrelation of residuals.

Jorgenson and Stephenson note the estimates of the rate of replacement investment in their equations (\(\delta\) in equation 2.32) are biased downwards (ranging from 0.012 to 0.034) since they are lower than the estimates they obtain in calculating their measures of capital stock. They attribute this finding to the fact that the rate of growth in the price of capital goods is biased upwards. They argue that this upward bias
results from the fact that their measure of the rate of growth of the price of capital goods is based on the price of inputs rather than the price of output from the capital goods producing industry and, in general, prices of output rise less quickly than prices for inputs. Jorgenson and Stephenson also test for the presence of aggregation errors in the industry groupings: total manufacturing, total durables and total non-durables. They find that aggregate industry groups cannot be used in regressions for industry investment without substantial aggregation error. The authors also report that the elasticity of output with respect to capital inputs, \( \alpha \) (contained within the parameters \( \varphi_i \) in equation 2.32) is 'distinctly on the low side' (p. 215). They were able to obtain estimates for \( \alpha \) by imposing a restriction that the sum of coefficients \( \varphi_i \) sum to unity. Estimates for \( \alpha \) range from 0.00516 for 'other non-durables' and 0.26394 for 'petroleum and coal products'. They suggest that the probable source of the downward bias in these estimates is the presence of errors of measurement in the observed values of changes in desired capital stock. They suggest further that the unsatisfactory nature of these results implies that the parameters of the production function should not be estimated using the neoclassical theory of investment, unless the accuracy of measurement of desired capital can be improved.

In general, Jorgenson and Stephenson conclude that 'a theory of investment behaviour based on the neoclassical theory of optimal accumulation of capital provides a highly satisfactory explanation of actual investment expenditures for our sample period' (p. 216). Moreover, they state that according to their analysis of the effects of policy instruments on investment, the role of the tax structure and factors affecting the user cost of capital are important. The authors encourage additional efforts to test the theory. They suggest the most important such test is to compare the results from their theory of investment behaviour with the results obtained from other approaches. Jorgenson's comparisons of alternative econometric models of investment behaviour are found in Jorgenson and Seibert (1968a) using firm level data and Jorgenson et al (1970a and 1970b) using data on industry groups within the manufacturing sector. We consider these investigations in Section 2.7.
2.3.5.2 Studies Critical of Jorgenson’s Approach

The version of the neoclassical model estimated by Jorgenson and Stephenson (1967a), and by Jorgenson and his other collaborators, consists of a distributed lag specification containing a composite term for changes in output and changes in user cost, i.e. $\Delta(y/c)$. This has been the subject of much criticism. In general, the estimated coefficients on this composite term will reflect a mixture of output and price effects and can generate misleading implications for policy evaluation. Consider a situation in which the relationship between output and investment is stronger than between user cost and investment. Estimated coefficients from a regression containing a composite term will exceed the coefficient on the user cost term when the change in output and the change in user cost variables are entered separately. Thus, any neoclassical model of investment behaviour that contains a composite term, and is used to assess the effects of policy (see, for examples, Hall and Jorgenson (1967, 1969 and 1971)), will tend to overstate the effects of changes in monetary and fiscal policies, operating through the user cost variable $c$, on investment. This result has been confirmed by Eisner and Nadiri (1968, 1970), Eisner (1969, 1970) and Chirinko and Eisner (1982, 1983).

Further criticism of the empirical models estimated by Jorgenson and associates has been levelled in Eisner and Nadiri (1968, 1970). Eisner and Nadiri (1968) set out to show that three substantial conclusions reached by Jorgenson and his associates are not sustained by Jorgenson’s own data. Eisner and Nadiri see the main conclusions as follows. First, Jorgenson and Stephenson (1967a, p. 216) state that the neoclassical theory of investment provides a ‘highly satisfactory explanation of actual investment expenditures’. Second, Jorgenson and Stephenson (1967a, p. 217) state that the tax structure and instruments that affect the cost of capital play a role in the determination of investment expenditures. Third, Jorgenson and Stephenson (1967b, p. 21) state that the time elapsed between the change in desired capital and first actual investment expenditures is approximately three quarters of a year. Using data supplied by Jorgenson, Eisner and Nadiri (1968) were able to show that these conclusions follow from the a priori constraints that Jorgenson assumed, or imposed, on parameters. Jorgenson assumes a Cobb-Douglas production function and this constrains the elasticities of capital stock with respect to output and with respect to relative prices to
be unity. Eisner and Nadiri show that when these assumptions, or restrictions, are empirically tested, they are contradicted and thus so are the derived conclusions that follow from them.

The analysis of Jorgenson's data leads Eisner and Nadiri (1968) to a number of specific conclusions among which are the following. Firstly, the role of relative prices, the importance of which the neoclassical theory was developed to show, is not confirmed. The elasticity of capital stock with respect to the ratios of the wholesale price index to various measures of the rental price of capital is far below the value of unity assumed by Jorgenson. In one case, it does not differ significantly from zero. Secondly, unconstrained estimates of the long run elasticity of capital stock with respect to the product of output and relative prices is also less than unity. Eisner and Nadiri suggest that this is a result of errors in Jorgenson's desired capital stock variables which have been introduced by the mis-specification of the role of relative prices. Thirdly, the elasticity of capital stock with respect to output is reasonably high, approaching unity in some instances. Eisner and Nadiri note that this is consistent with flexible accelerator models. Fourthly, Eisner and Nadiri's results contradict Jorgenson's assumption of a Cobb-Douglas production function, but are generally consistent with the implications of CES production functions with elasticities of substitution closer to zero than unity. Fifthly, estimates of Jorgenson and Stephenson of a delay of up to a year in the response of investment to changes in its determinants are contradicted by Eisner and Nadiri when the lag distributions are estimated freely from the data.

Eisner (1969, 1970) has also criticised the work of Hall and Jorgenson (1967, 1969) (which examine the role of taxation on investment) using arguments similar to those of Eisner and Nadiri outlined above. Eisner (1969) attempts to demonstrate that the substantive conclusions of Hall and Jorgenson stem directly from the assumptions underlying their model and have little to do with the data. Hall and Jorgenson's conclusions are as follows: Firstly, between 6.8 and 11.4 percent of various categories of gross investment, during the period 1954-1963, resulted from accelerated depreciation introduced as a policy measure in the US in 1954. Secondly, between 14.8 and 17.6 percent of net investment in equipment in 1963 was due to a reduction in tax depreciation in 1962. Finally, between 40.9 and 48.6 percent of net investment
or just over 10 percent of gross investment in 1963 could be attributed to the 1962 investment tax credit for equipment and machinery. Eisner outlines implications of the constraints imposed, or assumed, by Hall and Jorgenson and then re-estimates their models relaxing these constraints.

Eisner, in his conclusion, states that contrary to the conclusions of their articles, Hall and Jorgenson ‘have offered no empirical evidence of the role of tax factors in investment. Their specification of the investment function with the constraint that parameters of the price of output, output, and the reciprocal of the rental price or user cost of capital must all be identical, along with their specifications of an exact relation between the user cost of capital and the various tax incentives they consider, make it impossible for them to disentangle any possible influence of these tax incentives on investment. There is hence no justification for the quantitative estimates of these influences which they present in their article.’ (Eisner 1969, p. 386).

Bischoff (1969) enters into the Eisner-Nadiri-Jorgenson debate on the validity of the neoclassical model. Incredulous at the first of the conclusions reached by Eisner and Nadiri (1968) that the ‘role of relative prices, the critical element in the neoclassical approach, is not confirmed’ (p. 380), Bischoff (1969) undertakes what he refers to as ‘detective work aimed at finding out why Eisner and Nadiri obtained results contrary to the body of other research’ (p. 354). Bischoff notes that whilst none of Jorgenson’s many critics have been able to defend the precise manner in which he specified his model, their results are, without exception, favourable to the essence of the neoclassical model in that relative prices are found to matter. Bischoff’s analysis differs from Eisner and Nadiri’s in terms of the maintained hypothesis he adopts regarding the error process. Bischoff notes that ordinary least squares estimates of equations with lagged endogenous variables are not even consistent if the errors are not serially independent. Hence, Bischoff suggests that the assumption of serially independent errors is crucial if meaningful results are to be obtained. However, he shows that such an assumption may not be appropriate and this leads him to specify a general model that differs from the Eisner and Nadiri model only in the specification of the error process. Rather than assuming serially independent errors, Bischoff assumes a first order autoregressive error process. The resulting model encompasses the Eisner and Nadiri model as a testable special case. With two data sets, both similar
to that used by Eisner and Nadiri, Bischoff’s empirical results differ substantially. In applying the more general specification to the first data set, Bischoff finds that four of Eisner and Nadiri’s conclusions are overturned. For the second set of data at least two of the conclusions are reversed. Due to the noise level in these data sets, Bischoff suggests that firm conclusions regarding the real determinants of the demand for capital are difficult to draw.

In an attempt to draw less ambiguous conclusions, Bischoff applies the same model to a data set with a much lower noise level, namely aggregate demand for equipment in the period 1949-66. Using his general model he finds that both output and relative prices appear to have statistically significant effects on equipment demand and the point estimates of the parameters are almost precisely what one would expect if the data had been generated by a model including a Cobb-Douglas production function with constant returns to scale. The general model strongly rejects the stochastic specification used by Eisner and Nadiri. Further, the response of investment to changes in relative prices is slower than the response to changes in output and the hypothesis that both output and relative prices act with the same lag distribution is strongly rejected. Bischoff notes that had Eisner and Nadiri adopted the appropriate maintained hypothesis regarding the specification of the error term ‘it seems unlikely that their paper would ever have been written’ (p. 362). Since Bischoff conducts his empirical analysis with slightly different data sets and specifies models with slightly different lag structures, he is unable to replicate Eisner and Nadiri’s results exactly when estimating the special case of his own general model. In any event, the special case of the general model consistent with the Eisner and Nadiri model is rejected. Bischoff concludes that his results are favourable to the essence of the neoclassical approach to investment functions in that relative prices do matter, but notes that none of his work should be interpreted as support for the policy conclusions arrived at by Jorgenson and Stephenson (1968a).

2.3.5.3 Variants of the Jorgenson Approach

Variants of Jorgenson’s neoclassical model have been estimated by a number of other authors, partly in response to the criticisms of the pure version of the model. Gould
and Waud (1973), for example, estimated a reduced form version of the neoclassical model with Jorgenson's data and found that this performed better than, or at least as well as, the pure version of the model estimated by Jorgenson and Stephenson (1967a).

Gould and Waud derive an expression for desired capital stock that depends only on exogenous quantities that are unaffected by the firm's decisions or adjustment process, thus bypassing the econometric problems associated with the use of endogenous variables in dynamic modelling. These problems are discussed in Gould (1969) and above in Section 2.3.4. Jorgenson's investment equation is estimated with Jorgenson's measure of desired capital stock and then with the reduced form expression for capital stock. Using data up to the last quarter of 1962, Gould and Waud estimate models for the eleven industry classifications for the manufacturing sector established by Jorgenson and Stephenson (1967a) from 1947 to 1960. This allows them eight quarters of data for ex post forecast comparisons. Predictions made by the two models were compared using criteria such as the sum of squared forecast errors. Gould and Waud found that the reduced form model performed at least as well as the Jorgenson model, a hybrid model and autoregressive model (both of which were introduced as a means of facilitating comparisons between the main models under consideration). The authors note that this result is achieved despite a number of assumptions which gave an advantage to the Jorgenson model in predictive tests.

Boatwright and Eaton (1972) have also estimated a variant of Jorgenson's neoclassical model with UK data. Their point of departure from the pure version of the model is the assumption regarding the elasticity of desired capital stock with respect to relative prices. Recall, much of the criticism of Jorgenson's model stems from the latter's assumption that this elasticity is equal to unity (see the above discussion of Eisner and Nadiri (1968) and Eisner (1969)). Boatwright and Eaton attempt to explain the level of investment in plant and machinery in the UK manufacturing sector. They lay particular emphasis on measuring the impact of various policy measures designed to stimulate investment. In doing so, they distinguish between the nominal price of an asset as paid by the investor and the effective price after allowing for any applicable allowances. It is the effective price that is incorporated into the neoclassical type model of investment behaviour.
Boatwright and Eaton pay detailed attention to the structure of the lag response of investment to changes in its determinants. To this end, the distributed lag function is estimated by three independent techniques; the Almon, Pascal and Gamma distributions. The empirical evidence presented by the authors uncovers mixed support for the neoclassical theory of investment behaviour. When the lag structure is estimated by the technique introduced by Almon (1965), the assumption of a unit elasticity of desired capital stock with respect to changes in relative prices is rejected. Rather, the elasticity is found to be about 0.47 which implies, in common with the work of Eisner, the production function is of CES form. Boatwright and Eaton are satisfied with the shape and length of the lag distribution: it is smooth, unimodal and reaches a peak after ten quarters. The results obtained using a Pascal and gamma distributions are much less successful. Indeed results from the former are described as 'particularly disappointing, since they appear to reject the neoclassical theory of capital accumulation' (p. 409).

The work carried out by Bischoff (1970) provides a rare example of an attempt to estimate a relationship for investment in nonresidential construction. Bischoff notes that 'very little ink has been used up in explaining the fluctuations in non-residential construction' (p. 10). Further, he suggests that this is due to a lack of data distinguishing this form of investment. Even in Britain, where more data on non-residential structures is available, very few studies of investment in assets of this type exist. Bischoff states that studies almost invariably lump together investment in plant and equipment and lists a number of reasons why a separate study of nonresidential construction may be justified. These include the possibility of differential effects of tax policy and the fact that equipment investment may lend itself to a putty-clay approach since it is more likely to have been designed for a specific purpose.

Bischoff begins by estimating a basic accelerator model of nonresidential construction expenditure. Estimated using national accounts data for the sample period 1952-68, the resulting model performs well in terms of goodness of fit, but there is evidence of considerable unexplained serial correlation (the first order serial correlation coefficient is 0.87). Furthermore, Bischoff notes that the constant is quite large and the speed of adjustment appears too slow. In order to overcome these deficiencies,
Bischoff introduces a relative price term, $p/c$. Bischoff regards the resulting specification as a variant of Jorgenson’s neoclassical model.

Bischoff defines desired capital stock to be a simple but undefined linear function of output and relative prices. This avoids a composite term for output and relative prices which Jorgenson obtains in his expression for desired capital stock by assuming Cobb-Douglas technology. He also allows for separate lag distributions for the output and relative price variables. Taking what he calls an ‘eclectic approach’, Bischoff estimates this model with ten different formulations of the discount rate in his user cost of capital variable. The resulting equation is quite sensitive to which discount rate is used. Using an $F$-test to test the joint significance of the lagged user cost of capital variables, Bischoff finds that for six of the ten formulations the resulting equations do not lead to any significant improvement over the basic accelerator model. The equation generating the best results uses a dividend-price ratio for corporate stocks as its measure of the discount rate. In this formulation dividends are taken to represent a measure of the expected earnings of the corporation. In this model, the short-run elasticities of the stock of structures with respect to output and the relative price variables appear to be significantly below unity, but above zero. The speed and pattern of adjustment of investment to a change in one of its determinants are reported as being ‘relatively rapid and quite reasonable’ (p. 13) when compared with the adjustment speeds suggested by his accelerator model. The elasticity of capital with respect to output gives some indication of increasing returns. The effect of relative price, while quite small, is significant. The fit of the equation is good and, importantly, the degree of correlation in the error term is markedly reduced. In further analysis, Bischoff uncovers no support for the putty-clay hypothesis which we discuss in Section 2.4.

Variants of Jorgenson’s pure neoclassical model have also been estimated by Thurow (1969), Coen (1968, 1969, 1971) and Feldstein and Flemming (1971). Thurow (1969) notes that the Jorgenson’s introduction of lags into the investment equation implies that the economy is not in equilibrium, as the actual capital stock lags behind desired capital stock. In this situation the cost of capital and the marginal product of capital can diverge. Profit maximising firms invest to eliminate the gap between the marginal product of capital and the cost of capital. Thurow estimates a disequilibrium
investment function based on a standard Cobb-Douglas production function and Jorgenson’s definition of the user cost of capital. In allowing for disequilibrium behaviour, Thurow obtains an expression for desired capital stock which is a function of the difference between the after-tax return on capital and the cost of capital. The results of estimating these disequilibrium investment functions for investment in structures and in equipment were mixed.

Rather than assuming a Cobb-Douglas production function, Coen (1969) building on his earlier work (see Coen (1968)), employs a CES production function in his version of the neoclassical investment model. This allows the price elasticity of demand for capital, the crucial parameter in assessing the impact of tax policy, to be estimated. Using almost identical data to Hall and Jorgenson (1967), Coen concludes that the prices elasticity of demand for capital is significantly less than the value of unity assumed by Hall and Jorgenson. Thus the effects of tax policy in Hall and Jorgenson’s work are overstated. He concludes that the results of Hall and Jorgenson do not ‘stand up to close scrutiny, and their study must be regarded as inconclusive with respect to the effectiveness of tax incentives for investment’ (p. 378).

In separate paper (published in the same volume as Bischoff (1971a) and Hall and Jorgenson (1971)), Coen (1971) augments a version of the neoclassical model with a cash flow variable. The stated purpose of his study is to assess the impact of tax incentives on plant and machinery investment in the US manufacturing sector during the period 1954-66. Tax incentives are presumed to influence investment expenditure in two ways. Firstly, by reducing the implicit rental price of capital they increase the firm’s desired capital stock. Secondly, by increasing the flow of internal funds available for financing purchases of capital, they facilitate adjustments of capital stocks to desired levels. Coen, like others such as Bischoff (1969, 1971a), employs separate lag distributions for output and user cost variables and finds that the effect of a change in tax policy on investment is much lower than that found by Hall and Jorgenson (1971), whether or not the model contained the cash-flow variable (Hall and Jorgenson’s estimate was 6.9%, Coen’s estimates were 3.9% when cash flow is absent and 2.0% when included).

Feldstein and Flemming (1971) build on the work of Jorgenson and his collaborators taking into account the criticisms levelled by Eisner and Nadiri (1968). Bischoff
(1968, 1969) and Coen (1968, 1969). Unlike these studies, which apply modifications of the neoclassical model to US data, Feldstein and Flemming apply a modified version of the model to UK quarterly aggregate investment in equipment and structures over the period 1954-67. Like many of these US studies, Feldstein and Flemming are interested in the effects of tax incentives on investment expenditure. Again, their model differs from that of Jorgenson and his associates in that the assumption of Cobb-Douglas technology is relaxed and the elasticity of the desired capital stock with respect to tax induced changes in the user cost is not constrained to be unity, or even to be equal to the elasticity with respect to other changes in the user cost. In addition, the response of investment to changes in relative prices and the response to changes in output are not constrained to be identical. Furthermore, the impact of tax induced changes in retention behaviour is given explicit consideration.

The generalisations relating to the elasticity and the lag structure are found to be important. Feldstein and Flemming note that the constrained version of the model, which corresponds to the pure neoclassical model is inadequate and yields misleading results. In summary, estimates generated from the pure neoclassical model underestimate the effects of investment allowances, they ignore the effect of retained earnings and overstate the effect of other components of user cost.

In summary, the neoclassical model of investment behaviour has been applied to a number of data sets of different levels of aggregation, for investment in structures and investment in producers' durable equipment, in different countries, with a wide variety of results. Even using the same data set, different authors have arrived at markedly different conclusions. Investigators have criticised a number of the assumptions underlying Jorgenson's neoclassical model. Perhaps the most frequently recurring criticism concerns the assumption of a unitary elasticity of capital stock with respect to output and rental price which follow from Cobb-Douglas technology. (It is noteworthy that a unitary elasticity with respect to output follows from any linearly homogenous production function). These assumptions are relaxed in a variety of ways, usually resulting in an equation that allows the response of investment to changes in the cost of capital and changes in output to differ. Whilst many authors have found a significant role for relative prices, thus providing support for the neoclassical model (the credibility of which depends wholly on significance of
relative prices), output remains the dominant determinant of investment in empirical studies. The relative importance of output and user cost as determinants of investment is investigated further in Section 2.4.

We have considered neither the forecasting performance of these models nor their ability to explain investment behaviour relative to other models. We consider studies in which these aspects of the neoclassical model are examined in Section 2.7.

2.4 The Putty-Clay Model of Investment

The main distinction between the neoclassical and accelerator theories of investment is the underlying technology. In the accelerator model, a fixed coefficient production function implies relative prices do not matter, while the smooth production possibilities of the neoclassical production function implies the opposite. However, if we make a distinction between *ex ante* and *ex post* technologies, an intermediate possibility arises. A firm may be able to choose from a range of machines or buildings with different capital-labour ratios prior to purchase, but whence it has chosen and installed the machine or building, the firm may find it difficult to alter the capital-labour ratios. In other words, the set of possibilities open to a firm, *ex ante*, that is, when choosing the type of capital, may well be greater than the corresponding set of possibilities, *ex post*. Phelps (1963) coined the term 'putty-clay' to describe this type of technology in the sense that *ex ante* 'putty' becomes *ex post* 'clay'.

If one believes in this representation of a firm’s technology, then to model investment behaviour using either the fixed coefficient technology of the accelerator, or the smooth technology of the neoclassical model, is clearly inappropriate. In the fixed coefficient technology (clay-clay), there can be no substitution between factors either *ex post* or *ex ante*. This will give rise to right angled isoquants *ex ante* and *ex post*. The neoclassical production function, as we have seen above, allows for continuous factor substitution, but again assumes that the substitution possibilities are the same *ex post* as *ex ante*. According to this hypothesis, capital operates equally effectively doing any job and if expected output and demand for that product falls, it can be reshaped into other types of capital costlessly. In this sense, investment in the neoclassical model is fully reversible.
2.4.1 Putty-Clay Models as the Intermediate Case

Putty-clay models of investment differ significantly from the putty-putty models and from the clay-clay models in that the *ex ante* and *ex post* elasticities of substitutions are no longer assumed equal. *Ex ante*, smooth continuous substitution is possible in the sense that the firm can choose the capital-labour ratio it prefers, but *ex post*, the factor proportions are determined and fixed for the remainder of the economic lifetime of the capital. Therefore, the *ex ante* and *ex post* isoquants differ; *ex ante* isoquants are smooth, continuous and downward sloping, whereas *ex post* isoquants are right angles.

The economic implications of the putty-clay model are important. Unlike the clay-clay models relative prices matter. In addition, unlike the original Jorgenson model, expectations play a crucial role. Whereas in a putty-putty model capital equipment can be costlessly reshaped in the next period in accordance with a change in relative prices, the putty-clay model engenders real long-term decisions. There may be many efficient methods of altering capacity but the firm must choose that method that embodies the optimal capital-labour ratio and it can only do this by forming expectations about future relative prices. Faced with putty-clay technology, a firm that increases capacity by choosing a production technique with a high capital-labour ratio will have made a bad choice if wages are expected to fall and remain low throughout the lifetime of the capital. The putty-clay model is thus an intermediate case between the pure neoclassical and the accelerator models of investment, but in some respects represents a significant advance on both.

2.4.2 An Investment Function

The model is largely due to the work of Bischoff (1968, 1971a). Continuous substitutions between factors is possible only up to the point that new capacity is installed. Instead of assuming that firms adjust towards a desired capital stock, as in the neoclassical model, Bischoff assumes that firms adjust towards a desired level of productive capacity. Firms respond to changes in output prices relative to the user cost of capital by changing the capital intensity of only new net, or replacement, capacity
rather than the intensity of the entire capital stock. This argument results in a conceptual investment equation of the form

\[ I_t = (p_t/c_t)^* [Y_t - (1 - \delta)Y_{t-1}]^* \]  

(2.33)

where \((p/c)^*\) represents planned capital intensity, \((Y_t - Y_{t-1})^*\) represents planned net additions to capacity, and \(\delta Y_{t-1}\) represents replacement. Bischoff's (1968, 1971a) statistical specification of this relationship incorporates two separate lag distributions which allow the dynamic impact of the changes in relative prices, interest rates and tax policies captured in the relative price term to differ substantially from the dynamic impact of the output terms. It is given by

\[ I_t = \kappa + \sum_{i=0}^{\infty} \varphi_1 \left( \frac{p_{t-i-1}Y_{t-i}}{c_{t-i-1}} \right) + \sum_{i=0}^{\infty} \varphi_2 \left( \frac{p_{t-i-1}Y_{t-i-1}}{c_{t-i-1}} \right) + \delta K_{t-1} \]  

(2.34)

This is the equation for gross investment estimated in Bischoff (1971b). We consider the empirical performance of such equations in Section 2.4.4.

2.4.3 Implications of the Putty-Clay Hypothesis

As indicated above, the model has been used to help shed light on the debate about the relative importance of demand and prices as determinants of investment behaviour. In addition, given two separate lag distributions, the putty-clay model predicts that the timing of the response of investment will be different depending on whether the investment arises as a result of a change in demand or a change in relative prices. Compare a cost-minimising firm's reaction to a change in demand with its reaction to a change in relative prices. Suppose that both changes result in an equal increase in the firms' long-run desired capital stock. Faced with an increase in demand the firm can react immediately by purchasing more capital. However, for a change in relative prices, because the factor proportions are fixed \textit{ex post}, i.e. the capital-labour ratio is fixed \textit{ex post}, the firm must wait until existing machinery becomes economically or physically obsolete before purchasing new capital embodying the new optimal capital-labour ratio. Thus, for a cost-minimising firm confronted with putty-clay technology, we would expect to see the rate of investment react faster to changes in demand than to changes in relative prices.
It is easy to see how equation 2.34 embraces this possibility. Note that if one subtracts the derivative of \( I \) with respect to \( c/p \) in period \( t-1 \) from the derivative of \( I \) with respect to \( Y \) in period \( t-1 \), the difference is equal to the negative of the second term in equation 2.34. This quantity is positive if \( \varphi_{2t} < 0 \). Thus, the putty-clay hypothesis can be tested by testing the joint hypothesis that \( \varphi_{2t} < 0 \), since this implies that the response of investment to a change in user cost is less than the response of investment to change in output. In other words, the hypothesis that the relative price and output terms act with the same lag distribution is equivalent to the joint test that \( \varphi_{2t} = 0 \).

Evidence in support of differences in timing of response of investment to demand and relative price changes is presented in Eisner and Nadiri (1968 and 1970), Bischoff (1969 and 1971a) and Hausman (1973). We consider some of these studies in the following subsection. However, the evidence is not conclusive since Abel (1981) has shown that the response of investment to a change in relative prices can be slower than the response to a change in demand for an intertemporally optimising firm facing putty-putty technology, if we make the additional assumption that adjustments in investment are not costless.

2.4.4 Empirical Results with Putty-Clay Technology

In this section we consider the empirical results obtained with putty-clay models. We begin with an outline of the seminal works before considering a number of UK studies.

2.4.4.1 The Seminal Works

A vintage model of investment was first proposed by Bischoff (1968) for equipment expenditure. In this model the capital in which investment at any given point in time is embodied, is characterised by fixed proportions, although the proportions embodied in the equipment can be chosen from a set of alternatives described by an \textit{ex ante} production function allowing for continuous factor substitution. Firms are assumed to adjust towards a desired level of productive capacity, rather towards a desired capital stock. Moreover, firms respond to changes in output prices relative to the user cost of
capital by changing the capital intensity of only new net, or replacement, capacity rather than the intensity of the entire capital stock. This argument leads to a conceptual equation of the form given in equation 2.33 and an empirically testable equation given by 2.34. According to equation 2.33 investment in a given period is determined by the product of the optimal capital-output ratio in the same period (determined by the relative price of relevant inputs prevailing at that time, $p/c'$, and expected to prevail over the relevant future) and the gross increment to capacity that firms wish to provide for in that period, $[Y_c(1-\delta)Y_{c1}]^*$. Bischoff’s expression for the optimal capital-output ratio is based on two simplifying assumptions. First, he assumed that input prices were expected to remain constant over the life span of the currently acquired capital. Second, he assumed that capital remained in service until it completely depreciated physically. The principal conclusions from his analysis are as follows.

Firstly, since the relative price term appears as a significant regressor in the investment function, the efficacy of the neoclassical approach is substantially confirmed. Secondly, the model in which changes in relative prices and changes in output are entered as separate terms (referred to by Bischoff as the general neoclassical model) provides an explanation of equipment expenditures that is superior to that given by either the standard neoclassical model or the flexible accelerator model. Further, the response of investment to changes in relative prices is much slower than the response to changes in output, thus providing support for the putty-clay hypothesis. Thirdly, the long-run elasticity of equipment spending with respect to the rental price of equipment services is estimated, albeit as Bischoff notes without great precision, to be close to unity. Finally, variations in measures of the cost of capital seem to have the negative partial effect that theory suggests, but Bischoff has not been able to obtain stable estimates of the effects of these variables.

As mentioned in our discussion of empirical neoclassical models in Section 2.3.5, Bischoff (1969) has also presented evidence in support of the putty-clay hypothesis for equipment. However, Bischoff (1970) finds no evidence to support the hypothesis for US nonresidential construction. In these studies, however, the models are not derived under the assumption of putty-clay technology and consequently the hypothesis can not be rigorously tested.
Ando et al (1974) have noted that the second of the simplifying assumptions made by Bischoff (1968) in his derivation of an expression for the optimal capital-output ratio, namely that capital remains in service until it is completely physically depreciated, can only be justified if the first assumption, that input prices remain constant throughout the life-span of the capital good, holds. The principal objective of Ando et al is to generalise the analysis of Bischoff to allow for the fact that, typically, input prices are not expected to remain constant over time. The resulting investment function differs from that of Bischoff's only in the measurement of the cost of capital variable.

Ando et al (1974) use their resulting investment function to explain aggregate US investment under several assumptions concerning the unobserved expected rate of change of capital good's prices variable. The measure of the expected rate of change of prices that is most capable of explaining aggregate investment is one that contains a structural break: the expected rate of change of prices is constant for the first part of the sample period and for the later part of the period it is determined by the product of a variable calculated as a weighted average of past output price changes and another variable which takes a value of zero for low levels of output price inflation and a value of unity for high output price inflation. Ando et al were not able to find any other formulation that could account for investment orders and output for the period 1953-69 as a whole. The statistical properties of this equation are well defined and support the putty-clay hypothesis.

2.4.4.2 Empirical Results for the UK

King (1972) has developed an investment equation for the UK manufacturing sector from a vintage production model. One of the principal aims of the paper is to investigate the effect of investment incentives and taxation. King notes that the model proposed by Bischoff (1968) assumes constant returns to scale and that a constant proportion of existing capacity is scrapped each year. This means the age of the oldest capital does not appear as a variable in the Bischoff model. King develops an alternative theoretical model in which he assumes oligopolistic conditions in the product market, perfect capital markets, an *ex ante* Cobb-Douglas production function for each vintage, putty-clay technology *ex post* and cost minimising behaviour subject
to a predetermined level of total output, which is equal to the sum of outputs produced by all vintages in use. The resulting model, in which investment in a particular vintage is a log function of output produced by that vintage, the ratio of the real wage to the effective price of investment goods and the tax rate on company profits, is fitted to UK data for investment in plant and machinery in the manufacturing industry for the period 1948-68. King estimates the model using six different price indices for investment goods, most of which employ different discount rates to evaluate investment incentives. The results, in terms of the implications for government policy, are shown to depend heavily on the choice of discount rate. King does not draw any conclusions about the efficacy of the putty-clay theory from his results. He notes that although his model has a number of weaknesses, it succeeds in providing a framework through which the influence of government policy of investment may be examined.

Sarantis (1979) extends the theoretical model developed by King and attempts a similar analysis of the impact of investment incentives for ten separate UK two-digit manufacturing industries. The point of departure from the King model is King’s assumption of perfect capital markets. Sarantis notes that perfect capital markets are absent in the real world and suggests that increased borrowing costs may lead to the rejection of investment projects requiring external financing. As such Sarantis augments King’s model with a cash flow variable similar to that employed by Coen (1971), which provides a constraint on the volume of investment expenditure and is thus a determinant of its timing. This implies that the speed of adjustment is no longer constant, as in King’s model, but a variable depending on cash flow.

Rather than adopting King’s approach of experimenting with arbitrarily chosen discount rates, Sarantis chooses to measure firms’ annual rate of discount by the actual rates of return realised in each industry. Equations for each industry are estimated over the period 1951-74 and, in Sarantis’ view, these equations explain the investment behaviour in UK manufacturing industries satisfactorily. The cash flow variable is found to be significant and substantially affects the speed of adjustment of actual investment to its desired level in almost all of the industries considered. Relative factor prices have significant effects on investment and the response of investment to changes in factor prices appears to be longer than that to changes in output in most industries, thus supporting the putty-clay hypothesis. Sarantis states
that this also supports the view that the influence of relative factor prices should be tested separately from that of output in econometric models of investment behaviour. The long run elasticities of investment with respect to relative factor prices are greater than 0.5 in all industries and between 0.8 and unity in most industries. These equations are then used to assess the impact of incentives on investment at the industry level. Results indicate considerable inter-industry differences, which suggests to Sarantis that policy decisions based solely on estimates of aggregate investment equations could be misleading.

Anderson (1981) and Bean (1981) also examined the putty-clay hypothesis with UK data, although neither author develops models that are rigorously consistent with the hypothesis at a theoretical level. The former study concentrates on discriminating between putty-clay models, putty-putty models (of the Jorgenson variety) and clay-clay accelerator type models of ICCs aggregate investment using non-nested testing procedures suggested by Pesaran (1974) and Pesaran and Deaton (1978). Anderson is also interested in the role of financial factors and since he adopts a nominal measure of investment he is able to specify a general model in terms of output and financial factors (rather than the usual user cost term). Since putty-putty and clay-clay technologies are strongly rejected when nested within a general model containing separate lag distributions for output and financial terms, Anderson does not adopt non-nested test procedures to establish model superiority. The forecasting power of Anderson’s general model is found to be satisfactory and superior to both of the special cases. Anderson notes that while the dominant model is consistent with the putty-clay hypothesis, the fact that it is not formally derived from it means that it might also be consistent with a flexible accelerator model with credit rationing. Since the financing variable is significant, Anderson concludes that support exists for either a putty-clay or a credit rationing formulation.

Bean’s (1981) paper, published in the same volume of the *Economic Journal* has much in common with Anderson’s. In common with Anderson, Bean (1981) is interested in the role of financial factors as determinants of investment. He adopts a general model from which specific forms are tested, and he uses UK data. Bean estimates a relationship for real investment expenditure by the manufacturing sector. His general model suggests long lags between investment, output and capital costs,
but due to the computational limitations encountered in models with large numbers of
lagged variables, he opts to estimate such a relationship in two stages. First, he
estimates a general relationship between investment and output using the ‘general to
specific’ modelling approach, developed by Hendry and Anderson (1977), Hendry and
Mizon (1978) and Davidson et al (1978). The parsimonious model that he arrives at is
then augmented by a number of variables affecting the user cost of capital and an
additional capacity variable. The coefficients in the resulting equation are all correctly
signed and the statistical properties are well defined. In addition to entering these
variables separately, Bean estimates an additional relationship in which the variables
thought to affect the user cost of capital are entered in a single composite term. This
turns out to be Bean’s preferred model. Again, coefficients are correctly signed and
the statistical properties are satisfactory. Although Bean is unable to make any
statement about the elasticity of substitution in the underlying production function, an
examination of the response function suggests to Bean that the preferred model is
putty-clay in nature.

Using the non-nested test developed by Pesaran and Deaton (1978), Bean finds that
these other models are rejected by his own model. This result is to be expected given
the data-based approach to estimation. However, when the hypotheses are reversed,
his model is rejected by two of these other models. In his conclusion, Bean suggests
his data-based approach to modelling investment shows a significant role for the real
cost of capital and nominal interest rates in the determination of investment. The
failure of the other models to dominate Bean’s preferred specification in non-nested
tests can be seen as providing potential support for putty-clay technology.

We have now examined a number of studies in which investment equations are
derived under the assumption of putty-clay technology. We have also considered a
number of studies in which investment equations are empirically consistent with the
putty-clay hypothesis, but is not strictly derived from it. The general conclusion that
can be drawn from these studies is that the investment response to changes in relative
prices is slower than the response to changes in output. As in studies adopting the
neoclassical model, the results indicate that the effect of changes in output is
dominant. In the models presented by Bischoff (1968), Anderson (1981) and Bean
(1981), putty-putty and clay-clay models are rejected in favour of the putty-clay
alternative. We consider the studies which explicitly test the performance of the putty-clay model against alternative models of investment behaviour in Section 2.7 below. It is noteworthy that none of the studies reviewed in this section test the putty-clay hypothesis for investment in structures. However, in Section 2.3.5, we noted that Bischoff (1970) finds no evidence to support the putty-clay hypothesis with US data on investment in structures using putty-putty technology with separate lag distributions for output and relative prices.

In the neoclassical and putty-clay models of investment the adjustment of capital stock to its desired level (or intensity) is captured by a distributed lag relation. This is a rather ad hoc means of capturing the time structure of the investment problem but is necessary so as to avoid instantaneous adjustment. In the next section we consider models in which the adjustment process is incorporated into the theory.

2.5 Cost of Adjustment Models

In this section, we show two ways in which Jorgenson's neoclassical model can be modified to include adjustment costs. By suitably defining an adjustment cost function, we can obtain a specification for investment that is theoretically consistent, explicitly recognises the dynamics due to expectations and technology and isolates their influences. However, in this specification investment is related to an unobservable concept. More precisely, investment is found to be an increasing function of the ratio of the demand to supply price of capital. The main problem in empirical work, therefore, is that of relating the unobservable concept to observable variables. One way by which this might be achieved is considered more fully in Section 2.6.

2.5.1 The Nature of Adjustment Costs

Adjustment costs are the costs associated with the buying and selling of capital goods over and above the basic purchase price of these goods. The concept was introduced by Eisner and Strotz (1963). The subsequent literature makes a useful distinction between external and internal adjustment costs. External adjustment costs arise due to
the upward sloping nature of factor supply curves, while internal adjustment costs are attributable to the reduction in productivity which occurs within the firm when capital and/or labour stocks are changed. Consider the following specific examples of adjustment costs.

In the case of labour, examples of external adjustment costs include severance pay, expenditures on job advertisements and similar search costs, expenses for initial training etc. Similarly, changes in capital stock involve costs such as architects and planning fees, search costs associated with the gaining information on the latest technology available and so on. In contrast to these adjustment costs, internal adjustment costs generally arise because the absorption of additional capital and/or labour requires resources which might otherwise have been used in production. Thus, such costs arise from reorganising and retraining when new equipment is installed or additional labour is applied to the production process. For example, a firm may have personnel and training departments that are adequate for normal replacements of quits and retirements but if the firm increases its employment of labour, capital and/or labour will have to be devoted to both these departments. With a given quantity of inputs, the level of the firm's output must, therefore, fall. Similar internal costs associated with the adjustment of capital can be readily thought of. For example, a firm may have an adequate number of engineers responsible for the upkeep of existing capital, but if the capital stock is increased the firm will have to devote more labour and capital (in the form of tools etc.) to manage the installation of the new capital.

Studies in the investment field have generally focused on internal adjustment costs. More specifically, they have been dealt with in one of three ways. Firstly, they may be assumed to represent the lost output from disruptions to the production process as new capital goods are ‘broken in’ and workers retrained as in Lucas (1967a and 1967b), Gould (1968) and Treadway (1969). Alternatively, they may be thought of as the costs of additional labour in the form of engineers to install the new capital. Finally, they can be thought of as a wedge between the quantities of purchased and installed capital as in Uzawa (1969) and Hayashi (1982). These different rationales determine whether the price of output, labour or new capital is appropriate for valuing adjustment costs.

Jorgenson incorporated an assumption of convex costs of adjustments into his later work on the neoclassical theory of investment (see Jorgenson (1972, pp. 223-224)).
The attraction of the assumption for Jorgenson is that it rules out the possibility of instantaneous adjustment by the individual firm, a major problem in his original model. If adjustment costs are convex, i.e. increasing at an increasing rate, then it pays for the firm to adjust gradually. This is a result of the fact that more rapid adjustment is proportionately more costly than a slower adjustment. If, however, the marginal cost of adjustment were linear or concave, then there is no benefit to delaying adjustment and the firm would choose to adjust immediately. Convexity also forces the firm to think seriously about the future, as too rapid an accumulation of capital will prove costly. Alternatively, if the firm accumulates too slowly it foregoes profits. Thus, expectations are crucial since, by anticipating future changes in the economic climate, firms can react and gradually adjust their investment expenditures prior to the change in conditions, thereby reducing their total costs. The fact that a firm will choose to adjust gradually when there are convex costs of adjustment, provides a justification for modelling investment expenditures as a distributed lag formulation, thus allowing Jorgenson to by-pass one of the inconsistencies between his theoretical and empirical work.

Recall, in the neoclassical model the firm adjusts capital until the marginal cost of another unit of capital is equal to the marginal benefit (see equation 2.25 above). Equilibrium for the optimising firm faced with adjustment costs also occurs when the marginal cost of buying additional capital is just equal to the marginal benefit (or demand price). However, the marginal cost is no longer just the supply price of capital but also includes the marginal adjustment cost. Thus, adjustment costs introduce a wedge between the demand and supply price of a machine. The larger the gap between the demand price (or marginal benefit) and the price of the machine, the more the firm will invest, even though this incurs higher costs of adjustment. This is because the larger the gap between the demand price and the supply price of a capital asset, the greater the incentive to invest and hence the greater the marginal cost of investment the firm is willing to bear. Thus we can deduce that, in contrast to Jorgenson’s theoretical model, investment is an increasing function of the ratio of the demand to supply price of capital goods. We consider the relationship between supply and demand price more fully in the subsequent discussion of Tobin’s q model of investment.
2.5.2 An Investment Model with Adjustment Costs

The introduction of adjustment costs into the firm's optimisation problem has been pioneered by Lucas (1967a, 1967b), Gould (1968) and Treadway (1969). Recall, the aim of the perfectly competitive firm is to maximise the discounted value of future profits, i.e. its net worth, subject to a production function and the capital accumulation equation. Lucas, Gould and Treadway modify the profit function to incorporate a term to reflect the fact that the firm's actual output is below that which is technically feasible by an amount equal to the cost of installing new investment goods. Maintaining the notation defined above, their profit functions are of the form

\[ \pi(t) = p(t)[F(L(t), K(t)) - C(I(t), K(t)) - w(t)L(t) - z(t)I(t)] \]  (2.35)

where \( C(I(t), K(t)) \), the cost of adjustment function, is a measure of lost output from installing the new investment goods. This function depends on \( K(t) \) as well as on \( I(t) \) since the cost of installing \( I \) units of investment goods is likely to depend on the size of \( I \) relative to \( K \). As discussed above, it is assumed to be increasing and convex in \( I \) (that is, \( C_I > 0 \) and \( C_{II} > 0 \)), reflecting the fact that the cost of instalment per unit of investment will be greater, the greater the rate of investment for any given \( K \).

The firm's optimisation problem can be formally stated as

\[ \max_{I,J} \quad V(t) = \int e^{-\tau t} \left[ p(t)Y(t) - w(t)L(t) - z(t)I(t) \right] dt \]  (2.36)

subject to

\[ Y(t) = F[L(t), K(t)] - C[I(t), K(t)] \]  (2.37)

and

\[ \dot{K}(t) = I(t) - \delta K(t) \]  (2.38)

The current value Hamiltonian for this problem is given by

\[ H = p[F(L, K) - C(I, K)] - wL - zI + m[I - \delta K] \]  (2.39)

The timescripts have been dropped to simplify notation. \( m \), the current value multiplier, is equal to \( \lambda e^\tau \) where \( \lambda \) is the costate variable in optimal control problems. Whereas \( \lambda \) (used in the derivation of the neoclassical model) measures the marginal...
valuation of capital at the moment of planning (which, in more mathematical language, is the marginal effect of a change in \( K \) at \( t=s \) discounted back to \( t=0 \), \( m \) is the current marginal valuation of capital (which is simply equal to \( \lambda \) taken forward from \( t=0 \) to \( t=s \)). We choose to work with the current marginal valuation here for mathematical neatness.

The first order conditions for optimality are

\[
\frac{\partial H}{\partial L} = pF_I(L, K) - w = 0 \tag{2.40}
\]

\[
\frac{\partial H}{\partial I} = -pC_I(I, K) - z + m = 0 \tag{2.41}
\]

\[
\frac{\partial H}{\partial K} = pF_K(L, K) - pC_K(I, K) - m\delta = rm - \dot{m} \tag{2.42}
\]

\[
\frac{\partial H}{\partial m} = I - \delta K = \ddot{K} \tag{2.43}
\]

and the transversality condition is

\[
\lim_{T \to \infty} m(T)K(T)e^{-rT} = 0 \tag{2.44}
\]

Equation 2.40 can be rewritten as

\[
F_I(L, K) = \frac{w}{p} \tag{2.45}
\]

which simply says that the firm will hire labour up to the point where its marginal product is equal to its real wage. Notice that this is identical to the condition in equation 2.23 in the Jorgenson model. Interpretation of equation 2.42 is most easily grasped by writing it in integral form. First, note that using equation 2.37, equation 2.42 can be written as

\[
(r + \delta)m - \dot{m} = p\left[F_K(L, K) - C_K(I, K)\right] = p\frac{\partial Y}{\partial K} \tag{2.46}
\]

Multiplying both sides of equation 2.46 by \( e^{(r+\delta)t} \) we get

\[
\left[(r + \delta)m - \dot{m}\right]e^{(r+\delta)t} = \frac{d}{dt}(-me^{-(r+\delta)t}) = p\frac{\partial Y}{\partial K} e^{-(r+\delta)t} \tag{2.47}
\]

Integrating both sides of equation 2.47 between \( t=s \) and \( t = \infty \) gives
\[-[me^{-(r+\delta)t}]_s^\infty = -[m(\infty)e^{-(r+\delta)\infty} - m(s)e^{-(r+\delta)s}] = \int_s^\infty \frac{\partial}{\partial t} p \frac{\partial Y}{\partial K} e^{-(r+\delta)t} \, dt \] (2.48)

which reduces to

\[m(s)e^{-(r+\delta)s} = \int_s^\infty \frac{\partial Y}{\partial K} e^{-(r+\delta)t} \, dt \] (2.49)

Multiplying both sides by \(e^{(r+\delta)t}\) we get the following expression for the current marginal valuation of capital

\[m(s) = \int_s^\infty p(t) \frac{\partial Y(t)}{\partial K} e^{-(r+\delta)(t-s)} \, dt \] (2.50)

Thus, by writing equation 2.42 in integral form, we can interpret it more clearly: the marginal valuation of capital at time \(s\), \(m(s)\), is equal to the discounted stream to time \(s\) of the value of the marginal products of capital. Put another way, the change in the firm’s value, or net worth, \(V(t)\), brought about by the employment of one extra unit of capital is equal to the market value of the extra output that this unit yields. In short, \(m(s)\) is the marginal benefit of increasing capital stock by one unit.

Given this interpretation of equation 2.42, we can provide an interpretation of equation 2.41 by noting that the latter can be rewritten as

\[m = z + pC(I,K) \] (2.51)

This simply states that the firm will continue to invest until the marginal benefit of capital is equal to its marginal cost. Notice that the marginal cost of capital, given by the right hand side of equation 2.51, has two components: the first, \(z\), is simply the market price of investment goods; the second, \(pC(I,K)\), is the marginal adjustment cost, measured in terms of the market value of the output foregone, say for example, due to the disruption of the production process as new capital is installed.

We have already alluded to a relationship between the cost of adjustment model and Jorgenson’s neoclassical model. We take this opportunity to make this relationship explicit. The firm’s optimal employment of labour is identical in both models. Equations 2.25 and 2.45 state that the firm should hire labour up to the point at which its marginal product is equal to the real wage. We can also compare the optimal investment rules from the two models. Notice that optimal investment rule in the neoclassical model, given by equation 2.20, can be written as \(z = m\). This says that
firms should continue to invest until the marginal cost of an extra unit of capital stock (its supply price) is just equal to the marginal benefit of that unit $m$ (its demand price). When adjustment costs are introduced, the optimal investment rule given by equation 2.41 is also determined by the equality of marginal cost and marginal benefit, but now marginal cost incorporates an additional term: the marginal cost of adjustment, $pC(I,K)$. If there are no costs of adjustment, then $pC(I,K)=0$ and equation 2.41 reduces to 2.20. A similar relationship exists between the conditions for optimal capital stock in the two models. If there are no adjustment costs, the first order condition given by equation 2.42 reduces to 2.21 (since $pC(I,K)=0$). Thus, it is clear that the cost of adjustment model is consistent with the Jorgenson model modified to include adjustment costs.

2.5.3 The Derivation of an Investment Function

In order to determine an econometric specification for the cost of adjustment model we have to assume a particular form for the adjustment cost function. Recall, the cost of adjustment function, denoted $C(I,K)$, was assumed to be increasing and strictly convex in $I$. The function is also frequently assumed to be a quadratic in $I$, and linearly homogenous in $I$ and $K$. The following function satisfies these assumptions:

$$C(I(t), K(t)) = \frac{bK(t)}{2} \left( \frac{I(t)}{K(t)} - \zeta(t) \right)^2$$  \hspace{1cm} (2.52)

where $\zeta(t)$ can be regarded as a stochastic technology shock. Differentiating with respect to $I$ and dropping the timescripts we get

$$C'(I, K) = b\left( \frac{I}{K} - \zeta \right)$$  \hspace{1cm} (2.53)

Now recall this firm’s optimal investment rule given in equation 2.41. This can be rearranged into the following more convenient form

$$C(I, K) = \left( \frac{m}{z} - 1 \right) \frac{z}{p}$$  \hspace{1cm} (2.54)

Substituting equation 2.54 into 2.53, rearranging, and writing in discrete time, we get
\[
\left( \frac{1}{K} \right)_t = \frac{1}{b} \left( \frac{m_t}{z_t} - 1 \right) \frac{z_t}{p_t} + \zeta_t
\]  
(2.55)

Notice the error term is identical to the stochastic technology shock and therefore, unlike in the Jorgenson model, it follows from theory. Observe also that the model does not contain a constant. This is a consequence of the assumed form of the adjustment cost function. One way in which we could incorporate a constant into equation 2.55 is to add an additional term, linear in \( I/K \), to the cost of adjustment function 2.52.

This investment function suggests that whenever there is a discrepancy between \( m \) and \( z \), the firm has an incentive to change its capital stock, but its actions are tempered by the convex adjustment cost technology. The steeper the adjustment cost function, the larger is \( b \), and the more slowly investment responds. Thus, the introduction of adjustment costs have resulted in a wedge between the supply price, \( z \), and the demand price, \( m \), and this wedge eliminates the possibility of instantaneous adjustment of capital stock and therefore, infinite rates of investment. Rather, investment is an increasing function of the ratio of the demand price, \( m \), to the supply price, \( z \), of capital goods.

A final point to note about equation 2.55 is that the path of investment does not depend on capital stock or lagged variables. This is somewhat surprising given the dynamic adjustment costs facing the firm. However, the dynamics of the relationship are all contained in the contemporaneous \( m/z \) term. For empirical researchers the critical problem in developing an estimable equation from equation 2.55 is relating the unobservable \( m/z \) to observable variables. The \( q \) theory of investment provides one means by which this may be achieved. This uses information in financial markets to relate \( m/z \) to observables. We consider this theory more fully in Section 2.6, but we first consider an alternative representation of adjustment costs which will facilitate further analysis.

2.5.4 An Alternative Representation

In the model outlined above, adjustment costs are valued in terms of the price of output. An alternative way to introduce adjustment costs associated with investment
has been presented in Hayashi (1982) although, as Hayashi notes, this treatment is originally due to Uzawa (1969). Here, the firm maximises net worth, as defined in the Jorgenson model above, but the capital accumulation equation is modified such that

\[ \dot{K} = \psi(I, K) - \delta K \] (2.56)

Here, costs of adjustment are not measured in terms of lost output as in the Lucas, Gould and Treadway representation. Rather, costs of adjustment drive a wedge between the amount of capital purchased and the amount installed. In other words, $I$ units of investment do not necessarily turn into capital, only a fraction $\psi$ of the total investment in new capital does. $\psi(I, K)$ is assumed to be increasing and concave in $I$ and decreasing in $K$ (that is, $\psi_I > 0$, $\psi_{II} < 0$ and $\psi_K > 0$). This represents the presumption that the costs of instalment per unit of investment are higher, the higher the rate of investment for any given $K$. The concavity of installation function, $\psi$, also captures the irreversibility of investment: as $I$ becomes negative, $\psi$ drops sharply below zero.

Thus, with Hayashi type adjustment costs, the firm’s optimisation problem can be stated formally as

\[
\max_{L,I} V(t) = \int e^{-\gamma} \left[ p(t)Y(t) - w(t)L(t) - z(t)I(t) \right] dt
\] (2.57)

subject to

\[ Y(t) = F(L(t), K(t)) \] (2.58)

and

\[ \dot{K}(t) = \psi(I(t), K(t)) - \delta K(t) \] (2.59)

Dropping the timescripts, we can write the current value Hamiltonian for this problem as

\[
H = pF(L, K) - wL - zI + m \left[ \psi(I, K) - \delta K \right]
\] (2.60)

where $m$ is the current value multiplier. The first order conditions for this problem are
\( \frac{\partial H}{\partial L} = pF_L(L, K) - w = 0 \) \hspace{1cm} (2.61)

\( \frac{\partial H}{\partial I} = -z + m\psi_I(I, K) = 0 \) \hspace{1cm} (2.62)

\( \frac{\partial H}{\partial K} = pF_K(L, K) + m\psi_K(I, K) - m\delta = rm - \dot{m} \) \hspace{1cm} (2.63)

\( \frac{\partial H}{\partial m} = \psi(I, K) - \delta K = \dot{K} \) \hspace{1cm} (2.64)

and the transversality condition for this problem is

\[ \lim_{T \to \infty} m(T)K(T)e^{-rT} = 0 \] \hspace{1cm} (2.65)

Equation 2.61 is identical to the marginal productivity of labour conditions given in equations 2.19 and 2.40 for the Jorgenson and previous cost of adjustment model. (Note that equation 2.19 includes a discount factor representing the fact that we expressed the Hamiltonian for that problem in terms of the marginal value of capital rather than in terms of the current marginal valuation of capital used in the Hamiltonian for this model).

The interpretation of equation 2.63 is seen most clearly by rewriting it as

\[ (r + \delta - \psi_K)m - \dot{m} = pF_K(L, K) \]

\[ = p \frac{\partial Y}{\partial K} \] \hspace{1cm} (2.66)

and by following the procedure represented by equations 2.47 to 2.50, we can write it in integral form as

\[ m(s) = \int_{s}^{\infty} p(t) \frac{\partial Y(t)}{\partial K(t)} e^{-(r+\delta-\psi_K(t))(t-s)} \, dt \] \hspace{1cm} (2.67)

It is notable that this is identical to equation 2.50 but for the extra term, \( \psi_K(t) \), in the discount rate and the definition of \( Y \). Equation 2.67 states that \( m \) is the present discounted value of additional future profits that are due to one additional unit of capital. In other words, \( m \) is the marginal benefit of investing. Notice, that there are two benefits from increasing capital stock. The term \( pY_k \) represents the additional future profits that result from the increase in the productive capacity of the firm. The second benefit arises since \( \psi_K > 0 \). This implies that the fraction of investment, \( \psi \), that actually turns into capital increases as \( K \) increases, i.e. the cost of installing,
$I - \psi(I, K)$, is lower for higher $K$. Consequently, the discount rate in equation 2.67 decreases as $K$ increases, which implies that the present discounted value of $pY_K$ is higher for higher $K$.

To interpret equation 2.62, it is useful to rewrite it as

$$z + (1 - \psi_I)m = m$$

(2.68)

The first term here represents the purchase price of investment goods. The second term represents the adjustment cost associated with investment. If there were no adjustment costs so that $\psi(I, K) = I$, then the market value of the firm would increase by $m$ for one unit of additional investment. But capital stock increases only by $\psi_I$. Thus, the term $(1 - \psi_I)m$ represents the market value of the firm foregone due to the concave adjustment cost function. Thus, in short, equation 2.68 states that the marginal benefit of installing one unit of capital, $m$, is equal to the marginal cost of doing so, where marginal cost has two components: the purchase price of capital, $z$, and the adjustment cost associated with the investment, $(1 - \psi_I)m$.

Again, we highlight the relationship between this version of the cost of adjustment model and the Jorgenson model by noting that the first order conditions 2.61, 2.62 and 2.63 reduce to 2.19, 2.20 and 2.21 respectively, when there are no adjustment costs (i.e. when $\psi_I = 1$ and $\psi_K = 0$).

As in the cost of adjustment model of Section 2.5.2, we can derive an investment equation by assuming a functional form for $\psi(I, K)$ that is increasing and concave in $I$, and linearly homogenous in $I$ and $K$. Differentiating this with respect to $I$, substituting the resulting expression into equation 2.62 and rearranging to gain an expression for $I/K$ in terms of $m/z$, we get a function similar to equation 2.55.

As noted above, the critical problem in developing an estimable equation from equation 2.55 is relating the unobservable $m/z$ to observable variables. The $q$ theory of investment uses information in financial markets to relate $m/z$ to observables and it is to this that we now turn.
2.6 Tobin’s $q$ Theory

The literature on investment over the last three decades has been dominated by two theories, the neoclassical theory originating with Jorgenson and, more recently, the $q$ theory developed in a series of papers, Brainard and Tobin (1968), Tobin (1969, 1978) and Tobin and Brainard (1977). Since its development, $q$ theory has been shown to be equivalent to the neoclassical model modified to include costs of adjustment. We first outline the theory and then relate it to the cost of adjustment models.

2.6.1 An Intuitive Outline of $q$ Theory

The price investors are prepared to pay for an investment good (the demand price) depends on its expected profitability. The demand price of an entire firm is the market value of all its securities, i.e. the market value of all its debt and equity in the securities markets. The price that investors pay in order to acquire an investment good (the supply price) is determined by conditions in the market for investment goods. The supply price of an entire firm can be measured by assessing the replacement cost of all the firm’s assets. Equilibrium is attained when the supply price of goods and the demand price for investment goods are equal. Thus, in equilibrium, when there is no incentive to invest, the ratio of the demand price to the supply price is unity.

The ratio of the financial value of the firm to the replacement cost of its existing capital stock defines a concept called average $q$. That there exists a positive relationship between the stock market value of the firm and investment has been vividly articulated by Keynes (1936). He states (p. 151) that the ‘daily revaluations in the Stock Exchange, . . . , inevitably exert a decisive influence on rate of current investment. For there is no sense in building up a new enterprise at a cost greater than that at which a similar existing enterprise can be purchased; whilst there is an inducement to spend on a new capital project what may seem an extravagant sum, if it can be floated off on the stock exchange at an immediate profit’.

However, Tobin realised that average $q$ is a measure of the profitability of the firm’s existing capital stock and may not be a good predictor of investment which is likely to be determined by the profitability of acquiring new physical assets. Tobin noted that the relevant concept for investment was marginal $q$, defined as the ratio of market
value of new additional investment goods, their demand price, to their replacement cost, their supply price.

A firm operating in a relatively profitable environment may expect a return on increasing its capital that is greater than the cost of doing so. In this case, marginal $q$ would be greater than unity. According to Tobin, a firm should continue to invest until the incremental market value just equals the incremental cost of investing. That is, investment should continue until marginal $q$ equals unity. In an unprofitable environment, the expected profitability of increasing capital stock will be less than the cost of the new capital and thus marginal $q$ will be less than one. In order to maximise return to the shareholder, if that is the firm’s objective, it may be better served selling capital until marginal $q$ is equal to unity. In short, the implicit suggestion in this form of Tobin’s $q$, is that there are incentives for the firm to invest if marginal $q$ is greater than unity and to disinvest if marginal $q$ is less than unity. In other words, if the market value of an asset is higher than the cost of purchasing it, there will be an incentive to invest in that asset. An increase in the market valuation of houses relative to current cost of building will encourage residential construction. The incentive is the gain to be made in terms of increased profits, by the excess of the market value over the replacement cost.

While average and marginal $q$ will often move in the same direction, this will not always be the case. Consider a large permanent increase in the wage rate. Firms that have recently invested in technology that embodies a high capital-labour ratio will see their stock market values fall considerably. Consequently, average $q$ for these firms will fall below unity (assuming initial equilibrium). However, these firms and firms that did not undertake such investment have a substantial incentive to invest in new labour-saving technology, implying that marginal $q$ will be greater than unity. Although marginal and average $q$ are not typically equal, conditions have been identified under which the latter can be used as a proxy for the former. We now consider this relationship more formally.
2.6.2 The Equivalence of Marginal and Average $q$

That marginal and average $q$ are not always equal has presented a problem for empirical researchers. Marginal $q$ is not observable and many practitioners have simply regressed investment on the observable, average $q$. Hayashi (1982) has identified the formal conditions under which marginal and average $q$ are equal, that is, the conditions under which marginal $q$ can be inferred from the market data. The proof, given below, draws on the analysis of the Hayashi cost of adjustment model presented in Section 2.5.4. The marginal benefit from increasing capital stock was defined as

$$m(s) = \int_s^\infty p(t) \frac{\partial Y(t)}{\partial K(t)} e^{-(r+s-\psi)(t-s)} \, dt$$

(2.69)

It is the present discounted value of the extra output that is due to one additional unit of capital. The extra output, measured by $Y_K$, is sold in the output market at price $p$. Thus, the marginal benefit of increasing capital stock and the market value of one additional unit of capital are in fact equivalent terms. $m$, therefore, is the numerator of marginal $q$, which can now be formally defined using current notation as

$$q(t) = \frac{m(t)}{z(t)}$$

(2.70)

To reiterate, it is the ratio of the market value of additional capital to its replacement cost. Similarly, we can straightforwardly define average $q$, denoted $q^A$ hereafter, as

$$q^A(t) = \frac{V(t)}{z(t)K(t)}$$

(2.71)

Again, to reiterate, it is the ratio of market value of the entire firm to the replacement cost of all the firm’s capital assets. To show that $q(t) = q^A(t)$ all we need to do is show that $V(t) = m(t)K(t)$. Hayashi begins his simple proof by noting

$$\frac{d}{dt} [m(t)K(t)e^{-rt}] = [\dot{m}K + mK - rmK]e^{-rt}$$

(2.72)

Recall that profits in this model are defined as

$$\pi = pF(L, K) - wL - zI$$

(2.73)

If we assume $F(L, K)$ to be linearly homogenous in $K$ and $L$ and $\psi(L, K)$ to be linearly
homogenous in \( L \) and \( K \), then applying Euler’s theorem we can write the former as
\[ F(L,K) = F_L L + F_K K \]
and the latter as \( \psi(L,K) = \psi_L L + \psi_K K \). By substituting the marginal productivity of labour condition given in equation 2.61 into \( F(L,K) \), the profit function reduces to
\[ \pi = p F_K K - z I \quad (2.74) \]
Also, by substituting \( \psi(L,K) \) into the capital accumulation condition given in equation 2.59, we get
\[ \dot{K} = \psi_L I + \psi_K K - \delta K \quad (2.75) \]
Finally, note that condition 2.62 can be written as
\[ m = \frac{z}{\psi_I} \quad (2.76) \]
and by substituting this into 2.63 we get
\[ \dot{m} = (r + \delta - \psi_K) \frac{z}{\psi_I} - p F_K \quad (2.77) \]
Substituting these expressions for \( \dot{K}, m \) and \( \dot{m} \) into the right hand side of equation 2.72 we get
\[ \frac{d}{dt} \left[ m(t) K(t) e^{-rt} \right] = \left[ \left( r + \delta - \psi_K \right) \frac{z}{\psi_I} - p F_K K + \frac{z}{\psi_I} \left[ \psi_L I + \psi_K K - \delta K \right] - r \frac{z}{\psi_I} K \right] e^{-rt} \quad (2.78) \]
which after some simplifying reduces to
\[ \frac{d}{dt} \left[ m(t) K(t) e^{-rt} \right] = \left[ -p F_K K + z I \right] e^{-rt} = -\pi e^{-rt} \quad (2.79) \]
from equation 2.74. Integrating between \( t = 0 \) and \( t = \infty \) gives
\[ \left[ m(t) K(t) e^{-rt} \right]_0^\infty = - \int_0^\infty \left[ p F(L,K) - w L - z I \right] e^{-rt} \, dt = -V(t) \quad (2.80) \]
Evaluating the left hand side
\[ m(\infty) K(\infty) e^{-r\infty} - m(0) K(0) e^{-r0} = -V(0) \quad (2.81) \]
Using the transversality condition in equation 2.65, we can write
\[ m(0)K(0) = V(0) \]  

(2.82)

This immediately implies the equivalence of marginal and average \( q \). Dividing both sides by \( K \) and \( z \) and ignoring the timescripts, we get

\[ q = \frac{m}{z} = \frac{V}{zK} = q^t \]  

(2.83)

So, what assumptions were necessary to arrive at this conclusion? Firstly, the model assumes perfect competition so that the firm is a price-taker in the input and output markets. Secondly, the factor substitution possibilities in this model are the same \textit{ex ante} as \textit{ex post}, that is, production technology is putty-putty. Thirdly, capital was implicitly assumed to be homogeneous. Fourthly, it was necessary to assume that production and adjustment cost functions were linearly homogeneous in \( L \) and \( K \) in the case of the former and in \( L \) and \( K \) in the latter. Finally, the investment decision was implicitly assumed to be largely separate from the firm’s other real or financial decisions. In other words, unless these rather strong assumptions hold, marginal and average \( q \) are not equal, and hence investment will not be related to stock market value in the manner suggested by traditional \( q \) theory.

2.6.3 Marginal \( q \) and Adjustment costs

 Appropriately reformulated in terms of marginal \( q \), it is clear that for the rate of investment to be an increasing function of \( q \), some sort of adjustment costs must be assumed. If \( q \) exceeds unity the firm would want to buy capital and install it immediately if there were no adjustment costs. That is, there is a demand for an instantaneous jump in the capital stock and the consequent rate of investment would be infinite. Under these conditions, \( q \) will only deviate from unity in the instant. As \( q \) deviates from unity, the firm will invest or disinvest so that, by the end of an infinitesimal time period, it will be returned to unity. Thus, with no costs of adjustment the firm will react instantaneously to a change in \( q \), investing at an infinite rate to restore \( q \) to unity. To all intents and purposes, except in the instant, \( q \) will always equal unity and therefore investment can not be an increasing function of \( q \): investment is zero (except in the instant when it is infinite) and \( q \) is always unity (except in the instant).
However, if there are costs of adjustment associated with the installing capital and if these are increasing at the margin, investment will be an increasing function of $q$. In our discussion of the adjustment cost model we justified the assumption that adjustment costs were an increasing and convex function of the rate of investment. Given this, the total cost, that is the cost of purchasing and installing capital, will also be an increasing and convex function of the rate of investment. In our discussion of adjustment cost models the adjustment cost function is assumed to depend on $I$ and $K$, $C(I,K)$. Suppose now however, in order to aid simple exposition, adjustment costs are a function of $I$ only, i.e. $C(I)$. Since adjustment costs are increasing in $I$, $C(I)>0$. Further, since they are convex in $I$, we can also write $C'(I)>0$. Now the total cost of investment, the purchase cost plus the adjustment cost, can be written as $zI+C(I)$.

Further suppose, again without loss of generality, that the price of investment goods, $z$, is unity. In making such an assumption, the total cost of investment can be written as $I+C(I)$ and the marginal cost of investment is $1+C'(I)$. The optimal rate of investment is given by the rate where the marginal value of investment is equal to its marginal cost. Recall, marginal $q$ is the ratio of the marginal benefit of investment, $m$, to its replacement cost, $z$. Since we have assumed the price of investment goods, $z$, to be unity, the marginal benefit of investment (or the demand price) is simply equal to $q$. Thus, the optimal rate of investment is given when $q = 1 + C'(I)$. Rearranging, we get $I(q) = C^{-1}_{I}(q-1)$ and since $C(I)$ is an increasing function of $I$, $C^{-1}_{I}(q-1)$ will be an increasing function of $q$. If we assume that $C(0)=0$, so that $C^{-1}_{I}(0) = 0$, $I(q)$ will be an increasing function of $q$ with $I(1)=0$.

Thus, we have shown that for investment to be an increasing function of $q$, we must assume increasing and convex costs of adjustment. This assumption delivers a determinate rate of investment, that is, we no longer get the infinite or zero rate of investment that results in the absence of adjustment costs. From the above discussion it should be obvious that the cost of adjustment model and Tobin’s $q$ model both suggest that investment is related to the discrepancy between the demand price of an investment good (its marginal value) and its supply price (its replacement cost). In fact, the relationship between the two theories of investment behaviour goes far beyond this. Lucas and Prescott (1971), Mussa (1977), Abel (1979, 1980), Yoshikawa (1980) and Hayashi (1982) all demonstrate that adjustment cost technology and
optimising behaviour lead to a relationship between investment and marginal $q$. However, Abel and Hayashi have gone further and demonstrated that neoclassical theory, modified by the assumption of adjustment costs, and Tobin’s $q$ theory, are in fact equivalent. This result is implicit in the above discussion but we now make the equivalence explicit.

2.6.4 $q$ Theory and the Neoclassical Model with Adjustment Costs

Both of the cost of adjustment models presented in the previous section are identical to Jorgenson’s neoclassical model but for the fact that they have been modified by an assumption of adjustment costs. The first model, similar to those of Lucas (1967a, 1967b), Gould (1968) and Treadway (1969), incorporates costs of adjustment by assuming that the output produced by the firm is below that which is technically feasible by an amount equal to the disruption to the production process when new capital is installed. The cost of adjustment is measured in terms of the market value of lost output. The second model, similar to that of Hayashi (1982), incorporates costs of adjustment by assuming that not all investment results in new capital. In this case, adjustment costs are measured in terms of the value of investment that does not result in new capital. In both models the marginal benefit of investing can be derived from the first order conditions for maximisation as

$$m(s) = \int_s^\infty p(t) \frac{\partial Y(t)}{\partial K(t)} e^{-(r+\psi_K(t))(t-s)} \, dt \quad (2.84)$$

If output $Y$ is determined by the production function this equation actually represents the marginal benefit of investing when costs of adjustment are valued in terms of the investment not resulting in new capital. When the costs of adjusting capital stock are valued in terms of lost output, such that actual output is defined as $Y = F(L,K) - C(I,K)$, $\psi_K = 0$.

Further, we have derived expressions for the firm’s optimal investment rule from the first order conditions. The general form for this rule is given by

$$z + V' = m \quad (2.85)$$

where $V'$ denotes the firm’s market value, or net worth, forgone due to the adjustment
cost function. Where adjustment costs are measured in terms of lost output, then $V^f=C(I,K)$. In the model where the cost of adjusting is given in terms of value of investment not resulting in new capital $V^f=(1-\psi(I,K))m$. To simplify matters, we suppose that $V$ is a function of $I$ only, that is, $V(I)$. If we also take the price of capital goods, $z$, as the numeraire, thereby setting it to unity, the marginal cost of investing in these models is given by $1+V(I)$. Also, since we assume $z=1$, $q=m$ (in other words, since denominator of $q$ is unity, marginal $q$ is equal to the marginal benefit of investing, $m$). Thus,

$$q = m = \int \lim_{t\to\infty} \rho \frac{\delta Y}{\partial K} e^{-\left(r+\delta-w\right)I(t-s)} dt \tag{2.86}$$

Neoclassical theory of investment states that the firm will continue to invest until the marginal benefit of investing is equal to the marginal cost. When there are adjustment costs associated with investment, and these are of the form $V^f(I)$, the firm will continue to invest until

$$1 + V^f(I) = q \tag{2.87}$$

Rearranging, we get $l(q) = V^f^{-1}(q-1)$ and since $V(I)$ is an increasing function of $I$, $V^f^{-1}(q-1)$ will be an increasing function of $q$. If we assume that $V(0)=0$, so that $V^f^{-1}(0)=0$, $l(q)$ will be an increasing function of $q$ with $l(1)=0$. Hence, neoclassical theory augmented by assumption of adjustment costs implies a determinate rate of investment which is an increasing function of $q$. Thus, in general, equation 2.87 says that adjustment costs introduce a wedge between the demand price, $m$ (or $q$, if $z=1$), and the supply price or replacement cost, $z$.

2.6.5 The Derivation of an Estimable Investment Function

To derive an investment equation consistent with $q$ theory, reconsider the cost of adjustment investment equation given in equation 2.55. In the discussion of this equation, it was stated that in order to gain an operational, or estimable, equation for investment, we would need to relate the ratio $m/z$ to observable variables. It was stated that $q$ theory allows such a relationship to be made. From the above discussion of $q$ theory, and particularly from equation 2.70, it will be obvious that the ratio $m/z$ specifically defines marginal $q$. Thus, we can write equation 2.55 as
\[
\left( \frac{I}{K} \right)_t = \frac{1}{b} \left( q_t - 1 \right) \frac{z_t}{p_t} + \zeta_t \quad (2.88)
\]

As it stands, this equation is not estimatable since \( q \) is not observable in practice. If the conditions for equivalence between marginal and average \( q \), formalised by Hayashi and discussed above, hold, then we can replace marginal \( q \) in equation 2.88 with the observable \( q^d \). We can then estimate an investment equation of the form

\[
\left( \frac{I}{K} \right)_t = \frac{1}{b} Q_t + \zeta_t \quad (2.89)
\]

where

\[
Q_t = \left( q^d_t - 1 \right) \frac{z_t}{p_t} \quad (2.90)
\]

2.6.6 Strengths and Weaknesses of the \( Q \) Model

This remarkably simple equation for investment behaviour has a number of attractive features and has been very popular in the empirical literature. Firstly, the parameters of the equation are the structural parameters of the adjustment cost function and as such do not depend on the process by which firms form expectations of future variables. The relevant expectations in this model are captured through the measured regressor \( Q \) and as such this model, unlike all the previous models examined so far, will not be subject to the Lucas critique (see Lucas (1976), p. 42).

Secondly, the model solves the problem of unobservable expectations by equating a forward looking variable to one that is readily observed. Given \( Q \), we have a great deal of information about future conditions affecting investment without having to make specific assumptions about expectation formation or future conditions of supply and demand. For a forward looking firm constrained by adjustment costs, the rate of investment should be solely determined by contemporaneous \( Q \). If \( z \) is known at the beginning of the period and \( \zeta_t \) is due only to contemporaneous technology shocks then, because \( Q \) is dated at the beginning of the period, ordinary least squares will yield consistent estimates of \( b \).

A third attractive feature of this simple model is that the theory predicts that the \( Q \) variable should be a sufficient explanation of investment. This suggests that, in
general, the underlying theory can be tested by considering whether other potential regressors, such as output contain significant information in addition to $Q$. Evidence that a more general dynamic specification, or that other variables contain additional information, would suggest this simple model is too restrictive.

Finally, since the dynamics of the investment process are incorporated into the firm’s optimisation problem, this approach resolves a number of the theoretical issues from the neoclassical research programme. Like the neoclassical model it is derived directly from an optimisation problem. However, unlike neoclassical econometric investment equations, equation 2.90 is theoretically consistent. It recognises explicitly the dynamics due to expectations and technology and, moreover, isolates their influences. In addition, the error term follows explicitly from theory.

Despite these benefits, the usefulness of $Q$ theory is called into question by its generally disappointing empirical performance, a point that will be explored in the following subsection. First, we consider some of the practical difficulties encountered in implementing such theory.

The first of the practical difficulties with the $Q$ model is the possible mismeasurement of the three components comprising $q^4$. The first and most important measurement issue is that relating to the numerator of $q^4$, namely $V_t$. The reliability of financial asset prices in evaluating the underlying cash flows has been questioned in a number of studies, a survey of which is provided in Leroy (1989). Investor sentiment is often stated as the cause of differentials between market values and fundamentals. Broadly speaking, investor sentiment encompasses financial market phenomena such as excess volatility, mean reversion, fads or speculative bubbles. Sentiment creates a problem for the $Q$ model insofar as investment decisions are based on fundamentals. One of the ways in which the role of investor sentiment relative to fundamentals has been examined in the literature is to separate out the noise from that part of $q^4$ that contains an accurate signal of the firm’s fortunes. Engle and Foley (1975) estimate a band spectrum regression to separate the noise from the signal and enjoy some success with their adjusted asset price series which has a variance that is only 13% of the unadjusted series. We examine this study in more detail in Section 2.6.7.
The second measurement issue concerns the mismeasurement of the capital stock variable in the denominator of \( q^t \). Firstly, available estimates for capital stock are calculated using the perpetual inventory method with a fixed set of linear depreciation rates that may become extremely inaccurate over a period of major structural change. For example, developments in computing and rapid energy price rises may have made part of firm’s capital stock economically obsolete forcing them to accelerate depreciation. Thus, published capital stock series based on fixed depreciation rates may tend to overstate the replacement value of the existing capital stock. This tendency, should it exist, may, to some extent, be offset since the replacement value of the firm’s existing capital stock will not embody the value of the firm’s other assets, including intangibles such as brand loyalty or goodwill. Empirical analysis, however, has tended to find little support for the hypothesis of increased obsolesce (see Hulten, Robertson and Wykoff (1989) and Arnold (1985) as examples). Even when alternative estimates of \( K, \) are adopted they have only a modest effect on the empirical performance of the \( Q \) model (see Bond and Devereux (1989)).

A final possible source of mismeasurement lies with the tax and non-tax components on \( z_t \). However, since most of the variability in \( q^t,A \) comes from the numerator, it is unlikely that the mismeasurement of these terms, or for that matter \( K, \) will be of major importance.

The second practical difficulty with \( Q \) models, such as that in equation 2.90, concerns the conditions under which \( q^t \) can be used as a proxy for \( q \). Recall, there were five such conditions. Firstly, we need to assume perfect competition so that the firm is a price-taker in the input and output markets. Secondly, we need to assume that the factor substitution possibilities in this model are the same \textit{ex ante} as \textit{ex post}, that is, production technology is assumed to be putty-putty. The third assumption is that of homogeneity of capital. Fourthly, production and adjustment cost functions were assumed to be linearly homogeneous in \( L \) and \( K \) in the case of the former and in \( I \) and \( K \) in the latter. Finally, the firm’s investment decision is assumed to be independent of its other real or financial decisions. Unless these rather strong assumptions hold, marginal and average \( q \) are not equal, and hence investment will not be related to stock market value in the manner suggested by traditional \( q \) theory.
The extent to which these conditions can be relaxed within the $Q$ framework has been investigated by a large number of researchers motivated by the empirical difficulties with the $Q$ model. Chirinko (1993) details the results of some of these studies and concludes that the restrictiveness of the conditions permitting $q^4$ to proxy $q$ do not appear to be responsible for the $Q$ models empirical shortcomings. In none of the papers to which Chirinko refers does the empirical performance of the $Q$ model significantly improve when one or more of these conditions are relaxed.

2.6.7 Evidence on $q$ Theory

In contrast to the neoclassical and putty-clay models of investment, most $q$ theory studies have been concerned with estimation rather than an assessment of the effects of alternative policies. This is due, in part, to the fact that the calculation of the change in investment expenditure due to changes in tax parameters is not as straightforward in $Q$ models as it is for the neoclassical model and its derivatives. As Chirinko (1993, p. 1902) notes, in $Q$ models the analysis is conducted in two stages: the first stage is particularly involved because one has to quantify the response of asset prices to an alternative sequence of tax parameters that will affect current and future investment, which will feedback into $Q_t$. The second stage is accomplished by solving for investment, the capital stock and asset values simultaneously over an approximately infinite time horizon. Summers (1981) provides an example of this kind of study. That there are few studies of this type does not disadvantage this survey since it is the estimation of investment functions that is of principal interest.

We noted above two main caveats pertaining to the $Q$ model: the mismeasurement of the components of $Q$ and the conditions under which marginal $q$ and average $q$ are equivalent. However, neither appear to be responsible for its empirical shortcomings. The $Q$ model's empirical performance has generally been unsatisfactory and is reviewed in this section in terms of the statistical significance of the $Q$ variable, the fit of the equation and serial correlation in the residuals.
2.6.7.1 The Early Results

One of the earliest studies using the financial value of the firm as a determinant of investment is that presented by Grunfeld (1960). This study represents the first attempt to relate the financial value of the firm to investment, albeit in an accelerator type framework (see Section 2.2.4 for a discussion of Grunfeld's work on the accelerator). He suggests that the financial value of the firm, measured as the firm's discounted future earnings less the cost of future additions to capital stock, may be a better measure of expected profits than current realised profits. He notes that his $Q$ variable 'explains a larger proportion of investment behaviour than either lagged or current profits' (p. 233).

Another early study providing evidence in support of $q$ theory is that by Engle and Foley (1975). In common with most empirical studies, Engle and Foley's investment equation is not consistent with $q$ theory since investment is regressed on a distributed lag of $Q$ rather than only contemporaneous $Q$ as in equation 2.89. Furthermore, Engle and Foley also add variables measuring potential GNP and the gap between actual and potential GNP (i.e. capacity utilisation). Before running this regression the authors employ a band spectrum regression to separate the noise from the fundamental signal contained within their $Q$ variable. This approach involves decomposing the observed data into frequency components and then filtering the data to eliminate high frequency variations. The procedure, described by Granger and Hantanka (1964) as demodulation, low-pass filtering and remodulation, also allows them to eliminate very low frequencies. The variance of the resulting series of the asset prices is only 13% of the original series. The equation was estimated using quarterly seasonally adjusted data in constant dollars for both producers' durable equipment and business construction for the period 1953 to 1968.

The results show that the marginal propensity to invest in producer durables out of potential GNP is 10%, while for structures this propensity is about 3%. The modal lag between investment and a change in the price of capital goods is four quarters for durables and five quarters for structures and, since the lag distributions are roughly symmetrical, these are also the mean and medium of the lag patterns. This implies that a change in $Q$ will have its total effect in three years. The long run elasticities of investment with respect to $Q$ variable are said to be large. In the case of producer
durable equipment, this elasticity is slightly less than unity, whilst it is double that for structures. These results imply that a 10% increase in the $Q$ variable should result in an 8% increase in equipment spending and a 20% increase in expenditure in structures. Engle and Foley go on to compare the performance of this equation with alternative models of investment behaviour. We consider this aspect of this work in Section 2.7.

A further early study yielding similarly encouraging results is that by Ciccolo (1975). He also estimates a $Q$ model with distributed lags for $Q$, but without the capacity variable used by Engle and Foley (1975), and finds a significant role for the $Q$ variable in his equations of investment behaviour. However, these initially encouraging results with aggregate data have not been sustained in more recent work.

2.6.7.2 More Recent Results with US Data

Estimating equations for orders and investment of non-financial US corporations, with quarterly data for the period 1952-1976, which contain variables for capital stock, taxes and capacity utilisation in addition to a $Q$ variable, von Furstenburg (1977) arrives at the conclusion that ‘the use of $Q$ in equations for capital goods orders and investment must be regarded as optional’ (p. 388). He further notes that using ‘variables in addition to $Q$ is mandatory; otherwise, the resulting estimates are prone to be either inconsistent statistically or fraught with such serious autocorrelation of the error terms as to beg the question of how such a process can be generated or convincingly explained by the use of $Q$’ (p. 388).

Chirinko (1987) shows that if the firm actively participates in more than one financial market, $Q$ is likely to yield an uninformative and possibly misleading signal for investment expenditures. He modifies the theory to endogenise financial policy but when tested against a conventional model, Chirinko finds no support for his modification. Moreover, a number of the empirical problems inherent in the conventional model (significant lagged variables, residual serial correlation and slow speeds of adjustment) remain.

We have already discussed the theoretical work of Hayashi (1982) at some length in Section 2.5.4. This study also contains an empirical investigation in which an exact
relationship between marginal $q$ and average $q$ is derived. Marginal $q$, adjusted for tax variables, is then calculated from average $q$ and tested in a theoretically consistent empirical equation. Hayashi's equation, which is identical to that given in equation 2.89, is estimated by OLS for aggregate US annual investment as a share of capital stock for the period 1953-76. Although this $Q$ variable is found to be significant with a $t$-ratio of 4.64, it explains only 46% of the variation in the dependent variable and the Durbin-Watson statistic is very low indicating serial correlation in the residuals.

Similar results are obtained by Blanchard and Wyplosz (1981). They estimate a $Q$ equation with quarterly data on US gross private investment for the period 1953-1978. In their principal equation, estimated using two stage least squares, no assumptions are made about the error process and lagged $Q$'s and current and lagged output are included as additional independent variables. Current $Q$ is significant, but so is lagged $Q$ and lagged output. Only 57% of the variation in the dependent variable is explained by the variables in this specification and again the Durbin-Watson statistic (not strictly valid in the presence of a lagged dependent variable) is very low, 0.3, indicating serial correlation in the residuals.

A similar story is told by Summers (1981). This paper presents an analysis of the effects of tax policy on investment and valuation based on the $q$ theory of investment. Using a measure of $Q$ that is carefully adjusted to take account of tax variables, Summers runs a number of regressions for non-financial corporate investment over the period 1931-78. Experiments reveal that equations using the tax adjusted measure of $Q$ outperform those with standard measures of $Q$. Citing the work of Sargent (1980), Summers suggests that there are some reasons to doubt the exogeneity of $Q$. For example, shocks to the adjustment cost technology may induce spurious correlation between $Q$ and investment. For this (and other) reasons some of his equations are re-estimated using instrumental variables. The evidence in favour of the importance of tax variables is strengthened although the overall explanatory power is low for all specifications, with the $R^2$ ranging between 0.3 and 0.4.

In his comparison of the empirical performance of various investment theories, to be discussed at more length in Section 2.7, Clark (1979), again with US data, shows that a distributed lag $Q$ model does not perform adequately for equipment or structures, either within sample or out of sample.
2.6.7.3 Evidence from UK Studies

All studies discussed so far estimate $Q$ equations based on aggregate US data. A broadly similar pattern has been reported with aggregate UK data. Oulton (1981), with the principal aim of assessing the importance of $Q$ for investment in Britain, estimates an 8 period distributed lag $Q$ relationship for real gross quarterly investment by industrial and commercial companies for the period 1960-77. Delivery lags are used as a justification of a distributed lag term for $Q$. Since the residuals from the basic equations are found to be highly serially correlated the equations were re-estimated using the Cochrane-Orchutt technique. After this correction, the sum of the coefficients on the $Q$ variable are highly significant, as were the individual lag coefficients. The shape of the lag distribution resembled an inverted 'v' and the goodness of fit, as measured by the $R^2$, ranges from about 0.8 to 0.9. Oulton compares this model with an accelerator (amongst others) and finds that although the accelerator outperforms the $Q$ model in terms of explanatory power and significance, the accelerator variable loses its significance when included in equations with the $Q$ variable. Oulton further concludes that 'a $q$ theory of investment has to be taken seriously' (p. 200) and that empirical studies that ignore stock market prices 'are therefore omitting something important' (p. 200).

Jenkinson (1981) has also estimated a $Q$ model with UK data. His study examines the treatment of profitability factors in theoretical models of investment behaviour and compares the performance of these models. The models considered by Jenkinson are the accelerator, Jorgenson's neoclassical model and a $Q$ model. We compare the relative performance of these models in Section 2.7. At this point we consider only those results relating to the empirical performance of the $Q$ model. Although Jenkinson acknowledges that the introduction of lags into the investment function is arbitrary, he estimates a distributed lag relationship for industrial and commercial companies' gross investment (measured as real investment in manufacturing, distribution and non-financial services) for the period 1967-76. The residuals were found to be serially correlated (the Durbin-Watson statistic was 1.17), and as such corrected estimates are presented. The sum of the coefficients on the lagged $Q$ terms are found to be significant with a $t$-ratio of 3.99 and the shape of the lag coefficients is plausible. The $R^2$ on the equation corrected for first order serial correlation is 0.945.
Jenkinson concludes that the results from this study, like those of Oulton (1981), indicate that the $Q$ model performs reasonably well with UK data, and consequently seems worthy of further consideration.

Another empirical investigation applying $q$ theory to UK aggregate investment data is that carried out by Porteba and Summers (1983). The authors adjust $Q$ to incorporate taxes at both the corporate and personal level and carry out tests of two views of dividend behaviour, which can be derived as special cases of a general model by making different assumptions about the firm’s financial margin. The competing views are tested by comparing the performance of investment equations estimated under each theory’s predictions. Porteba and Summers note that according to $q$ theory, lagged values of $Q$ should have no impact on investment since all the information used in agents’ calculation of lagged $Q$ can be used in the calculation of contemporaneous $Q$. However, their preliminary results indicate that investment activity is highly serially correlated, with a Durbin-Watson statistic of 0.49 and 0.65, depending on which hypothesis of dividend taxation is assumed. Moreover, the overall fit of the equations is poor. In order to guard against the prospect that residual serial correlation could contaminate inferences about the competing hypotheses of dividend taxation, the authors choose to over fit the model and apply common factor restrictions (as suggested by Hendry and Mizon (1978)). The resulting parsimonious model contained just one lag of $Q$ in addition to current $Q$. Porteba and Summers express some concern about the significance of the lagged $Q$ term. They justify its presence by suggesting that the annual investment series used to construct their dependent variable will include some projects for which the decision to invest was taken in the year prior to recording the expenditure. In other words the lagged $Q$ term may be picking up these decision lags. On estimating this model the fit of the equations improve (the $R^2$'s improve to about 0.9 for both dividend tax hypotheses).

Porteba and Summers note that their results support earlier findings (citing Oulton (1981) and Jenkinson (1981)) that the $Q$ model can be quite powerful in explaining the observed investment behaviour of British industry. The authors state that a 10% increase in the stock market value will lead to a 15% increase in the investment rate. Further, they note that the coefficients on the $Q$ variables in their equations are larger than those estimated by Oulton and Jenkinson. They suggest that this may be a result
of the fact that their study uses annual rather than quarterly data, the larger sample period or better estimates of the tax effects. The fit of these equations is also said to be better than that of equations in earlier studies. Part of the reason given for the larger coefficient on the $Q$ variable is that annual data on $Q$ are less contaminated by short term fluctuations in market value than quarterly observations. Measurement error and noise is concentrated at high frequencies and therefore longer term movements are more likely to represent investors' underlying views, i.e. the fundamentals. As we have discussed above, Engle and Foley have estimated an investment function for the US using the band spectral regression technique. Porteba and Summers apply this approach to their British investment data. They alternately choose to eliminate those components of $Q$ which occurred at periodicities of less than three and less than five years. The results indicate that the low frequency relationship between investment and $Q$ is stronger than the relationship which is observed with quarterly or annual data.

Dinenis (1989) assesses the short-run and long-run impact of the measures introduced in the 1984 Budget on private non-residential fixed investment in the UK. Whilst noting that the relationship between investment and $Q$ ought to be contemporaneous and static, Dinenis introduces \textit{ad hoc} delivery and expectational lags to estimate a general dynamic model. The parsimonious representation of this general model contained a lagged rather than a contemporaneous $Q$ and a first order autoregressive error term. The implied rate of depreciation in this model is 3.7%. The coefficient on lagged $Q$ is 340, implying very large adjustment costs and therefore a slow response of investment to a change in $Q$. Thus, the effect of $Q$ on investment is small.

In summary, the encouraging results of early $q$ theory investigations of aggregate investment behaviour have not been sustained in more recent studies. All studies discussed here have found significance of other explanatory variables, residual serial correlation and poor fit of equations when the model is correctly specified. Some studies have also found the $Q$ variable to be insignificant. Studies purporting to have found a role for $Q$ have usually specified investment functions that are not consistent with theory.
2.6.7.4 Empirical Results with Disaggregated Data

In principle, there are distinct advantages in exploiting data on individual firms. In the first place it allows theory, developed in the context of the ‘representative’ firm, to be tested at the level at which it is formulated, so reducing the econometric problems introduced by aggregation across firms. Biases resulting from aggregation across firms are eliminated. Secondly, some variables can be measured more accurately at the firm level. This is certainly true of the market value of the firm and also for the effective factor prices that it faces. Thirdly, the estimates can be obtained using both time series and cross section variation across the data. This should contribute to their precision and also allows consistent estimation in the presence of correlated company specific effects. Finally, heterogeneity in, for example, firms’ effective tax rates can be explicitly taken into account. These factors may lead one to anticipate greater empirical success of an application of the $Q$ model to micro data. Although there is some evidence to suggest that the statistical significance of $Q$ increases at lower levels of aggregation, most authors have found that the empirical problems with aggregate $Q$ models are also present in studies using disaggregated data. Readers interested in empirical results with disaggregated data are referred to studies by von Furstenburg, Malkiel and Watson (1979, 1980), Chappel and Cheng (1982), Salinger and Summers (1983) and Fazzari, Hubbard and Petersen (1988) for the US, Hayashi and Inoue (1991) for Japan, Funke, Ryll and Willenbockel (1989) for West Germany, Blundell, Bond, Devereux and Schiantarelli (1992), Devereux and Schiantarelli (1990) and Bond and Devereux (1989) for the UK.

2.6.7.5 Final Remarks on Empirical Results from $Q$ Models

Apart from the statistical significance of $Q$ and goodness of fit, a complementary approach for evaluating the $Q$ model is to compare its theoretical implications with the empirical results. Implicit in the above review, are three persistent discrepancies. Firstly, the dynamics of the $Q$ model appear to be inadequate. Results usually show serially correlated residuals and significant lagged dependant and lagged $Q$ variables. This review has tried to highlight the fact that these lags are frequently justified by the assumption of delivery lags and that these assumptions, or ex post rationalisations, are
wholly inappropriate in the context of theoretical $Q$ models, or adjustment cost models more generally, in which dynamic elements appear explicitly in the firm's optimisation problem. Secondly, the $q$ theory implies that no other variables should have a systematic relation to investment, but quantity variables such as output, capacity utilisation, sales, profits and cash flow are frequently significant. In empirical work, however, $Q$ is often measured with error. If quantity variables such as output are correlated with this measurement error, then one would expect them to appear significant in a $Q$ equation.

The validity of these two criticisms depends on the properties of the error term and the estimation technique. If the error term contains a technology shock correlated with endogenous liquidity and output, then these terms would be expected to be significant in a model estimated by ordinary least squares. Their significance should disappear when the models are estimated with instrumental variables that are orthogonal to the error process. It is noteworthy that significant lagged variables are consistent with a serially correlated technology shock.

The third and perhaps most important criticism of the $Q$ model is that estimated adjustment costs are implausibly large. For example, the coefficient $d(I/K)/dQ$ in Summers' (1981) preferred equation is 0.031 which implies an adjustment cost parameter of 32. Summers notes (p. 101) that such a value implies that the actual capital stock would have moved only three quarters of the way to its long run steady state, twenty years after an unexpected rise in inflation from 0 to 8%. Many studies generate even larger values for the adjustment cost parameter (for example, in the case of Dinenis (1989), discussed above, the coefficient on $Q$ is 0.00294 which implies an adjustment cost parameter of 340).

The significance of variables such as output and liquidity in $Q$ equations suggests that quantity variables are important determinants of investment. Indeed, as with much work carried out using the neoclassical model and its many variants, the weight of evidence points to only a modest response of investment to changes in price variables, such as interest rates and taxes, and a much larger response to changes in quantity variables. In an attempt to clarify this impression, we now review a number of studies that have sought to compare the empirical performance of alternative models of investment behaviour. It is hoped that this will also bring together the empirical issues
implicit in the above discussion, thereby providing a platform for the estimation of investment functions in this work.

Thus, despite its theoretical attractiveness the usefulness of the $Q$ model has been called into question by these empirical failures. The $Q$ concept is only one of the means by which an estimable investment equation can be derived from a cost of adjustment model, in which investment is a function of the unobservable shadow price of capital. The empirical failures of $Q$ models has led to recent interest in alternative approaches to relating the unobservable shadow price of capital to observables. The empirical performance of such approaches are discussed in Chirinko (1993) but are not considered here.

2.7 Comparing Alternative Models of Investment

A number of researchers have attempted to form a consensus model of investment demand by evaluating the performance of alternative models on the same set of investment data. To the extent that we are interested in comparing the performance of alternative investment models, these studies have much in common with this work. These studies have typically sought to measure and rank model performance on the basis of within or out of sample goodness of fit criteria. In this section we consider several studies of this type. Rather than presenting an exhaustive review of these papers, many of which have already been referred to above, we choose to provide a brief outline of the models considered, stating the type of investment being modelled, the sample period over which models are estimated, the criteria by which within and out of sample performance is assessed and the actual rankings of models. Although the theoretical consistency of estimated equations and the associated empirical deficiencies are discussed in passing, this is not of primary concern in this section since many of the issues are covered above. We do not consider any studies attempting to compare models undertaken prior to the innovation of Jorgenson’s neoclassical model.
2.7.1 Jorgenson’s Model versus the Rest

One of the earliest series attempts at model comparison was carried out by Jorgenson and Seibert (1968a) in an attempt to increase the credibility of the neoclassical model as an explanation of corporate investment behaviour. They compare two versions of the neoclassical model (one including a capital gains term in the user cost variable and the other excluding it) with an accelerator, an expected profits and a liquidity model. They concentrate on annual time series data over the period 1949-63 for a small but representative sample of firms from the 1962 Fortune Directory of the 500 largest US industrial corporations. Each model is estimated for each of fifteen firms with the lag distribution being determined from general Pascal distributed lag functions. Relative performance is assessed in a number of ways, but Jorgenson and Seibert argue that the best measure of performance is provided by the residual variance for the best fitted distributed lag function corresponding to each theory. Using this criteria the neoclassical models comes out on top, followed by the expected profits, accelerator, and liquidity theories in that order. Jorgenson and Seibert note that whilst their measures of performance sharply discriminate between the neoclassical models and the other models, the discrimination of the two versions of the neoclassical models is less sharp and requires further examination.

Elliot (1973) notes even if criticisms of the Jorgenson neoclassical model levelled by Eisner (1969 and 1970) are ignored, the evidence in favour of the neoclassical model provided by Jorgenson and Seibert (1968a) is weak, since it is based on a sample of only 15 firms. He suggests that greater significance could be attached to their results if the time series sample were expanded to include more firms, or if model comparison were conducted on a cross-sectional basis. Elliot extends the Jorgenson and Seibert analysis to 184 firms inside and outside manufacturing and conducts analysis of the data for each year between 1953 and 1967 and for each of the 184 firms separately. Testing identical models to Jorgenson and Seibert and using similar criteria for ranking performance, he finds that the enlarged time series comparisons reveal that only small differences can be identified in the overall performance of the liquidity, accelerator and neoclassical models. Thus, Elliot concludes that the major conclusion of Jorgenson and Seibert, that the neoclassical model is a more effective explanation of investment, does not stand up to the enlarged test he applies.
Jorgenson, Hunter and Nadiri (1970a, 1970b) compare the empirical performance of the neoclassical model of investment with that of three other quarterly models for individual manufacturing industries in the US, over the period 1948-1960. They compare the performance of the model estimated by Jorgenson and Stephenson (1967a) given by equation 2.34 above with other models of investment estimated by Anderson (1964), Eisner (1962) and Meyer and Glauber (1964). Goodness of fit and absence of autocorrelation of errors are the basis for comparison in Jorgenson et al (1970a). The Jorgenson and Stephenson model is specified such that the Eisner model, a version of the accelerator, can be obtained as a special case. Jorgenson and Stephenson’s model is found to outperform the others in terms of the stipulated criteria. Eisner’s model performs second best and, according to Jorgenson et al, the models of Meyer and Glauber and of Anderson do less well and appear to be mis-specified.

In Jorgenson et al (1970b) the same models are assessed with regard to predictive performance. The authors assess predictive ability and performance of these models by testing for a structural break at the beginning of the forecast period and by comparing within and out-of-sample errors. According to these criteria Jorgenson et al find that Eisner’s model outperforms the others, closely followed by that of Jorgenson and Stephenson. The superiority of the Eisner model provides further tentative evidence that quantity variables, such as output and profits, are more important determinants of investment than price variables.

We have commented on the Gould and Waud (1973) study at some length in Section 2.3.5 above. Here, we comment only on their results pertaining to the comparison of their model with that of Jorgenson and Stephenson (1967a). Gould and Waud estimate a reduced form variant of the standard neoclassical model in which desired capital stock is determined by only exogenous variables. Using the same quarterly data as Jorgenson and Stephenson on eleven manufacturing industries, Gould and Waud find that the reduced form model performed at least as well as Jorgenson and Stephenson’s model, a hybrid model and an autoregressive model. (The latter two models were introduced as a means of facilitating comparisons between the main models under consideration).
2.7.2 More Recent Studies Making Systematic Comparisons

None of the studies discussed so far in this section have included the $Q$ model among the models compared. In this section we review those studies in which the $Q$ model features. In addition, all of the studies considered in this section (except that conducted by Jenkinson (1981)) compare the performance of these models when applied to investment in structures. Moreover, many focus on the relative forecast performance of these models. Bischoff (1971b) is one of the earliest such studies, providing the inspiration for several others. He estimates five models of investment behaviour with a view to using these to forecast non-residential fixed investment expenditure for the early 1970's. The five models are the generalised accelerator model, the cash flow model, the $Q$ model (which he refers to as the securities value model), the standard neoclassical model and a putty-clay model. These models are estimated with quarterly data for the period 1953-1968 for investment in equipment and non-residential structures separately. In all equations investment is modelled as a distributed lag function of its determinants using the Almon polynomial lag technique. The fits of the equations were good, but artificially enhanced by a correction for serially correlated errors.

Bischoff uses these equations to forecast eight periods beyond the estimation period and these forecasts are compared with actual data by means of the root mean square error criteria. Bischoff notes that the root mean squared errors are generally larger than the standard errors of regressions for both equipment and structures. For structures, the standard neoclassical and generalised accelerator models provide the best forecasts. The standard neoclassical model in particular, follows the movement in the actual series very well, although it does predict a peak in the series one quarter too early. The cash flow model provides the worst out of sample performance, despite it providing a good explanation of structures investment within sample. The $Q$ equation for structures tends to over predict actual investment and the estimated peaks are predicted one period late. The root mean squared errors and the mean errors from the securities value equations exceed those on nearly all other equations. Thus, the general conclusion of these tests is that the output based models, the accelerator, the standard neoclassical and the putty-clay models perform best over the given forecast period, though no single model is clearly superior to the others.
We briefly reviewed the work by Engle and Foley (1975) in Section 2.6.7. They estimate a distributed lag \( Q \) model augmented with a capacity utilisation variable. In the latter part of their paper they compare the empirical performance of this equation, for equipment and structures, with the performance of the five models estimated by Bischoff (1971b) discussed above, over the same sample period, and using the same data. The authors note that given the differences between the fits of the equations are quite small, the fit provides little basis for choosing a preferred model. As such, they examine the differences between equations in terms of the shape of the lag distributions and forecasting performance. They draw attention to the fact that the shape of the lag distributions for Bischoff’s models often exhibit peaks in the first quarter declining thereafter, with later quarters assuming negative coefficients. This implies some overshooting in investment. The lag distributions of the Engle and Foley \( Q \) equation are more typical, rising to a peak in the fourth quarter for equipment and fifth quarter for structures before falling to zero on the eleventh lag. Their \( Q \) type equations, for both equipment and structures, are said to be superior to all of Bischoff’s equations when forecast performance is assessed in terms of root mean squared error and mean bias. As such, their \( Q \) equations are regarded as providing much better explanations of investment behaviour than Bischoff’s, although it must be noted that their equations also contain a capacity utilisation variable which may be thought of as capturing an output effect. Bischoff himself noted that the equations which performed best in his analysis contained measures of output (see Bischoff (1971b) p. 35). Thus, the performance of Engle and Foley’s \( Q \) equations may well have been enhanced by the inclusion of this variable.

But for the inclusion of an accelerator term in the cash flow model, Clark (1979) has estimated the same models as those studied by Bischoff (1971b). These five models are estimated with investment expenditure data for equipment and structures separately for the period 1954-73. The equations are then used to forecast from 1973 to 1978 and their relative performance is assessed against the actual data. Clark notes that for equipment, the output-based equations fit the data best within sample whereas the securities value equation fits the data least well. The within sample fit of equations for structures are significantly worse, with the standard errors of regression on average twice those for equipment. For structures, visual inspection of the actual and forecast
data reveals that all models over predict investment. With the exception of the hybrid accelerator cash flow model, this over prediction is rather severe. Towards the end of the forecast horizon, predictions begin to get closer to the actual values as the structures investment recovers from the low of 1975. The actual forecast standard errors for structures are much larger than those estimated within sample. The $Q$ equation performs better than the neoclassical, putty-clay and accelerator models, but poorly compared with the hybrid accelerator-cash flow model. Clark concludes that output is clearly the primary determinant of non-residential fixed investment and that the relative price of capital services is not very helpful in explaining quarterly data on fixed business investment. He then goes on to re-estimate the accelerator model and accelerator-cash flow models over the entire sample period, to provide forecasts through to 1981 for both investment equipment and structures.

Bernanke, Bonn and Reiss (1988) cite the work of Clark (1979) as an example of research aiming to form a consensus model of investment demand by evaluating the performance of alternative models on the same set of data. However, they note that given the arbitrariness of within or out-of-sample goodness of fit criteria used to rank model performance, it is hardly surprising that investigators have been in disagreement on the merits of alternative models. With the exception of the cash flow model, which is omitted, Bernanke et al estimate the same models as Bischoff (1971b) and Clark (1979), again with US data, with a view to discriminating between them using the more rigorous techniques of non-nested testing. This was used in addition to criteria such as root mean squared error, Durbin-Watson and $R^2$ statistics and tests for structural stability that have frequently been used to discriminate between models in earlier studies. These conventional criteria indicate that no model uniformly outperforms the other models. Of the four models, the accelerator and the putty-clay models appear to provide the best explanation of net investment in equipment within sample. For net investment in structures, the $Q$ model appears to provide the best fit. Out of sample, the ranking of the models is less clear. When these models are compared on the basis of non-nested tests that take into account serial correlation in the residuals, all the models are rejected by at least one of the other models. However, Monte Carlo simulations carried out by Bernanke et al indicate that the non-nested
tests they employ are biased in favour of a rejection of the null, and as such they state that test results need to be viewed with caution.

Kopcke (1985) compares the accelerator, cash flow, neoclassical and $Q$ models, along with an autoregressive model of investment. The models are compared in terms of within sample fit and out-of-sample forecasting performance. Kopcke's neoclassical model allows for the fact that investment can respond to changes in output differently to changes in user cost and thus may be used to cast light on the putty-clay hypothesis. The models are estimated with 24 years of quarterly data from 1956 to 1979 for US investment expenditure in equipment and non-residential structures separately. These models were then used to forecast both series from 1980 to 1984 and these forecasts were compared with actual data. Both within and out-of-sample performance was evaluated using mean absolute error and root mean squared error statistics. All of Kopcke’s equations fit the data very well within sample and on the basis of these criteria no equation dominates. When the models are used to produce static forecasts, i.e. one-step-ahead forecasts, no single model significantly outperforms the others. Differences in the performance of the models are revealed most clearly in the dynamic forecasts. For equipment, the cash flow equation performs best, followed by the $Q$ equation. Both equations are significantly better than the neoclassical model which provides the next best dynamic forecasts. For structures, the $Q$ equation performs worst on the basis of the adopted criteria, which is consistent with the results of Bischoff (1971b). The forecast statistics indicate that the autoregression, accelerator and neoclassical model, in that order, provide the best forecast of investment in structures, but visual inspection of the actual and forecast data indicates that none of the models fit the cyclical course of the data well. When investment expenditure in equipment is added to that in structures to form a total investment series, the cash flow and neoclassical models perform best, although both under predict the series substantially.

In the studies investigating the comparative performance of investment models discussed so far, only Engle and Foley (1975) uncover support for the performance of the $Q$ model relative to other models. A similar absence of support for the $Q$ model emerges from a study by Poret and Torres (1989). They compare the performance of ‘standard’ aggregate investment equations for five OECD countries with that of $Q$
type equations. The two simple specifications used as ‘standard’ investment equations are the accelerator model and a profitability model (in which investment is a function of the growth rate of capital stock lagged one period, the rate of return on capital and the expected real rate of return on financial investments). Both models are estimated with biannual OECD data for each country over the period 1971-85. The authors note that it is difficult to discriminate between the models. Both models tend to under predict at turning points. They combine these models to form a hybrid profit-accelerator model and compare this with a $Q$ type model containing variables measuring current $Q$ and capacity utilisation (the latter justified on the grounds that it might capture any possible difference between marginal and average $q$). The equation also contained a lagged dependent variable. In nearly all cases the $Q$ variable is significant, but so are the lagged dependent variable and the capacity utilisation variable. Moreover, in non-nested tests the $Q$ type model was rejected by the hybrid model for three of the five countries, whilst it is unable to reject the hybrid model for any country when the null and alternative hypotheses were reversed. In none of the five countries does the $Q$ type investment equation outperform the ‘standard’ models in forecasting exercises. However, none of these models are rigorously derived from theory and many of the estimated equations contain serially correlated errors. As such, one ought to be cautious in attaching significance to these findings.

There is also an absence of strong evidence supporting the $Q$ concept in model comparisons with UK data. Jenkinson (1981) uses quarterly data to estimate a $Q$ model, a putty-clay type model and an accelerator model for aggregate investment undertaken by UK industrial and commercial companies over the period 1967-76. Although Jenkinson is not able to say that the $Q$ model out performs the other two models, he does state that the results of the $Q$ and neoclassical model, assessed in terms of goodness of fit criteria, are at least as good as those derived from the accelerator model.

In conclusion, we can say that although a number of authors attempt to form a consensus model of investment by evaluating the performance of alternative models on the same set of data, these attempts are not particularly successful. In some cases, authors are not able to discriminate between models, either within sample or out of sample, and in other cases in which the ranking of alternative models is determined,
the margin of determination is not convincing. Moreover, other studies using slightly different equations, estimation techniques or sample periods find alternative model rankings. In general, however, the evidence suggests that, on balance, accelerator models and putty-clay type models provide the best within sample explanation of investment behaviour. Further, the accuracy of forecasts generated from these models tends to be superior to those generated from other models. The performance of $Q$ models has generally been disappointing, despite the fact that theoretical rigour is often sacrificed in favour of econometric results.

2.8 Other Models of Investment

In this section we provide a short account of a number of other theories of investment behaviour. These theories fall conveniently under one of three headings; the Keynesian theory, profit models and theories emphasising the irreversibility of investment. We attempt to identify similarities between these theories and those considered earlier in this chapter. We begin with an outline of the Keynesian theory of investment.

2.8.1 An Outline of Keynesian Theory

A significant element in the theory of investment set out by Keynes (1936), is the emphasis on the potential importance of the behaviour of the capital goods producing industry. Subsequent theories, which we have outlined above, have focused almost exclusively on the demand side. Keynes's theory also stresses the role of expectations in determining the level of investment.

Keynes followed a conventional path in obtaining a demand function for capital which was a decreasing function of the rate of interest. A profit maximising firm will hire an input up to the point where the marginal product of that input is equal to its price. More precisely, a firm will employ capital up to the point where the present discounted value of its future marginal products is equal to its price. The present discounted value of the future marginal products can be defined as
where $\pi_{t, i}$ are the expected returns net of price in period $i$, $r$ is the rate of interest and $n$ is the lifetime of the asset. This can be thought of as the demand price for the asset. If the demand price exceeds the cost of buying one unit of capital, the firm should invest in the asset.

An alternative approach for assessing investment decisions of firms is the internal rate of return method. The internal rate of return, $\rho$, is that rate of return that would make the present value of future marginal products equal to the cost of buying the machine. That is,

$$PV_t = \sum_{i=1}^{n} \frac{\pi_{t, i}}{(1 + \rho)^i} = z_t$$  \hspace{1cm} (2.92)

where $z$ denotes the cost of buying unit of capital, which can be thought of as the supply price. Thus, the internal rate of return is that $\rho$ which equates $PV$ and $z$. Keynes referred to $\rho$ as the marginal efficiency of capital, $MEC$. According to this method of investment appraisal, the firm should invest if the $\rho$ exceeds the marginal cost of borrowing funds, which under the assumption of perfect capital markets will be equal to the rate of interest, $r$. Keynes's theory of investment can now be summarised by the equilibrium condition that investment be carried out up to the point at which the $MEC$ is equal to the rate of interest. In other words, the firm should continue to invest until the demand price equals the supply price.

There are a number of problems with this approach to investment. The first problem originates from criticisms of the internal rate of return criterion as a means of investment appraisal. Hirshleifer (1970) documents the relative merits of the internal rate of return and the present value method. We note only that the internal rate of return may not be unique, or indeed a real number, since the method requires solving a polynomial equation in $\rho$.

The second problem relates to the derivation of a macro demand for capital function from a micro demand for capital function. Recall, a negatively sloping demand for capital function for a profit maximising firm was obtained on the assumption of diminishing marginal productivity, i.e. that $\rho$ declines as $K$ increases. However, we
cannot use this argument to obtain a macro demand function. The approach typically taken in the literature is just to assume that the macro demand for capital function is also inversely related to the rate of interest.

Keynes’s theory of investment as laid out above relates the interest rate to the demand for capital and not the demand for investment. It is the derivation of an investment function from the demand for capital function which gives rise to the third problem. When a firm’s actual capital stock is less than its desired capital stock it will try to make up the difference immediately resulting in a rate of planned investment almost equal to infinity. The rate of actual investment will be limited by how quickly the firm can be supplied with the capital goods it requires. Note, a similar dichotomy exists between Jorgenson’s theoretical and empirical work. This was discussed in Section 2.3.4.

At a macro level desired investment would also be infinite but actual investment will be limited by the rate of production of the capital goods industry. It is in this element of the model that we see most clearly that Keynes had a two sector model of investment in mind. The supply curve in this model is usually assumed to be upward sloping. Since the MEC schedule measures the demand for capital stock and the supply schedule is a flow concept, we cannot simply put the two schedules together to define an aggregate investment function. An investment function is derived by defining a concept known as the marginal efficiency of investment, MEI. The marginal efficiency of investment is the rate of return to an increase in capital stock, allowing the costs of producing the capital goods to vary. The MEI schedule is inversely related to the rate of interest since as \( I \) increases the cost of buying the machine, \( z \), increases driving down the rate of return. A diagrammatic derivation of the MEI schedule is given in Junankar (1972). Note that the investment function is not a demand function but rather depends on both the supply of capital goods and demand factors captured by the MEC schedule. The problem is that, although it is possible to derive an aggregate investment function, it is not possible to derive an investment function for the firm. Witte (1963), amongst others, has attempted to remedy this but the resulting work has tended to contain a number of theoretical inconsistencies.

As it stands, it would be inaccurate to suggest that the theory outlined above encompassed all of Keynes’s views on investment since he stressed the role of
expectations. It is in the demand price term that expectations enter this theory. Since the rate of return is calculated on the basis of expected future returns, changes in expectations would change the demand price of capital assets and hence the equilibrium rate of investment at any given rate of interest. In deriving the negative relationship between investment and the rate of interest, it is assumed that expectations are held constant. In addition, as noted in our discussion of Tobin’s $q$, Keynes was acutely aware of the relationship between the stock market value of the firm and investment (see Section 2.6.1). Furthermore, the fact that Keynes speaks of equilibrium being reached when investment has been undertaken up to the point where the demand price equals the supply price of capital goods, that is, to the point where the ratio of the former to the latter is equal to unity, has led many to regard Keynes as the originator of $q$ theory. It is in this sense that we must make a distinction between Keynes’s theory of investment and what has become known as the Keynesian theory of investment, outlined above.

2.8.2 Profit and Liquidity Theories of Investment

As we noted above, models developed subsequent to Keynesian theory have tended to focus on demand factors as determinants of investment. The accelerator model, although chronologically preceding the Keynesian model, also focuses exclusively on the demand side. The class of models that we consider in this section is very closely related to the accelerator. We first consider the profit theory of investment.

Recall in the basic accelerator model the optimal capital stock is some function of output and investment is some function of the change in output. The profits theory of investment postulates that the optimal capital stock is some function of the level of profits. By assuming the entrepreneurs derive utility from the size of their firm, Klein (1948) obtains an investment function, which depends of the level of profits. However, as noted by Eisner and Strotz (1963), this version of the profits theory is inconsistent with the profit maximisation, if we assume decreasing returns to scale, since the larger the firm’s profits the more the firm expands and accepts lower profits. In an alternative version of the model, optimal capital stock is some function of expected profits. Typically, expected profits are in turn assumed to be some function
of current and past profits. To the extent that profits are some function of output or sales, this version of the profits theory will be empirically indistinguishable from the accelerator model.

Another version of the profits theory is the liquidity version, sometimes referred to as the cash-flow model of investment. In this version, it is assumed that capital markets are imperfect so that the cost of externally borrowed funds is greater than funds generated internally. The importance of such imperfections has been argued by Meyer and Kuh (1957) and Duesenberry (1958). Given these imperfections, the higher a firm’s internal cash flow, the lower the cost of capital and hence the larger the firm’s optimal capital stock. Thus, this model posits that optimal capital stock is some function of internal cash flow, where internal cash-flow is largely determined by the level of profits. Therefore, the firm’s optimal capital stock is some function of profits and if profits depend on output, the cash-flow model will also be empirically indistinguishable from the accelerator model of investment.

Since the earliest econometric studies of Tinbergen (1939), Klein (1951) and Meyer and Kuh (1957), liquidity variables have also been used to augment other models of investment. Liquidity has frequently appeared in versions of the neoclassical model, although the empirical performance of such models has tended to be weaker than the performance of accelerator-type models containing liquidity variables (see Sinai and Eckstein (1983) for an exception). In reviewing the results of a number of models, Jorgenson (1971, p. 1133) concludes that variables associated with internal finance do not appear as significant determinants of the level of capital stock in any model in which output also appears as a significant determinant. Contrary to this conclusion, many subsequent studies find liquidity to be a very significant regressor. However, the theoretical basis for including such variables is largely non-existent. Moreover, the development of such theory has been discouraged in the light of the Modigliani and Miller theorem. Modigliani and Miller (1958) prove that if there is no possibility of default on loans and no transactions costs or taxes, the firm will be indifferent as to its method of financing investment and it can make its investment decisions independently of its financing decisions.
2.8.3 Irreversibility and the Investment Process

A fundamental issue implicit in all of this discussion on the various theoretical models of investment behaviour is the translation of the demand for capital stock into a demand for an investment flow. The significance of current and lagged quantity variables in the empirical literature suggests that these dynamics are very important to understanding investment behaviour. One way in which the translation problem has been solved in the literature is by relying on internal adjustment costs. Models of this kind were considered in Section 2.5. A recent literature has explored an alternative means of overcoming the translation problem. As with the cost of adjustment models, this also incorporates dynamics into the optimisation problem.

This literature examines the investment dynamics that arise from an irreversible investment and the ongoing resolution of uncertainty that together, give value to postponing investment decisions. Postponing an investment will be desirable if the improved information that becomes available as a result is more valuable than the return forgone by not investing immediately. Thus, there is an opportunity cost of investing today, as opposed to delaying investment to learn more about prospective returns, which drives a wedge between the benefits and costs characterising the optimal investment policy. According to this view, the optimal investment rules derived for the adjustment cost models, and given in equations 2.41 and 2.68 above, will not hold since they should contain an additional term representing the value of waiting.

Analogies have been drawn between an irreversible investment opportunity and a financial call, or stock, option. A stock option gives the holder, for some specified time period, the right to pay a given price for a share in stock that has some value. Exercising the option is irreversible and, although the stock can be sold to another investor, one can not retrieve the option or the money paid to exercise it. Similarly, a firm with an investment opportunity has the option to spend money now, or in the future, in return for a capital good of some value. Again, the asset can be sold to another firm, but the option to invest and the money invested is not retrievable, i.e. it is irreversible. As with the stock option, this option to invest is valuable, in part because the future value of the capital good is uncertain. If the value of the capital good increases the net payoff from investing increases. If the value falls, the firm need
not invest at all losing only what it spends to obtain the opportunity to invest, i.e. what it spends to obtain the option. When a firm makes an irreversible investment it forfeits its option to invest. It gives up the possibility of waiting for new information that might affect the desirability of investment or its timing. The lost option value is an opportunity cost that must be included as part of the cost of investment.

The literature has been recently surveyed by Pindyck (1991) and Dixit (1992). In his survey paper Pindyck notes several reasons why investment in capital goods is often sunk and therefore irreversible. The first is the fact that capital is often firm or industry specific, that is, it can not be used productively by a different firm or by a different industry. Although a steel plant could, in principle, be sold to another company, the value of the plant will be approximately the same for other firms in the industry so there is likely to be little gain in selling it. If the price of steel falls so that the plant, ex post, turns out to have been a bad investment, it is likely to be viewed as a bad investment by other firms in the industry. Thus, the ability to sell the plant will not be worth much. Another reason why investment in capital should be viewed as sunk cost and therefore irreversible is related to the ‘lemons’ problem (see Akerlof (1970)), which suggests that the equilibrium price of used goods will be less than that of new goods due to asymmetry of information on the quality of the used good between the buyer and seller. The resale value of office equipment, cars, computers etc., which are not firm specific, will be well below their purchase cost, even if new. Irreversibility can also arise as a result of government regulations or institutional arrangements. Capital controls, for example, may well prevent foreign or domestic firms from selling their capital stock and reinvesting abroad.

Firms do not always have possibilities of delay. In some cases, it will be imperative that the firm invests quickly to pre-empt investment by (potential) competitors. It is argued however, that delay is often feasible albeit at the cost of forgone cash flows. The costs must be weighted against the potential benefits of waiting for new information. The irreversibility literature suggests that this opportunity cost of waiting may be large, and that optimal investment rules that ignore the opportunity cost may be in substantial error. However, as Eberly (1992) points out, the empirical evidence supporting the analytical results in the literature is conspicuously scarce.
Nevertheless, the theory provides a useful alternative to cost of adjustment models as a means of deepening knowledge of the dynamics underlying investment.

2.9 An Overview of the Investment Literature

In this section we make the relationships between the alternative theories of investment explicit and summarise empirical findings to date. We also outline some important issues which, although peripheral to this study, have been much written about.

2.9.1 A Theoretical Overview

The economics literature on the determinants of investment is large and can be confusing. It contains many competing theories, some of which emphasise real factors and some financial factors. In fact, as shown by Sampson and Skinner (1996), theories are not mutually exclusive, each one simply ignores or emphasises its own particular set of factors. Above we have outlined the four dominant theories of investment using a framework in which the differences and similarities between them can be easily identified. We take this opportunity to make the connection between theories explicit.

Recall, Jorgenson’s neoclassical model is based on the maximisation of net worth over an infinite time horizon, subject to a standard neoclassical production function and the constraint that the rate of growth of capital stock is equal to net investment. Jorgenson assumes perfect competition in all markets (including the markets for new and used capital) and an absence of adjustment costs. The resulting first order conditions for maximisation are given by equations 2.19 to 2.22. Given that \( m = \lambda e^n \), equation 2.20 can be written as \( z = m \). This defines the firm’s optimal investment rule and it states that the marginal cost of investing in one unit of new capital is equal to the associated marginal benefit. The optimality conditions for labour and capital are given in equations 2.19 and 2.21 respectively.

We noted in Section 2.3.4 that, due to the lack of adjustment costs or delivery lags in Jorgenson’s theoretical model, the firm is always optimally adjusted. This implies an infinite rate of investment which presents a problem when it comes to deriving an
empirical equation consistent with the theoretical model. Jorgenson bypassed this problem using the \textit{ad hoc} assumption on delivery lags in his empirical work. This resulted in equation 2.32 in which investment was related to optimal capital stock through a distributed lag relation.

In the discussion of equation 2.32 in Section 2.3.3, it was noted that if relative prices are assumed to be constant, Jorgenson's version of the neoclassical model reduces to the flexible accelerator given by equation 2.9. If delivery lags are also absent, the neoclassical model reduces to a simple accelerator of the form of equation 2.2. It is perhaps surprising that this single assumption differentiates the empirical neoclassical equation from the accelerator model of investment, given the vastly different theories that underly these models. It should be noted, however, that Grossman (1972) has derived the accelerator model from an optimisation model.

By assuming that factor proportions are variable only up to the point of installation, and that the firm's objective is to choose a capital intensity so as to maximise net worth, Bischoff (1968 and 1971a) derives an investment equation in which the response of investment to changes in user cost is not constrained to be equal to the response of investment to changes in output. This can be thought of as a modification to the basic neoclassical model in which the importance of relative prices can be tested independently of the role of output.

The introduction of costs of adjustment into the neoclassical optimisation overcomes the problem of instantaneous adjustments in capital stock and infinite investment. Two alternative models were presented in Sections 2.5.2 and 2.5.4. In both models, the firm's objective is to maximise the present discounted value of future profits subject to production function and capital accumulation constraints. In the first of these models, adjustment costs are valued in terms of the price of output lost due to adjusting capital stock. The first order conditions for this model are given in equations 2.40 to 2.43. In the second, adjustment costs are assumed to force a wedge between the quantity of capital purchased and that installed. The first order conditions for this model are given in equations 2.61 to 2.64. The connection between these models and Jorgenson's neoclassical model can be seen most easily by comparing the respective first order conditions.
Let us first compare the first order conditions of the first cost of adjustment model (in equations 2.40 to 2.43) with those of the neoclassical model (equations 2.19 to 2.22). The only difference in first order conditions of these two models is the presence of the terms \( C(I,K) \) and \( C_x(I,K) \) in the cost of adjustment model. The conditions for optimal employment of labour, given by equation 2.40 and 2.19, are identical. That is, the firm’s optimal employment of labour is unaffected by adjustment costs. The optimal investment rule, given by equation 2.41, like 2.20, implies the equality of the marginal benefit and marginal cost of investment. However, the marginal cost of investment now includes an additional term, namely the marginal adjustment cost, \( C_x(I,K) \). If there are no adjustment costs, such that \( C(I,K)=0 \) and therefore \( C_x(I,K)=0 \), then equation 2.41 reduces to 2.20. Moreover, if \( C(I,K)=0 \) then \( C_x(I,K)=0 \), and the first order condition given by 2.42 reduces to 2.21, we can derive the Jorgenson’s term for the user cost of capital. Thus, it should be clear that this formulation of the adjustment cost model is equivalent to Jorgenson’s model modified to include adjustment.

Similar logic can be used to show the connection between the second adjustment cost model and the neoclassical model. The first order conditions for the former (given in equations 2.61 to 2.63) reduce to those of the latter (given in equations 2.19 to 2.22) if we assume no adjustment costs, such that \( \psi(I,K) = I \), and therefore \( \psi_I(I,K) = 1 \) and \( \psi_K(I,K) = 0 \). In so doing, Jorgenson’s expression for the user cost of capital can be derived from equation 2.63.

Given this rather close relationship between the first order conditions of the cost of adjustment and neoclassical models, it may at first sight appear odd that the final investment equation for the cost of adjustment models differs so fundamentally from Jorgenson’s final specification. However, the explanation for these differences is straightforward. Jorgenson’s empirical investment equation (given by equation 2.32) is derived under the assumptions of Cobb-Douglas technology and delivery lags. The former assumption is somewhat restrictive and the latter is \( ad hoc \) and inconsistent with his theory. Neither of these two assumptions are required for the derivation of the cost of adjustment investment function given by equation 2.55. However, the explanatory variable in equation 2.55 is unobservable and difficult to relate to measured variables.
The relationship between cost of adjustment models and accelerator models has been analysed in detail by Nickel (1978, pp. 30-31) and Junankar (1972, appendix to chapter 4). Both Nickel and Junankar show that the neoclassical and accelerator models can be derived from an adjustment cost model if technology is appropriately specified.

The critical problem in empirical work with cost of adjustment models, is that of relating the unobservable regressor to observable variables. Tobin’s \( q \) theory uses information in financial markets to make this relationship. Although the theory was developed independently from the neoclassical and cost of adjustment models, Abel (1979, 1980) and Hayashi (1982) demonstrate that the neoclassical theory, modified by the assumption of adjustment costs, and \( q \) theory are, in fact, equivalent. We illustrate the argument in Section 2.6.4. In simple terms, marginal \( q \) is defined as the ratio of the market value of one extra unit of capital, or its demand price, to its replacement cost, or supply price. In the notation of this chapter, the numerator is denoted by \( m \) and the denominator is denoted by \( z \). Thus, marginal \( q = m/z \) which is essentially the regressor in the cost of adjustment model given by equation 2.55. As noted in Section 2.5.3 above, \( m/z \) is not measurable. However, Hayashi (1982) derives conditions under which average \( q \), which is measurable and makes use of information in financial markets, can be used as a proxy for marginal \( q \). These conditions are discussed in Section 2.6.2. Thus, if these conditions hold, average \( q \) can be used to operationalise the cost of adjustment investment function given by equation 2.55.

In Section 2.8, we examined Keynesian theory, profit and liquidity models and the theory of irreversibility. These theories can also be tied in loosely with other theories of investment. In Section 2.8.1 we discussed, elements of the Keynesian theory of investment. Although this theory is, to a large extent, distinct from other theories of investment, there is some connection with the \( q \) theory of investment. It was noted that Keynes recommended that investment should be carried out up to the point at which the demand price for capital equals its supply price. The ratio of the former to the latter defines marginal \( q \). This, and Keynes’ awareness of the effects of the stock market on investment, has led many commentators to suggest that Keynes was the originator of \( q \) theory. In Section 2.8.2, it was noted that profits and liquidity theories of investment were typically very closely related to the accelerator model of
investment. In fact, to the extent that liquidity or cash-flow are determined by profits and profits are a function of output or sales, these models are indistinguishable from the accelerator model of investment. The theory of irreversibility, discussed in Section 2.8.3, like the Keynesian theory is largely distinct from the mainstream investment literature. However, it has much in common with cost of adjustment models. For example, both seek to shed light on the dynamics of investment behaviour and how these dynamics can be incorporated into the firm’s optimisation problem. In addition, as in the cost of adjustment models, there is a wedge between the benefits and costs characterising the optimal investment rule. However, in the cost of adjustment models this wedge is created by the costs of adjustment, whereas in the irreversibility literature, the wedge is due to the value that the firm assigns to waiting for improved information on the prospective returns. Proponents of the irreversibility theory of investment argue that cost of adjustment models are deficient to the extent that they take no account of the opportunity cost of immediate investment.

In summary, the models considered in this section are not mutually exclusive. Rather, they emphasise or ignore a particular set of factors. Tobin’s q theory, which some argue has its origins in Keynes, can be thought of as one particular type of cost of adjustment model, which emphasises the effect of financial markets on investment. If the neoclassical model, which emphasises the role of relative prices but ignores adjustment costs, is actually augmented with adjustment costs, then it is equivalent to q theory. Jorgenson’s neoclassical model reduces to the flexible accelerator if the role of relative prices is ignored, and to the simple accelerator if delivery lags are also absented. When Jorgenson’s model is modified to allow a different response of investment to relative prices than to output, the putty-clay model results. Finally, to the extent that cash-flow is determined by profit, and profit by output, the liquidity and profit models are indistinguishable from the accelerator.

2.9.2 An Overview of Empirical Results

Given that only a few studies focus exclusively on structures investment, a review of such studies would be unrepresentative of the important findings in the literature as a whole and would not provide a secure platform for estimating models in this thesis.
As such, the review has been broadened to include studies of investment in equipment and studies of aggregate nonresidential fixed investment.

In general, the results of studies attempting to model investment expenditure have been disappointing. Whilst proponents of each theory can point to a number of studies which purport to uncover evidence in favour of their model, none of the models considered here have performed consistently and systematically well in tests against actual data or against alternative models.

Despite the theoretical simplicity of the flexible accelerator hypothesis, it has stood up reasonably well to empirical testing against both actual data and other models of investment behaviour when implemented correctly. In this model, investment is a function of current and lagged changes in output. According to this theory, changes in relative prices do not affect investment. Moreover, the adjustment process is rather *ad hoc* and usually modelled as a distributed lag relation. As Griliches (1967) states ‘while widely and variously used, most distributed lag models have almost no or only a very weak theoretical underpinning. Usually the form of the lag is assumed *a priori* rather than being derived as an implication of a particular behavioural hypothesis’ (p. 42). As a result of these deficiencies, many researchers disregard the accelerator as a credible theory of investment behaviour. The role of liquidity has also been investigated within this framework also with some reasonably encouraging results. That liquidity mattered is not much disputed in this part of the literature. The critical question was whether liquidity had an effect separate from that of output. In any event, prior to 1960, the empirical work points to the dominance of quantity variables, such as output and liquidity, over price variables, such as interest rates. Since there was no role for relative prices, there existed no channel through which tax policy could affect investment.

These issues are addressed by the neoclassical revolution in the 1960’s but with mixed success. In addition to the theoretical deficiencies of the neoclassical model discussed in Section 2.3.4, there are a number of criticisms of an empirical nature, such as the *ad hoc* adjustment process and those relating to the assumption of Cobb-Douglas technology, *ex ante* and *ex post*, which have undermined the efficacy of the pure neoclassical model. These problems make it difficult for researchers to determine the precise nature of the role of relative prices. Whilst the work of Jorgenson and his
collaborators points to a significant role for relative prices, a number of authors, using Jorgenson's own data, find evidence to the contrary when one or more of Jorgenson's strong assumptions are relaxed. The assumption of Cobb-Douglas technology results in investment being a function of a composite term for changes in output and user cost. This implies an identical response of investment to changes in output and changes in relative prices. If the effect on investment of changes in output is stronger than the effect of changes in user cost, estimated coefficients on the composite term will be larger than the coefficient on the user cost term in equations in which changes in output and changes in user cost are entered separately. As such, versions of the neoclassical model that contain a composite term will overstate the effects of changes in monetary and fiscal policy which operate through the user cost of capital.

By specifying a production function that allow vintage effects to influence the relationship between past investments and the capital stock, changes in output and changes in user cost can enter the investment equation separately. As a result, if capital is putty-clay in nature, the response of investment to changes in user cost is no longer constrained to be identical to the response to changes in output. The empirical work using the putty-clay model is, in general, more encouraging than the results of work adopting the pure version of the neoclassical model and findings are consistent with \textit{a priori} expectations. Relative prices are important, the response of investment to changes in user cost is slower than the response to changes in output and, in general, the effect of changes in policy have been smaller than in the Jorgenson model. The effect of changes in relative prices, although significant, is not large relative to the effect of changes in output. The putty-clay model has also performed relatively well in tests against other models of investment behaviour. However, the criticisms levelled at the accelerator and the Jorgenson model, regarding the \textit{ad hoc} nature of the adjustment process, apply equally to this explanation of investment behaviour.

One of the most appealing aspects of cost of adjustment models is that they are not subject to this criticism: the adjustment process is built into the firm's optimisation problem. Another attractive feature of these models is that they are not subject to the Lucas Critique. Lucas (1976) argued that in formulating plans, economic agents necessarily look into the future and thus the decision rules guiding their actions
depend on parameters describing expectations of future variables, as well as parameters of technology. Lucas views economic policy as the selection of rules that generate paths of policy variables, rather than the selection of arbitrary paths. Thus, as Lucas (1976) notes, ‘any change in policy will systematically alter the structure of economic models’ (p. 126) and the estimated coefficients of these models can no longer be regarded as structural, that is, they are no longer invariant to policy regimes. The important implication for policy analysis is that these econometric equations will prove unstable in precisely those situations in which they are called upon to analyse the effects of proposed policies. Thus, quantitative policy analysis can only proceed if the econometric specification of models permits the expectations parameters, which will vary with alternative policies, to be identified separately from technology parameters which are invariant to policy changes. Cost of adjustment models focus on the modelling and isolation of dynamics arising from expectations.

Despite their theoretical appeal, we have seen in this review that the usefulness of these models is called into question by their generally disappointing empirical performance. Although there are two caveats pertaining to the $Q$ model, the first relating to the possible measurement of the components of $Q$ and the second concerning the conditions under which average $q$ can be used as a proxy for marginal $q$, empirical evidence suggests that neither of these are responsible for its poor performance. Whilst the $Q$ variable is usually significant, equations generally fit the data poorly with low $R^2$s. The presence of serially correlated errors and the significance of lagged dependant and $Q$ variables indicate that the dynamics appear to be inadequate. Further, according to theory no other variables should have a systematic relationship with investment, but quantity variables, such as output and liquidity, are frequently statistically significant. Finally, $Q$ equations generate estimates of costs of adjustment parameters that are unfeasibly large.

Although a number of the theoretical deficiencies inherent in the early theories of investment have been addressed in more recent models, the empirical performance of investment models has not improved. While a number of theories have been shown to provide an adequate explanation of investment when applied to a particular set of data, over a particular sample period, no single model consistently provides an adequate explanation of investment behaviour. Moreover, in studies that attempt to
rank the performance of alternative models, no single model consistently emerges as the winner. As Feldstein (1982) notes 'no matter how precisely coefficients of a particular specification appear to be estimated . . . . estimating alternative models to study the same question can be a useful reminder of the limits of our knowledge' (p. 831). This suggests that much work remains for investment researchers. The underlying sentiment is captured by Eisner (1974) who writes that the 'estimation of investment functions is a tricky and difficult business and the best posture for any of us in that game is one of humility' (p. 101).

Although there is clearly no uniformity in the results of these studies, it is possible to draw one major conclusion from the evidence presented above. This is that the response of investment to changes in price variables tends to be small and unimportant, relative to the effects of quantity variables. This suggests that the direct response of investment to changes in fiscal parameters is likely to be modest, as is the direct response to changes in the tools of monetary policy, such as interest rates. The recurring significance of financial factors in the literature would suggest that liquidity effects are important, although the underlying theoretical justification for including these variables in models of investment behaviour is not well developed. Moreover, the precise channels through which these variables operate is not clear. This suggests that future research should expand the view of the firm and the margins along which it operates.

2.9.3 Some Peripheral Issues

There are three issues on which much has been written but which we have not received much attention in this review. First is the issue of the treatment of replacement investment. In the accelerator and neoclassical models of investment it was assumed that capital depreciates at a constant geometric rate. In making this assumption we are able to treat replacement investment as a constant proportion of capital stock. In his original version of the neoclassical model, Jorgenson assumed geometric depreciation of capital and has since strongly defended it. The validity of the geometric depreciation assumption has been the subject of numerous empirical investigations (see Feldstein and Foot (1971), Eisner (1971), Jorgenson (1971),
Hulten and Wykoff (1981a), Coen (1975), Pakes and Griliches (1975), Felstein and Rothschild (1974) and Nickel (1978)). Among the alternative decay specifications are the 'one hoss shay' and straight line decays. Summaries of this debate can be found in Jorgenson (1989) and Hulten (1991). It is worth emphasising, however, that given our interest in new construction output, replacement investment is a peripheral issue in this thesis. Replacement investment corresponds to that part of construction output due to repair and maintenance. The discussion of the treatment of replacement investment is not entirely without relevance to this thesis since, as we will see in Chapter 4, the derivation of measures of capital stock depends crucially on assumed pattern of depreciation.

The second issue, receiving only scant attention in this review, relates to the time structure of the investment process. Final investment expenditure takes place some time after the fundamental determinants of investment change. In large construction projects, the total lag between a change in the environment that prompts consideration of an investment possibility and the final investment expenditure, can be several years. After a change in variables affecting investment, time is spent considering whether the investment ought to be undertaken. The amount and timing of investment can be affected by the time taken to gain planning permission and putting out to tender. There will often be a lag between the eventual placing of an order and the start of the construction process as workers are hired, machinery is moved on site, or orders for materials are placed. There then follows a construction lag, that for large projects can last a number of years, during which time there is a flow of investment expenditures. All this takes place before the possible benefits of the investment can be realised.

Despite being inconsistent with theory, the lag between a change in the fundamentals and the consequent change in investment expenditure in the accelerator and neoclassical models, is often captured by a distributed lag relation. However, theory provides no guidance on the factors governing the shape and length of the distributed lag specification. In practice, therefore, shape and length tend to be chosen on an *ad hoc* basis, either by estimation or by imposing a particular form *a priori*. A literature has emerged on the time structure of the investment process, notable contributions to
which have been made by Mayer (1960), Jorgenson and Stephenson (1967b) and Jorgenson (1971).

Models developed subsequent to the neoclassical model have tended to derive the optimal adjustment path for capital stock explicitly from the firm's optimisation problem. Therefore, the use of distributed lag specifications to capture the time structure of investment is not an issue in these models. Since most researchers in the investment field agree that future research ought to be undertaken with models in which dynamics are derived explicitly from the firm's optimisation problem, the treatment of the time structure of investment and the estimation of distributed lag formulations are unlikely to be at the forefront of forthcoming discussion in the investment literature.

Another issue, to which we have paid little attention in this review, is that of the appropriate level of aggregation. To a large extent, this reflects the fact that the issue has received only scant attention in the literature: most studies completely ignore the issue of aggregation. The inconsistency between the behaviour of the individual firm, on which much of the theory of investment is based, and aggregate behaviour has been highlighted by Geweke (1985). There is, of course, no reason why the individual firm should provide the basic framework for analysis. Since much of the interest in investment emanates from macroeconomists attempting to gain an improved understanding of the relationship between policy, investment and the economy, the 'representative firm' may provide a more appropriate unit of analysis. Moreover, there may be additional advantages to estimating aggregate investment equations: microeconomic data tends to be more poorly measured and microeconomic equations tend to be more poorly specified than their macroeconomic counterparts.

In short, there is very little definitive evidence that supports empirical investigation at the firm level as opposed to an aggregate level, or vice versa. As such, the appropriate level of aggregation remains an open question. Comparing parameter estimates at different levels of aggregation can itself be very informative. Given our interest in industrial and commercial construction output, the investment relations estimated in this thesis are estimated on an aggregate basis.
2.10 Models of Construction Activity: Studies from Another Literature

In this section we examine a growing body of research published in the building and surveying literature in which econometric techniques are used to derive equations for various measures of construction activity. Many of these studies are motivated by a desire to improve understanding of the determinants of the construction cycle and seek to explain its volatility. Others seek to gain improved information about future conditions in the construction market. Given our interest in modelling and forecasting new private industrial and commercial building activity, it would seem prudent to review some of this literature.

It is noteworthy that whilst the models of construction activity estimated in the building and surveying literature are often intuitively sensible, they tend not to be consistent with economic theory. The variables posited as determinants of construction activity are frequently chosen on an *ad hoc* basis. Whilst the resulting equations are often representative of construction market characteristics, they lack theoretical rigour and to some extent this undermines the credibility of empirical results. The credibility of results is further undermined by that fact that, typically, little or no attention is paid to econometric issues. As such, in reviewing these studies we do not consider their findings in any great detail, but merely outline the underlying methodology and the inherent weaknesses.

In a series of papers, Barras and Ferguson (1985, 1987a, 1987b) investigate the incidence and causes of building cycles in Britain. Unlike much of the work on building cycles some attention is paid to the underlying theory. In Barras and Ferguson (1985) spectral techniques and turning point analysis are used to identify the main post war cycles. They find evidence of demand cycles with a period of 4 to 5 years and supply cycles with a period up to 9 years. When sector data is aggregated the shorter demand cycles are less easily identified. Drawing on work developed in Barras (1983) and Barras and Ferguson (1985), Barras and Ferguson (1987a) present a theoretical framework suitable for dynamic modelling of these cycles in which some of its distinctive characteristics are incorporated. The framework combines an accelerator type model with economic variables, such as GDP and interest rates. The former is intended to capture the long production lags in building activity, whereas the influence of the economic factors is assumed to be exogenous. They restrict attention
to private sector building since public sector activity is characterised by periodic shifts in government policy which can not easily be analysed within the same type of modelling framework as that suitable for the private sector. In particular, they focus on the private industrial, commercial and residential sectors.

Whether activity in the residential sector can be adequately represented within the same theoretical framework as that of the industrial and commercial sectors is open to considerable doubt. Whilst, to some extent at least, demand for residential space may be viewed as investment demand, it is unlikely that residential demand fits into the neat framework presented by Barras and Ferguson (1987a). Certainly, in the economics literature, none of the theories of (fixed business) investment, including the accelerator model detailed above, have been applied to residential structures. Variables such as personal disposable income, mortgage rates and house prices seem to be more important factors affecting the decision to invest in residential construction. Swan (1970) provides a full documentation of factors affecting home building in the US.

Barras and Ferguson (1987a) develop a theoretical model in which the private sector property market is compartmentalised into an investment market and a user market. They note that equilibrium in these two markets does not imply that the quantity of development starts will equal the quantity completed. Two reasons for this short-run disequilibrium are posited. Firstly, inherent delays in the construction process introduce lags between starts and completions. Secondly, the quantity demanded by users at the market price or rent will not typically equal the quantity demanded by investors at the same prices or rents. Barras and Ferguson note that this disequilibrium can only exist in the short-run since investors will tend to adjust their plans so that all new floor space developed will be let. Thus, the long-run equilibrium level of floor space is assumed to be a function of user activity. Starting from a flexible accelerator type model Barras and Ferguson develop an error correction type model which combines short-run dynamics and long-run equilibrium relationships between variables. This econometric relationship is combined with univariate time series methods using a transfer-function model.

The results of this analysis are presented in Barras and Ferguson (1987b). For each of the private industrial, commercial and housing sectors, the authors estimate transfer
functions for new orders and output. User activity is determined by variables such as GDP, industrial production, consumers expenditure and personal disposable income. The models also include property market variables such as rents, capital values and yields, a number of building cost variables, such as a construction price index and a land price index and a number of measures of investor finance. Given that long runs of property market data do not exist, the models are estimated over two sample periods: models estimated over a long sample period exclude property market variables. Their results are summarised as follows. Activity in the user market plays an important role in explaining construction in the industrial and commercial sectors. For the industrial sector, the level of manufacturing output provides the best proxy for user activity, whereas, because of the heterogeneous nature of commercial building, GDP offers the best proxy. Only for housing does the investment market play an important role in determining the level of building activity. Consistent with the results in Barras and Ferguson (1985), the proposition that a 9 year supply side cycle is inherent in the data is supported. This is attributed to the lag between construction starts and completions. Evidence for the shorter demand side cycle, reflecting fluctuations in the business cycle, is less strong but nevertheless present. Activity in the investment market also influences the level of building activity although the effect is not as large as activity in the user market. Development costs, such as interest rates and construction costs, seem to exert the weakest influence on industrial and commercial building cycle. For residential investment, however, this effect is larger. Barras and Ferguson are unable to draw firm conclusions as to the effects of property market variables. This is essentially a result of the rather short data set.

Wheaton and Torto (1990) provide another example of research in which the accelerator model is adopted. Survey data on industrial building is compared with data on industrial employment and output to ascertain the character of industrial space demand and the role of replacement investment. This part of their analysis suggests that the production of industrial space should be measured as an investment decision. The model is described as a variation on the accelerator model. However, rather than desired capital stock being a function of output, Wheaton and Torto define desired capital stock as a function of expected future output in the manufacturing and wholesale sectors and the effective cost of capital. Current and lagged output (itself
proxied by employment) are used to predict future output. The effective cost of capital is measured as the real interest rate divided by one minus the effective tax rate. A number of equations containing different combinations of variables and lag structures are estimated and appear to be statistically well behaved.

Wheaton and Torto state that industrial completions are adequately explained by changes in output (proxied by employment) and the after-tax cost of capital. They state that this is consistent with the general findings in the economics investment literature. Further, they note that the lag between starts and completions appears to be longer than those typically found in the investment literature and this leads to short-run imbalances between supply and demand. Changes in wholesale output or employment generate a relatively rapid increase in desired capital, while movements in manufacturing output or employment and the effective cost of capital influence desired capital with a two year lag. The model is used to derive a measure of this imbalance between supply and demand and this is found to be consistent with vacancy rate data.

An investigation into cycles in the US office market is presented in Wheaton (1987). He develops a structural system of equations in order to draw some inferences as to the possible causes of its cyclical behaviour. In addition, the model is used to generate forecasts under different assumptions about the macro economy. Wheaton emphasises the importance of long-term contracts, suggesting that these prevent rents from adjusting to clear the market instantaneously. He incorporates this effect into the model by adding a difference equation in rents to the supply and demand equations. Desired demand is assumed to be a function of office employment, the real rental rate for new space, employment growth and the level of occupied space. The construction, or supply equation, on the other hand is assumed to be a function of the real rental rate, the vacancy rate, the stock of space, employment growth and the cost of construction.

The three equation system was estimated using OLS on biannual data from 1967 to 1986. The final specification is arrived at after much trial and error, especially with regard to the lag structure. Wheaton suggests that the results are encouraging given the model's simple structure. The coefficients on both the demand and supply equations are significant and of the correct sign. The employment growth variable
seems to command a much larger influence on the demand equation than expected but further testing seems to support its importance. Wheaton addresses the problems of serial correlation, endogeneity and non-normality and concludes these to be inconsequential. Wheaton uses this model to generate forecasts for three alternative macroeconomic scenarios. The dominant result from a base forecast and under two alternative scenarios, growth and recession, is that the high vacancy rates in the US office market at the time of writing were expected to persist for longer than in previous periods of high vacancy rates.

A number of other studies in the surveying literature also develop models of construction activity with a view to forecasting. These studies generally aim to generate forecasts for construction workload. Examples of this type of study are provided by Tang, Karasudhi and Tachopiyagoon (1990), Akintoye and Skitmore (1994) and Oshobajo and Fellows (1991). Tang et al estimate demand functions for three types of construction, namely residential, non-residential and 'other' construction in Thailand. We comment only on results pertaining to non-residential construction. Demand for non-residential construction is hypothesised to be a function of expected profits (proxied by the ratio of corporate savings to the construction cost index), industrial production, value of exports, number of tourist arrivals and gross domestic product. The justification for the latter two variables is not obvious and none is provided. Aside from the fact that the choice of these variables is rather ad hoc, there are a number of other reasons why the results of this study should be treated with caution. For example, estimating any relationship with what appears to be only 10 years of annual data, and in so doing applying tests at the 20% level of significance, is unlikely to yield the ‘true’ relationship. The forecasts generated from this study must be placed within very wide confidence limits.

In the study by Akintoye and Skitmore (1994) construction new orders are modelled and the resulting equations are used to forecast construction demand. The authors estimate equations for private sector housing, industrial and commercial new orders by OLS. The equations are estimated using quarterly data for the UK from 1974 to 1988. The criticisms made of Tang et al, regarding the ad hoc choice of variables, apply here. However, there are other problems which stem from their use of leading indicators and the seasonality of their data. Akintoye and Skitmore speak of
combining ‘leading indicators for the purposes of explaining trends in construction
demand’ (p. 3). The leading indicators of the macro economy published by the Office
of National Statistics are chosen such that these series systematically lead turning
points in economic activity by a consistent period. Thus, a leading indicator of
construction new orders ought to contain turning points that consistently lead new
orders. It is our belief that the authors have applied the term ‘leading indicators’ to
variables that do not have this quality. Variables such as gross national product and
unemployment are described as leading indicators of construction new orders but
some doubt must exist as to whether these do in fact lead new orders. Evidence
suggests that GNP is at best coincident with new orders and that unemployment may
lag new orders by as much as two quarters. In fact, total new orders, which includes
new orders from sectors other than construction, form part of the leading indicator
series of the economic cycle published by the Office of National Statistics. Whilst we
recognise that certain characteristics of the construction sector may result in
construction new orders being placed later than new orders in other sectors, we feel it
unlikely that the lag will be sufficiently long to result in construction orders lagging
GNP. New commissions, architects’ workloads and work at the production drawing
stage may provide examples of variables that could be considered as leading
indicators of new orders. In short, we believe that the authors use of the term ‘leading
indicator’ is inappropriate.

With regard to the seasonality in their data, casual inspection of the data set (presented
in Appendix B of their paper) reveals a mixture of seasonally adjusted and unadjusted
series. For example, the GNP series appears to be seasonally adjusted whereas the new
orders data seem to be unadjusted. This is contrary to the statement made in the paper
asserting that the data is unadjusted. It is well known in the econometrics literature
that the application of seasonally adjusted data in econometric analysis increases the
possibility of obtaining mis-specified models with spurious dynamics and poor
forecasting performance. The use of unadjusted data necessitates that the model be
modified to take account of the seasonality. This often takes the form of incorporating
seasonal dummies or seasonally differencing the data. For models incorporating both
adjusted and unadjusted series, the inclusion of seasonal dummies will lead to
enhanced empirical performance. However, Akintoye and Skitmore take no account
of the seasonality. These factors have undoubtedly contributed to the model's poor forecasting performance noted by the authors.

Oshobajo and Fellows (1991) identify variables that indicate trends in contractors' workload and use these as leading indicators to build models for forecasting construction new orders. The indicators are Royal Institution of Building Architects (RIBA) new commissions, RIBA work at the production drawing stage and interest rates. The time series properties of these indicators, and variables measuring activity in construction, were then analysed and the results used to formulate models. More specifically, Oshobajo and Fellows aim to determine the importance of the seasonal, cyclical, trend and irregular components of each series. They find that the underlying pattern is often obscured by the large irregular element. Oshobajo and Fellows estimate a univariate time series model and a model based on the classical decomposition for new orders and compare the results of these models with those derived from a leading indicator model. On the basis of the cross-correlation analysis, only the interest rate variable is considered to be an appropriate leading indicator of new orders, the lead time being between 1 and 2 quarters. As such, the interest rate variable is the only variable used in the estimation stage of the leading indicator model. The reported diagnostics are said to be adequate and the models are believed to fit the data 'sufficiently'. Details of the relative forecasting performance of these models are conspicuously scarce. However, Oshobajo and Fellows note that the use of the leading indicator model 'improved forecast accuracy by 5 to 12%' (p. 241).

2.11 Concluding Remarks

In this chapter we have examined the vast investment literature. The first part of this review has been organised around the four dominant theories of investment. We have presented an outline of each theory and considered equations of the type estimated in empirical studies. We have also described some of the empirical results associated with each of these theories. In the second part of the chapter we consider other models of investment behaviour. These are included on the grounds that they have contributed to the overall theory of investment. The theory of investment is disparate and confusing. In developing this theory we have used a common framework so that the
similarities and differences between theories can be easily identified. In the third part of this chapter we provide an overview of the investment literature in which the relationships between theories of investment are made explicit and in which empirical findings relating to these theories are summarised. In the final part of this chapter we have examined a number of studies from the building and surveying literature, which attempt to estimate models of construction activity. In many ways this growing body of research has much in common with the objectives of this thesis. However, typically, there are a number of deficiencies inherent in research from this literature. Firstly, in most studies, very little attention is paid to theory in model specification. Secondly, once the model has been specified there is very little consideration of econometric issues.

A discussion of the estimation issues has been largely absent from this review. In particular, problems associated with the use of nonstationary data in estimating economic relationships has not received mention. This is, in part, due to the fact that this issue has received little attention in the investment literature. Before we proceed to the estimation stage of this thesis, we must consider this and wider econometric issues associated with the investment equations to be estimated in this thesis. This forms the subject of the next chapter.
Econometric and Forecasting Issues

3.1 Introduction

In the first part of this chapter we concern ourselves with issues related to the estimation of models of private industrial and commercial construction output. The latter part of the chapter is concerned with issues related to forecast evaluation methods. To the extent that the construction of new buildings represents capital formation we can model construction output as an investment problem. Some of the models to be developed in this thesis are consistent with the traditional models of investment. The survey of the literature on investment models contained within Chapter 2 is largely devoid of discussion of econometric issues and to a large extent this reflects an absence of discussion in the literature. In addition to addressing some of the issues related to the estimation of investment models, we also outline the approach to estimation to be adopted in this thesis.

The main issue to be addressed in the estimation of the accelerator, neoclassical and putty-clay models is the treatment of the time structure of the investment process. These theories define an optimal capital stock (or capital intensity in the case of the putty-clay model) but have little to say about how firms adjust their actual level of capital to the optimal level. As we have seen in Chapter 2, to avoid instantaneous adjustment and an infinite rate of investment, empiricists implicitly assume that there are delivery lags in the investment process such that investment in any one period is made up of a fraction of investment goods ordered and delivered this period plus delivery of investment goods ordered in previous periods. These dynamics are typically captured in a distributed lag model. We consider issues related to the estimation of distributed lag models in Section 3.2.

There is no discussion of the issue of non-stationarity of economic data in the investment literature. This is an important issue since the distribution of conventional test statistics in econometrics is calculated under the assumption that the residual series is stationary. Moreover, the regression of one non-stationary time series on another may lead statistical analysis to suggest a relationship between series even where one does not exist. This is known as the spurious regression problem and since
many economic time series are non-stationary, it has important implications for economic modelling. Given this, one might be surprised to find a conspicuous absence of such testing in the empirical investment literature. However, this absence is due, in part, to the fact that much of the literature was developed prior to realisation of the importance of this issue. In this thesis we examine the data for non-stationarities and, in this respect, this represents an advance on the existing investment literature. In Section 3.3 we introduce the concepts of stationarity, integration and unit roots and in Section 3.4 we outline a number formal and informal tests for non-stationarity.

In order to avoid the potential problem of spurious regressions we aim to estimate investment models that are consistent with the results of these tests. It has been suggested that first differencing may go some way to rectifying the spurious regression problem. Many series will be stationary after applying such a transformation in which case valid statistical inferences may be drawn. However, models containing only differenced series are lacking some meaningful economic information and long-run interpretation. Granger (1981) offered cointegration as an alternative solution which, in certain cases, enables the retention of long-run relationships even when the model is expressed in first differences. In this thesis we examine the data for cointegration and, to the extent that much of the empirical literature on investment predates the concept of cointegration, this also represents an advance on the existing investment literature. In addition, cointegration analysis has not previously been applied in work which aims to model construction activity. We outline the concept of cointegration and present tests for cointegration in Section 3.5.

As is clear from our review of the empirical literature associated with each of the models of investment in Chapter 2, no single model dominates the others in terms of within sample or out of sample performance. Moreover, in general, the performance of investment models has been poor relative to the performance of models of other macroeconomic aggregates, such as consumption. In Chapter 2 we outlined some of the problems specific to each of the investment models. The explanation for the poor performance of these investment models may lie with the inadequacy of investment theory or with the empirical implementation of these theories. We can examine the adequacy of the determinants of investment suggested by theory in isolation by using
a relatively non-structural, data based approach to modelling. Sims (1980) has
developed such an approach in response to the potential problems that may arise when
estimating structural equations. In estimating equations of this kind, interpretation of
parameters may be blurred and identification compromised if, for example, an
exogenous variable has multiple interpretations. Sims believed that the restrictions
needed to identify the econometric structure are ‘incredible’ and this belief provided
the basis for the new methodology of vector autoregressive (VAR) modelling. VAR
analysis provides a largely statistical representation of a set of variables with
minimum use of \textit{a priori} restrictions. This procedure involves regressing
contemporaneous values of each variable on lagged values of all the variables in the
model. No variables are treated as exogenous and no variable is excluded from any of
the equations in the model. This implies that everything causes everything else and
there is no room for assuming anything more than very general economic principles as
a starting point. Sims’ methodology is often referred to by its critics as atheoretical
macroeconometrics. However, the standard defence is that there is literally no variable
that may not enter the consumer’s demand or labour supply function and, therefore,
there can be no variable that can be excluded from the maximisation of lifetime utility
problem. In applying such a formulation to investment, one need not worry about the
dynamics of the relationship between investment and its determinants which have
been the source of much empirical difficulty. Instead, one can focus on assessing
whether the determinants of investment posited by theory provide a sufficient
explanation of investment behaviour.

One of the criticisms of this approach is that variables entering into the system must
be stationary in order that the spurious regression problem can be avoided. Given that
many economic variables are non-stationary and must be first differenced to become
stationary, many VARs contain first differenced data. As such, there is a loss of
long-run information. Johansen (1988) suggested a reparameterisation of a pure VAR
which allows long-run information to be retained. The resulting system is sometimes
referred to as a vector error correction model (VECM). In this thesis we use the
Johansen procedure to estimate VECM models consistent with dominant theories of
investment. In so doing we are able to assess whether the poor performance of
investment models is due to the determinants of investment posited by theory being
insufficient explanators of investment, or due to inadequate empirical implementation of these theories. In short, issues relating to the empirical implementation of these theories are by-passed when the VAR or VECM approach to modelling is adopted. We present an outline of the Johansen procedure in Section 3.4.4.

VAR models are particularly useful for forecasting exercises, since the forecaster does not need to worry about the economic theory underlying the model (this allows empirical specifications to differ from those suggested by theory). Moreover, the forecaster does not need to make assumptions about the values of exogenous variables in the forecast period. This is in contrast with standard econometric forecasting where forecasts have to be conditioned on knowledge of the exogenous variables.

An additional benefit to the use of VAR models in forecasting exercises arises from the fact that such models represent a more flexible approach to modelling. Few would argue that the use of supplementary information, such as leading indicators, in a modelling exercise would not lead to improved forecasts. The inclusion of leading indicators in structural models, however, may not be justifiable if these indicators are inconsistent with theoretical priors. However, these variables can be readily incorporated into a VAR model, since in using the atheoretical models the forecaster does not need to worry about the economic theory underlying the model.

Despite the benefits of VAR modelling, there is a conspicuous absence of studies adopting this approach in the investment literature. Moreover, the approach has never been used to forecast private industrial and commercial construction output or construction activity more generally. In addition to developing VAR models consistent with the dominant theories of investment, we develop a number of VAR models for construction output and augment them with variables thought to be leading indicators of construction output. The forecasting performance of these models and the single equation investment models are compared with each other and with the performance of a benchmark ARIMA model.

This brings us to the issues surrounding the evaluation of forecasting performance. There are a number of studies in the empirical investment literature which aim to systematically compare the forecast performance of alternative investment models. We considered these studies in Section 2.7. On the whole the methods of comparison
in these studies are crude, being based on statistics summarising the sample evidence (e.g. the mean square forecast error) and visual inspection of plots of forecasts and forecasts errors. In this thesis we adopt more rigorous methods of comparing forecasts. In Section 3.5 we discuss the underlying objectives of evaluation, some frequently used measures of forecast accuracy and some of the problems associated with these measures. Of the available measures we select three that are appropriate given the aims of this work. In Section 3.6 we outline attempts to create conditions under which forecasts derived from different generating mechanisms can be fairly compared and we discuss the parameters of the forecasting contest.

3.2 Issues Related to the Estimation of Distributed Lag Models

In the accelerator, neoclassical and putty-clay models, investment is related to its determinants through some distributed lag relation. However, these theories provide no guidance on the factors governing the shape and length of distributed lag specification. In practice therefore, shape and length tend to be chosen on an *ad hoc* basis, either by estimation or by imposing a particular form *a priori*. There are a number of problems associated with the estimation of distributed lag models. We outline some of these problems in this section and describe the procedure to be used to estimate the investment functions in this thesis.

Consider the following distributed lag model:

\[ y_t = \alpha + \sum_{i=0}^{k} \beta_i x_{t-i} + u_t \]  

(3.1)

The first problem encountered when estimating such a model is that of determining the lag length \( k \). Since theory usually has little to say about \( k \) most researchers have adopted a data based approach to determine it. Schmidt and Waud (1973) suggest choosing the lag length so as to maximise \( \bar{R}^2 \). However, Frost (1975) found that this method leads to a substantial upward bias in the lag length. It is well known that the \( R^2 \) criterion implies that a regressor is retained if the F-statistic on that regressor is greater than unity. Frost suggests that the bias can be corrected for by retaining the regressor if the F-statistic is greater than two rather than unity. However, this procedure is rather arbitrary. We choose instead to adopt the procedure suggested by
Akaike (1974). That is to say, we choose \( k \) so as to minimise the Akaike Information Criterion, AIC. The AIC is defined as \( \ln \hat{\sigma}^2 + 2N/T \), where \( \hat{\sigma}^2 \) is the estimated residual variance, \( N \) is the number of parameters and \( T \) is the number of observations.

A second problem is that due to high correlations between \( x_t \) and its lagged values, i.e. multicollinearity, parameter estimates are not reliable. One commonly adopted solution to this problem has been posited by Almon (1965, 1968). Almon suggests that the distributed lag parameters \( \beta_i \) should be constrained to lie on a polynomial of degree \( r \). That is to say

\[
\beta_i = \eta_0 + \eta_1 i + \eta_2 i^2 + \ldots + \eta_r i^r
\]  

(3.2)

If \( r=2 \) then equation 3.1 above becomes

\[
y_t = a + \sum_{i=0}^{k} \left( \eta_0 + \eta_1 i + \eta_2 i^2 \right) x_{t-i} + u_t
\]

\[
= a + \eta_0 z_0 + \eta_1 z_1 + \eta_2 z_2 + u_t
\]  

(3.3)

where

\[
\begin{align*}
z_{0t} &= \sum_{i=0}^{k} x_{t-i}, \\
z_{1t} &= \sum_{i=0}^{k} ix_{t-i}, \\
z_{2t} &= \sum_{i=0}^{k} i^2 x_{t-i}
\end{align*}
\]  

(3.4)

Thus, Almon suggests regressing \( y_t \) on the constructed variables \( z_{0t}, z_{1t} \) and \( z_{2t} \). Once estimates of the \( \eta \)'s have been obtained, equation 3.2 can be used to get estimates of \( \beta \). The procedure reduces the number of parameters in the distributed lag of \( x_t \) from \( k+1 \) to \( r+1 \).

It is straightforward to determine \( r \), the degree of polynomial. Anderson (1966) suggests that \( y_t \) should be regressed on \( z_{0t}, z_{1t}, z_{2t}, z_{3t}, \ldots \) as defined in equation 3.4, dropping higher terms sequentially. The highest order term should be dropped if the \( t \)-ratio is insignificant and then the regression reestimated. The process should continue until the \( t \)-ratio on the highest order term is significant.

Often, in addition to constraining the \( \beta \) to lie on a polynomial, researchers impose endpoint constraints such as \( \beta_1 = 0 \) and \( \beta_{k+1} = 0 \). It is sometimes argued that these endpoint constraints are responsible for the giving the lag distributions plausible
shapes (see Schmidt and Waud (1973)). However, instead of imposing endpoint constraints \textit{a priori}, the restrictions can tested using a simple $F$-test.

So, to summarise, the lag length of the distributed lag term in the accelerator, neoclassical and putty-clay models are determined using the minimum AIC. The lagged coefficients are constrained to lie on an Almon polynomial and the degree of polynomial is determined according to the procedure suggested by Anderson (1966). The Almon restrictions and the endpoint constraints are tested using an $F$-test.

3.3 Stationarity, Integration and Unit Roots: An Introduction

As noted above the empirical investment literature gives little consideration to the issue of non-stationarity. In this thesis we consider the issue explicitly and test for its presence. Before outlining the tests for non-stationarity to be applied in this work it is appropriate to define two concepts: stationarity and integration. A stationary series can be defined as one whose mean and variance remain constant (and finite) over time. More accurately, such a series is said to be weakly stationary. The stronger condition of strict stationarity requires that existing higher order moments are also constant over time. In what follows, the term stationarity is assumed to mean weak stationarity. It is noteworthy that a \textit{white noise} process is a weakly stationary process which has a zero mean and is uncorrelated over time. Further, a white noise process will be strictly stationary if the process if normally distributed.

An integrated series is defined as a non-stationary series that can be transformed into a stationary series simply by differencing it $d$ times. Such a series is said to be integrated of order $d$, denoted $I(d)$. Using this notation, a stationary series is said to be $I(0)$. Testing whether an individual time series is $I(1)$, as opposed to $I(0)$, is the problem commonly referred to as that of testing for a ‘unit root’. A series integrated of order $d$ is said to have $d$ unit roots. Since an integrated series is one that may be made stationary by differencing, some non-stationary series are by implication not integrated processes. That is, there exists non-stationary series that however differenced, can not be transformed into a stationary series.

The distribution of conventional test statistics in econometrics is calculated under the assumption that the residual series is stationary. Since, in general, any linear
combination of non-stationary series is itself non-stationary, it is essential to
determine the order of integration for the variables comprising a linear regression. In
the special case when the non-stationary behaviour in one series exactly offsets that in
another, a linear combination of the two variables will be stationary, and the variables
are said to be cointegrated. We consider the concept of cointegration more fully in
Section 3.4. Since two series of different orders of integration can not possibly
cointegrate, such that the linear combination is stationary, testing for unit roots is a
fundamental premise of cointegration analysis. Many of the problems that may arise
in regressions that include non-stationary variables have been noted by Granger and
Newbold (1974). They note that regressing a non-stationary series on another will lead
to statistical analysis to suggest a relationship between the series, even when one does
not actually exist. This is known as the spurious regression problem and, since most
economic time series are non-stationary, it has disturbing implications for
macroeconomic modelling. Estimating such a model will generally lead to a badly
defined error and mean. Consequently, since conventional asymptotic theory assumes
stationarity of all explanatory variables (possibly around a deterministic trend), such
estimation is invalid.

The definition of stationarity, and indeed the whole problem of investigation, becomes
much more complex when a time series contains a seasonal component. When using
seasonal data it may be necessary to seasonally difference to achieve stationarity (that
is, if \( x_t \) is a seasonal time series measured four times per annum, it may be appropriate
to transform data by \( y_{t-4} \) rather than by \( y_{t-1} \) to achieve stationarity). A seasonal
component of a time series may be described by a deterministic process, a stationary
stochastic process, or an integrated process in much the same way as a time series
with no seasonal component. Although it is common for seasonality to be modelled
deterministically, or as a stationary process, it may be the case that the seasonal
component drifts over time, in which case the process contains an integrated seasonal
process and seasonal differencing, as advocated by Box and Jenkins (1970), may be
necessary to achieve stationarity.

Thus, we can extend the definition of non-stationarity to include seasonal data as
follows. A series is said to be seasonally integrated of order \((d,D)\), denoted \( S_l(d,D) \), if
it can be transformed to a stationary series by applying $s$-differences (that is, by applying the filter $y_t - y_{t-s}$) $D$ times in addition to first differences $d$ times.

As with a simple integrated series, such as an $I(1)$ series, a seasonally integrated series retains the effects of a shock indefinitely and has a variance that increases linearly with time. However, a seasonally integrated series does not behave like an $I(1)$ process in all respects. For example, shocks to the system will alter the seasonal pattern of the series so that the sequence of observations corresponding to each quarter (assuming the series is measured four times per annum) may evolve in different ways. First differencing such a seasonally integrated series is not sufficient to transform it into a stationary series.

Many tests of the order of integration of a time series have been developed over the last twenty years. In the next section three such tests are briefly outlined. All three tests are performed on the data in this thesis.

3.3.1 Formal Tests for Non-Stationarity

In this section we outline three tests commonly used to determine the order of integration of a time series. The tests proposed by Dickey and Fuller (1979) and Phillips and Perron (1988), and described in Section 3.3.1.1, are tests for a simple unit root. The test offered by Hylleburg, Engle, Granger and Yoo (1990) and described in Section 3.3.1.2 provides a framework for testing for the presence of seasonal unit roots.

3.3.1.1 Testing for Simple Unit Roots

As noted above, testing whether an individual series is $I(1)$, as opposed to $I(0)$, is the problem that is commonly referred to as that of testing for a unit root in a time series. Two basic tests of the order of integration of a time series have been developed by Dickey and Fuller (1979) and Phillips and Perron (1988). These tests are based on the OLS estimation results from a suitably specified regression equation. For a time series, $y_t$, the general form of the ‘augmented’ Dickey-Fuller (ADF) regression is given by
\[ \Delta y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 t + \sum_{j=1}^{k} \gamma_j \Delta y_{t-j} + \epsilon_t \] (3.5)

where \( \epsilon_t \) for \( t=1\ldots T \) \( (T \text{ denotes the sample size}) \) is assumed to be white noise and \( t \) denotes a time trend. The number of lagged terms, \( k \), is chosen to ensure that the errors are uncorrelated. The basic test for non-stationarity is based on the \( t \)-ratio associated with \( \alpha_1 \). Rejection of the null hypothesis, that \( \alpha_1 = 0 \), implies stationarity. Failure to reject the null hypothesis may well imply that \( y_t \) is non-stationary. However, it is well known that the power of the test statistics (their ability to correctly reject a false null) is weakened if the model is over parameterised. The appearance of the time trend in Dickey-Fuller regression presupposes that \( y_t \) contains a deterministic trend. If \( y_t \) does not contain a deterministic trend, then the Dickey-Fuller regression given above is over specified. In this case, failure to reject may be due to the lower power of the test statistics in the over specified model. It is therefore important to test the appropriateness of the time trend in the Dickey-Fuller regression.

If it is not possible to reject the null of \( \alpha_1 = 0 \), one must test the joint hypothesis that \( \alpha_1 = \alpha_2 = 0 \) before drawing any firm conclusions as to the series' order of integration. Rejection of the joint hypothesis implies that the time trend, \( t \), is appropriate in the Dickey-Fuller regression and that the failure to reject the initial hypothesis, \( \alpha_1 = 0 \), does indeed suggest that \( y_t \) is non-stationary. If the joint hypothesis that \( \alpha_1 = \alpha_2 = 0 \) can not be rejected, then the time trend is inappropriate. The Dickey-Fuller regression should then be re-estimated without the time trend and the hypothesis that \( \alpha_1 = 0 \) tested again. Rejection of the null in favour of the alternative hypothesis (that \( \alpha_1 < 0 \)) implies that \( y_t \) is stationary.

Given that the failure to reject the null hypothesis, \( \alpha_1 = 0 \), (at either stage) implies that \( y_t \) is non-stationary, standard critical values (calculated on the assumption that all variables in the regression equation are stationary) will be invalid. However, appropriate critical values have been calculated by a number researchers (see Fuller (1976), Dickey and Fuller (1981), and Guilkey and Schmidt (1989), for examples).

The selection of \( k \) is an important consideration. It is generally safer to take \( k \) to be a fairly generous number, since if there are too many lags the regression is free to set
them equal to zero, albeit at the cost of test statistics having lower power. With too few lags, on the other hand, residual autocorrelation may remain and thereby invalidate test statistics. One can, of course, perform tests for autocorrelation on the estimated residuals from the regression in order to check the acceptability of the premise that these residuals are white noise. Alternatively, model selection criteria, such as the Akaike Information Criterion (AIC) and the Schwarz Criterion (SC) may be used to determine \( k \). Given that the SC punishes extra parameters more harshly than the AIC and is therefore more likely to yield a preferred model with some autocorrelation in the residuals, the lag length on the Dickey-Fuller regression \( k \) is determined here using the AIC. Although there is no shortage of degrees of freedom in these regressions, the lag length is constrained to a maximum of \( k=9 \), on the grounds that none of the series under consideration here are likely to be generated by an autoregressive process of order greater than nine.

As an alternative to the inclusion of lagged terms, Phillips and Perron suggest a non-parametric correction for serial correlation (see Phillips (1987), Phillips and Perron (1988), and Perron (1988)). The approach begins by calculating the above unit root tests from the regression equations with \( k=0 \). The statistics are then modified after estimation in order to take account of the effect that autocorrelated errors will have on the results. The critical values are the same as those used for the Dickey-Fuller test. The transformed statistics use an estimate of the error variance that is constructed from the estimated residuals \( \hat{\varepsilon}_i \) as:

\[
\frac{1}{T} \sum_{t=1}^{T} \hat{\varepsilon}_i + \frac{2}{T} \sum_{s=1}^{Q} \omega(s, Q) \sum_{t=r+1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_{t-s} \tag{3.6}
\]

where \( Q \) is a truncation lag parameter and \( \omega(s, Q) \) is a window given by

\[
\omega(s, Q) = 1 - s/(Q + 1) \tag{3.7}
\]

The selection of \( Q \) is an important consideration and is determined here by the highest significant lag (up to a maximum of \( Q = T^{1/2} \) ) from either the autocorrelation function, ACF, or the partial autocorrelation function, PACF, of the first differenced series.
Both the Dickey-Fuller and the Phillips-Perron (PP) tests have power functions that are relatively flat in the region of the null. This is to say they over accept the null (of non-stationarity) when the data is in fact stationary. In effect the test are biased towards the null. We attempt to allow for this to some extent by choosing a test size of 10% rather than the more traditional 5%. It is noteworthy that the power of the two tests differs depending on the type of data generating process that the series is thought to have. Since the PP tests have been derived to expand the class of model to which unit root tests of this type can be applied, it is not surprising that the power of this test is higher than the ADF test when dealing with mixed or moving average processes. Although the converse is true of the ADF test with autoregressive processes, the power of both tests is still low.

There is a problem with these tests when testing for multiple unit roots, \( d > 1 \), in a series. The sequence of testing, which starts with a test for a single unit root in the undifferenced series and then proceeds to test for a second unit root (that is, tests the first-differenced series) if the first null (of a unit root in the levels) is not rejected, and so on, is not a valid testing sequence. The reason why such a testing procedure is invalid is that the tests described above take the complete absence of unit roots as the alternative hypothesis. Dickey and Pantula (1987) suggest an alternative sequential testing procedure for unit roots which takes the largest possible number of unit roots under consideration as the first maintained hypothesis and then decreases the order of differencing each time the current null is rejected. This process continues until the null hypothesis is not rejected. Dickey and Pantula state that, on the basis of their simulation study, this sequential approach is considerably more powerful than the sequence that takes the smallest number of roots, i.e. one, as its first maintained hypothesis. This approach to testing for multiple unit roots is adopted in this work. The largest number of roots, to be taken as the first maintained hypothesis, is assumed to be two on the grounds that an economic time series integrated of order greater than two is implausible.

The unit root tests described in this section may not perform well on data that contain a seasonal component. However, many economic time series are seasonal and indeed, as we will discuss in Chapter 4, there are good reasons for using unadjusted data wherever possible. In order to successfully apply these tests to seasonal data, it is
appropriate to remove the seasonal component of a series prior to testing. To this end, each of the series to be analysed in this thesis will be regressed on a constant and three seasonal dummies prior to testing. If the seasonal dummies from this regression are found to be significant, then the series will be assumed to have a (deterministic) seasonal component. For those series found to have a deterministic seasonal component, the unit root tests were applied to the saved residuals from these regressions rather than to the actual series. This method of removing the deterministic seasonality prior to performing unit root tests has been justified by Dickey, Hasza and Fuller (1984), Dickey, Bell and Miller (1986) and Osbourne, Chui, Smith and Birchenhall (1988).

3.3.1.2 Testing for Seasonal Unit Roots

Tests for seasonal integration have been proposed by Hasza and Fuller (1982), Dickey, Hasza and Fuller (1984), Osbourne, Chui, Smith and Birchenhall (1988), Hylleburg, Engle, Granger and Yoo (1990) and Engle, Granger, Hylleburg and Lee (1993), amongst others. The test proposed by Hylleburg et al (1990) (hereafter referred to as the HEGY test) has the advantage over earlier tests of allowing both deterministic and stochastic seasonality to be tested within the same model. The principal difference between these two types of seasonality is that shocks in the deterministic seasonal model die out in the long run, while in the stochastic seasonal model they have a permanent effect. For this reason, deterministic seasonality may be regarded as a rather poor approximation to stochastic seasonality. In addition, the HEGY test gives a greater insight into the nature of the seasonality. For example, one can distinguish between different types of seasonal cycles. With quarterly data, a seasonal cycle in which the process returns to its original value on the cycle with a period of two (i.e. a semi-annual cycle) can be distinguished from a seasonal cycle in which the process repeats with a period of four (i.e. an annual cycle). Moreover, the presence of a simple (non-seasonal) unit root can also be tested within the HEGY framework.

For quarterly data, the most general representation of the HEGY test is given by
\[
\Delta_4 y_t = \sum_{i=1}^{4} \eta_i Q_{it} + \sum_{i=1}^{4} \Theta_{i, \tau_{i-r}} + \sum_{i=1}^{k} \tau_i \Delta_4 y_{t-i} + \epsilon_t
\]

where \( k \) is the number of lagged dependent variables, chosen to ensure stationary residuals, \( Q_{it} \) denote seasonal dummies, and the \( \Theta_{i, \tau} \) variables are constructed from the series \( y_t \) as

\[
\begin{align*}
\Theta_{1,i} &= (1+L)(1+L^2)y_t = y_t + y_{t-1} + y_{t-2} + y_{t-3} \\
\Theta_{2,i} &= -(1-L)(1+L^2)y_t = -y_t + y_{t-1} - y_{t-2} + y_{t-3} \\
\Theta_{3,i} &= -(1-L)(1+L)y_t = y_{t-1} + y_{t-2} \\
\Theta_{4,i} &= -(L-1)(1+L)y_t = \Theta_{3,1} = -y_{t-1} + y_{t-3}
\end{align*}
\]

where \( L \), the lag operator, implies \( Ly_t = y_{t-1} \) and \( L^3 y_t = y_{t-3} \). This model is estimated by ordinary least squares. If the null hypothesis of stochastic (rather than deterministic) seasonality is true, then all the \( \eta \)'s will be equal to each other and all the \( \Theta_{i, \tau} \)'s will not differ significantly from zero. If, however, the \( \eta \)'s are different and at least one of the \( \Theta_{i, \tau} \)'s are non zero, then there is evidence to suggest that the series contains a combination of both deterministic and stochastic seasonality. Each of the \( \Theta_{i, \tau} \)'s has a different interpretation. For example, if only \( \Theta_1 \) is negative, then there is no evidence of non-seasonal stochastic stationary component, that is, there is no evidence of a component corresponding to an I(0) process. If only \( \Theta_2 \) is negative, then there is no evidence of a semi-annual cycle. The coefficients \( \Theta_3 \) and \( \Theta_4 \) relate to the annual cycle and may be tested jointly. A rejection of both the nulls that \( \Theta_2 = 0 \) and \( \Theta_3 = \Theta_4 = 0 \), implies that there are no seasonal unit roots in \( y_t \). In other words, for rejection of all seasonal unit roots we require \( \Theta_i = 0 \) for \( i=2,3,4 \). Rejection of null hypothesis of \( \Theta_i = 0 \) (for all \( i=1...4 \)) implies that the series is stationary.

The value of \( k \), the number of lagged dependant variables, is chosen to be as small as possible (to maximise the power of the test) whilst ensuring that the residuals are stationary. In this work, \( k \) is determined by the AIC (up to a maximum permitted lag length of \( k=9 \)).

It is worth noting that the HEGY test is low in power (even lower than the ADF and Phillips-Perron tests). For this reason, the size of the test is increased and, as such,
10% critical values are reported in Chapter 4. Critical values for this test are provided in Hylleburg et al (1990). The fact that the HEGY test has lower power than the ADF and Phillips-Perron tests provides a justification for using the latter tests of a simple unit root, even though the HEGY procedure itself enables such as a test. The HEGY test may suggest evidence of a simple unit root, where the more powerful ADF and Phillips-Perron tests do not. This has implications for the order in which these tests should be performed. The HEGY procedure must be conducted first, and the results of the tests for a simple unit root (based on the null that $\vartheta_1 = 0$ in equation 3.8) are then checked against results of the ADF and PP tests. This approach is adopted here.

That more than one type of unit root test is to be used raises the possibility that conflicting results from these tests, regarding a series order of integration, may emerge. When such conflicts occur, we draw on other statistical procedures to help determine the appropriate order of integration. These are described in the following subsection.

3.2.2 Complementary Methods for Determining $d$

The autocorrelation function and the spectral density are used in the thesis to assist in determining the order of integration of series in cases where the formal unit root tests, described above, give conflicting results. The spectral density is also used in conjunction with the HEGY test to identify the presence of seasonal unit roots in a series. Before outlining how the autocorrelation function and the spectral density are used here, we begin with some definitions.

3.3.2.1 The Autocorrelation Function

For any given time series, $y_t$, the sample autocovariance can be estimated using the formula,

$$c(\tau) = \frac{1}{T} \sum_{t=1}^{T} (y_t - \bar{y})(y_{t+\tau} - \bar{y}) \quad \tau = 0, 1, 2, ...$$

(3.9)

where $T$ is the number of observations and $\bar{y}$ is the series mean. The sample autocorrelations are calculated as
\[ \rho(\tau) = \frac{c(\tau)}{c(0)} \]  

(3.10)

The sample autocorrelation function measures the degree to which one value in a series is correlated with previous values. In other words, it measures the `memory` of the process. A plot of the sample autocorrelations is sometimes referred to as the sample correlogram. In this work the acronym ACF denotes the sample correlogram. The ACF can provide important information concerning the nature of the underlying generating process, particularly if the underlying process is believed to be a autoregressive (AR) or moving average (MA) process. In such cases the ACF tends to display distinctive patterns. In general, MA processes have an ACF which is suddenly cut off. The point at which it does so is determined by the order of the MA process. AR processes, on the other hand, tend to decay and the manner in which they do so is determined by the order of the process and the value of the coefficients. The ACF of a mixed process tends to display a mixture of characteristics consistent with the autoregressive and moving average components.

Rather than using the ACF to identify the type and order of the underlying data generation process, it is used here to determine whether a series is stationary or non-stationary when the results from the unit root tests conflict. For a stationary time series, the ACF will decline at higher lags. If the ACF shows slow decay, there is evidence of non-stationarity. In Chapter 4 the ACF is only estimated for those series for which the order of integration can not be easily determined from the results of the simple unit root tests.

3.3.2.2 The Spectrum and Spectral Density Function

Whilst the autocorrelation function will tend to be effective in exposing a single cycle in a time series if that series contains only one cycle, it will tend to break down if the series contains several cycles. Analysis in the frequency domain, via the spectral approach, on the other hand, allows us to examine the relative importance of all cycles in the process simultaneously. Indeed, the motivation for analysis in the frequency domain lies in its ability to identify the cyclicality inherent in a series. Since the spectral approach is used rather infrequently in applied econometric work, some space
is dedicated here to an outline of the underlying theory. More detailed accounts of analysis in the frequency domain and its wider applications in economics can be found in Harvey (1993), Granger and Hatanaka (1964) and Granger and Newbold (1986).

In theory, transferring from the time to the frequency domain is a simple matter of making use of the complex Fourier transformation on the series autocovariance

\[ f(\lambda) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \gamma(\tau) e^{-i\lambda\tau} \quad -\pi \leq \lambda \leq \pi \]  (3.11)

where \( f(\lambda) \) is the theoretical power spectrum and \( \lambda \) is frequency measured in radians per unit time. Since the autocovariance function is symmetric, it can be shown that the above function defining the theoretical power spectrum reduces to

\[ f(\lambda) = \frac{1}{2\pi} \left\{ \gamma(0) + 2 \sum_{\tau=1}^{\infty} \gamma(\tau) \cos \lambda \tau \right\} \quad 0 \leq \lambda \leq \pi \]  (3.12)

for any real process. Whereas the autocovariance function depends on time, the above transformation gives us a function dependent on frequency. Since \( f(\lambda) \) can be regarded as a continuous function, we are not restricted to processes that contain only cycles.

Consider, for example, a white noise series which contains no cyclical components. The autocovariances of such a function are \( \gamma(\tau)=0 \) for all \( \tau\neq0 \) and the variance of the function \( \gamma(0)=\sigma^2 \). On substituting these values into \( f(\lambda) \) above gives the following theoretical spectrum for a white noise process.

\[ f(\lambda) = \frac{\sigma^2}{2\pi} \]  (3.13)

This theoretical power spectrum is plotted against frequency in Figure 3.1. The process may be regarded as consisting of an infinite number of cyclical components, all of which have equal weight.

Compare this with a process that contains only a discrete number of cyclical components. The spectrum of such a series will not be continuous, but will exhibit large discrete spikes at the particular frequencies corresponding to these cyclical components. The theoretical power spectrum allows one to simultaneously identify the cyclical components in the series. Moreover, since larger spikes represent more
Figure 3.1: The spectral density of a white noise process

\[ f(\lambda) = \frac{\sigma^2}{2\pi} \]

Figure 3.2: The spectral density of an MA(1) process (coefficient = 0.5)

\[ f(\lambda) \]

\( \lambda \in [0, \pi] \)
dominant cycles in the series, the theoretical power spectrum can be used to indicate the relative importance of cycles.

Consider, as a further example, the following moving average process (also considered by Harvey (1993)),

\[ y_t = \varepsilon_t + 0.5\varepsilon_{t-1} \quad (3.14) \]

where \( \varepsilon \sim \text{iid}(0,\sigma^2) \). Now \( \gamma(0) = E[y_t y_{t+1}] = E[(\varepsilon_t + 0.5\varepsilon_{t+1})(\varepsilon_{t+1} + 0.5\varepsilon_t)] = 5\sigma^2/4 \) and \( \gamma(1) = E[y_t y_{t+2}] = E[(\varepsilon_t + 0.5\varepsilon_{t+1})(\varepsilon_{t+1} + 0.5\varepsilon_{t+2})] = \sigma^2/2 \). Substituting these values into equation 3.12 gives

\[ f(\lambda) = \frac{\sigma^2}{8\pi} (5 + 4 \cos \lambda) \quad (3.15) \]

The theoretical power spectrum \( f(\lambda) \) for this moving average process is plotted against frequency \( \lambda \) in Figure 3.2. Notice that \( f(\lambda) \) is high for low frequencies and falls gradually as the frequency increases.

Now consider a simple moving average process which contains a single cyclical component of perfect regularity. A difficulty arises in graphing \( f(\lambda) \) in this case since the process will be partly continuous, from that pertaining to the MA process, and partly discrete corresponding to the cycle. However, in economics we are unlikely to encounter a series such as this since, although cycles exist in economic data, they never occur with perfect regularity. A partly cyclical process, such as those frequently encountered in economics, will tend to have a theoretical spectrum of the form displayed in Figure 3.3.

This plot suggests that the economic time series contains three important cycles \( \lambda_1, \lambda_2 \) and \( \lambda_3 \). Both the height of the spikes and their sharpness contain important information about these cycles. As stated above, the more dominant cycle in a time series will tend to be reflected in a higher spike. The sharpness of the spike, on the other hand, represents the regularity of the cycle. In general, the cycles show up as peaks in a continuous spectrum being spread about a frequency ‘band’ rather than being concentrated at one particular frequency.

In our discussion of spectral analysis so far we have talked of the theoretical power spectrum. We now turn to the problem of estimating the power spectrum. As with its
counterpart in the time domain, namely the ACF, the theoretical and estimated functions can differ (often substantially) and this is largely a result of data constraints. For empirical work in economics, we only ever have a finite number of observations available and, as a result, we can only estimate autocovariances up to lag $T-1$. Thus, given this, it would seem wise to estimate

$$f(\hat{T}) = \frac{1}{2\pi} \left\{ c(0) + 2 \sum_{\tau=1}^{T-1} c(\tau) \cos \lambda \tau \right\}$$

(3.16)

where $c(\tau)$ is the estimate of $\gamma(\tau)$. This differs from the theoretical power spectrum given in equation 3.12 in that $T-1$ replaces $\infty$ as the upper limit of the summation. However, there is a problem with this estimator of the spectrum: although the spectrum is asymptotically unbiased, it is not consistent, i.e. the covariance of the estimator does not tend to zero as the sample size $T$ increases. Thus, no matter how large the sample size, $f(\hat{T})$ will not converge to $f(\lambda)$. In practice, the estimated power spectrum when plotted against frequency will tend to be very jagged and irregular. Out of this arises the danger that some peaks, due simply to sampling, may be identified as cycles.

Figure 3.3: The spectral density of a partly cyclical process
In an attempt to solve this problem, researchers have developed the following modified estimator.

$$f(\hat{\lambda}) = \frac{1}{2\pi} \left[ c(0)w(0) + 2 \sum_{\tau=1}^{T-1} c(\tau)w(\tau) \cos \lambda \tau \right]$$

(3.17)

Notice that the two terms inside the brackets are now weighted by the function $w(\tau)$, known as the lag window. In general, $w(\tau)$ tends to decline as $\tau$ moves from zero to $M$ and is zero for $\tau > M$. Thus, higher order autocovariances receive less emphasis which has the desired effect of improving the resolution of the estimator, $f(\hat{\lambda})$. A number of researchers have posited a variety of different forms for this lag window, among the most common of which is that introduced by Blackman and Tukey (1958) which takes the form

$$w_M(\tau) = \frac{1}{2} \left[ 1 + \cos \frac{\pi \tau}{M} \right] \quad \tau \leq M$$

(3.18)

It is with a form of this estimator that we work. This weighted or smoothed estimate of the spectrum can be shown to be a consistent estimator of $f(\lambda)$. Its variance can be shown to vary positively with the ratio $M/T$. That is to say, the smaller $M$ and the larger $T$, the lower the variance. A problem arises here since as $M$ decreases, the ability to distinguish between adjacent frequencies diminishes. In other words, the frequencies can be ‘smudged’. There is, therefore, a trade-off between minimising the variance of the estimator to produce more stable estimates and maximising the ability to discriminate between adjacent frequencies. Whilst, in general, the choice of $M$ will depend on the type of spectrum being estimated, a suggested rule of thumb is to keep the ratio $M/T$ below 0.3.

It can be shown that a theoretical power spectrum is undefined for a non-stationary time series (see Harvey (1993), p. 184). However, inadequacies in ‘real’ data usually enable the power spectrum of a non-stationary time series to be estimated. In estimated power spectra, non-stationarities appear as large (but not infinite) spikes. Thus, if an economic time series contained a simple unit root, one would expect to find a large spike in the estimated spectrum at the zero frequency. Similarly, should the time series contain a seasonal unit root, then one would expect the estimated spectrum to contain large spikes at one, or more, of the seasonal frequencies. It is
worth emphasising that these spikes would be infinitely large if the data were 'perfect'. However, large spikes are also indicative of a strong regular cycle in a stationary time series. In using the estimated spectrum to identify possible non-stationarity in data, one must be mindful of this source of potential misinterpretation. Relatively high power at the seasonal cycles may be indicative of deterministic seasonality or seasonal non-stationarity. For this reason it would seem sensible to remove the deterministic seasonality prior to estimating the spectrum. If the power at the seasonal frequencies remains high (relative to other frequencies), this can be taken as evidence of non-stationarity.

It has been noted by Granger and Hatanaka (1964) that, assuming the economic structure of a time series does not change too rapidly over time, it is possible to use the spectrum to examine subtle non-stationarities in that time series. From the above discussion, it should be clear that a series can be envisaged as being made up of a sum of frequency bands. For a stationary series the amplitudes of these frequency bands will be constant over time. Conversely, the amplitude of at least one frequency band making up a non-stationary time series will change over time. Thus, by examining the shape of the spectrum over time, it is possible to determine whether the importance of one, or more, of the components of a series have altered over time. This property of spectra is also used to identify potential non-stationarities in the data. In this work, the period over which we have data for a particular time series is divided into two sub-periods. Spectra are estimated for each of the sub-periods and compared to see how the shape of the spectrum has changed. Of course, spectra will need to be scaled as if there were equal power in each of the two sub-periods. In actual fact, the power (i.e. variance) becomes considerably larger with later years and so the power in the second sub-period will be larger than that in the first. Comparing the spectrum over different sub-periods is particularly useful for identifying seasonal unit roots in the data. For example, if the seasonal pattern is gradually changing over time, so that the relative importance of this component of the time series alters in some way, one would expect this to be reflected in the spectra. More precisely, if the seasonal cycle was becoming more important over time, one would expect to see spikes at the seasonal frequencies that are larger in the spectrum for the second sub-period relative to those in the spectrum for the first sub-period.
It is in these ways that we use the spectrum to identify (the nature of) non-stationarities in the data used in this thesis. It is noteworthy that the power spectrum is often standardised by dividing through by the variance of the series. This standardised function is generally known as the spectral density and it is this concept that is used in this work.

3.4 The Concept of Cointegration

Granger and Newbold (1974) noted that problems may arise in regressions which include variables that are non-stationary. More accurately, the problem of regressing a non-stationary series on another will lead to statistical analysis to suggest a relationship between the two series even when one does not actually exist. This is known as the spurious regression problem and since most economic time series are non-stationary, it has disturbing implications for economic modelling. Estimating such a model will generally lead to a badly defined error and mean, rendering most conventional statistical inferencing invalid.

It has been suggested that first differencing may go some way to rectifying the problem. Many series will be stationary after applying such a transformation in which case valid statistical inferences may be drawn. However, models containing only differenced series will result in a loss of meaningful economic and long-run interpretation. Granger (1981) offered cointegration as an alternative solution which, in certain cases, enables the retention of long-run relationships even when the model is expressed in first differences.

Consider a pair of I(1) series, \(x_t\) and \(y_t\). Any linear combination of these is usually I(1). However, should the non-stationary behaviour in \(y_t\) exactly offset that of \(x_t\), then the linear combination given by

\[ u_t = y_t - ax_t \]  \hspace{1cm} (3.19)

will be I(0) and a very special relationship is said to exist. If this is so the variables are said to be cointegrated, and \(y_t = ax_t\), can be thought of as a long-run equilibrium. \(u_t\) can be thought of as the deviation of \(x_t\) and \(y_t\) from their long-run equilibria. If \(x_t\) and \(y_t\) were not cointegrated, i.e. \(u_t\) was I(1), then since \(u_t\) will wander far and wide from its
mean, the equilibrium concept cannot be applied. Strictly speaking of course, a non-stationary series has no properly defined mean.

In short, only if the variables are cointegrated can valid inference be carried out on the levels equation, and only if they are cointegrated will there exist any meaningful long-run relationship between them. If the I(1) variables are not cointegrated there will be no equilibrium relationship and the best we can do is to estimate a difference equation.

3.4.1 The Two-Step Procedure

Engle and Granger (1987) used the properties of the cointegrating parameter, $a$, which converges to its true value faster than standard econometric parameters, to propose a two-step estimating procedure for dynamic modelling. In simple terms, this involves carrying out the static cointegrating regression

$$y = ax_t + u_t$$

(3.20)

Using the OLS estimate $\hat{a}$, and then adding the residuals $u_t$ (the equilibrium error) to the dynamic first difference model

$$\Delta y_t = b_1 \Delta x_t - b_2 (y_t - a x_t)$$

(3.21)

This is an error correction model (ECM). Only if $x_t$ and $y_t$ are cointegrated, and thus the resulting $u_t$ is stationary, does equation 3.21 make sense. Granger (1981) noted this relationship between the error correction model and cointegration. It has since proved extremely useful in economic theory due to the fact that it provides an alternative to working in differences with no long-run interpretation.

It is noteworthy that an ECM generated using the Engle and Granger two-step procedure will not necessarily yield the same results as other modelling strategies such as general-to-specific. In the second stage of the two-step estimation procedure, all of the regressors are stationary whereas under the general-to-specific methodology this may not be the case for small samples. In addition, the small sample problem may result in misspecification bias in the first stage of the procedure, because short-run dynamics are not considered.
Although the two-step procedure based on OLS estimation has the major advantage of being relatively simple and intuitive, it does suffer a number of disadvantages. One disadvantage is that the distribution of the test statistics, such as the augmented Dickey-Fuller test, will, in general, be slightly different in any particular application. Although variations on statistics have been developed to suit particular situations (for example, the existence of a non-zero mean or drift parameter), the procedures do not cater for every circumstance and are often laborious to calculate. In general, the critical values can only be taken as a rough guide.

3.4.2 A Simple Alternative Test

An approach to testing for evidence of a long-run relationship between non-stationary variables has been suggested by White (1993). Test statistics are constructed from the residuals of a cointegrating regression. For \( M+1 \) time series, each of which is I(1), two common forms of cointegrating regression are:

\[
y_t = \beta_0 + \sum_{i=1}^{M} \beta_i x_{it} + u_t \quad (3.22)
\]

\[
y_t = \beta_0 + \beta_1 t + \sum_{i=1}^{M} \beta_i x_{it} + u_t \quad (3.23)
\]

Notice that equation 3.23 contains a trend term. The choice of regressand is somewhat arbitrary and different choices can be considered. A test for no cointegration is given by a test for a unit root in the estimated residuals, \( \hat{u}_t \). The augmented Dickey-Fuller regression is

\[
\Delta \hat{u}_t = \alpha_0 \hat{u}_{t-1} + \sum_{i=1}^{k} \alpha_i \hat{u}_{t-i} + \varepsilon_t \quad (3.24)
\]

There are a number of test statistics associated with this equation. The test statistic used here is a \( t \)-test for \( \alpha_0 = 0 \). Critical values for this test can be found in Davidson and MacKinnon (1993). If the test statistic is less than (i.e. more negative than) the critical value then there is evidence of cointegration. This implies that the residuals from the cointegrating regression (which contains only I(1) variables) are I(0). If the residuals are found to be I(0), there is evidence of a long-run relationship between the I(1)
variables. The lag length $k$ in equation 3.24 is determined according to the minimum AIC.

3.4.3 The Non-Uniqueness Problem

A more fundamental problem with the two-step procedure concerns the number of cointegrating combinations that exist between a set of variables. Consider the variables $x_t$ and $y_t$, which are again assumed to be integrated of order one. Now if $x_t$ and $y_t$ cointegrate with the parameter $a$, then

$$u_t = x_t - ay_t \sim I(0) \tag{3.25}$$

as stated earlier and can be shown to be unique. To see this, suppose we had a cointegrating parameter $b$, such that

$$\omega_t = x_t - by_t \sim I(0) \tag{3.26}$$

Adding and subtracting $by_t$ to equation 3.25 gives

$$u_t = x_t - (a - b)y_t - by_t \tag{3.27}$$

That is,

$$u_t = \omega_t - (a - b)y_t \tag{3.28}$$

By assumption, $u_t$ and $\omega_t$ are both $I(0)$, while $y_t$ is $I(1)$. Therefore, equation 3.28 can hold only if $a = b$, that is, if $a$ is unique. In the above example, uniqueness is demonstrated for two variables, $x_t$ and $y_t$. Unfortunately, once we consider more than two variables, it is no longer possible to demonstrate the uniqueness of the cointegrating vector. Instead, it turns out that if we have a vector of $N$ variables, each integrated of the same order, there can be up to $N$ cointegrating vectors. Thus, if we cannot rule out cointegration between a set of three or more variables, we have no guarantee that the least squares estimate is an estimate of a unique cointegrating vector. In a system with three variables, for example, it is quite possible that there are two statistically significant distinct cointegrating vectors and our least squares estimate is a linear of combination them.
3.4.4 The Johansen Procedure

Johansen (1988) suggests a method for estimating all the distinct cointegrating relationships which exist within a set of variables, and constructs a range of test statistics. The method begins by expressing the data generation process of a vector of $N$ variables, $X_t$, as an unrestricted vector autoregression in the levels of the variables

$$X_t = \Pi_1 X_{t-1} + \ldots + \Pi_k X_{t-k} + \mu + \phi D_t + U_t$$  \hspace{1cm} (3.29)

where each of the $\Pi_i$'s is an $N \times N$ matrix of parameters, $X_t$ is a $N \times 1$ vector of variables, the error $U_t$ is $\text{IN}(0, \Omega)$, $D_t$ is a vector of centred seasonal dummies which sum to zero over the full year, and $\mu$ is a constant. The system of equations in 3.29 can be reparameterised in ECM form such that

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-k} + \mu + \Phi D_t + U_t$$  \hspace{1cm} (3.30)

where

$$\Gamma_i = -(I_n - \Pi_1 - \ldots - \Pi_i), \hspace{1cm} (i = 1, \ldots, k - 1)$$  \hspace{1cm} (3.31)

and

$$\Pi = -(I_n - \Pi_1 - \ldots - \Pi_k)$$  \hspace{1cm} (3.32)

$I_n$ denotes the identity matrix. Notice that equation 3.30 is expressed as a traditional first differenced VAR model but for the term $\Pi X_{t-k}$. In fact, Johansen (1991) has demonstrated that a cointegrated system of variables can be represented in three main forms: the vector autoregressive, error correction and moving average forms. These representations are isomorphic to each other. In other words, Johansen (1991) proves that the Granger Representation Theorem (Engle and Granger (1987)) holds when $N>2$.

The main question here is whether the coefficient matrix $\Pi$ in equation 3.30 contains information about the long-run relationship between the variables in the vector $X_t$. There are three possible cases:

i) $\text{Rank (}$\Pi$\text{)} = N$, that is, the matrix $\Pi$ has full rank. This implies that the vector process $X_t$ is stationary and that model 3.29 should be estimated in levels.
ii) Rank (\(\Pi\))=0, that is, the matrix \(\Pi\) is the null matrix. This implies that there are no cointegrating vectors and consequently no long-run relationship between variables. The model should be estimated as a traditional first differenced VAR with no long-run component.

iii) \(\text{Rank (}\Pi\text{)}=\rho\), where \(0<\rho<N\). This implies that there are \(N\times\rho\) matrices \(\alpha\) and \(\beta\), such that \(\Pi=\alpha\beta\). That is, there are \(\rho\) cointegrating vectors.

It is the third case that is of most interest. The cointegrating vectors contained within the matrix \(\beta\) have the property that \(\beta X_t\) is stationary even though \(X_t\) itself is non-stationary. It is in this sense that equation 3.30 above can be thought of as an error correction model. The null hypothesis therefore, is that there are \(\rho\) cointegrating vectors. The test of this hypothesis is a test of whether

\[
\Pi = \alpha\beta'
\]  
(3.33)

where \(\beta\) can be interpreted as a matrix of \(\rho\) cointegrating vectors and \(\alpha\) is a matrix of weights attached to these vectors in each ECM. (Recall, the reparameterised VAR implies a system of \(N\) ECMs).

Johansen provides the maximum likelihood inference on \(\rho\), \(\alpha\), and \(\beta\). The log-likelihood function for the system expressed in equation 3.30 is

\[
L = -\frac{T}{2} \ln 2\pi - \frac{T}{2} \ln |\Omega| - \frac{1}{2} \sum_{t=1}^{T} U_t' \Omega^{-1} U_t
\]  
(3.34)

The goal is to choose \((\Omega, \Gamma, \Pi, \mu, \Phi)\) so as to maximise the log-likelihood function given by equation 3.34, subject to the constraint given by equation 3.33. The exposition below draws from the solution to this problem given in the work of Banerjee et al (1993, Section 8.2).

The first step is to concentrate \(L\) with respect to \(\Omega\). This yields the conventional result that \(\hat{\Omega} = T^{-1} \sum_{t=1}^{T} U_t U_t'\). Next, we remove the known \(I(0)\) variables from equation 3.30 to focus on the matrix of interest, \(\Pi\), which requires concentrating \(L\) with respect to \(\Gamma\). The effects of the lagged \(\Delta X_t\) terms can be removed from \(\Delta X_t\) and \(X_{t-k}\) by regression. That is, \(\Delta X_t\) and \(X_{t-k}\) should each be regressed on \(\Delta X_{t-1}, \ldots, \Delta X_{t-k+1}\) to obtain residuals \(R_{0t}\) and \(R_{kt}\), respectively. Thus, \(R_0\) and \(R_t\) are defined as
\[ R_{ot} = \Delta X_t - \sum_{i=1}^{k-1} \hat{\Gamma}_i \Delta X_{t-i} \]  
(3.35)

and

\[ R_{kt} = X_{t-k} - \sum_{i=1}^{k-1} \hat{\Gamma}_i \Delta X_{t-i} \]  
(3.36)

Notice, the ECM in 3.30 can be rewritten as

\[ \Delta X_t + \alpha \beta' X_{t-k} = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + U_t \]  
(3.37)

Removing the effects of \( k-1 \) lags of \( \Delta X_t \) on \( \Delta X_t \) and \( X_{t-k} \) just leaves \( U_t \). Thus, after such a correction, equation 3.37 can be written, in current notation, as

\[ R_{ot} - \alpha \beta' R_{kt} = U_t \]  
(3.38)

Returning to the main argument, the concentrated likelihood function \( L^*(\Pi) \) now depends only on \( R_{ot} \) and \( R_{kt} \) and takes the form

\[ L^*(\Pi) = \kappa - \frac{T}{2} \ln \left| \sum_{t=1}^{T} (R_{ot} - R_{kt})(R_{ot} - R_{kt})' \right| \]  
(3.39)

Next, we calculate the second-moment matrices of these residuals and their cross products as

\[ S_{oo} = T^{-1} \sum R_{ot} R_{ot}' \quad S_{ko} = T^{-1} \sum R_{kt} R_{ot}' \]  
(3.40)

\[ S_{ko} = T^{-1} \sum R_{kt} R_{ot}' \quad S_{kk} = T^{-1} \sum R_{kt} R_{kt}' \]  
(3.41)

In so doing, we can rewrite the log-likelihood function as

\[ L^*(\Pi) = \kappa_0 - (T/2) \ln \left| S_{oo} - \alpha \beta' S_{ko} \beta \alpha' + \alpha \beta' S_{kk} \beta \alpha' \right| \]  
(3.42)

If \( \beta \) were known, an estimate of \( \alpha \) could be found in the usual way, i.e. from a regression of \( R_{ot} \) on \( \beta R_{kt} \). However, in practice \( \beta \) is unknown and must be solved for.

Differentiating equation 3.42 with respect to \( \beta \) gives the following expression for \( \hat{\alpha} \) in terms of \( \beta \),

\[ \hat{\alpha}(\beta) = S_{ok} \beta (\beta' S_{kk} \beta)^{-1} \]  
(3.43)

Substituting this estimate of \( \alpha \) into equation 3.42 above and simplifying, we get...
For a constant matrix $S_{o0}$, maximising equation 3.44 is equivalent to minimising

$$|\beta'(S_{kk} - S_{ko}S_{oo}^{-1}S_{ok})\beta|$$

(3.45)

To locate the minimum, one proceeds as with LIML and imposes the normalisation $\beta'S_{kk}\beta = I_n$. The MLE now requires that one minimises the numerator of equation 3.45 with respect to $\beta$ subject to this constraint. This involves finding the saddle-point of the Lagrangean,

$$|\beta'(S_{kk} - S_{ko}S_{oo}^{-1}S_{ok})\beta| - \gamma[\text{trace}(\beta'S_{kk}\beta - I_n)]$$

(3.46)

Thus, the minimisation problem reduces to a generalised eigenvalue problem in which we need to solve a set of equations of the form

$$(\lambda S_{kk} - S_{ko}S_{oo}^{-1}S_{ok})\beta = 0$$

(3.47)

where $\lambda$ is obtained by solving

$$|\lambda S_{kk} - S_{ko}S_{oo}^{-1}S_{ok}| = 0$$

(3.48)

for the $\rho$ largest eigenvalues $\lambda_1 > \lambda_2 > \ldots > \lambda_\rho > 0$. The columns of $\beta$ are the corresponding eigenvectors,

$$\hat{\beta} = (\hat{\nu}_1, \ldots, \hat{\nu}_\rho)$$

(3.49)

Once $\hat{\beta}$ has been calculated, all the other maximum likelihood estimators can be obtained. For example, using equation 3.43 and the restriction $\beta'S_{kk}\beta = I_n$, $\hat{\alpha}$ is given by

$$\hat{\alpha} = S_{ok}\hat{\beta}$$

(3.50)

and thus

$$\hat{\Pi} = \hat{\alpha}\hat{\beta}$$

(3.51)

Now, we can obtain the complete set of eigenvectors $V=(\nu_1, \ldots, \nu_\lambda)$, corresponding to the eigenvalues $\Lambda=(\lambda_1, \ldots, \lambda_\lambda)$, by solving

$$(\lambda S_{kk} - S_{ko}S_{oo}^{-1}S_{ok})V = 0$$

(3.52)
subject to the normalisation $V' S_{kk} V = I_n$. From equation 3.52,

$$
\Lambda S_{kk} V = S_{ko} S_{oo}^{-1} S_{ok} V
$$

(3.53)

Since $V' S_{kk} V = I_n$, equation 3.52 reduces to

$$
V' \Lambda S_{kk} V = V' S_{ko} S_{oo}^{-1} S_{ok} V = \Lambda
$$

(3.54)

It is noteworthy that $\Lambda$ is ordered such that the first $\rho$ elements (denoted $\Lambda_\rho$) are the largest eigenvalues and the remaining $N-\rho$ elements are the smallest. The sub-matrix $\Lambda_\rho$ corresponds to $\beta$ which is a sub-matrix of the eigenvector matrix $V$.

Using equation 3.45 and the restriction $\beta' S_{kk} \beta = I_n$, the concentrated likelihood function given in equation 3.44 can be written as

$$
L^*(\beta) = \kappa_2 - (T/2) \ln |\hat{\beta}' (S_{kk} - S_{ko} S_{oo}^{-1} S_{ok}) \hat{\beta}|
$$

(3.55)

$$
L^*(\beta) = \kappa_2 - (T/2) \ln |\hat{\beta}' S_{kk} \hat{\beta} - \beta' S_{ko} S_{oo}^{-1} S_{ok} \beta'|
$$

(3.56)

Using 3.54, the concentrated log-likelihood function 3.56 can be rewritten as

$$
L^*(\beta) = \kappa_2 - (T/2) \ln |I_n - \Lambda_\rho|
$$

(3.57)

which reduces to

$$
L^*(\beta) = \kappa_2 - \frac{T}{2} \sum_{i=1}^{\rho} \ln (1 - \lambda_i)
$$

(3.58)

When $\Pi$ is not constrained to equal $a\beta'$, the unrestricted maximum of the log-likelihood function is given by

$$
L^*(V) = \kappa_3 - \frac{T}{2} \sum_{i=1}^{N} \ln (1 - \lambda_i)
$$

(3.59)

Since the $\rho$ largest eigenvalues deliver cointegrating vectors, and since $\lambda_{\rho+1}, \ldots, \lambda_N$, should be zero for non-cointegrating combinations, tests of the hypothesis that there are at most $\rho$ cointegrating vectors, and thus $N-\rho$ unit roots, can be based on twice the difference of between the unrestricted and restricted log-likelihoods. That is, the test of at most $\rho$ cointegrating vectors $0 \leq \rho \leq N$ is calculated as.
\[ L^{**}(V) - L^{**}(\beta) = -T \sum_{i=\rho+1}^{N} \ln (1 - \lambda_i) \]  

(3.60)

for \( \rho = N-1, N-2, \ldots, 0 \). This is known as the trace statistic, the critical values of which are given in Johansen (1989) and Osterwald-Lenum (1992). An alternative statistic, known as the maximal eigenvalue statistic, is given by

\[ \lambda_p^{\text{max}} = -T \ln(1 - \lambda_{p+1}) \]  

(3.61)

for \( \rho = N-1, N-2, \ldots, 0 \). Both tests are employed in the same sequence, \( N-1, N-2, \ldots, 0 \), with the number of cointegrating vectors being selected as one more than the first significant statistic.

The two tests do not necessarily give the same result due to their low power. As such, the theoretical priors of the econometrician often supplement the results of formal testing. Further, the number of cointegrating vectors may be determined, in part, by the feasibility of the eigenvectors. Whereas the response of some researchers faced with more than one cointegrating vector is to select that which makes most economic sense, an alternative is to partition the variables into endogenous and weakly exogenous variables, by testing the restrictions on the \( \alpha \) matrix which might then allow the estimation of a single equation. It is noteworthy that the validity of the concept of testing for weak exogeneity remains contentious. An applied example of the procedure is given in Johansen (1992).

Such a procedure is not of interest here, given our interest in forecasting, since the forecasts for one series requires past values of all other series in the model and thus the estimation of the system as a whole. However, for purely illustrative purposes, we outline the process by which exogeneity may be tested. Consider the hypothesis

\[ H_0 : \alpha_1 = 0 \]  

(3.62)

The appropriate \( A \) and \( B \) matrices for testing this restriction (shown here for an \( n \) equation system) are
The solutions to the eigenvalue problem given in equation 3.52, for the restriction that $\alpha_i=0$, will yield alternative eigenvalues. The test statistics for this hypothesis about $\alpha$ is then given by:

$$lr = T \sum_{r=1}^{N} \ln \left( \frac{(1-\hat{\lambda}_{ir})}{(1-\hat{\lambda}_{ir}^*)} \right)$$

(3.63)

where $\hat{\lambda}_{ir}$ is the unrestricted eigenvalue and $\hat{\lambda}_{ir}^*$ is the eigenvalue generated when the restriction $\alpha_i=0$ is imposed. If the restriction is found to hold for any $\alpha$, the implication is that the error correction term is insignificant in equation $i$ of the system. Thus, it is argued by some that it would be legitimate to drop that equation from the system. This is not, however, appropriate when the model is to be used for forecasting, since the forecasts for one series requires past values of all other series in the model and thus the estimation of the system as a whole.

As well as enabling the estimation of all distinct cointegrating relationships which exist in a set of variables, the Johansen procedure allows the correction of the cointegrating relations for short-run dynamics. The model here describes an inherent tendency to move towards long-run equilibrium without ever reaching it due to the fact that shocks act to repel it from its long-run path. Graphs of the cointegrating relations corrected for short-run dynamics will often appear more stationary than the cointegrating relations themselves. Attention is drawn to this fact in the applied example of the Johansen procedure given by Johansen and Juselius (1990).

Finally, it is noteworthy that when more than one cointegrating vector is found to exist it is, in general, not possible to assign a structural interpretation to them. This point has been forcibly made by Wickens (1993). He notes that one of the aims of VAR analysis is to provide a statistical representation of a set of variables which makes minimum use of a priori restrictions. In particular, there is no necessity, or presumption, that the variables could form a complete structural model on their own.
As a result, it is unusual to give a structural interpretation to the estimates of VAR models. The vector error correction model is usually formulated with similar disregard for structural considerations. Indeed, one of the main reasons for using a vector error correction model is that it is free from structural considerations. However, attempts are commonly made to give structural meaning to cointegrating vectors. Wickens (1993) notes that when the vector error correction model corresponds to a complete structural system, interpretation of multiple cointegrating vectors is, in principle, possible, although difficult without additional information. However, in general, the vector error correction model does not correspond to a complete structural system. If only exogenous variables are omitted, then the cointegrating vectors can be given a structural interpretation. If, on the other hand, endogenous variables are omitted then, in general, it is not possible to give a structural interpretation to the cointegrating vectors. Typically, we do not know whether or not a vector error correction model embodies a complete structural system.

As we have already noted VARs and VECM models are particularly useful for forecasting exercises, since the forecaster does not need to make assumptions about the values of exogenous variables in the forecast period. This is in contrast with standard econometric forecasting where forecasts have to be conditioned on knowledge of the exogenous variables. An additional benefit to the use of VAR models in forecasting exercises arises from the fact that such models represent a more flexible approach to modelling. Variables such as leading indicators can be readily incorporated into a VAR model, since in using atheoretical models the empirical model can differ from the underlying economic theory.

Despite the benefits of VAR modelling, there is a conspicuous absence of studies adopting this approach in the investment literature. Moreover, the approach has never been used to forecast private industrial and commercial construction output or construction activity more generally. In addition to developing VAR models consistent with the dominant theories of investment, we develop a number of VAR models for construction output and augment them with variables thought to be leading indicators of construction output. The forecasting performance of these models and the single equation investment models are compared with each other and with the performance of a benchmark ARIMA model.
This brings us to the issues surrounding the evaluation of forecasting performance. As we have already noted, there are a number of studies in the empirical investment literature which aim to systematically compare the forecast performance of alternative investment models. We considered these studies in Section 2.7. On the whole the methods of comparison are crude, being based on statistics summarising the sample evidence (e.g. the mean square forecast error) and visual inspection of plots of forecasts and forecasts errors. In this thesis we adopt more rigorous methods of comparing forecasts. We now set out some of the issues surrounding forecast evaluation and describe the methods of forecast evaluation to be adopted in this thesis.

3.5 Issues in Forecast Evaluation

Having discussed the estimation issues we must consider some of the underlying issues in forecast evaluation. We discuss some basic requirements of evaluation exercises and outline a number of frequently used measures of forecast accuracy. We then discuss some of the merits and limitations of these measures and define the parameters of the forecast comparison exercise to be adopted in this work.

3.5.1 Some Basic Requirements of a Forecast Evaluation Exercise

Practitioners frequently argue over the merits of particular models on the basis of theoretical specification, but given that it is generally difficult to make a clear case for one formulation over another, and given that of any set of economic forecasts are inevitably less than perfectly accurate, to have some method of forecast evaluation is of paramount importance.

Evaluation of forecast performance should, and does, generally take place at two levels. The first is the subjective level in which forecasts are examined for large errors or defects at turning points in an attempt to determine their inadequacies. If the practitioner has failed to allow for circumstances that were known, or at least could have been foreseen at the time the forecast was made, then it is beneficial to alter the forecast generating mechanism in some way to prevent the same mistake being made repeatedly. The danger in this approach, however, is the inherent tendency for the forecaster to explain away events that could not possibly have been foreseen at the
time the forecast was made. This point emphasises the need for an objective method for evaluating forecast performance.

Granger and Newbold (1986) have suggested three basic requirements for an objective evaluation exercise. Firstly, it should allow the analyst to assess whether one set of forecasts is any ‘better’ than its competitors. Secondly, it ought to allow the researcher to assess the ‘worth’ of a particular set of forecasts. Finally, it should be enable the researcher to judge whether a particular set of forecasts could be improved upon by modifying the forecast generating mechanism. As will been seen, these requirements are not easily fulfilled.

Granger and Newbold’s discussion of these goals is based on a quadratic cost of error function. Such a cost function has obvious advantages: it is mathematically more tractable than other functions, it bears an obvious relationship with the least squares criterion, often used to generate forecasting models, and it is not an unreasonable assumption a priori. The mean squared forecast error (MSFE) criterion, an obvious example of a quadratic loss function, can defined for n forecasts as

$$MSFE = \frac{\sum (F_t - A_t)^2}{n}$$

where $F_t$ denotes forecast values and $A_t$ denotes actual values for $t=1,...,n$. It is frequently used as the basis for assessing the quality of two or more sets forecasts. In empirical work, the forecast yielding the lower average squared error is usually judged superior. It is often desirable, however, to determine whether the forecast generating procedure is significantly better than the other(s) on the usual criteria of statistical significance. One might be tempted to compare variances using an $F$-test. However, Granger and Newbold note that comparing forecast generating procedures in this way can be misleading. Firstly, the assumption that the forecast errors gained from one forecast generating mechanism are uncorrelated with those of another is unreasonable. Second, multiple-step-ahead forecast errors are not generally white noise. In fact, even for optimal forecasts, the $h$-step ahead forecast errors will, in general, constitute a moving average process of order $h-1$. An optimal forecast is defined here as a prediction that makes use of all relevant information available at the time the forecast is made. Given this definition, it makes little sense to treat forecast optimality as a
useful working concept, since no economic forecast can really be judged the best that could possibly have been made given all the information in the universe.

As a result of these two general properties of forecast errors, the complexities to making valid comparisons between forecast generating procedures on the usual criteria of statistical significance have, until recently, been quite formidable. We will consider a relatively simple (and new) procedure by which such comparisons can be made in Section 3.5.3.

Granger and Newbold (1986) note that assessing the worth of a particular set of forecasts is also rather difficult. When there exists no competitor against which to judge a set of forecasts, Granger and Newbold suggest that it may be useful to construct one. Increasingly, forecasts are compared with so-called naive forecasts, i.e. forecasts generated without the benefit of economic theory. For example, Theil (1966) compared the mean squared forecast error of a forecast with a ‘no change rule’ in which the forecast values are given by the last observed value. We discuss this concept in Section 3.5.2. In addition, more sophisticated competitors have frequently been developed in the form of ARIMA models. Once constructed, the practitioner is confronted with the difficulties associated with assessing whether one set of forecasts is ‘better’ than a competing set.

However, Granger and Newbold question the value of constructing a naive competitor with which to compare against the model of interest. They regard the task of outperforming a naive model as rather undemanding and suggest that it would be more fruitful to consider a combination of the forecast of interest with a forecast from a naive competitor, such as an ARIMA model. If the variance of the combined forecast error is not significantly less than that of the forecast of interest, then the competing ARIMA forecast would appear to have no additional useful information. Granger and Newbold refer to this as ‘conditional efficiency’ and urge researchers to be dissatisfied with their models if the forecasts they generate are not conditionally efficient with respect to the ARIMA forecasts. Nelson (1972) suggests a framework for testing conditional efficiency. If we denote the set of one-step-ahead forecasts from the model of interest as $F^1_i$, the corresponding forecasts from an ARIMA model as $F^2_i$, and the observed time series as $A_i$, then conditional efficiency can be tested using the regression
The coefficient $\beta$ can be estimated using OLS and its difference from unity tested. If $\beta$ is insignificantly different from unity the ARIMA forecasts contain no additional useful information. That is, the set of forecasts from the model of interest are conditionally efficient with respect those from the ARIMA model. It is noteworthy that by reversing the definitions of $F^1_t$ and $F^2_t$ we can test whether the set of forecasts generated by the model of interest contains information in addition to the set generated by the ARIMA model. Of course, something is seriously wrong if the set forecasts from the model of interest contains no additional useful information. Note that this approach to assessing the worth of a set of forecasts is not restricted to the case where the competing set of forecasts are generated by an ARIMA model. Indeed, any two competing sets of forecasts can be judged in this way.

A number of measures of forecast quality have been developed that are not dependant on comparisons with competing forecasting models. However, there are many pitfalls associated with this line of attack. Granger and Newbold describe a procedure developed by Mincer and Zarnowitz (1969) which attempts to define a concept of forecast 'efficiency'. Mincer and Zarnowitz argue that this concept can be tested by estimating the regression

$$A_t = \alpha + \beta F_t + \varepsilon_t$$

by OLS. If the hypotheses of $\alpha = 0$ and $\beta = 1$ can not be rejected then, in the terminology of Mincer and Zarnowitz, the forecast is said to be 'efficient'. However, Granger and Newbold raise a fundamental objection to this concept of 'efficiency'. Whilst their criterion is clearly desirable, it says nothing about the variance of the forecast errors. A large number of sets of forecasts with differing error variances may satisfy this condition, but it does not make sense to say that all of these sets of forecasts are optimal. Only that set of forecasts satisfying the criterion and having minimum error variance can be considered optimal and typically, the minimum error variance is unknown. Thus, if the definition of forecast efficiency is to contemplate the existence of an optimal forecast, the condition that $\alpha = 0$ and $\beta = 1$ is a necessary, but not sufficient condition for forecast efficiency.
Granger and Newbold also note two practical objections to the Mincer and Zarnowitz test. First, $F_t$ and $\varepsilon_t$ are likely to be correlated for suboptimal forecasts, in which case estimates $\alpha$ and $\beta$ in equation 3.66 will be inconsistent. Second, for the usual test procedures to be valid, $\varepsilon_t$ must be white noise and this is not necessarily so for suboptimal one-step-ahead forecasts, and generally not so even for optimal forecasts of more than one step ahead.

In the absence of a competing forecast, attempts at evaluation have centred on comparisons of actual and forecast time series. The main problem with this approach is that one has no idea of the minimum attainable forecast error variance. This is not a serious problem if all series are equally difficult to forecast. However, this is not the case. The level of consumption, for example, is much less difficult to predict than change in stock market prices. Given this, a forecaster ought to be satisfied with forecasts of the latter that appear to be significantly worse than those of the former. However, provided these difficulties are borne in mind, it is possible to develop some measures which, under certain conditions, do convey information about the quality of a forecast. Granger and Newbold consider some of these measures. We do not discuss these in any great detail here since in our forecasting exercise we have a number of competing forecasts available.

In their discussion of the third requirement of a forecast evaluation exercise, namely that the exercise should enable the analyst to judge whether a particular set of forecasts could be improved upon by modifying the forecast generating mechanism, Granger and Newbold focus on the use of diagnostic checks. The emphasis is not on their use to assess the quality of a set of forecasts, but rather to suggest possible strategies for improving the forecast generating mechanism. In this context, the concept of conditional efficiency (described above) may be of some use. If a set of forecasts is found to be conditionally inefficient with respect to a set of forecasts generated by a univariate ARIMA model, the implication is that the forecaster has not optimally taken account of all the information provided by past values of the data. In this case, an examination of the lag and error structure in the relevant equations of the model may well suggest remedies for the defect.
Theil (1961) has suggested another means by which the source of forecast errors can be analysed. This is based on a decomposition of the mean squared forecast error into three components. Defining the mean squared forecast error as in equation 3.64, the decomposition may be written as

$$MSFE = (\bar{A} - \bar{F})^2 + (S_F - rS_A)^2 + (1 - r^2)S_A^2$$  \hspace{1cm} (3.67)

where \(\bar{A}\) and \(\bar{F}\) are the means of \(A\) and \(F\) respectively, \(S_A\) and \(S_F\) are the sample standard deviations and \(r\) is the simple correlation between \(A\) and \(F\). Dividing through by \(MSFE\) gives

$$1 = UM + UR + UD$$  \hspace{1cm} (3.68)

where

$$UM = (\bar{A} - \bar{F})^2/MSFE, \hspace{0.5cm} UR = (S_F - rS_A)^2/MSFE,$$

and

$$UD = (1 - r^2)S_A^2/MSFE$$  \hspace{1cm} (3.69)

Interpretation of \(UR\) and \(UM\) is helped by reconsidering equation 3.66, for which the least squares estimates are given by

$$\hat{\beta} = S_{AF}/S_F^2 \hspace{0.5cm} \text{and} \hspace{0.5cm} \hat{a} = \bar{A} - \hat{\beta}\bar{F}$$  \hspace{1cm} (3.70)

where \(S_{AF}\) is the sample covariance of \(A\) and \(F\). Note that if \(\hat{\beta} = 1\) and \(\hat{a} = 0\) then \(UM=0\) and moreover, given that \(S_F - rS_A = S_F(1 - \hat{\beta})\), \(UR=0\). This suggests the following interpretation for \(UM\), \(UR\) and \(UD\). \(UM\) is the proportion of MSFE due to the mean error or bias in the forecast, \(UR\) is the proportion of the MSFE due to the coefficient \(\beta\) differing from unity and therefore \(UD\) is the proportion not explained by the mean or slope error. \(UD\) is sometimes known as the unexplained error. In this sense, it is desirable for a set of forecasts to have a low mean and slope error and a high disturbance or unexplained error. As noted above, we can not say that a set of forecasts with zero mean and slope error are optimal, since we are usually unable to say whether the set of forecasts has the minimum possible forecast error variance. If \(UM=UR=0\), then
\[ MSFE = (1 - r^2)S^2_A \]  

(3.72)

However, there may be a number of sets of forecasts satisfying this criterion, each generating different MSFE. Under certain conditions this criterion allows us to select between alternative sets of forecasts, but it does not allow us to determine whether a set of forecasts is optimal since typically we do not know the minimum possible MSFE given all the relevant information. It is noteworthy that although \( U_M = U_R = Q \) is satisfied when \( \alpha = 0 \) and \( \beta = 1 \) in equation 3.66, the practical objections to estimating such a relationship using OLS, outlined above, apply equally here. However, if used correctly, Theil’s decomposition of the MSFE can provide insights in the sources of forecast errors and a means by which forecast quality can be assessed in the absence of competing forecasts.

As an alternative to such decompositions in the absence of competing forecasts, Granger and Newbold suggest an analysis of the time series properties of the forecast error itself. Such an examination may provide useful, and sometimes decisive, conclusions on forecast quality and the sources of forecast errors. For example, the forecast error can easily be tested for a zero mean. In addition, an optimal \( h \)-step ahead forecast may have significant error autocorrelations up to order \( h \) and zero autocorrelations beyond \( h \). If such a pattern is not exhibited in the sample autocorrelation function, the forecast error must be correlated with something that was known at the time the forecast was made and hence the forecast could have been improved upon. However, as Granger and Newbold note it is rare that a sufficiently long error series exists to permit an analysis of a correlogram or spectrum.

As is clear from the above discussion, evaluating the quality of a set of forecasts is not straightforward. In this study we have available sets of forecasts from a number of competing models. In Section 3.5.2 and 3.5.3, we consider practical means by which the accuracy of different sets of forecasts can be compared. As a starting point we also consider a number of frequently used descriptive measures of forecast accuracy, some of which assume alternative cost of error functions. Alternative cost of error functions are considered in Granger (1969). Much discussion has focused on the relative merits of these descriptive measures of forecast quality and a basic introduction to these issues is given in Holden, Peel and Thompson (1990). There are many potential...
pitfalls for the practitioner and great care must be exercised in designing a framework within which forecast comparisons can be made. In Section 3.6 we outline the framework for comparison adopted in this thesis.

3.5.2 Some Descriptive Measures of Forecast Accuracy

We begin by considering a number of frequently used measures of forecast accuracy. The mean squared forecast error, defined in equation 3.64, is perhaps the most popular of the descriptive measures for assessing the accuracy of a set of forecasts. It is based on a quadratic cost of error function. We noted some of the merits of a quadratic loss function in Section 3.5.1. The MSFE assigns the same cost to a positive error as to negative error. Larger errors are penalised more heavily than smaller errors because the errors are squared. Since the units of the MSFE are the square of the units of $F_h$, it is common to take the square root of the MSFE to give a measure called the root mean squared forecast error (RMSFE). An alternative way of expressing the MSFE is to define $e_t$ as the forecast error $(F_t - A_t)$ and then equation 3.64 can be written as

$$MSFE = \frac{1}{n} \sum e^2_t = \frac{1}{n} \sum (e - \bar{e} + \bar{e})^2 = \frac{1}{n} \sum (e - \bar{e})^2 + \bar{e}^2$$ (3.73)

The first term is the variance of the forecast error and the second term is the square of the error mean. Therefore, the MSFE is an increasing function of the variance and the mean of the error. Since an optimal forecast will be unbiased and efficient, the mean error will be zero and variance of the forecast will be small, resulting in a small MSFE.

The mean absolute forecast error (MAFE), defined by

$$MAFE = \frac{1}{n} \sum |F_t - A_t|$$ (3.74)

measures the absolute size of the error and hence large errors are not penalised by the effects of squaring. As such, the measure corresponds to a linear loss function. As with the MSFE, positive and negative errors are assigned equal cost.
Analogous to the MSFE and the MAFE are the mean square percentage forecast error (MSPFE) and the mean absolute percentage forecast error (MAPFE). These measures are respectively defined by

$$\text{MSPFE} = \frac{100 \sum (A - F)^2}{n|A|}$$  \hspace{1cm} (3.75)

and

$$\text{MAPFE} = \frac{100 \sum (A - F)}{n|A|}$$  \hspace{1cm} (3.76)

As their names suggest, these measures take the error as the percentage of actual value. As such, for a given absolute forecast error, both criteria penalise more heavily when the value of the actual series is low. A variation of the MAPFE is the median absolute percentage forecast error (MdAPFE). This is simply the median value of the absolute percentage errors and it is preferred when these errors have a skewed distribution, in which the mean is distorted by a few extreme outliers.

One of the problems with the MSFE and MAFE is that they are both affected by the units of measurement of the data. Theil (1961) has attempted to develop a unit free measure of forecast quality. This measure is known as the inequality coefficient and is defined as

$$U_1 = \frac{[\sum (F_i - A_i)^2/n]^{1/2}}{[\sum F_i^2/n]^{1/2} + [\sum A_i^2/n]^{1/2}}$$  \hspace{1cm} (3.77)

Notice that the numerator is the RMSFE and the denominator is the sum of the root mean squares of the forecasts and actuals. The advantage of the inequality coefficient lies in the fact that it lies between zero and one. If the forecast for each period in the forecast horizon is correct, $U_1$ takes on the value zero. If, on the other hand, the forecasts are all zero and the actual values are non-zero, or vice versa, $U_1$ takes the value of unity. As such, a low value for $U_1$ indicates an accurate forecast. This measure of forecast accuracy has been criticised on the grounds that it is difficult to interpret and has the potential to given misleading results (see Granger and Newbold (1986), p. 279). For this reason, Theil (1966) proposed using an alternative statistic, $U_2$, defined by
in which the MSFE is scaled by the mean squares of the actual outcomes. The \( U_2 \) statistic was initially posited for forecast changes, but it is commonly used for forecast of levels also. Again, the minimum value is zero (implying perfect forecasts), but there is no maximum value. A value for \( U_2 \) equal to unity simply corresponds to all forecasts being equal to zero. This is of no particular relevance when data is in levels where a zero forecast is unlikely, but for the forecast of changes a value of unity is given by the no-change forecast.

All the criteria defined above are generally used as descriptive statistics for summarising the characteristics of the sample evidence. In this sense, the forecast yielding the lowest value for a given criterion is judged the most accurate forecast. In addition, these measures are said to be parametric in the sense that they depend on desirable properties of means and variances which occur when the underlying distribution is normal. For example, the use of the MSFE implicitly assumes that each of the forecast errors has the same mean and variance and that these are constants. It can be argued that forecast comparisons over time are misleading since a new situation arises each time a forecast is made. As time passes, the forecasting technique may become more sophisticated, new data becomes available on the accuracy of past forecasts enabling persistent errors to be identified and new personnel may be brought into a forecasting team. The result of these factors is that the forecast error is unlikely to be well behaved, i.e. the mean and variance are unlikely to be constant over time. Such criticism has led to the development of non-parametric measures of forecast accuracy which do not assume a normally distributed population. Two such measures are the sign test and the rank test.

The former test is based on the percentage of times that a particular forecast is better than an alternative competing forecast. In the rank test, the original numerical measure of accuracy, which might be the MSFE for a number of forecasts for example, is replaced by a ranking which is then tested for significance. While these tests have some attractions, particularly when forecasting ordinal series, they ignore important information which is readily available: both the sign and the rank test ignore the numerical size of errors.
It is well known that the rankings across models are not invariant to the criterion used to measure forecast accuracy. The ranking of models may be sensitive to the variables used in these models, to the length of the forecast horizon, or to whether the models are expressed in levels or first differences. Given this, it is important to know which measure of forecast accuracy is most 'appropriate' given the circumstances surrounding a particular forecasting exercise. Armstrong and Collopy (1992) provide an empirical comparison of six error measures, judged against a number of criteria such as reliability, construct validity, sensitivity and outlier protection. The six measures are the RMSFE, MAPFE, MdAPFE, two versions of the relative absolute error, (RAE, calculated as the absolute forecast error over the forecast error obtained from a random walk), and the sign test (in which the alternative forecast generating mechanism is also a random walk).

In their preliminary discussion, Armstrong and Collopy (1992) note that it is important that any measure of forecast accuracy used across time series be unit free. For this reason data was scaled prior to analysis. Although Theil's $U_2$ statistic provides a unit free measure of forecast accuracy, Armstrong and Collopy (1992) do not include this measure among those they consider, quoting a survey conducted by Carbone and Armstrong (1982) which states that only two per cent of academics and practitioners use this statistic as their preferred measure of forecast accuracy. Instead, Armstrong and Collopy (1992) include the relative absolute error (RAE) among those measures they consider on the grounds that it is an easily understandable alternative to the $U_2$ statistic.

The results of the Armstrong and Collopy (1992) study are quite consistent with previous findings and are summarised as follows. In an attempt to assess their reliability, each measure is repeatedly applied in a number of forecasting exercises. Those measures found to consistently rank models correctly are deemed most reliable. Armstrong and Collopy find that the sign test performs best but neither the RMSFE or the MAPFE do particularly well. However, both the RMSFE and the MAPFE perform well on construct validity criteria, that is, they do actually measure what they purport to measure. As one would expect, RMSFE is not rated highly in terms of its ability to be unaffected by outliers. This is due to the fact that it is based on a quadratic loss function and so the errors are squared. The MdAPFE, on the other hand, performs
well in terms of the effect of outliers, but this is to be expected since it was originally developed from the MAPFE specifically because of its relative insensitivity to outliers. In terms of the sensitivity of measures (to changes in parameter estimates, for example), Armstrong and Collopy find that RMSFE and the MAPFE perform relatively well.

Although their study was reasonably comprehensive, Armstrong and Collopy admit that it is far from exhaustive. For example, a number of commonly used measures of forecast accuracy, designed specifically to counteract weaknesses in the measures analysed, were not included in the study. In addition, the results of their study do not necessarily apply when examining forecasts of a single time series, rather than a set of forecasts from a number of time series. Although the study summarises the stock of knowledge on the relative merits of measures of forecast accuracy, as Ahburg (1992) notes, it does not provide a single answer to the question of which measure is the most appropriate construct with which to evaluate forecast accuracy. Of course, there is no single answer.

The appropriate cost of error function used should be related to actual costs of making a forecast error. If large errors are proportionately more costly to the forecaster than small errors, then a quadratic cost of error function is appropriate. However, there are a number of available measures of forecast accuracy consistent with a quadratic cost of error function. Which of these is appropriate will depend on the precise nature of the forecasting exercise. Of course, in many instances, for example macro forecasts made by independent institutions, the costs of making an error are notional rather than real since decisions are not generally made on the basis of forecasts. In this case, the choice of cost of error function is somewhat arbitrary.

Armstrong and Collopy (1992) find that the RMSFE is found to perform reasonably well against other measures in all tests except that of reliability and outlier protection. However, Chatfield (1992) notes that many of the reliability problems associated with the RMSFE are only arise when one attempts to average through a number of series, say in a system. This is due to the measure’s scale dependancy. When one considers an individual time series, the reliability problems of the RMSFE are much less severe and, moreover, when multiple time series are considered, the measure can be adapted to avoid scale dependancy. Wallis et al (1987) avoids the problem of
scale-dependency when comparing forecasts for four different variables from a number of macroeconomic models by dividing the RMSFE for each variable by its average value. This gives a relative root mean square forecast error which will, of course, be low for accurate forecasts. Taylor (1992) suggests that taking logarithms of the data and then using the RMSFE overcomes many problems of the RMSFE, including that of reliability. It is noteworthy that this is roughly equivalent to using the root mean square percentage error (RMSPFE). Fildes (1992) and Ahburg (1992) suggest that an alternative adaptation of the RMSFE, namely the geometric mean squared forecast error (GMSFE), also overcomes the reliability problem related to scale-dependence.

Whilst the RMSFE is the most widely used measure of forecast accuracy, Ahburg (1992) notes that the MAPFE statistic is the most popular unit free measure. In the study by Armstrong and Collopy (1992) the MAPFE statistic performs relatively well. Theil’s $U_2$ statistic, which is a much less widely used unit free measure of forecast accuracy, is regarded by many (Collopy (1992), Ahburg (1992) and Fildes (1992), for example) as more reliable than MAPFE and RMSFE statistics, especially when dealing with more than one time series. The difficulty in interpreting the statistic, often suggested as the reason for its infrequent adoption, should not in itself lead a researcher to reject it as a measure of forecast accuracy, since this does not detract from its reliability in ranking models.

3.5.3 Practical Methods for Comparing Forecasts

As noted above, the criteria defined above are descriptive statistics that summarise the characteristics of sample evidence. In this sense, the forecast yielding the lowest value for a given criterion is judged the most accurate forecast. However, one is often interested in whether one forecasting procedure performs significantly better than another under the usual criteria of statistical significance. Ashley, Granger and Schmalansee (1980) have developed a test by which two or more MSFEs can be compared to see if one is significantly smaller than the other(s). The starting point is the recognition of the fact that the MSFE is calculated as the sum of the forecast error
variance and the mean as shown in equation 3.73. Reproducing this in slightly different notation gives

\[
MSFE(e) = \text{var}(e) + \text{mean}(e)^2
\]  

(3.79)

where var(e) is the sample variance and mean(e) is the sample mean of the forecast error. For two sets of n forecasts with white noise errors, \(e_1\) and \(e_2\), the difference between their MSFE is given by

\[
MSFE(e_1) - MSFE(e_2) = \text{var}(e_1) - \text{var}(e_2) + (\text{mean}(e_1)^2 - \text{mean}(e_2)^2)
\]  

(3.80)

Defining \(s\), as the sum of the errors \(e_1\) and \(e_2\) and \(d\), as the difference between errors \(e_1\) and \(e_2\), we can write the covariance between \(s\) and \(d\) as

\[
\text{cov}(s, d) = \text{var}(e_1) - \text{var}(e_2)
\]  

(3.81)

Therefore, the difference between the two MSFEs can be related to the covariance of \(s\) and \(d\) and the mean errors in the following manner.

\[
MSFE(e_1) - MSFE(e_2) = \text{cov}(s, d) + (\text{mean}(e_1)^2 - \text{mean}(e_2)^2)
\]  

(3.82)

Thus, if the forecasts \(F_2\) are more accurate than \(F_1\), MSFE(\(e_2\)) will be smaller than MSFE(\(e_1\)) and thus the right hand side of equation 3.82 will be positive. Therefore, a test of whether the MSFE is smaller for \(F_2\) can be based on whether the combination of the covariance and the difference between the squares of the errors is positive. This can be done by estimating the relationship

\[
d_t = \beta_1 + \beta_2(s_t - \bar{s}) + u_t
\]  

(3.83)

Ashley et al (1980) point out that \(u\) is correlated with \(s\), but they argue that the resulting bias in \(\beta_2\) is likely to be negligible in moderately sized samples. The least squares estimators of the parameters are

\[
\beta_1 = \bar{d} \quad \text{and} \quad \beta_2 = \frac{\text{cov}(s, d)}{\text{var}(s - \bar{s})}
\]  

(3.84)

where the hat denotes an estimated term. If there is no difference between the two MSFE, then \(\beta_1\) and \(\beta_2\) will be close to zero. A rejection of the test of the joint hypothesis that \(\beta_1 = \beta_2 = 0\) implies that the difference between the two MSFEs is
statistically significant. If the forecasts $F_2$ have a smaller MSFE than $F_1$, then either $\beta_1$ or $\beta_2$ or both will be positive. If the coefficients are significantly negative, then one can conclude that the forecasts $F_2$ are not more accurate than $F_1$. The individual coefficient can be tested by the standard $t$-test with $n-2$ degrees of freedom. However, the conditions under which this procedure can be validly applied are unlikely to hold in practice. For example, $e_{1t}$ and $e_{2t}$ are unlikely to be white noise for suboptimal one-step-ahead forecast and will generally not be white noise even for optimal forecasts of more than one step ahead.

Diebold and Mariano (1995) develop a much more versatile test which can be used to compare a set of forecasts of more than one step ahead. We consider the case in which the forecast performance is judged using the MSFE, although the test presented by Diebold and Mariano allows for more general cost of error functions. The null hypothesis is that of equality of expected forecast performance

$$E[e_{1t}^2 - e_{2t}^2] = 0$$  \hfill (3.85)

Defining

$$D_t = e_{1t}^2 - e_{2t}^2$$ \hfill (3.86)

the test is based on the observed sample mean

$$\bar{D}_t = n^{-1} \sum_{t=1}^{n} D_t$$ \hfill (3.87)

A difficulty arises here since $D_t$ is likely to be autocorrelated and indeed for optimal $h$-step ahead forecasts, the sequence of errors follows a moving average process of order $h-1$. This result can be expected to hold for any reasonably well conceived set of forecasts. It is therefore assumed that all autocorrelations of order $h$ or higher in the series $D_t$ are assumed to be zero. In this case, it can be shown that the variance of $\bar{D}_t$ is asymptotically given by

$$V(\bar{D}_t) \approx n^{-1} \left[ \gamma_0 + 2 \sum_{k=1}^{h-1} \gamma_k \right]$$ \hfill (3.88)

where $\gamma_k$ is the $k$th autocovariance of $D_t$ which can be estimated as
Given this, the Diebold-Mariano test statistic is given by

\[ S_1 = \left[ \hat{V}(\hat{D}) \right]^{-1/2} \hat{D} \]  \hspace{1cm} (3.90)

where \( \hat{V}(\hat{D}) \) is obtained by substituting equation 3.89 into 3.88. Under the null hypothesis, this statistic has an asymptotic standard normal distribution so that in practice tests can be implemented very easily. Simulation evidence presented by Diebold and Mariano (1995) suggests that for moderately sized samples the test is somewhat over-sized. However, Harvey, Leybourne and Newbold (1997) suggest a simple modification to this test that considerably improves its performance. The modified test statistic is defined as

\[ S_1^* = \left[ \frac{n + 1 - 2h + n^{-1} h(h - 1)}{n} \right]^{-1/2} S_1 \]  \hspace{1cm} (3.91)

and Harvey et al suggest that this statistic should be compared with critical values from a \( t \)-distribution rather than a normal distribution. They also note that whilst some size distortion remains, the test is certainly adequate for practitioners.

As noted above, the test is not restricted to forecast comparisons with a quadratic cost function. By defining equation 3.85 and 3.86 in terms of some function of \( e \), say \( g(e) \) the tests can be applied to other standards of forecast accuracy such as the MAPFE. The procedure is identical to that described above but for the definition of \( D_\tau \).

Another practical method for comparing forecasts relates to the concept of conditional efficiency, as described by Granger and Newbold (1986) and discussed in Section 3.5.1. Here, the forecast of interest can be combined with a competing forecast from a univariate ARIMA model. If the variance of the combined forecast error is not significantly less than that of the forecast of interest, then the ARIMA would appear to contain no additional useful information. Granger and Newbold urge researchers to be dissatisfied with their model if the forecasts it generates are not conditionally efficient with respect to the ARIMA model. The concept of conditional efficiency can be tested for one-step-ahead forecasts by estimating equation 3.65 using OLS and testing the hypothesis that \( \beta \) equals unity. If \( \beta \) is significantly different from unity, the forecasts
of interest are not conditionally efficient with respect to the ARIMA forecasts. Unfortunately, this procedure is inappropriate for forecasts of more than one period ahead, since in general the corresponding forecasts errors are not well behaved.

The concept of conditional efficiency and the procedure suggested by Diebold and Mariano (1995) and modified by Harvey et al (1997) are the main methods by which forecast comparisons are made in this thesis.

3.5.4 Appropriate Criteria for Assessing Forecasts of Private Industrial and Commercial Construction Output

Given our interest is in forecasts of a single time series, namely private industrial and commercial construction output, the problems with descriptive measures of forecast quality discussed above, related to scale dependency and units of measurement, are not of first order importance. However, when forecasts are generated from different models, it is important that like with like comparisons are made. More accurately, care must be taken when comparing forecasts of the actual level of private industrial and commercial construction output generated from one model with the forecasts of the logarithm, or the first differences, of this quantity generated using an alternative model. Some of the problems associated with the application of descriptive measures of forecast accuracy across different time series also apply to comparisons of forecasts based on different transformations of the same time series. Therefore, if a particular model forecasts something other than the actual level of private industrial and commercial construction output, say for example, the log or first differences of the series, it will be necessary to transform forecasts from this model in such a way that they correspond to the actual levels of the series. For example, if a model generates forecasts for the logarithm of the series, then the exponent of the forecasts (and outcomes) are taken prior to calculating the value of the forecast accuracy criterion. In so doing, the accuracy of forecasts from this model can be directly compared with those from a model of the actual level of the series by comparing the values of the criterion.

In making such transformations prior to calculating the value of the criteria, potential problems associated with units of measurement and scale dependency are avoided.
The MSFE and MAPFE statistics, for example, can be validly used as a means of assessing the accuracy of alternative forecasts. Recall, that the former measure is based on a quadratic cost function, whereas the latter is based on a linear cost function. To this extent, the two measures may deliver alternative ranking of models if forecast errors contain outliers. Given that these measures are perhaps the two most widely used objective measures of forecast accuracy, and that one may be more useful than the other in certain circumstances, we use both as a means of assessing the accuracy of forecasts of private industrial and commercial construction output in the following sections. In addition to these criteria, we adopt a third objective criterion by which forecast accuracy may be judged. This is Theil’s $U_2$ statistic. By using three measures of forecast accuracy rather than one, we are more likely to arrive at the ‘correct’ ranking of models.

In order to test whether one set of forecasts is significantly more accurate than another, we use the approach suggested by Diebold and Mariano (1995), modified by Harvey et al (1997) and discussed in Section 3.5.3. We also make use of the concept of conditional efficiency to assess the quality of one-step-ahead forecasts from the econometric investment models and the VAR models.

In addition to these objective measures of forecasting accuracy, we adopt a more informal approach to forecast evaluation based on a graphical analysis of the forecast and outcome data. Of course, much of the information contained in the time series plot of the outcome and forecast data is summarised by the objective measures. However, in cases where the ranking of forecasts is not agreed upon by the objective measures, the time series plot may provide additional information regarding the causes of alternative rankings which may help determine the ‘correct’ ranking.

### 3.6 The Forecasting Contest

In Chapter 5, several models of private industrial and commercial construction output will be estimated. In Chapter 6 these models will be used to generate forecasts of private industrial and commercial construction output for the period 1994q1 to 1996q4. The forecasts for the twelve quarter horizon are made using only the information on private industrial and commercial construction output that was
available at the end of 1993. As discussed in the previous section, the accuracy of forecasts over the twelve quarter forecasting horizon is assessed using the MSFE, the MAPFE, the $U_2$ statistics, the concept of conditional efficiency, the modified Diebold-Mariano test and some informal analysis of time series plots of forecasts and forecast errors.

In essence, the models of private industrial and commercial construction output to be estimated in Chapter 5 fall into one of three categories. First, we estimate models of private industrial and commercial construction output consistent with the traditional models of investment considered in Chapter 2. These can be thought of as econometric models. Second, we estimate ARIMA models of private industrial and commercial construction output (one of which will form the benchmark model), which are based on time series techniques. Models of this kind are commonly referred to as time series models. The third set of models to be estimated in Chapter 5 are less theoretically rigorous and make use of VAR analysis.

For each type of model, estimation is to be carried out with a different transformation of the private industrial and commercial construction output series. For example, in the VAR models the data for private industrial and commercial construction output is logged, whereas for the econometric models the data was not logged. Care must therefore, be taken when comparing forecast performance. For example, forecasts made by an econometric model actual data can not straightforwardly be compared with the forecasts generated by a VAR model using logged data. In order to allow fair comparisons of forecast accuracy to be made when using different types of models (or more generally, different transformations of the data) all forecasts are transformed in such a way that they are comparable with the level of actual private industrial and commercial construction output prior assessing accuracy.

The forecasting performance of a particular model is first compared with the performance of other models of the same type. That is, the accuracy of a particular econometric model for example, is first compared with the performance of the other econometric models estimated in this work. The performance of these models are then compared with the forecast performance of the benchmark ARIMA model. Only then are comparisons between the forecast performance of econometric models and VAR models made.
Comparisons of performance over the 12 quarter forecast horizon are made in terms of three sets of forecasts: one-step-ahead (or static) forecasts, four-step-ahead forecasts and twelve-step-ahead (or dynamic) forecasts. For one-step-ahead forecasts, each quarterly forecast of private industrial and commercial construction output is calculated with full knowledge of private industrial and commercial construction output in the previous quarter. This experiment, however, provides little understanding of how well a forecaster would have fared by using the model to forecast private industrial and commercial construction output for the period 1994 to 1996 from the vantage point of the fourth quarter of 1993. Nevertheless, the static forecast’s error performance can be compared to the model’s error performance during the period of estimation. In other words, static prediction gauges whether the equation is able to track the data during the forecast horizon as well as it does during the estimation interval. The dynamic twelve-step-ahead forecasts do not benefit from knowing private industrial and commercial construction output in previous periods. Unlike the static forecasts, the dynamic forecast can not be kept from straying far from actual private industrial and commercial construction output by constantly checking its previous errors and incorporating corrections for them in subsequent forecasts. Actual data for private industrial and commercial construction output are not used to derive forecasts. Instead, the forecasts use previous estimates of private industrial and commercial construction output to derive new forecasts. Dynamic forecast errors therefore can not be compared easily to static forecast errors. In addition, given a forecast horizon of 12 quarters, there is only one twelve-step-ahead forecast from each model. Therefore, it is difficult to make comparisons of forecasts based on the usual criteria of statistical significance. The performance of the dynamic forecast suggests how well a model might estimate private industrial and commercial construction output several years into the future. The four-step-ahead forecasts use estimates for private industrial and commercial construction output in the previous three quarters and actual data before that. These forecasts are likely to give an indication of models’ performance over a longer horizon and since, we are able to construct nine four-step-ahead forecasts for each model (given the 12 quarter forecast horizon), comparisons based on statistical significance are possible.
Given that we have the out-turn data for private industrial and commercial construction output over the 1994q1 to 1996q4 forecast horizon, the one-step-ahead and multiple-step-ahead forecasts are said to be *ex post* (within sample). More importance is attached to the dynamic forecasting performance of these models, since *ex ante* (out-of-sample) we are usually interested in forecasts of a series more than one quarter ahead.

In Chapter 6 of this thesis we will compare the forecasts generated by the econometric and VAR models with the forecasts generated by the benchmark ARIMA model over alternative horizons. However, whilst we may be able to determine a 'winning' model using the concepts of conditional efficiency and the modified Diebold-Mariano test, we can be sure that the forecasts from this model will not be optimal since economic forecasts are necessarily suboptimal. Given that the residuals from a regression of the actual data on the forecast data (as in equation 3.66) are unlikely to be white noise, we can not even test whether the forecasts fulfil the necessary conditions for optimality described by Mincer and Zarnowitz (1969) and discussed in Section 3.5.1.

### 3.7 Concluding Remarks

In the first part of this chapter we have outlined issues related to the estimation of models of private industrial and commercial construction output. We focus on the time structure of the investment process in the traditional models of investment and on the issues of non-stationarity, spurious regressions and cointegration. We have also described the approach to estimation to be adopted in this thesis. In addition to estimating models consistent with the traditional theories of investment, we will also estimate models of private industrial and commercial construction output using a more flexible approach to modelling, namely VAR analysis. We have outlined this procedure and discussed its benefits in a forecasting exercise in Section 3.4.4. This approach has not hitherto been applied either in the investment field or in the modelling of construction activity. The latter part of the chapter is concerned with issues related to the evaluation of forecasts. As noted in Section 3.4.4, the methods of evaluation of forecast performance in the empirical investment literature are somewhat rudimentary. In this thesis we will be considering the relative forecast
performance of the econometric investment models and VAR models using modern sophisticated tools of analysis. We have discussed the criteria by which forecast accuracy is assessed in this thesis and describe the parameters of the forecasting contest in Sections 3.5 and 3.6.

In the following chapter we describe the data, its sources, problems in constructing variables and the limitations of the resulting series. In addition, we perform some preliminary statistical analysis as a precursor to estimating models of new private industrial and commercial construction output. This statistical analysis focuses primarily on determining the order of integration of each of the time series to be used in this work. To do this we employ the techniques described in Section 3.3 above.
4 Data Description and Preliminary Statistical Analysis

4.1 Introduction

The objective of the first part of this chapter is to describe the data necessary to estimate models of private industrial and commercial construction output. To the extent that construction output represents capital formation, a number of the models to be developed in the next chapter are consistent with investment theory. To this end, some of the variables discussed in this chapter are also consistent with investment theory. Recall, from the outline of investment theory given in Chapter 2, that there are four main theories of investment; the accelerator, neoclassical, putty-clay and Tobin's $q$. In addition to these dominant theories, there are other theories of investment (also outlined in Chapter 2) which have not received the same attention in the literature. These theories suggest alternative determinants of investment. Moreover, there are a number of other variables, that are not entirely consistent with any one theory of investment, that have frequently been found to be significant in the empirical literature. These variables, and others thought to be indicators of construction output or investment, may legitimately be included in a VAR model. We therefore consider these variables in this chapter.

A number of variables emerge from the dominant theories as important determinants of investment. In the accelerator model, investment is determined by changes in the level of output. The neoclassical model posits a role for the user cost of capital, in addition to that for output, although in the pure version of the model Jorgenson's maintained hypothesis of a Cobb-Douglas production function and subsequent derivation of desired capital stock does not allow a direct test of what he wants to prove: that relative prices matter. That is, the impact of user cost on investment can not be observed in isolation since both user cost and output are constrained to enter into the investment model as a composite variable. The user cost of capital and output are also important in the putty-clay model but, unlike the neoclassical model, the user cost of capital is allowed to take on a role that is independent of output. Tobin stresses the importance of information in the financial markets in his model of investment behaviour. Thus, according to these four core theories, output, the user cost of capital
and \( q \) are important determinants of investment. The definitions of these variables, the sources of the data, details of how these variables are constructed and the limitations of the resulting measures are provided in Section 4.2. These variables form the basis of four single equation models of investment behaviour consistent with the four dominant theories of investment.

In addition to these variables, another set of variables emerge as important determinants of investment from the empirical literature and other theories of investment. These include liquidity, profitability, capacity utilisation and business optimism. Since in this work investment is measured by construction output, there will also be information content in variables such as new orders. The definitions of these variables and the sources of the data etc. are provided in Section 4.3.

The data is collected in current prices (unless otherwise stated) and on a quarterly basis. All data is seasonally unadjusted, since it has been shown that the use of adjusted data in econometric analysis increases the possibility of obtaining misspecified models with spurious dynamics and poor forecasting performance. Two of the most influential pieces of work on the subject of seasonal adjustment are contained in Wallis (1974) and Sims (1974). Wallis (1974) studies the effect of official seasonal adjustment procedures on the relations between variables. He considers the effects of applying various filters to one or more variable in a lag relation and concludes that ‘the indiscriminate use of filter, or the non-availability of unadjusted data, will inevitably lead to mistaken inferences about the strength and dynamic pattern of the relationships’. Sims (1974) examines the use of published seasonally adjusted data in regression work and concludes that such work will be deficient since seasonal adjustment filters will not eliminate all of the power at the seasonal frequencies. Bell and Hillmer (1984) provide a survey of the major issues involved in the seasonal adjustment of time series data. In addition, to outlining the work of Wallis (1974) and Sims (1974), Bell and Hillmer survey work that has concentrated on forecasting accuracy when using seasonally adjusted data. They note that using adjusted data has a severe disadvantage in that forecast error variances and forecast intervals can not be estimated precisely.

In the latter part of the chapter we examine the time series properties of all the variables described in Sections 4.2 and 4.3 as a foundation for later analysis. In
particular, we scrutinise the data for evidence of non-stationarity. As noted in Chapter 3 (see, for example, Section 3.4), the distribution of conventional test statistics in econometrics is calculated under the assumption that the residual series is stationary. Moreover, we noted that the regression of one non-stationary time series on another may lead statistical analysis to suggest a relationship between series even where one does not exist. This is known as the spurious regression problem and since many economic time series are non-stationary, it has important implications for economic modelling. Thus, testing for evidence of non-stationarity is a necessary precursor to model estimation. Despite this, there is a conspicuous absence of such testing in the empirical investment literature. This statistical analysis is conducted in Sections 4.4. Section 4.5 concludes this chapter with a summary of its findings.

4.2 Definitions, Sources and Limitations of Core Data

In this section we develop data series necessary for the estimation of investment models consistent with the four dominant theories outlined in Section 2.2 to 2.6. In particular, we develop measures of real net investment, measured by real new private industrial and commercial construction output, real private sector output, real capital stock, the real user cost of capital and a $Q$ variable. The effects of corporate taxation and investment incentives built into the tax system complicate the derivation of measures of the real user cost of capital and $Q$ considerably. We begin by considering new private industrial and commercial construction output as a measure of net investment in buildings.

4.2.1 Construction Output as a Measure of Net Investment, $\Delta K$

To the extent that construction output represents capital formation, we can model new private industrial and commercial construction output as an investment problem. We explore the relationship between published construction output and investment data in Section 4.2.1.3. It is private industrial and commercial construction output that is the dependent variable in the models to be estimated in Chapter 5 and, moreover, it is this series that we wish to forecast. In the investment models to be estimated in the next chapter, we treat the construction output series as a measure of investment.
4.2.1.1 An Introduction to Construction Output Data

The Department of the Environment (DoE) define the level of construction output as the amount chargeable to customers for building and civil engineering work done in a relevant period (Housing and Construction Statistics (HCS), 1985-1995, p. 209). A distinction is made between new construction output (which includes extensions and major alterations, i.e. improvements, to existing buildings, site preparation and demolition, in addition to the construction of new buildings) and repair and maintenance. The value of new construction work is further disaggregated by type of work: housing, infrastructure, other public work, private industrial and private commercial work. Thus, in confining our interest only to the value of new private industrial and commercial construction work, other components of construction output namely, repair and maintenance, housing, public sector work and infrastructure, are ignored.

Private industrial work is dominated by the construction of factories and warehouses. Private commercial work is dominated by office buildings and buildings for retail distribution. Work is defined as private if commissioned by a private owner or organisation, or by a private developer and includes work carried out by firms on their own initiative. Thus, no construction commissioned by a public sector organisation or institution, no matter what its intended use, can be categorised as private sector work.

To the extent that the factors driving private industrial building and private commercial building are similar, and distinct from factors driving house-building or infrastructure work, these categories are modelled in aggregate.

It is important to distinguish between private industrial and commercial construction output, as defined by the DoE and outlined above, from work that might be commissioned by industrial and commercial companies (ICCs) as defined in the United Kingdom National Accounts (referred to hereafter as the Blue Book). Industrial and commercial companies are defined in the Blue Book 1997 (p. 192) as being comprised of all corporate bodies other than public corporations, banks and other financial companies and institutions. Property companies are included in the definition of ICCs. Private industrial and commercial construction output, however, captures all work of a non-housing, non-infrastructure nature commissioned by the private sector. Although the private sector is dominated by ICCs, it also includes the
personal sector (which consists mainly of households, individuals resident in the UK, and unincorporated businesses), and financial companies and institutions (FCIs). Both the personal sector and FCIs commission new construction. Of course, the vast majority of personal sector building takes the form of new housing or extensions etc. to existing housing commissioned by individuals and households. However, unincorporated businesses commission new building work for industrial or commercial purposes, the value of which is not insignificant. FCIs, whilst occupying commercial buildings, also commission new work in buildings for commercial use that they themselves do not intend to occupy. Such an investment will be undertaken speculatively when rents gained from letting such property and the prospective capital gains are expected to be favourable relative to other investments, such as gilts or securities. Thus, private industrial and commercial building is undertaken not only by ICCs, but by the personal sector and FCIs also.

Recall, the definition of construction output includes the amount chargeable to customers for civil engineering work done in the relevant period. To the extent that civil engineering work is not likely to be determined by the same set of factors as building work, this could be viewed as problematic. However, in restricting attention to private industrial and commercial work, this potential problem is bypassed. Almost all civil engineering work relates to infrastructure (new or otherwise). In considering only private industrial and commercial construction output, infrastructure is excluded and thus so is civil engineering work.

4.2.1.2 New Private Industrial and Commercial Construction Output

Data on both private industrial and private commercial construction output has been published on a quarterly basis since 1955. The data is currently published by the DoE in HCS (see Tables 2.3-2.4 in part 2 of the June 1997 issue). Historically, construction output has been published by type of work. Until 1992 five types of work were identified; public housing, private housing, other public work, private industrial and private commercial building. A sixth type of work, infrastructure, was defined and introduced in 1992. Prior to 1980, most infrastructure work was carried out by the public sector and, as such, was included as ‘other public work’. However, with the
implementation of the Conservative government’s privatisation programme came an increasing amount of infrastructure work commissioned by newly privatised industries. Currently, the majority of infrastructure work is carried out by the private sector.

In 1993, the DoE published revised quarterly series’ (in current prices) back to 1980 to take account of this reallocation of infrastructure (see *HCS*, June 1993, part 2). Revised output price indices, used to convert construction output data from a current to a constant price basis, were also published back to 1983 (see *HCS*, June 1993, part 2). Revised constant price series for construction output taking account the reallocation of infrastructure were also published (*HCS*, December 1993, part 2). As a result of the introduction of infrastructure as a separate category, there is a substantial discontinuity in the ‘other public work’ data at 1980, as post 1980 infrastructure work (previously published under ‘other public work’) is now published under its own heading. Since public sector and infrastructure work are of no interest here, this in itself presents no problem. However, the amount of private sector infrastructure work in 1980, although small, was not zero. Thus, post 1980, the series for private industrial construction output and private commercial construction output do not contain work on infrastructure, but before 1980 these series do contain the small amount of work on infrastructure carried out by the private sector. Thus, there is a small discontinuity in these two series at 1980. This discontinuity was corrected for by rescaling the pre-1980 data by the percentage difference between the old and newly defined series in the first quarter of 1980.

Estimates are collected in current price terms. These are revalued in 1990 prices using output price indices published by the DoE in *HCS* (June 1997, part 2, Table 2.1). It is the real value of new private industrial and commercial construction output that is of interest in this thesis. We discuss the merits of the output price index used to convert current price private industrial and commercial construction output data into constant prices in Section 4.2.2. This series constitutes the net investment variable that forms the focus of interest this thesis. The resulting series, which is denoted $\Delta K$, in what follows, is shown in Figure 4.1. Note that real net investment increased steadily between 1955 and 1970. Figure 4.1 also shows a very sharp increase in real investment between 1985 and 1990. In some of the models to be estimated in Chapter
Figure 4.1: Private industrial & commercial construction new orders and output

(£ million, constant prices)

construction output, $\Delta K$

new orders, $O$

Figure 4.2: The rate of investment in industrial and commercial buildings
5, it is the rate (rather than the level) of net investment that is of interest. The rate of net investment is calculated by dividing the value of investment by the value of capital stock. The measure of capital stock adopted in this work is derived and described in Section 4.2.5. The rate of investment is plotted in Figure 4.2. A noteworthy feature of this chart is the marked trend decrease in the rate of investment since 1975. Notice also that this series is rather volatile.

4.2.1.3 Limitations of the Series as a Measure of Net Investment

In the UK, additions to the nation's wealth are divided into three categories: the acquisition of fixed assets, the acquisition of stocks and work in progress, and net investment abroad. The first two of these three forms of investment are in tangible assets and comprise capital formation. Gross Domestic Fixed Capital Formation (GDFCF) is the standard description of the addition to the nation's stock of fixed assets. That is, it constitutes the first of the three forms of investment. It is defined in the *Blue Book 1997* (p. 191) as expenditure on fixed assets such as buildings, vehicles, plant and machinery, either for replacing or adding to the stock of existing fixed assets. Expenditure on maintenance and repairs (depreciation expenditure) is excluded, whilst that on improvement is included (in practice, however, it is often difficult to distinguish between maintenance and improvement). GDFCF is valued at the cost to the purchaser including the costs directly connected with acquisition and installation.

In studies of investment behaviour, the dependant variable is often investment in fixed capital. In the *Blue Book*, investment in new fixed capital is defined by GDFCF. Since GDFCF does not include expenditure on maintaining existing capital (replacement investment), it is a measure of net investment. Published data is disaggregated by type of asset and by sector. The type of asset most closely corresponding to our series for net investment is that described in the *Blue Book* as private sector GDFCF in 'other buildings and works'. The term 'works' refers to expenditure on civil engineering projects.

As stated in Section 4.2.1.1, DoE data distinguishes between new construction work and repair and maintenance. The series derived in Section 4.2.1.2 is for new
construction work alone. Although the GDFCF series also excludes depreciation, there are a number of important differences between the two series and it is to these that we now turn. The general differences between construction output and investment expenditure data are discussed in Fleming (1986, p. 138). Here, we consider the differences between the narrower concepts of private industrial and commercial construction output, as defined in HCS, and private sector GDFCF in other buildings and works, as defined in the Blue Book. First, there is a difference in geographical coverage. Quarterly estimates of construction output, made by the DoE are based on returns from a sample of contractors for Great Britain. The United Kingdom National Accounts are, of course, compiled for the whole of the UK. Thus, if there were no other differences between series, the value of construction output would be less than private sector GDFCF in new buildings and works by an amount equal to the value of output of new construction work in Northern Ireland. Second, unlike GDFCF data, construction output statistics do not include an estimate of professional fees such as those of architects, designers and consultants. Third, GDFCF estimates for buildings and other works include estimates of the reduction in builders' stock of completed buildings and of the reduction of work in progress on uncompleted buildings, whereas estimates of the value output of new construction work do not.

These differences in the two series are a direct result of differences in the definition. However, other differences exist which relate to the collection methods used for each series. For example, estimates of GDFCF are collected by sector, asset type and industry. To obtain sector estimates GDFCF by asset type, estimates of GDFCF in dwellings, other new buildings and works, plant and machinery etc. are initially aggregated within each industry and then across industries before being allocated to a sector. As a result, the coverage is reasonably wide and detailed: GDFCF is available for six institutional sectors (ICCs, local authorities etc.) cross-classified by five types of asset (plant and machinery, dwellings etc.) in the Blue Book 1997 (see Table 13.5, p. 156). As noted above, private sector GDFCF in other buildings and works, includes expenditure on civil engineering work. However, civil engineering and work on new buildings are likely to have different determinants. Since estimates of private sector expenditure on the former can not be separated out from the latter, this presents a problem to the empirical researcher. To the extent that the private industrial and
commercial construction output series relates only to buildings, this series may provide a better measure of investment in buildings than the GDFCF data. It is noteworthy that all of the differences discussed so far mean that, *ceteris paribus*, estimates for private industrial and commercial construction output are lower than the corresponding estimates of private sector GDFCF in other buildings and works. (Compare the two series in Figure 1.5).

There are also differences in timing between the two types of data; actual expenditure may not be coincident with the actual execution of work. In addition, on both the expenditure and output sides, there are certain problems relating to the timing of the recording of the data. Strictly speaking, investment, or more precisely changes in a nation’s productive capacity, would be best measured if the acquisition of fixed assets was recorded at the time these assets become available for use. Recall, construction output is the amount chargeable to customers for building work done in the relevant period. Thus, these estimates do not track changes in the productive capacity, since purchasers of new buildings make progress payments to contractors and these payments lead the entry of the capital investment (the new building) into the nation’s productive capacity. However, these problems apply equally to GDFCF and are described in *United Kingdom National Accounts: Sources and Methods 1985* (see CSO (1985), p. 187). This contrasts with investment in plant and machinery (and to a lesser extent, small scale building work) since payments often lag delivery of goods as purchasers delay the settling of accounts in order to maximise their financial capital.

Thus, in this sense, neither private industrial and commercial construction output or private sector GDFCF in other buildings and works actually measure investment. The fact that payments for new buildings lead the entry of new buildings into the nation’s capital stock has an important implication: there is a shorter lag between changes in the determinants of investment and the recording of investment than between changes in the determinants of investment and the realisation of investment (when the building is ready for use). In other words, the investment is recorded some time between the placement of the order and the investment’s realisation.

This, of course, raises the question as to whether an investment model is suitable for the purposes of modelling the value of new work on industrial and commercial buildings undertaken for private sector clients. If the purpose of this thesis was to
model investment demand, then new orders would certainly be a more appropriate dependant variable than construction output. The time lag between a change in economic factors prompting investment and the placement of a new order is much shorter than the lag between a change in the economic factors and the recording of construction output, or the subsequent realisation of that investment. This is evident in Figure 4.1: the new orders series consistently leads the output series. However, the purpose of this thesis is not to model investment demand, but to model and forecast construction output. Since GDFCF is usually modelled as an investment problem, there would seem to be no argument against modelling construction output within the same framework: the difficulty of the lag between the change in economic factors prompting investment and the recording of data apply equally to both variables. It is noteworthy that modelling construction output as an investment supply problem would be inappropriate since, due to progress payments, the recording of investment (in either the construction output or GDFCF data) precedes the time at which the capital good is ready for use.

A limitation of the construction output data relates to its geographical coverage. As mentioned above, the DoE collect data on construction output for Great Britain. Most other official economic data, however, is collected on a United Kingdom basis. Unfortunately, separate estimates of economic variables for Northern Ireland do not, in general, exist. This creates an inconsistency in coverage between the investment series and its explanatory variables which is virtually impossible to reconcile with any degree of accuracy. Given that investment in industrial and commercial buildings in Northern Ireland is very small relative to that in Great Britain, and that the contribution of Northern Ireland to economic aggregates is unlikely to be idiosyncratic, this inconsistency is ignored.

In summary, geographical coverage aside, private industrial and commercial construction output may be a more appropriate measure of investment in buildings than private sector GDFCF in other building and works. The fact that estimates of construction output are calculated and collected from surveys conducted on a quarterly basis is also advantageous, since GDFCF estimates are derived from annual or biennial census inquiries. Confronted with the more specific task of modelling the value of output on new private industrial and commercial building work, it is certainly
the most appropriate construct with which to work. Finally, it merits note that estimates of construction output for private sector dwellings are used as the starting point for both quarterly and annual estimates of GDFCF in dwellings by the private sector since, unlike GDFCF for other assets, there is not a more suitable source from which to estimate expenditure by private purchasers on new dwellings.

4.2.2 The Price of Investment Goods, \( z \)

The current value of new private industrial and commercial construction output, \( z\Delta K \), is transformed into a constant (1990) price series, \( \Delta K \), using an output price index, \( z \). More specifically, \( z \) is the price index relating to private industrial and commercial construction output. The DoE publish a family of six output price indices in HCS (see June 1997, part 2, Table 1), one for each of the following; private housing, public housing, infrastructure, other public work, private industrial building and private commercial building. \( z \) was derived using a weighted average of the last two of these output price indices. Both of these series contained minor discontinuities in 1980q1 which emerged as a result of the DoE’s decision to introduce a separate category for infrastructure in published tables. This problem is discussed in more detail above in Section 4.2.1. The discontinuities are resolved in the same way as the discontinuities in the construction output data are resolved, that is, the pre-1980 data was rescaled by the percentage difference between the value of the series under the old and new definition in 1980q1. These two series were then combined to produce an output price index for private industrial and commercial construction output. The weights for each series were determined by the respective values of work done in each sector in 1990.

The constant price series is somewhat less reliable than the current price series because of the well known problems in measuring the price of capital goods. Although all constant price investment series suffer from this problem, it is particularly acute for investment in buildings. The main difficulties lie in the heterogeneous nature of capital goods, since although some capital goods are of a standard type, many more are unique. The heterogeneous nature of each building renders the measurement of price changes extremely difficult. The DoE’s output price indices (which are also used to deflate estimates of GDFCF in dwellings and in other...
buildings and works) are based on tender price movements (described in *Economic Trends* (*ET*), July 1978). They are suitably lagged to reflect the time taken between tender stage and the construction work and adjusted to take account of the cost of variation of price clauses which are a feature of many major construction contracts. These tender price indices are calculated from samples of contracts which are repriced at the prices ruling in the base year. As a result, it is not possible to do more than indicate general trends in investment in buildings, or for that matter investment in any asset, at constant prices.

4.2.3 Private Sector Output, $Y$

Our investment series relates to private industrial and commercial buildings. According to the theory developed in Chapter 2, the main determinant of investment in the accelerator or putty-clay models will be the output of those firms undertaking investment. Thus, $Y$ denotes the output of the private sector and not aggregate output. Deriving such an output measure from national accounts data is not without its difficulties. An output-based measure of gross domestic product (GDP) broken down by industry is published in the *Blue Book* (for example, see Table 2.1 in the 1997 edition). This shows the composition by industry group of GDP and the forms of factor incomes originating in each industry. The total of factor income generated in each industry is identical to net output or value added of the industry, that is to say, the excess of the value of the goods and services produced by the industry over the cost of goods and services supplied by other industries or imported for use in that production, before provision is made for the depreciation of fixed assets. In this context, the value of goods and services is reckoned at factor cost, meaning that it excludes any taxes on expenditure paid by the producer and any subsidy received by the producer from general government is not deducted.

Unfortunately, this industry group decomposition of GDP is insufficient to identify private sector output. Although, in the national accounts, a large amount of the public sector is contained within two industry groups, namely ‘public administration etc.’ and ‘education and health’, it is not completely contained within them. For example, both the manufacturing and the energy and water supply industries have had
considerable public involvement in the past. In addition, there is likely to be some private sector involvement in the ‘education and health’ industries. The matter is further complicated by the extensive privatisation programme implemented over the course of the 1980s. In that decade, many publicly owned firms and industries were transferred to the private sector. As such, any attempt to create a reasonable measure of private sector output must take account of these factors, but the lack of sufficiently disaggregated output data is likely to make this task extremely complex and yield an inaccurate final measure.

However, a sectoral decomposition of an income-based measure of GDP is also published in the *Blue Book* (see Table 2.5 in the 1997 edition). Here, GDP at current factor cost is split by sector of employment and, where possible, by type of factor income. There are five sectors of employment; the personal sector, ICCs, FCIs, public corporations, central government, and local authorities. Each sectors’ total factor income is split (where relevant) into the following types of income; income from employment, income from self employment, rent, and gross trading profits/surplus. These factor incomes are distributed according to the sector in which they are earned and not according to the sector to which they are paid. The total of factor incomes originating in each sector is equal to the net output (or value added) of that sector, that is, it shows the contribution of each sectors’ productive activity to GDP. In other words, it represents the excess of the value of the goods and services produced by the sector over the cost of goods and services supplied by other sectors or imported for use in production.

A measure of the output of the private sector can be obtained simply by aggregating total factor incomes for the personal sector, ICCs and FCIs. Unfortunately, this data is not available on a quarterly basis. Annual estimates are available from various issues of the *Blue Book*. Annual data, back to and including 1963, for total factor income of the personal sector, ICCs and FCIs are also available on the *Office of National Statistics (ONS) Database* (with identifying codes GICW, CAJN, and GIDD respectively). Annual estimates prior to, and including 1963, were collected from the eight editions of the *Blue Book* up to and including the 1974 edition. Due to data revisions, there was a small discontinuity in all three series in 1963 (the data for 1963 and later years having been taken from the *ONS Database*, which is regularly revised,
and the data for 1963 and earlier years having been taken from printed editions of the *Blue Book* which are not). A continuous annual series for private sector output (at current factor cost) is derived for the period 1955 to 1996, by aggregating total factor income for the three sectors and splicing the pre-1963 private sector output series with the post-1963 output series to eradicate the discontinuity.

The problem of obtaining a quarterly measure of private sector output from the annual observations remains. The solution adopted here makes use of an expenditure-based measure of GDP. A seasonally unadjusted, quarterly, expenditure-based measure of GDP at market prices is obtained by deducting the value of imports of goods and services from total final expenditure. By further subtracting taxes on expenditure and adding in subsidies, we arrive at a quarterly measure of GDP at factor cost. The quarterly observations inherent in this current factor cost measure of GDP are used to obtain a quarterly series for private sector output.

Over the period between 1955 and 1996 the private sector has typically been responsible for between 70% and 80% of total output. Since the private sector’s contribution to total output is so large, there is no reason to suppose that the seasonal pattern of private sector output will differ substantially from the seasonal pattern of total output. As such, the seasonal pattern of the expenditure-based measure of GDP was ‘imposed on’ the private sector output series. More accurately, the quarterly estimates of the expenditure-based measure of GDP (total output) in any one calendar year are multiplied by the ratio of private sector output to GDP in that year. Seasonally unadjusted current price data on total final expenditure, imports of goods and services and the factor cost adjustment are given in *Economic Trends Annual Supplement (ETAS) 1997* (see Table 1.3) and are also available on the *ONS Database* (with respective identifying codes DJAK, DJAG, and CTGV).

A constant (1990) price series, \( Y \), is obtained by dividing the current price series by \( p \) (which is described in Section 4.2.4). From this series for private sector output, the new private industrial and commercial construction output, \( \Delta K \), is deducted. This is because the inclusion of the value of private industrial and commercial construction output would lead to an artificially high correlation between the \( \Delta K \) and \( Y \), since the former series is contained within the latter. As such, failure to exclude construction output would have an adverse impact on econometric analysis. The problem of
spurious correlation in this context is discussed by Gordon and Veitch (1986). Of course, deducting construction output from private sector output implies that the private sector output series can not explain that part of construction output commissioned by construction sector firms. However, this part of construction output is likely to be relatively small.

The resulting series is shown in Figure 4.3, along with the expenditure-based measure of GDP at constant factor cost (also derived in Section 4.2.4). Notice that the two series track one another closely. In the period between 1975 and 1981, the contribution of private sector output to total output was generally less than 70%. This relative weakness of private sector output is shown in Figure 4.3 by the slight divergence in private sector output from the total output series. The contribution of private sector output to total output steadily increased throughout the 1980s (partly as a result of the privatisation process and the other less interventionist government policies) and this is represented by a narrowing of the gap between the two series in Figure 4.3.
4.2.4 The Price of Private Sector Output, $p$,

Just as $Y$ should represent the output of the private sector, $p$ should represent the price of private sector output. Ideally, $p$ would be the implicit price deflator for private sector output, constructed using real and nominal private sector output data. The series for total factor income of the personal sector, ICCs, and FCIs used to obtain annual estimates for private sector output $Y$, are only available at current factor cost (i.e. on a current price basis). As such, an implicit price deflator for private sector output can not be constructed in this fashion.

However, since private sector output has typically contributed between 70% and 80% to total output, an implicit price deflator for the latter will provide a good proxy for the price of the former. Recall, from the derivation of the series for private sector output in Section 4.2.3, total output is approximated by an expenditure-based measure of GDP at factor cost. The implicit price deflator for this series is constructed by dividing the current price series by the constant price series. The derivation of the current price series is discussed in the previous section. Quarterly unadjusted constant (1990) price data for total final expenditure, imports of goods and services, and the factor cost adjustment are published in ETAS 1997 (see Table 1.3). The data for these series are also available back to 1955 on the ONS database (the identifying codes for total final expenditure, imports of goods and services and the factor cost adjustment are given by DJDA, DJCY and DJCU respectively). Subtracting imports of goods and services and the factor cost adjustment from total final expenditure gives an expenditure-based measure of GDP at constant factor cost. Dividing the current price series by the constant price series gives the implicit price deflator for GDP at constant factor cost. This implicit price deflator represents the price of total output in the economy and is used here as a proxy for the price of private sector output.

4.2.5 Estimates of Capital Stock, $K$,

Recall from Section 2.6 (equation 2.89), the $q$ theory of investment posits a relationship between the rate of investment and $Q$. In order to calculate the rate of investment, we need a measure of the capital stock. Since we are dealing with investment in industrial and commercial buildings undertaken by the private sector, $K$
is defined as the stock of privately owned industrial and commercial buildings. Estimates of such a narrowly defined construct do not exist and the derivation of estimates is not straightforward. In this section, we derive three measures of $K$ and examine the merits of each.

4.2.5.1 Gross versus Net Capital Stock: Definitions

Estimates of UK capital stock are presented in the Blue Book. Estimates are given both before and after depreciation. These estimates are denoted gross and net capital stock respectively. Both are aggregates of the value of assets in use. Gross capital stock values the assets at their full replacement cost, i.e. their cost to replace anew. Net capital stock is gross capital stock less accrued capital consumption, i.e. their cost to replace in their current condition. Whereas gross capital stock is a one dimensional concept because it measures the quantity of assets in place, net capital stock is two dimensional because it takes into account not only changes in the quantity of assets in place, but also changes in their unexpired lives. Whereas for the gross capital stock the whole of the original value of a fixed asset is deemed to remain in stock until the year of its retirement, for net capital stock the original value of the asset is deemed to decline gradually over its service life. It is in this sense that net capital stock is a measure of the cost of replacing fixed assets with assets of identical condition. This value may be said to represent the wealth that is tied up in fixed capital which has not been consumed, i.e. the value of the services still to be consumed. Strictly speaking, this is not a market valuation since, for much plant and machinery, there is no second-hand market: plant and machinery tend to go into a factory and remain there until it is scrapped. In the case of buildings, where there is an active second hand market, the market does not distinguish between the structures and the land on which they are situated.

Thus, in empirical work the appropriate measure of capital stock depends on the assumption made about the decay pattern of fixed capital. If one assumes that the productive capacity of a machine is the same throughout its life, such that it provides the same amount of services for each time period until it expires or is scrapped, then a gross measure of capital stock is most appropriate. If, on the other hand, one assumes
constant exponential decay, such that, as a fixed asset ages, its value depreciates by a constant percentage of the written down value at the beginning of the time period, then a net measure of capital stock is appropriate. Estimates of capital stock are derived using a straight-line depreciation assumption in the *Blue Book*. This assumption holds that a fixed asset depreciates by a constant amount each year (equal to one $\frac{1}{n}$th of its cost if the asset is expected to last $n$ years).

Despite the fact that a straight-line decay pattern is assumed in the derivation of estimates of capital stock in the *Blue Book*, recent empirical evidence provides support for the constant exponential decay assumption. Using highly general Box-Cox transformations to examine alternative decay specifications, Hulten and Wykoff (1980, 1981a and 1981b) present evidence that upholds the exponential decay hypothesis and clearly rejects the 'one-hoss shay' (which assumes that an asset provides the same amount of services each period until it expires or is scrapped) and straight-line alternatives. This work contradicts results from an earlier body of research (see, for example, Hall (1971), Feldstein and Foot (1971), Eisner (1972), Feldstein and Rothschild (1974), Bitros and Kelejian (1974) and Coen (1975)) in which little support for the exponential decay pattern is uncovered.

Whatever the merits of alternative decay assumptions, constant exponential decay is by far the most widely used deterioration specification in modern studies of investment behaviour. This is largely due to its convenient simplicity and its ability to track very closely the age-price profiles of a variety of used fixed assets. It also follows that since there is more empirical support for exponential decay than for one-hoss shay decay, most analysts employ a net rather than a gross measure of capital stock. This current convention is adopted in this thesis. A further justification of this assumption is provided by renewal theory. It has been shown that, when there is a large number of different types of assets, the decay pattern of these assets will, on average, be exponential, even if individual assets have alternative decay patterns.

4.2.5.2 Capital Stock in the National Accounts

A current price measure of net capital stock, hereafter referred to as capital stock, is presented, broken down by six sectors and by five types of asset in the *Blue Book*.
1997 (Table 14.7). Of interest in this thesis is a measure of the value of the stock of privately owned industrial and commercial buildings. The private sector is comprised of the personal sector, ICCs and FCIs. Annual estimates of the value of the stock of non-residential structures owned by the private sector can be derived by summing individual annual estimates of the stock of ‘other buildings and works’ for the personal sector, ICCs and FCIs (the respective ONS Database identifiers are EXHC, EXHD and EXHE and they are available at current replacement cost back to 1947). This measure is denoted $K^{bb}$.

There are a number of problems with this series. Firstly, it is only available on an annual basis. Secondly, it comprises more than the stock of privately owned industrial and commercial buildings: it also includes the stock of infrastructure or civil engineering structures owned by the private sector. Since the measure of investment used here excludes investment in infrastructure, it follows that any series used to explain it should also be net of infrastructure. In order to capture the relationship between investment and its determinants as accurately as possible, our capital stock series should ideally only measure the stock of privately owned industrial and commercial buildings. Finally, estimates of capital stock published in the Blue Book are substantially less reliable than estimates of other economic variables. This is due partly to the fact that they have been calculated using the perpetual inventory method. As such, they depend to a great extent on assumptions made about the lives for different categories of asset; there is very little hard information to support these assumptions. Indeed, in the Blue Book (see, for example, p. 215 of the 1997 edition) it is stated that some capital stock estimates may not be accurate to within 20%. One particular period which causes concern for capital in general is 1974/75; the period of the first oil price shock. Baily (1981) and Summers (1981) suggest that a quadrupling of the oil price in this period reduced the value of existing capital stock which was too heavily dependent on oil. As a result, depreciation in this period is underestimated in national accounts and, by implication, the value of the capital stock is overestimated. However, examining the market price of used capital goods before and after the energy price shocks, Hulten, Robertson and Wykoff (1989) find no evidence of increased obsolescence. The increased obsolescence argument may also be applied to the revolution in office computing equipment in the 1980s. It is noteworthy that this
problem is unlikely to affect industrial and commercial buildings directly, but may have an indirect effect. There is unlikely to be a direct effect since buildings are not particularly dependent on oil and are not easily substituted by computers. However, to the extent that oil dependent plant and machinery and the computers are accommodated in buildings (that may be purpose built), increased obsolescence of these machines may increase obsolescence of buildings. Some of the more general problems related to the measurement of capital stock are discussed in Ward (1976) and Griffin (1979).

4.2.5.3 Three Alternative Measures of Capital Stock

Three alternative measures of capital stock are constructed which are believed to have some advantages over the *Blue Book* estimates. These alternative measure make use of construction output data published by the DoE in *HCS*.

4.2.5.3.1 The Perpetual Inventory Approach and $K^a$

In addition to making use of data relating to construction output, the first alternative measure of capital stock, denoted $K^a$, makes use of the perpetual inventory approach. In this approach an estimate of the previous period’s net capital stock at current replacement cost is adjusted for depreciation, inflation, investment and disposals during the current period to obtain an estimate for the current period. Thus

$$z_t K_t = z_t I_t + \left( \frac{p_t}{p_{t-1}} \right) (1 - \delta) z_{t-1} K_{t-1}$$

(4.1)

where, in general terms, $z_t I_t$ denotes the value of gross investment in current prices, $z_t K_t$ denotes the current value of this period’s net capital stock at current replacement cost, $z_t$ measures the prices of investment goods and $p_t$ measures the price of output. $z_t I_t$ is defined here, more specifically, as the value of total construction output (new work plus repair and maintenance) relating to industrial and commercial building commissioned by the private sector. $z_t$ is defined here as the output price index for new work in private industrial and commercial buildings (and is derived in Section 4.2.2). $p_t$ is defined as the output price index for private sector output (and is derived...
in Section 4.2.4). Given these definitions of $z_l$, $z$, and $p$, it follows that $zK$, measures the value of the stock of privately owned industrial and commercial buildings. This method of estimating a series for net capital stock requires a benchmark starting value for $zK$.

This measure of capital stock has several potential advantages over *Blue Book* estimates. Firstly, it is available on a quarterly basis since gross investment is available quarterly. Thus, the *ad hoc* procedure of interpolating quarterly estimates from annual data is avoided. Secondly, if the gross investment data and the benchmark starting value for capital stock exclude the value of private sector civil engineering work (that is, infrastructure) then the derived estimates for capital stock would not include infrastructure and, as such, would correspond to our investment series more accurately than the *Blue Book* estimates. Thirdly, this measure of capital stock can be transformed from current to constant price form using the appropriate deflator, $z$.

In practice, however, there are a number of difficulties in realising these potential advantages. Firstly, published estimates of the value of repair and maintenance undertaken by the private sector are not split in the same way as estimates of new work undertaken by the private sector. Whereas new work undertaken by the private sector is split between housing, infrastructure, industrial buildings and commercial buildings, repair and maintenance undertaken by the private sector is split between housing and non-housing. That is, estimates of the value of private sector repair and maintenance on industrial buildings, commercial buildings and infrastructure are lumped together under one heading, namely private sector non-housing repair and maintenance. Thus, in the absence of an estimate of the value of work done repairing and maintaining infrastructure, the gross investment series (the sum of new work and repair and maintenance), necessary for the derivation of the estimates of capital stock in the perpetual inventory method, will include repair and maintenance on infrastructure. As a result, the measure of gross investment is inconsistent with that of net investment. More importantly, the derived measure of capital stock will not fully exclude all infrastructure and, as such, will also be inconsistent with the net investment series. In this respect, this measure of capital stock is not much better than the *Blue Book* estimates.
In an attempt to overcome this limitation, a measure of the value of work done repairing and maintaining infrastructure is imputed from the data on the value of new work carried out on infrastructure (published in HCS, June 1997, part 2, Table 2.4b). It seems reasonable to assume that the ratio of repair and maintenance on infrastructure to total repair and maintenance on non-residential structures will be approximately equal to (but slightly lagging) the ratio of investment in new infrastructure to total investment in new nonresidential structures. Thus, the series for non-housing repair and maintenance is multiplied by unity minus the ratio of investment in new infrastructure to total investment in new nonresidential structures. This gives an imputed series for the value of repair and maintenance carried out on industrial and commercial buildings. Given this series for the value of repair and maintenance undertaken on industrial and commercial buildings, a series for gross investment in industrial and commercial buildings (net of infrastructure) is obtained by adding to it the net investment series $\Delta K$ (the value of new work in industrial and commercial buildings). The series for non-housing repair and maintenance is published in HCS (see, for example, Table 2.3 in part 2 of the June 1997 issue).

Gaining an initial benchmark value for capital stock with which to start this perpetual inventory procedure is also problematic. It would be inappropriate to take as a starting value an estimate of capital stock for a particular year from the Blue Book since, as discussed above, these estimates do not exclude infrastructure. Moreover, the Blue Book estimates measure the stock of capital in the UK, whereas as the construction output data published by the DoE in HCS are for Great Britain. Thus, an estimate for capital stock taken from the Blue Book would have to be adjusted to exclude the stock of infrastructure and the stock of all nonresidential structures in Northern Ireland. Unfortunately, data on the stock of infrastructure in the United Kingdom and the stock of nonresidential structures in Northern Ireland is not available and can not be easily imputed. A starting value can, however, be derived by other means. The benchmark period is assumed to be 1955q1 and the starting value for this period is assumed to be equal to this quarter’s estimate of the value of the stock of industrial and commercial buildings given by the second alternative to the Blue Book estimate. The procedure used to derive this series is described in Section 4.2.5.3.2.
In so far as this alternative measure of the capital stock is derived using the perpetual inventory method, it shares some of the deficiencies of the Blue Book estimates. Reference has already been made to the problems inherent in using this procedure. Estimates of gross capital stock rely heavily on largely unsubstantiated assumptions regarding the length of assets’ lives and the rate of retirements. In the National Accounts, the lifespan of a commercial building is assumed to be 80 years and until 1983 the lifespan of industrial buildings was also assumed to be 80 years. These assumptions are based on the rather arbitrary assumptions on asset lives made by Dean (1964). Subsequent to 1983, it is assumed that industrial buildings erected between 1890 and 1930 had progressively shorter life spans and since 1930 the average expected lifespan of an industrial building has been fixed at 60 years. With measures of net capital stock, the rate of retirements is of no direct interest since it is the assumed constant rate of depreciation and not retirements that affect net capital stock. King and Fullerton (1984) show that if economic depreciation is truly straight line, the relationship between asset lives and the rate of exponential decay is given by

\[
\delta \approx \frac{2}{n}
\]  

(4.2)

where \( n \) is the lifespan of the asset. With asset life of 80 years, as assumed by Dean (1964), equation 4.2 implies a constant exponential rate of depreciation of 2.5% per annum. Thus, one disadvantage of estimating net capital stock using the perpetual inventory method is the inherent assumption of a constant rate of depreciation. We have suggested above that the energy price shocks in the 1970s may have resulted in increased obsolescence and an understated rate of depreciation at this time. A similar argument can be applied to the office computing revolution in the 1980s. By implication net capital stock derived using the perpetual inventory method with a constant rate of depreciation will be overestimated in these periods. Reference has also been made to the fact that Hulten, Robertson and Wykoff (1989), in examining the market price of used capital goods before and after the energy price shocks, find no evidence of increased obsolescence. In any case, these shocks would only alter the rate of depreciation for a short period, and would not lead to an increase in the rate of depreciation or acquisition subsequent to the shock. This measure of the stock of privately owned industrial and commercial buildings, \( K^a \), is a real construct and is shown in Figure 4.4.
4.2.5.3.2 The Repair and Maintenance Approach and $K_t^\beta$

The second alternative to the net capital stock estimates published in the *Blue Book* makes use of repair and maintenance data. Assuming, as most researchers do, that capital is accumulated according to the relationship

$$A^\ast_t = Y_t^\alpha - M_t^\beta$$ (4.3)

a measure of capital stock can be easily derived. Rearranging the above relationship we can obtain

$$K_{t-1} = \frac{\Delta K_t - I_t}{\delta}$$ (4.4)

Thus, net capital stock at the end of the last period is equal to the difference between gross and net investment, which is replacement investment, divided by the rate of depreciation. Repair and maintenance data, which is available on a quarterly basis in *HCS* (see, for example, Table 2.3 in part 2 of the June 1997 issue), provides a measure of replacement investment. Given this, a measure of last period’s net capital stock can
be derived by dividing the current period’s repair and maintenance estimate by an assumed constant rate of depreciation.

There are two important problems with this measure of capital stock. First, as discussed above, the repair and maintenance data published in HCS is not available at the required level of disaggregation. For the private sector, it is split between repair and maintenance on housing and that on non-housing. Since the non-housing component includes repair and maintenance of privately owned infrastructure, the derived measure of capital stock will include the value of the stock of privately owned infrastructure. This problem is discussed above in Section 4.2.5.3.1 and is solved in the same way here: the private non-housing repair and maintenance series is multiplied by unity minus the ratio of private work on new infrastructure to total private investment in new structures. This gives an imputed estimate of the repair and maintenance of industrial and commercial buildings and thus excludes the repair and maintenance of infrastructure. The second problem with this measure relates to the depreciation rates. Even if one assumes a constant rate of depreciation and accepts the inherent problems of such an assumption, the capital stock series depends crucially on the choice of depreciation rate. If the chosen depreciation rate is lower (higher) than the actual depreciation rate, the derived capital stock series will overestimate (underestimate) the actual series by a fixed percentage of the actual series. The actual rate of depreciation was determined using the method of King and Fullerton (1984) given in equation 4.2 above where $n$, the lifespan of buildings, is taken to be 80 years, as in work of Dean (1964). This yields an annual rate of exponential depreciation of 2.5%. King and Fullerton (1984) also assume a lifespan of 80 years. Bond and Devereux (1988) and Blundell et al (1992) cite King and Fullerton (1984) as justification for their choice of 2.5% for the annual rate of exponential depreciation of industrial buildings. This estimate is slightly lower than those used in other studies in the investment literature, but is not inconsistent with them. These studies are typically of aggregate investment or investment in plant and machinery. Given that the lifespan of plant and machinery is typically shorter than the life-span of buildings, one would expect to observe a higher assumed rate of depreciation in these studies. The real value of the stock of industrial and commercial buildings derived using this method is denoted $K^p$. It is plotted in Figure 4.4.
4.2.5.3.3 The Net Investment Approach and $K'$

One other problem with $K'$ is that it is over identified. It is possible to derive a measure of capital stock using equation 4.3 without imposing a fixed rate of depreciation. Rewriting equation 4.3 as

$$K_t = K_{t-1} + I_t - \delta K_{t-1}$$

$$= K_{t-1} + \Delta K_t$$ (4.5)

it is immediately apparent that there is no need to make use of information on depreciation in order to construct a measure of capital stock. Indeed, all that is necessary is last period’s capital stock and the current period’s net investment. This measure, denoted $K'$, does not suffer from the disadvantages of $K^a$ and $K^d$. Although derived using the perpetual inventory formula, it does not rely on data on repair and maintenance and is not derived under the assumption of a constant rate of depreciation. In fact, an implied rate of depreciation can be derived with the use of the repair and maintenance data. The rate of depreciation is found by dividing this period’s value of repair and maintenance by last period’s capital stock (that is, $\delta = I_t - \Delta K / K_{t-1} = \delta K_{t-1}/K_{t-1}$). The implied rate of depreciation is shown in Figure 4.5.

The first point to note about Figure 4.5 is that the implied rate of depreciation has not been constant throughout the sample period. The rate of depreciation in 1955q1 is defined to be 0.6% per quarter (corresponding to 2.5% per annum). From 1955, the implied rate of depreciation falls steadily to a minimum in 1975. Between 1975 and 1979 the implied rate of depreciation increases from 0.20% per quarter to 0.31% per quarter. This would appear to support the hypotheses of Baily (1981) and Summers (1981) on increased obsolescence. Between 1979 and 1981, the rate falls again before rising gradually throughout the 1980s. This supports the idea that the office computer revolution in the 1980’s indirectly increased the rate of obsolescence of buildings.

The only problem is that of finding a starting value of capital stock with which to begin this procedure. The starting value chosen here is the same as that chosen for $K^a$: the value of $K^d$ in 1955q1. This measure of the real value of the stock of privately owned industrial and commercial buildings, $K'$, is plotted in Figure 4.4 along with $K^a$ and $K^d$. 

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Figure 4.5: An implied rate of depreciation for industrial and commercial buildings

(per cent per quarter)

Figure 4.6: Annual measures of the stock of privately owned industrial and commercial buildings

(£ billion)
4.2.5.4 A Comparison of the Alternative Measures of Capital Stock

Annualised estimates of $K^a$, $K^β$, and $K^γ$ are plotted in Figure 4.6 along with the Blue Book measure of capital stock, denoted $K^{BB}$, which is only available annually. The first point to note about Figure 4.6 is that as expected $K^{BB}$ exceeds $K^a$ (although at times the margin is small) and $K^β$. This is largely due to definitional differences: $K^{BB}$ includes the stock of infrastructure and the stock of structures in Northern Ireland, whereas both $K^a$ and $K^β$ do not. The divergence between $K^β$ and $K^{BB}$ through time is much more pronounced than the divergence between $K^a$ and $K^{BB}$. $K^γ$ on the other hand, consistently exceeds $K^{BB}$ throughout the 1970s and early 1980s, despite the fact that $K^{BB}$ includes the stock of infrastructure and the stock of structures in Northern Ireland. This is largely due to the fact that $K^{BB}$ is derived under the assumption of a constant rate of depreciation in the national accounts.

It is also noteworthy that $K^β$ exhibits far more variation than any of the other measures. This is due to the fact that $K^β$ is derived using data for repair and maintenance on industrial and commercial buildings and this is somewhat cyclical. Moreover, the perpetual inventory method is well known for generating series which are rather smooth. $K^a$ and $K^γ$ are very smooth even when compared with the $K^{BB}$ which is also generated using the perpetual inventory method. Variations in $K^β$ mirror the variations in the Blue Book estimates quite closely although, on the whole, variations in $K^β$ tend to be more violent than variations in $K^{BB}$. With the exception of 1974 and 1980, in which it exhibits slight downturns, $K^{BB}$ has risen continuously but at varying rates throughout the sample period. $K^β$ on the other hand, exhibits more downturns, some of which are rather substantial and prolonged (e.g. those in 1974 and 1980). The principal difference in the paths of $K^β$ and $K^{BB}$ occurs in 1989/90 when the former takes a dramatic downturn and the latter increases at an accelerated rate. The value of work done repairing and maintaining existing industrial and commercial buildings fell quite sharply as the effects of the recession took hold.
4.2.5.5 Deriving a Measure of Capital Stock: A Summary

In summary, three alternatives to the estimates of net capital stock published in the *Blue Book* have been derived. A comparison of these three measures is presented in Figure 4.4. All three measures have been derived by applying the perpetual inventory formula to the available data in different ways. They all have the advantage that they are available on a quarterly basis and exclude the stock of privately owned infrastructure and the stock of structures in Northern Ireland. $K^a$ is derived from a direct application of the perpetual inventory formula, assuming a constant rate of depreciation. This procedure requires a benchmark starting value. The starting value was assumed to be equal to the value of $K^a$ in 1955q1. $K^b$ is derived by rewriting the perpetual inventory formula such that last period’s capital stock is equal to replacement investment divided by the rate of depreciation. Since this measure does not depend on the value capital stock in a previous period, there is no need for a benchmark starting value. Again, the rate of depreciation is assumed to be constant. In a sense, both of these measures of capital stock are over identified in that they assume a constant rate of depreciation when such an assumption is not necessary. For $K^c$, the perpetual inventory formula is rewritten such that capital stock in the current period is equal to last period’s capital stock plus net investment in the current period. As with $K^a$, a benchmark starting value is required and this is also taken to be the value of $K^c$ in 1955q1. A distinct advantage of this measure is that it is not necessary to assume a constant rate of depreciation. Indeed, an implied rate of depreciation can be derived with the use of repair and maintenance data. As shown in Figure 4.5 and described in Section 4.2.5.3, the assumption of a constant rate of depreciation may be inappropriate. There is an additional disadvantage with both $K^a$ and $K^b$ series resulting from the fact that estimates of the value of the work done repairing and maintaining existing privately owned infrastructure had to be imputed and deducted from total private non-housing repair and maintenance.

Of the three measures, $K^a$, $K^b$ and $K^c$, the latter is considered superior. Since net investment is defined as the change in capital stock, it is important that when first differences of these series are taken, the resulting series resemble net investment, $\Delta K$. First differences of $K^b$ are shown in Figure 4.7. Notice that this series is extremely
Figure 4.7: First differences of capital stock

Figure 4.8: First differences of capital stock
volatile (due to the seasonality in the expenditure on repair and maintenance data) and
bears only a relationship in trend with $\Delta K$, plotted in Figure 4.1. First differences of $K^a$
and $K^f$ are shown in Figure 4.8. Notice that the first changes in $K^f$ are definitionally
equal to the net investment series, $\Delta K$, given in Figure 4.1. Taking first differences of
$K^a$, results in a series that is rather similar to $\Delta K$. The difference in the two series is
accounted for by the fact that $K^a$ is derived assuming a constant rate of depreciation.
Since $\Delta K^f$ is definitionally equal to $\Delta K$, $K^f$ is the preferred measure of the stock of
privately owned industrial and commercial buildings. Hereafter, $K^f$ is referred to as $K$.

Preliminary statistical analysis is conducted with $K$. In such analyses, it must be
remembered that estimates of capital stock are notoriously unreliable: it has already
been noted that government statisticians concede that Blue Book estimates of capital
stock may not be accurate to within 20% (see CSO (1985), p. 201). As such, all
measures of capital stock come with something of a health warning. However, since
our measure of the stock of privately owned industrial and commercial buildings is
constructed using construction output data, it is likely to be measured with less error
than published estimates of capital stock in general.

4.2.6 Deriving a Measure of the User Cost of Capital, $c$

As was demonstrated in Section 2.5, the neoclassical model of investment posits a
role for the user cost of capital. In this section we develop a measure of the user cost
of capital for privately owned industrial and commercial buildings. The resulting
measure is adjusted to take account of corporate taxation and the effect of various tax
incentives associated with investment in buildings.

4.2.6.1 A Basic Measure

Unlike the labour input, for which wage rate data is typically available, the user cost
of capital is not directly observed. Although some types of investment goods have
active rental markets (e.g. aeroplanes and some industrial, retail and commercial
buildings), in many cases firms purchase capital inputs and consume them entirely by
themselves. An implication of this is that because of a lack of data, firms must
typically infer indirectly the user cost of capital that they implicitly charge themselves to use their own investment goods. If the second hand market for investment goods, which certainly exists in the case of buildings, is assumed to be perfectly competitive such that firms are indifferent between owning and renting capital, the implicit user cost of capital that firms charge themselves must be equal to the price that firms could get if they were to rent their capital to others.

Hall and Jorgenson (1967) have emphasised that the user cost of capital must incorporate at least four factors. First, there is the opportunity cost of having funds tied up in investment goods. Denoting the price of investment goods by $z_t$ and the rate of interest by $r$, the opportunity cost of having funds tied up in a unit of investment goods is given by $z_t r_t$. Second, assuming that investment goods depreciate at a constant geometric rate of $\delta \%$, then the renter of investment goods would need to compensate the owner for depreciation. The per unit depreciation cost is given by $z_t \delta$. Third, a measure of the user cost of capital should incorporate capital gains. Investment goods experience price changes over time that result in capital gains or losses to their owners. If the expected change in the price of investment goods is denoted $\hat{z}_t$, then a basic representation of the user cost of capital is given by the sum of the above three factors. That is

$$c_t = z_t \left( r_t + \delta - \frac{\hat{z}_t}{z_t} \right)$$  \hspace{1cm} (4.6)

The fourth factor emphasised by Hall and Jorgenson (1967) as affecting firms’ user cost of capital is the effect of taxation. This element of the user cost of capital is discussed in Section 4.2.6.3.

4.2.6.2 Measurement Issues and $c_t$

The measurement of each component of the user cost of capital formula has been the subject of controversy. Before we consider the way in which the user cost of capital is affected by taxation, we should examine some of this controversy. Suitable measures of these components, given the particular investment problem in this thesis, and the related data constraints are also discussed. First, as noted in the discussion of the derivation of capital stock above, the appropriate decay pattern of capital goods has
been the subject of much debate. Recent work by Hulten and Wykoff (1981a and b) and Hulten, Robertson and Wykoff (1989), in contrast to earlier work, finds in favour of a constant geometric decay specification. Since the constant geometric decay assumption is by far the most common in applied investment analysis and since this convention is followed in the derivation of the capital stock variable, it is adopted in this derivation of the user cost of capital without further justification. Such a decay specification has been assumed (without testing) in many UK investment studies with the user cost of capital (see Boatwright and Eaton (1971), Feldstein and Flemming (1971), Bean (1981), and Jenkinson (1981) for examples). In deriving alternative measures of capital stock in Section 4.2.5, $\delta$ was estimated to be 0.025 based on the assumption of an 80 year lifespan for industrial and commercial buildings. $\delta$ is also assumed to take the value of 0.025 for the purposes of measuring user cost. As noted in Section 4.2.5 this value is broadly consistent with values of $\delta$ assumed in other empirical studies of investment behaviour in the UK.

The measurement of $r$, has also been the subject of some controversy. Firms have available various possible sources of funding from which to finance investment, including cash flow and external debt or equity. Since capital markets are imperfect, each of these sources of finance has a distinct cost and it is not entirely clear how these costs should be combined into a single measure of $r$. In practice, US researchers have used a variety of variables as their measure of $r$. These include a corporate bond yield (e.g. the Baa yield), weighted averages of debt and equity costs, and ex post average internal rates of return. UK researchers have also used a variety of measures. For example, Boatwright and Eaton (1971) use the Financial Times dividend yield on ordinary industrial shares, Feldstein and Flemming (1971) use a weighted average of equity and debenture yields, Bean (1981) uses the rate of bank borrowing calculated as two points above the base rate, and Jenkinson (1981) uses the gross redemption yield on short dated five year British government stock. Given the difficulties of measuring the cost of debt and the cost of equity, and the problem of choosing weights with which to combine these costs into a single average measure of the opportunity cost of capital, we follow Jenkinson (1981) and adopt the gross redemption yield on short dated British government stock as the preferred measure of $r$. This data is available from Financial Statistics and on the ONS Database and has
the identifying code AJLV. Pre-1963 data is only available from *Financial Statistics*. Here, gross redemption yield data is available on a monthly basis back to 1960 but only on an annual basis from 1955 to 1959. Between 1960 and 1963, quarterly estimates were constructed by averaging monthly data for each quarter. For the period 1955 to 1959, quarterly estimates were gained by interpolating the annual estimates such that the average of the quarters in any year was equal to the published annual estimate.

Much debate has also centred on the measurement of the capital gains term in the user cost formula. The problem arises because this term should in fact be regarded as an *expected* (annual) capital gains term, and so the empirical researcher must deal with the issue of measuring unobserved expectations. In practice, some researchers have assumed perfect foresight and replaced expected with the realised prices for new investment goods (e.g. Christensen and Jorgenson (1969)), while others have assumed static expectations in either the levels or the changes of investment good prices, and yet others assume nonstatic expectations. An empirical comparisons of alternative expectation formations is given in Ando *et al* (1974).

Researchers in the UK have addressed this difficulty in a variety of ways. Boatwright and Eaton (1971), for manufacturing investment in plant and machinery, and Feldstein and Flemming (1971), for aggregate investment in equipment and structures, ignore the effects of capital gains. Even for investment in plant and machinery, which tends to enter a factory and remain there until the end of its useful life and for which second-hand markets rarely exist, this is likely to be inappropriate. In the absence of second-hand markets, firms do not invest in plant and machinery in periods of rapidly rising prices in order to realise capital gains after the price has finished rising. In this case, the inclusion of the capital gains term with the weight of unity, implied by equation 4.6, will result in a user cost of capital that is too low and a level of desired capital stock that is too high in periods of rising prices. The capital gains term cannot be ignored (i.e. given a weight of zero) however, since in periods of rising (or falling) prices, firms will have an incentive to bring forward (or postpone) investment expenditure. The essence of this argument is given in Nickell (1978, p. 265).

For buildings, which do have active second hand markets, expected capital gains often provide an incentive to invest, especially for speculative property companies which
intend to let their buildings on the rental market. Indeed, when capital markets are perfect as Jorgenson assumes, a weight of unity on the capital gains term is appropriate. One might be tempted to argue that transaction costs and other capital market imperfections mean that companies' investment plans will not be as responsive to changes in expected capital gains as Jorgenson's theory predicts. However, to use these costs of adjustment as an argument for imposing a weight of less than unity on the capital gains term would be inappropriate and inconsistent with the essence of the Jorgenson model which depends heavily on the assumption of perfect capital markets.

Notwithstanding, it may be desirable to impose a weight of less than unity in practice. This is because in periods of rapidly increasing prices, when inflation exceeds the nominal rate of interest, the very large capital gains will lead to the user cost of capital being close to zero and consequently a level of desired capital stock that is unrealistically high to have been generated solely by firms' desire to realise capital gains. In short, firms do not generally buy fixed capital assets to make capital gains. Indeed, in 1974q2 and 1980q1 the rate of inflation in the price of new industrial and commercial buildings in the UK (as measured by the industrial and commercial construction output price index \( z \)), exceeded 40% and 30% respectively. Given respective nominal rate of interests of 12% and 15% for these periods (as measured by the gross redemption yield on short dated five year British government stock, \( r \)) and an assumed rate of depreciation of \( \delta=2.5\% \), the user of cost of capital, and therefore the desired capital stock, would be negative. The implication here is that firms should adjust their capital stocks infinitely. Thus, in order to avoid this undesirable characteristic of Jorgenson's model, a weight on the capital gains term of less than unity is necessary.

It is worth emphasising that a weight of less than unity on the expected capital gains term implies that capital markets are imperfect. To the extent that firms can not sell unwanted (specialised) plant, machinery and buildings on second-hand markets or are faced with transactions costs, Jorgenson's neoclassical model (which depends so heavily on the assumption of perfect capital markets), might not be an appropriate framework with which to analyse firms' investment decisions.
A weight of less than unity has been imposed on the expected capital gains term by Jenkinson (1981) in his measure of the user cost of capital. He employs the actual current rate of inflation in investment goods prices as his measure of expected inflation and the value of the weight on this variable is determined using a grid search procedure. The grid search was conducted over the range from zero to a half. A weight of more than one half would have lead to a negative real interest rate, implying that firms should expand their capital stock infinitely. Rather worryingly, the best equations, both in terms of goodness of fit and absence of serial correlation, had a zero weight on the inflation variable.

Bean (1981) takes an alternative approach to measuring capital gains. He employs two measures of expected capital gains. The first consists of a distributed lag of past investment goods inflation and the second uses the CBI Industrial Trends Survey question relating to the expected trend over the next four months of the price of domestic orders. He avoids weighting the resulting measures of capital gains by entering all the components of the user cost of capital into his neoclassical type model separately.

The measure of expected capital gains adopted in this thesis is a one-period-ahead forecast of the inflation in the price of investment goods, generated from an ARIMA model. No evidence of a unit root was found in the actual inflation series using an augmented Dickey-Fuller regression with five lags. (This lag length was determined by the Akaike Information Criterion, see Akaike (1973)). A number of ARMA models were fitted to the inflation series and an ARMA(9,1) was found to perform adequately against a variety of portmanteau test statistics. The one-period-ahead forecasts generated by this model closely track the actual current rate on inflation in the price of investment goods.

In order to avoid negative values for the user cost of capital, the approach suggested by Nickell (1978) is also followed here and a weight of less than unity is imposed on the capital gains term. The value of the weight was chosen to be large as possible (i.e. as close to unity, its theoretical value in the Jorgenson model) whilst ensuring a positive user cost of capital throughout the sample period. This resulted in a weight of 0.2 on the capital gains term. If firms can sell all unwanted capital assets on second hand capital markets (and if these markets are perfect) then they will invest when
there are capital gains to be made. In this case, a coefficient of unity is appropriate on
the capital gains term. A weight of less than unity, although not consistent with
Jorgenson’s model, implicitly recognises capital market imperfections such as
transactions costs and the fact that firms can not always sell unwanted capital on
second hand markets.

An alternative approach to the problem of a negative user cost of capital makes use of
dummy variables when \( c < 0 \). This might be justified on the grounds that the measured
cost of capital is a poor proxy for the \textit{ex ante} cost of capital. However, this results in a
loss of important information and is therefore unsatisfactory. Thus, imposing a weight
of less than unity would seem to be the most appropriate means of dealing with a
negative cost of capital and is certainly superior to ignoring capital gains altogether.

4.2.6.3 Adjusting \( c_t \) for Taxation

As mentioned earlier, the basic measure of the user cost of capital needs to be
adjusted for taxation. Since various types of taxes affect the after tax return on an
investment, the inclusion of taxation into the user cost formula is a complex issue.
The existence of various types of investment tax allowances on different assets
complicates matters further. Taking the effects of taxation into account, a tax adjusted
user cost of capital is written as

\[
\hat{c}_t = \frac{z_t(1-A_t)(r_t(1-\tau_t)+\delta - \hat{z}/z_t)}{(1-\tau_t)}
\]

where \( A_t \) is the present value of the allowances associated with investment in period \( t \)
and \( \tau_t \) is the rate in corporate income taxation in period \( t \). The economic interpretation
of equation 4.7 can be easily seen. The basic hypothesis regarding the impact of
investment incentives is that they make their impact by modifying the price of the
asset as seen by the investor. Given the value of the tax savings which the investor can
gain by buying the investment asset, an effective price of investment goods is given by
the product of the nominal price index, \( z_t \), and the value of tax savings, \( (1-A_t) \). Thus, if
the value of investment incentives rises and the nominal price index remains constant,
the effective price to the investor will fall since the investor is now receiving a greater
financial incentive than he did prior to the increase in the value of investment.
incentives. The rate of corporate taxation also makes its impact by modifying the investors' perception of the asset price. $r_i$ is adjusted by the rate of corporate income tax since it is the after tax nominal interest rate that determines the after tax opportunity cost of capital (the concept of interest to the investor). The user cost of capital formula has been adjusted for taxation in this way in a number of studies of UK investment (see Feldstein and Flemming (1971), Boatwright and Eaton (1971), Bean (1981) and Jenkinson (1981) for examples).

With the exception of $r$, and $A_i$, all terms in the tax-adjusted user cost formula given in equation 4.7 have been previously defined. It is to the measurement of the corporate rate of taxation, $r$, that attention is now turned.

4.2.6.3.1 Measuring the Rate of Corporate Taxation, $r$

The system of company taxation in the UK has changed several times in the post-war era, each time altering the extent to which it discriminated against distributed profits. Until 1958, companies in the UK paid (in addition to a standard rate of income tax) a tax on profits. The rate of profits tax was higher on that portion of profits distributed than on that retained. A second system, in force between 1958 and 1965, abolished this differential element in profits taxation: both retained and distributed profits were taxed at a single rate, in addition to the tax on income at the standard rate. The two rate system of taxation of profits reappeared with the introduction of corporation tax in 1965. Under this system corporate income was taxed at a single rate of corporation tax but shareholders were charged income tax on dividends and capital gains tax (introduced at the same time) on realised capital gains. This had the effect of raising the tax burden on dividends relative to retentions. This classical system of corporate taxation was replaced in 1973 with an imputation system in order to alleviate part of the double taxation of dividends. The imputation system gives shareholders credit for tax paid by the company, and this credit may be used to offset their income tax liability on dividends. Part of the company’s tax liability is ‘imputed’ to the shareholders and regarded as a prepayment of their income tax on dividends. The company pays tax on its profits at the rate of corporation tax, and any profits that are subsequently distributed are regarded as having already paid income tax at a certain
rate, which is known as the ‘rate of imputation’. In practice, the rate of imputation is set equal to the basic rate of income tax. Shareholders have to pay additional income tax if the marginal rates of income tax exceed the standard rate, while if their marginal rate of income tax is below the standard rate they receive a rebate from the Inland Revenue. This prepayment of shareholders income tax, paid when profits are distributed, is called advance corporation tax (ACT). Total company taxes minus ACT is known as mainstream corporation tax. The various systems of corporate taxation that have operated in the UK in the post-war period are described in depth by King and Fullerton (1984) and by Kay and King (1990).

It has been argued by King and Fullerton (1984) that the number of changes to the corporate tax system in the UK renders it highly misleading to represent the effects of the corporate tax system by a single rate, namely the statutory rate (p. 40). There is a conceptual problem as to which is the relevant tax rate to use in an investment model: the tax rate on total profits, that on distributed profits or that on undistributed profits. Since the relative tax rates on these concepts have changed several times in the post-war period, it is important to determine which is most appropriate in the context of the firm’s investment decision. Feldstein and Flemming (1971) used the tax rate on total profits as measured by the ratio of retentions plus net dividends to taxable income. King (1972) argues that this measure is inappropriate since managers evaluate investment projects with reference only to their effect on retained profits. The tax penalty on distributed profits would be regarded by managers as something that might be passed on to the shareholder. Taxation on retentions, on the other hand, fall on profits available to the management for fixed investment. Thus, the appropriate rate of taxation is that on retained profits. King also provides empirical evidence in support of a rate on retained profits.

Having decided on the most appropriate concept of corporate taxation, attention is now turned to how that rate might be measured. Since 1965, the rate of corporate taxation on retained profits is given by the rate of corporation tax. The rate of corporation tax is not a good measure of the corporation tax liability. For example, a corporate liquidity crisis in 1975, in which the tax payments due would have led to a number of large firms experiencing serious financial difficulties, led to the introduction of a temporary system of ‘stock relief’. This had the effect of eliminating
most of the tax liability of the UK manufacturing sector. Further, throughout the 1970’s capital allowances were so generous that companies with large capital expenditure programs were able to offset almost all of their tax liability against these allowances. The result was that corporation tax revenue as a percentage of total corporate income was extremely low throughout this period. Thus, although the rate of corporation tax in 1975, for example, was 52%, taxes actually paid, as a proportion of taxable corporate income was closer to 10%. This is not to say that an average measure of corporate tax liability is a more appropriate ‘rate of corporate taxation’ than the rate of corporation tax. The neoclassical model of investment behaviour states that the firm will employ capital up to the point where the marginal product of capital is equal to its marginal cost. Since the user cost of capital is a marginal concept, it is the marginal tax rate on retained profits that is important in the firm’s investment decision. Even with the generous capital allowances in the 1970’s, the vast majority of firms paid some corporation tax and, as such, their marginal rate of taxation of retained profits was equal to the rate of corporation tax.

Along with the introduction of ACT in 1973, the Conservative government introduced a new lower rate of corporate taxation for small companies. (Special rates also apply to cooperative and building societies and to insurance companies). In 1990, small companies in the UK whose total profits were less than £200,000 were taxed at a lower rate of 25%, as opposed to the rate of 34% for large companies. Since the floor of the full rate is low and the difference in rates is relatively small, we shall take the basic statutory rate of 34% as the marginal rate of taxation of the corporate sector in 1990.

Thus, the rate of corporate taxation, \( \tau \), since 1965 is assumed to be equal to the corporation tax rate. This rate is published on an annual basis back to 1969 in Inland Revenue Statistics (IRS) 1997 (Table A.4). The annual rate prior to 1969 can be found in the supplementary notes of IRS 1972 or in the annual Report of the Commissioners of Her Majesty’s Inland Revenue (various issues). Changes to the rate of corporation tax usually take effect from the beginning of the fiscal year. Therefore, changes in the rate of corporation tax are assumed to take effect from April 1st.

Obtaining a rate of corporate taxation on retained profits for years previous to 1965 is slightly more complex. As we have already noted, between 1958 and 1965 companies
were taxed at a standard rate of income tax on taxable income and profits, retained or otherwise, were taxed additionally. Thus, the marginal tax rate on retained profits is equal to the standard rate of income tax plus the rate of profits tax. Again, since tax rates are set for the fiscal year, we assume for years in which there is a change in tax rate, the rate in the first quarter of a year is given by the old tax rate and the tax rate in subsequent quarters of that year is equal to the new rate. For the period from 1955q1 to 1958q1, the rate of corporate taxation is calculated as the standard rate of income tax plus the rate on retained profits. The rates of income tax (for 1955 to 1965), retained profits tax (for 1955 to 1958) and the total profits tax (for 1958 to 1965) can be found in respective issues of the annual *Report of the Commissioners of Her Majesty’s Inland Revenue* (various issues). The derivation of this series for corporate taxation is also described in King (1977, Appendix A). Data on all the tax rates described above have been collated by King for all years between 1947 and 1975 and are presented along with the derived rate of corporate taxation for fiscal and calendar years over this period (see King (1977), p. 258-259).

The resulting series for the rate of corporate taxation since 1955 is shown in Figure 4.9. Since 1955, the rate steadily increased to a peak of 56.25% in 1964 before falling to 40% at the beginning of 1965. It remained around this level until 1973 when the rate was increased to 52%. It remained at 52% for ten years until 1983 and since then it has gradually fallen, reaching a post-war low of 31% in 1997.

4.2.6.3.2 Quantifying the Present Value of Investment Incentives, $A_i$

The effective rate of corporation tax on income available for investment in fixed assets depends critically upon the depreciation allowances granted on fixed investment. Investment allowances have been very generous in the past, particularly when concern about the need to stimulate investment grew in the 1960s. Allowances were so generous that companies could write off their capital expenditures against tax much more rapidly than the investment asset itself deteriorated. For example, at their most generous, allowances for plant and machinery between March 1972 and March 1984 permitted companies to write off the whole of any expenditure on plant and machinery against tax in the first year after the investment.
Since we are concerned with investment in new industrial and commercial buildings, it is the allowances associated with this type of investment to which we restrict our attention. Allowances on industrial buildings have also been generous: between 1981 and 1984 for example, companies could write off 75% of their capital expenditure on industrial buildings against tax immediately after purchase. In addition, special cash grants were available to companies investing in assisted regions. For commercial buildings, however, no depreciation allowances have ever been given (except for hotels and commercial buildings in enterprise zones in the early 1980s) and this is because such assets are assumed to retain their value. The result is that throughout the 1970s and early 1980s, investment by most industrial companies qualified for immediate expensing or greatly accelerated depreciation. When combined with the fact that nominal interest payments were tax deductible, this means that the treatment of such investment (where it was debt financed) was extremely generous.

In the 1984 reform of corporation tax, there was a significant reduction in the rate of corporation tax (from 52% to 35% staggered over a transitional period from 1984 to 1986). This was coupled with a reduction in the value of allowances so that they were more closely related to the true rate of economic depreciation. The reduction in the
rate of corporation tax also had the effect of reducing the attractiveness of debt finance. Although these reforms reduced the variability in tax rates for different types of asset, it was achieved at the cost of increasing the overall effective tax rate on new investment. The effects of this tax reform on investment are examined by Dinenis (1989).

Since investment in commercial buildings has never received depreciation allowances, we can ignore investment of this type in calculating the present value of investment allowances. It will be necessary to combine the incentives available on industrial buildings with the zero allowances on commercial buildings in order to measure the value of allowances available on investment in industrial and commercial companies. We discuss how we might combine the series shortly.

Most capital assets attract capital allowances whereby the depreciation of the asset is an allowable expense in compiling the tax liability of the firms. Four main types of allowance have been in operation in the post war period: investment allowances, initial allowances, writing down allowances and investment grants. Investment allowances permit the firm the right to set against tax a given proportion of the initial cost of an asset. In essence, this is a gift from the Inland Revenue and a firm is not required to depreciate the asset by this amount for the purposes of calculating other future tax allowable depreciation charges. The initial allowance is a special allowance that permits an asset to be depreciated for tax purposes by an exceptionally large amount in the first year (compared with the subsequent years). The asset, however, must be written down by this amount in calculating future allowable depreciation. Writing down allowances (or annual allowances as they are sometimes referred to) are the annual amounts by which an asset may be depreciated each year for tax purposes. The most commonly used method of calculating writing down allowances is that whereby an asset is depreciated by a fixed proportion, specified for different types of assets, of its written down value at the end of the preceding year. This is known as the reducing balance method. Generally speaking, a firm can choose between this method and a straight line method in which the asset is written down by a given proportion each year of the asset’s initial costs. In the case of industrial buildings and some other assets, however, the firm has no such choice and must adopt the straight line method of annual depreciation for tax purposes.
In 1966, investment allowances were replaced by a system of cash investment grants. These took the form of a cash payment equal to a fixed proportion, usually 20%, of the cost of the asset and were mainly awarded to the manufacturing, construction and extractive industries. The investment grant must be deducted from the cost of the asset before capital allowances are calculated. Certain items of capital expenditure (including industrial buildings) were not eligible for the investment grant. This system was discontinued in 1970, but various grants have been available for industrial investment since then and these have varied by type of asset and by region. Due to a lack of data, the effects of regional grants on investment expenditure have been ignored and since industrial (and commercial) buildings did not qualify for an investment grant during the period in which this system was in operation, these can also be ignored.

Attention is now turned to the calculation of the present value of investment incentives. The data on the rates of allowances since 1955 has been collected from a number of sources. The relevant allowances for industrial buildings consist of investment allowances, initial allowances and writing down allowances which are respectively denoted \( i^\pi, i^\xi \) and \( i^\varepsilon \). Data for rates of initial and writing down allowances back to 1966 are published in *IRS 1996* (Table A.3). The rates of investment allowances are given for the period from April 1954 to January 1966 (the latter date corresponding to the abolition of investment allowances) in *National Accounts: Sources and Methods 1968* (see CSO (1968), p. 389). The rates of initial allowance and writing down allowance for the period from 1954 to 1968 are also given there (see p. 389 and p. 388, respectively). In what follows, it is assumed that the firm has sufficient taxable income to make full use of such allowances and that the firm expects the current statutory tax and discount rates to remain unchanged. Further, it is assumed that there is no lag in the payment of taxes or in the receipt of due allowances.

The present value of allowances on a unit investment accruing in the year in which investment is undertaken is given by

\[
\left( i^\pi + i^\xi + i^\varepsilon \right) \tau
\]

For the straight line method of tax depreciation, in which the writing down allowance
is a constant proportion of investment expenditure, the present value of allowances on a unit investment received in the first, second and third year after purchase are \( \pi(i^w/(1+r)) \), \( \pi(i^w/(1+r)^2) \) and \( \pi(i^w/(1+r)^3) \) respectively. However, straight line depreciation implies that a building only receives the writing down allowance for a fixed number of years, \( N \), which may be less than the lifespan of the building, \( n \). \( N \) is determined such that total allowances do not exceed the initial investment expenditure, that is

\[
i^u p^I I + i^u p^I I + (1 + N)i^w p^I I = p^I I \quad (4.9)
\]

and so

\[
N = \frac{(1 - i^u - i^w)}{i^w} \quad (4.10)
\]

Thus, the present value of allowances on a unit investment received in year \( N \) is \( \pi(i^w/(1+r)^N) \). Whilst the lifespan of the building may exceed \( N \), no allowances are received in years subsequent to \( N \) and thus the present value of allowances received in these years is zero. Thus, the present value of future depreciation allowances on a unit investment in industrial buildings in period \( t \), denoted \( A'_t \), is given by

\[
A'_t = \tau_t \left[ (i^u_t + i^y_t + i^w_t) + \sum_{j=1}^{j} \frac{i^w_t}{(1 + \rho_t)^j} \right] \quad (4.11)
\]

where \( j \) is the lesser of \( N \) years or the building’s lifespan, \( n \), and \( \rho_t \) is the rate of discount. Given that allowances have been rather generous in the past, it is assumed that \( N \) is less than the lifespan of the building and thus that \( j = N \). Equation 4.11 can be written as

\[
A'_t = \tau_t \left[ (i^u_t + i^y_t + i^w_t) + i^w_t \frac{(1 + \rho_t)^N - 1}{\rho_t(1 + \rho_t)^N} \right] \quad (4.12)
\]

This equation represents the present value of future depreciation allowances on a unit investment in industrial buildings in period \( t \). The actual price of industrial buildings must be modified by this amount in order to obtain a measure of the effective price. In other words, \( A'_t \) measures the reduction in the price of new industrial buildings which firms receive because of investment incentives on a typical pound of investment.
Some controversy surrounds the choice of discount rate, $\rho$. Boatwright and Eaton (1971) for example, assume a constant after tax rate of discount of 8%. They argue that their results were not significantly affected by changes in this rate. Feldstein and Flemming (1971) also assume a constant rate of discount of 10%. King (1972) and Sumner (1974) show that the present value of incentives is very sensitive to the value of the discount rate; the higher the value of $\rho$, the lower the value of $A_t$, and the smaller the impact of incentives. Sarantis (1979) uses a vintage model of investment to analyse the effects of investment incentives in ten two-digit manufacturing industries. In calculating the present value of tax savings due to allowances he argues that it is more realistic to measure $\rho$ by the actual rates of return realised in each industry. This was measured by the average post tax rate of return of quoted UK manufacturing companies over his sample period. Bean (1981) and Jenkinson (1981) use the method for calculating the present value of allowances developed by Mellis and Richardson (1976) and in this work the discount rate is taken to be constant at 10%. Bond and Devereux (1989), Dinenis (1989) and Blundell et al (1992) all use the current rate on British government consols to discount future allowances in their $q$ models. In support of this approach, Summers (1987) presents survey evidence that suggests these discount rates are usually equal to post tax real financial cost of investment. The real financial cost of investment has been proxied by the gross redemption yield on short dated five year British government stock in this work and so this is taken as the appropriate rate of discount in the calculation of the present value of investment incentives.

Given the rates of allowances and the dates of changes in these allowances, it is possible to construct quarterly series for each of these allowances. Where a change in the rate occurred within a quarter, the rate was determined by a weighted average of the old and new rates, the weights being determined by the number of the days at each rate as a share of the total number of days in a quarter (which was taken to be 91). Given quarterly series for initial, investment and writing down allowances, and series for the discount rate, the rate of corporate taxation and the number of years for which an investment receives an allowance, a quarterly series for the present value of future allowances on a unit investment in industrial buildings was constructed in accordance with equation 4.12.
In order to obtain a measure of the present value of incentives on total buildings, it is necessary to combine the present value of incentives on industrial buildings with the zero value of incentives on commercial buildings. Bond and Devereux (1989) and Blundell et al (1992) use calculations made by Devereux (1986) and take a weighted average of the two series, assuming that industrial buildings constitute sixty five per cent of the total value of buildings. Thus

\[ A_t = a_t A_t^I + (1 - a_t) A_t^C \]  

(4.13)

where \( a_t \) is constant at 0.65 and \( A_t^C \) is the present value of allowances on commercial buildings (which is zero for all \( t \)). Although the ratio of value of industrial buildings to total non-residential buildings is not constant over time, the variation is likely to be small and slow. An alternative means of combining the value of incentives on industrial and commercial buildings makes use of investment (or, more precisely, construction output) data. The weight associated with the present value of allowances on industrial buildings, \( a_t \), is determined as the value of net investment in industrial buildings (see HCS, June 1997, part 2, Tables 2.3 and 2.4) as a share of the value of total net investment in all non-residential buildings (measured by the value of new private industrial and commercial construction output, \( \Delta K \)). It is this procedure for combining the two series that is adopted here. The present value of allowances on industrial buildings, along with the present value of allowances on industrial and commercial buildings, are shown in Figure 4.10. It is clear that these incentives amount to a substantial inducement to productive investment. Until 1985, the value of these incentives has always exceeded 10% and at their most generous (for example, in the early to mid 1960s and the late 1970s to early 1980s) the value of these incentives was almost 20%. Thus, the value of incentives allowed firms to recoup almost 20% of the price of a typical investment. Since 1983, the value of these incentives have decreased considerably due, in part, to the decrease in corporation tax and, in part, to the fact that the value of investment in industrial buildings as a share of total investment in buildings has been decreasing since this time. As a result, the effective price of investment in buildings differs substantially from the actual price of buildings. Figure 4.11 shows the effective and actual price of investment in buildings (in real terms). It is immediately apparent that the value of incentives was considerable prior to 1983.
Figure 4.10: The present value of allowances on a unit investment

Figure 4.11: The actual and effective real price of investment in buildings
4.2.6.4 The Resulting Measure of User Cost

Having described the derivation of each of the components of user cost in some detail, attention is now turned to the resulting measure of the user cost. This measure was constructed according to equation 4.7 with a weight of 0.2 on the expected capital gains term $z_t$. The resulting series is shown in constant prices (derived by deflating the nominal price series by $p$) in Figure 4.12, along with a measure of the real user cost which excludes capital gains altogether (included for comparison). Since these estimates may be of some general interest and since they are rather tedious to calculate, they are presented in Table 4.1. With the exception of the period between 1970 and 1982, the two series follow similar paths. Before 1970 and after 1982, expected capital gains are relatively small. However, expected capital gains between 1970 and 1982 (even after multiplication by 0.2) are quite substantial and this is reflected in the divergence of the two series. In 1974q2, when the value of expected capital gains peaked, the measure of real user cost including expected capital gains takes on a value very close to zero. It is this observation that prevents the weight on the expected capital gains term from taking on a value greater than 0.2 (recall, the weight was chosen to be as close to unity as possible, whilst avoiding a negative real interest rate which would imply that firms should expand their capital stock infinitely). If one adheres rigidly to the Jorgenson definition of user cost with a coefficient of unity on the expected capital gains term (this series is not shown in Figure 4.12), then the estimated cost of capital in 1974q2 is -0.74. This implies a negative desired capital stock which is a rather unwelcome characteristic of the Jorgenson model. Imposing a weight of 0.2 on the capital gains term would seem to be a more reasonable way around this problem than ignoring capital gains altogether. It is noteworthy that since the user cost of capital is close to zero in 1974q2, it follows that desired capital stock in this period will be very large. This might cause problems in the preliminary statistical analysis conducted later in this chapter and in the estimation of the neoclassical model.
4.2.7 Deriving a Measure of $Q$.

Recall, the cost of adjustment model developed in Section 2.5 resolves the inconsistency between Jorgenson’s theoretical model and his empirical implementation of that model. However, the resulting cost of adjustment equation relates investment to an unobservable concept, the shadow price of capital $m$. As discussed in Section 2.6, Tobin’s $q$ theory provides a means by which this unobservable concept can be related to measurable variables summarised within $Q$. In this section we develop a measure of $Q$. Again, the effects of corporate taxation and investment incentives in the tax system complicate the derivation of this measure considerably. We also consider some of the limitations of the resulting series.

4.2.7.1 A Recapitulation of the Basic Model

In Chapter 2 (equation 2.89), it was shown that the standard $q$ model states that given a suitably defined adjustment cost function, the optimal rate of investment has a one-to-one relationship with $Q$ defined as
\[ Q_t = (q_t - 1) \frac{z_t}{p_t} \]  

(4.14)

where \( z_t \) is the price of investment goods in period \( t \), \( p_t \) is the price of output in period \( t \) and \( q_t \) is the ratio of the shadow value of an extra unit of capital to the replacement cost of that unit (uninstalled). If certain conditions derived by Hayashi (1982) are fulfilled, then marginal \( q \) is equivalent to average \( q \), \( q^a \), which (if stock markets are efficient) is observable. Under these conditions, \( q_t \) is defined, in discrete time when capital is valued at the end of the period, as

\[ q_t = \frac{\text{\( V_t \)}}{1 - \delta} \frac{\text{\( z_t K_{t-1} \)}}{\text{\( K_{t-1} \)}} \]  

(4.15)

where \( V_t \) denotes the stock market value of the firm, \( \delta \) is the rate of economic depreciation and \( K_{t-1} \) is the stock of all physical capital at the end of period \( t-1 \).

4.2.7.2 Extending the Basic Model

This basic model can be extended to allow for the effects of corporate taxation whilst retaining the relationship between investment and \( q^a \). In order to achieve this, a number of adjustments need to be made to equation 4.15. First, the effective output price becomes \((1 - \tau) p_t \), where \( \tau \) is the rate of corporate taxation and is defined above in the derivation of a measure of the user cost of capital (see Section 4.2.6.3.1). Second, the price of capital goods is modified by a factor \((1 - A_t)\), where \( A_t \) is the present value of investment incentives which arise as a result of a unit increase in investment, to give an effective price of capital goods \((1 - A_t) z_t \). \( A_t \) has also been derived above in the discussion of the user cost of capital variable (see Section 4.2.6.3.2).

Here, as in the user cost of capital, it is assumed that firms earn sufficient taxable income to make full use of such allowances and that the firm expects the current statutory tax and discount rates to remain unchanged. Further, it is assumed that there is no lag in the payment of taxes or in the receipt of due allowances. Finally, market value of the firms \( V_t \), reflects the present value of all future depreciation (writing down) allowances that can be claimed on past investment. This is denoted \( B_t \) and must be deducted from the numerator of \( q_t \) to find the market valuation of capital stock.
Table 4.1: The real user cost of capital and tax-adjusted Q for companies and financial institutions
Year
1955Q1
1955Q2
1955Q3
1955Q4
1956Q1
1956Q2
1956Q3
1956Q4
1957Q1
1957Q2
1957Q3
1957Q4
1958Q1
1958Q2
1958Q3
1958Q4
1959Q1
1959Q2
1959Q3
1959Q4
1960Q1
1960Q2
1960Q3
1960Q4
1961Q1
1961Q2
1961Q3
1961Q4
1962Q1
1962Q2
1962Q3
1962Q4
1963Q1
1963Q2
1963Q3
1963Q4
1964Q1
1964Q2
1964Q3
1964Q4
1965Q1
1965Q2
1965Q3
1965Q4
1966Q1
1966Q2
1966Q3
1966Q4
1967Q1
1967Q2
1967Q3
1967Q4
1968Q1
1968Q2
1968Q3
1968Q4

c,
0.07217
0.07568
0.07773
0.07898
0.05939
0.06755
0.07209
0.07396
0.08039
0.07994
0.08705
0.07468
0.07412
0.07141
0.07841
0.08226
0.08324
0.08112
0.08228
0.08308
0.08481
0.07973
0.08419
0.08198
0.08158
0.07943
0.08038
0.08669
0.07113
0.07374
0.07201
0.07358
0.06848
0.07312
0.06994
0.06414
0.08663
0.07798
0.07459
0.07423
0.08966
0.08548
0.08444
0.07808
0.08011
0.07555
0.08111
0.07770
0.08096
0.09205
0.09473
0.09051
0.08779
0.07799
0.07773
0.08246

Q.
-0.3502
-0.5267
-0.6134
-0.7494
-0.7774
-0.8222
-0.8194
-0.8160
-0.7959
-0.8022
-0.8443
-0.7959
-0.7224
-0.8263
-0.2076
-0.1145
0.2396
0.1455
0.3956
0.3294
0.4256
0.3728
0.5276
0.1877
0.1743
-0.0900
-0.0872
-0.0973
0.1131
0.0057
0.5874
0.6444
0.9037
0.8246
0.9609
0.9807
0.8670
0.4323
0.3591
0.2125
0.2781
0.4592
0.3127
0.1528
0.0282
-0.2710
-0.0936
-0.0632
0.1106
0.1992
0.3950
0.5223
0.7777

Year
1969Q1
1969Q2
1969Q3
1969Q4
1970Q1
1970Q2
1970Q3
1970Q4
1971Q1
1971Q2
1971Q3
1971Q4
1972Q1
1972Q2
1972Q3
1972Q4
1973Q1
1973Q2
1973Q3
1973Q4
1974Q1
1974Q2
1974Q3
1974Q4
1975Q1
1975Q2
1975Q3
1975Q4
1976Q1
1976Q2
1976Q3
1976Q4
1977Q1
1977Q2
1977Q3
1977Q4
1978Q1
1978Q2
1978Q3
1978Q4
1979Q1
1979Q2
1979Q3
1979Q4
1980Q1
1980Q2
1980Q3
1980Q4
1981Q1
1981Q2
1981Q3
1981Q4
1982Q1
1982Q2
1982Q3
1982Q4

c,
0.09097
0.10033
0.09896
0.09938
0.09395
0.07657
0.07050
0.07358
0.07014
0.07002
0.05715
0.04540
0.03982
0.05254
0.07463
0.06433
0.04460
0.03751
0.04595
0.03140
0.01546
0.00470
0.04237
0.07701
0.10686
0.10752
0.11457
0.11501
0.12040
0.12178
0.14603
0.14925
0.11554
0.09633
0.10111
0.09912
0.09410
0.11072
0.11252
0.11328
0.10244
0.06854
0.06559
0.07059
0.07685
0.06495
0.08225
0.09920
0.13621
0.16057
0.19547
0.22155
0.21654
0.19242
0.15433
0.13241

Q,

0.6434
0.4117
0.2764
0.2319
-0.0486
-0.2058
-0.2601
-0.1999
-0.2555
-0.2453
-0.1667
0.0651
0.0773
0.1815
0.1294
0.1664
-0.0650
-0.4831
-0.6041
-0.6911
-0.9402
-1.1899
- .3335
- .4130
- .3669
- .4234
- .4556
- .3358
- .3146
- .3769
- .3933
- .3331
- .3146
- .2463
- .1567
- .0023
- .0329
- .0237
- .0899
- .1014
- .1890
- .0208
- .0048
-0.7930
-0.7773
-0.7927
-0.8923
-0.8397
-0.9952
-0.8910
-0.9680
-0.8938
-0.9853
-0.8049
-0.7889
-0.6081

Year
1983Q1
1983Q2
1983Q3
1983Q4
1984Q1
1984Q2
1984Q3
1984Q4
1985Q1
1985Q2
1985Q3
1985Q4
1986Q1
1986Q2
1986Q3
1986Q4
1987Q1
1987Q2
1987Q3
1987Q4
1988Q1
1988Q2
1988Q3
1988Q4
1989Q1
1989Q2
1989Q3
1989Q4
1990Q1
1990Q2
1990Q3
1990Q4
1991Q1
1991Q2
1991Q3
1991Q4
1992Q1
1992Q2
1992Q3
1992Q4
1993Q1
1993Q2
1993Q3
1993Q4
1994Q1
1994Q2
1994Q3
1994Q4
1995Q1
1995Q2
1995Q3
1995Q4
1996Q1
1996Q2
1996Q3
1996Q4

c,

Q,

0.14805
0.14214
0.14649
0.13835
0.13113
0.13426
0.13667
0.12429
0.13234
0.12846
0.11576
0.10773
0.10576
0.09986
0.12081
0.14299
0.12139
0.10823
0.11952
0.11767
0.10844
0.10158
0.10636
0.10134
0.10491
0.11000
0.11493
0.12450
0.14582
0.15031
0.15263
0.14788
0.14313
0.13579
0.13832
0.13017
0.12427
0.11412
0.11541
0.09230
0.08328
0.07913
0.07225
0.06368
0.06463
0.07554
0.07970
0.07331
0.06398
0.05828
0.05418
0.05620
0.06335
0.07121
0.07423
0.07795

-0.6999
-0.5640
-0.5420
-0.3767
-0.4203
-0.3635
-0.3942
-0.2184
-0.2042
-0.1370
-0.1009
0.1717
-0.1254
-0.1648
-0.1965
-0.1029
-0.0768
0.0734
0.2207
0.3616
0.2028
0.1101
0.0180
0.1371
0.0667
0.0954
0.1165
0.1177
-0.0040
-0.1027
-0.2912
-0.2978
-0.3032
-0.2574
-0.2767
-0.0693
-0.1440
-0.0998
-0.1850
0.0654
0.2249
0.4103
0.4331
0.6242
0.4382
0.3764
0.2917
0.3153
0.3388
0.3220
0.3356
0.5389
0.6633
0.6875
0.7619
0.9329

Note: the construction of the real user cost of capital and tax-adjusted Q is described in Section 4.2.6
and 4.2.7, respectively.

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A further adjustment of equation 4.15 is necessary to account for an inconsistency in the coverage of the numerator and denominator of \( q_t \). The denominator of \( q_t \) measures the replacement value of the stock of fixed capital and as such does not include the inventories and work in progress. On the other hand, the numerator \( V_t \) measures the market value of the firm which will include the value of inventories and work in progress. One approach to the treatment of inventories is to add together replacement value of inventories and the physical capital stock and consider this a measure of the total replacement value of the firm’s physical assets. This approach, used by Porteboa and Summers (1983), involves adding the value of inventories to the denominator of equation 4.15. The alternative approach, used by Bond and Devereux (1989) and adopted here, is to treat inventories as liquid assets. This implies that there are no adjustment costs incurred in changing the level of inventories. Given this assumption, a valuation of the firm’s fixed capital can be derived by subtracting inventories from \( V_t \) in equation 4.15.

Incorporating all these factors yields a tax adjusted \( Q \), which is given as

\[
Q_t = \left[ \frac{V_t - B_t - Z_t}{(1 - \delta)z_t(1 - A_t)K_{t-1}} - 1 \right] \frac{z_t(1 - A_t)}{p_t(1 - \tau_t)}
\]  

(4.16)

where \( Z_t \) is the value of inventories.

4.2.7.3 Deriving the Ingredients of \( Q_t \)

With the exception of \( V_t, B_t \), and \( Z_t \), all variables in equation 4.16 have been derived above. In order of create a series for \( Q_t \), we need to obtain data on these three variables. We consider the derivation of \( B_t \) first.

4.2.7.3.1 The Present Value of Allowances Remaining on Past Investment, \( B_t \)

In the context of the investment problem addressed in this thesis, \( B_t \) represents the present value of remaining writing down allowances on past investment in industrial and commercial buildings. Since commercial buildings receive no allowances, the problem reduces to one of calculating the present value of remaining allowances on past investment in industrial buildings, \( B'_t \). The calculation of \( B'_t \) is slightly more
complex than the calculation of $A'$, the present value of incentives of current investment in industrial buildings. In deriving $B'$, we adopt the notation used in the derivation of $A$. Consider an investment period $s$, where $s < t$. The present value of investment allowances on this investment at period $t$ is

$$B'_{t-s,t} = \sum_{k=0}^{s + \mathbb{N}_s - t} \frac{\tau_t i_s z_s I_s}{(1 + \rho_t)^k}$$  \hspace{1cm} (4.17)$$

where $s + \mathbb{N}_s - t$ represents the number of periods after $t$ for which an investment made in period $s$ receives a writing down allowance. The sum of all such allowances includes all periods, $s$, for which $s + \mathbb{N}_s > t$, i.e. all periods $s$ in which an investment made in period $s$ receives a writing down allowance after period $t$. If the total of such periods is denoted $F$, then the present value of depreciation allowances on past investment in industrial buildings, $B'$, is

$$B'_t = \sum_{s=0}^{F_t} B'_{t-s,t}$$  \hspace{1cm} (4.18)$$

Thus, the present value of depreciation allowances remaining on past investment in industrial and commercial buildings is

$$B_t = B'_t + B'_c = B'_t$$  \hspace{1cm} (4.19)$$

since $B'_c$ is zero. There are a number of computational difficulties involved in constructing $B_t$ to which attention is now turned.

These difficulties arise as a result of the fact that the construction of $B_t$ requires data for investment and the rate of writing down allowance in period $t - \mathbb{N}_s$. Consider a period in which an investment is entitled only to an annual writing down allowance of 2%. In such a period, equation 4.10 generates a value of $\mathbb{N}_s$ equal to 49 years. This implies that investment undertaken in any particular year will receive a writing down allowance in that year and in 49 subsequent years. Prior to 1944, the only allowance available on investment in industrial buildings is the writing down allowance and the rate was set at 2%. An investment made in any year prior to 1944 (and after 1878 when wear and tear allowances were first introduced) will receive a writing down allowance for 49 subsequent years. By implication, in 1955 for example, the present value of investment allowances remaining on past investment will depend in part on
the value of investment undertaken 49 years previously, i.e. in 1906. Thus, in order to construct the present value of investment allowances remaining on past investment data on investment in industrial buildings (and the associated rate of the writing down allowance) is required back to 1906.

Unfortunately, a series for investment in industrial buildings is not available back to 1906. Pre 1955 investment data of any kind is rather sketchy and generally regarded as somewhat unreliable, especially prior to 1948. Further, of the series that do exist, all are annual. Given the lack of quarterly data for this period and the computational complexities involved in its construction, \( B \) is first constructed on an annual basis and then interpolated to arrive at quarterly estimates.

Annual estimates for investment in industrial buildings were derived as follows. For the period 1948 to 1955, estimates are taken from the *Blue Book 1959* (Table 47, p. 50). This series measures investment in ‘other buildings and works’ carried out by the private sector. As such, it includes investment in commercial buildings and infrastructure. The series is spliced with the post 1955 series for investment in industrial buildings, as measured by the associated construction output series (see *HCS*, June 1997, part 2, Tables 2.3 and 2.4). Redfern (1955) presents data for total investment in ‘other buildings and works’ for the period 1924-53 (excluding the period 1939-47). This includes private sector investment in commercial buildings, infrastructure and all investment in buildings and infrastructure carried out by the public sector. In the absence of relevant data prior to 1924, it is assumed that real investment in ‘other buildings and works’ between 1906 and 1923 is constant and equal to the level of real investment in 1924. (This assumption is not very plausible, particularly since the First World War occurs in this period). A current price series can be derived using Redfern’s price indices for ‘buildings and works’. Thus, from Redfern (1955) a current price series for investment in ‘other buildings and works’ for the period 1906 to 1948 (excluding the period 1939-47) has been derived. Although discontinuous, this series is spliced with the post 1948 private investment in industrial buildings data. The war years present a further problem. However, information in Dean (1964, Table 2) allows us to overcome this problem. Dean presents estimates for privately financed fixed investment in buildings for the period 1939-45. For 1941, 1942 and 1943 this series takes values of zero. Thus, zero values can be imposed on
the series for privately financed investment in industrial buildings in these years. Estimates for 1939, 1940, 1944 1945, 1946 and 1947 are linearly interpolated.

This rather crude series for pre-1955 private investment in industrial buildings used to construct $B$, should be regarded as providing no more than an indication of the general trend in such investment. Nevertheless, the use of this series will result in a measure of $B$, that is superior to one obtained from ignoring pre-1955 data altogether.

The pre-1955 rates of initial and investment allowances (introduced in April 1944 and April 1954 respectively) were obtained from National Accounts Statistics: Sources and Methods 1955 (see CSO (1955), p. 330). The rate of annual writing down allowance is obtained from The Capital Allowances Act 1968 and The Income Tax Act 1945.

The resulting measure of $B$, is plotted in Figure 4.13. Notice that the series falls quite substantially in the early 1980s. This fall reflects rather generous initial allowances available on investment in industrial buildings in the 1970s and early 1980s. Generous initial allowances have the effect of reducing the total number of years that an investment receives a writing down allowance, thereby reducing $B$,.
4.2.7.3.2 The Market Value of Firms, $V_t$

Following Oulton (1981), Jenkinson (1981) and Dinenis (1989), the market value of firms was constructed by capitalising the streams of dividend and debt interest payments at appropriate rates. In practice, there are a number of complications associated with this method of calculating $V_t$. The first arises from the fact that foreign owned subsidiaries have capital located in the UK. Earnings from this capital do not remain in the UK but contribute towards the profits of the foreign owned parent. Any measure of firms’ market value gained simply by summing dividend and interest payments will exclude the value of foreign owned capital located in the UK, since dividend payments data do not include payments by foreign owned companies to their foreign shareholders. Thus, in order to gain a measure of the market value of all capital located in the UK, we must add the value of foreign owned capital to dividend and interest payments. The second problem arises due to the fact that UK companies have capital located abroad from which they earn income. This income will affect the value of shareholders’ dividend payments. This is a problem since the denominator of $q$ includes only capital assets located within the UK. Similarly, firms earn income from financial assets (as well from physical capital) and again the denominator of $q$ includes only physical assets. In order that $V_t$ measures the value of domestically located physical capital, it is assumed that income from financial assets and that from abroad constitute the same proportion of firms’ assets as they did of the firms’ income.

Given these complications, the market value of all quoted companies and financial institutions is calculated as

$$V_t = \theta \left[ \frac{DPO_t}{r^o_t} + \frac{DPP_t}{r^p_t} + \frac{DIP_t}{r^D_t} + \frac{PDA_t}{r^f_t} \right]$$ \hspace{1cm} (4.20)

where $DPO_t$, $DPP_t$, $DIP_t$, and $PDA_t$, respectively denote firms’ annualised dividend payments on ordinary shares, annualised dividend payments on preference shares, annualised debt interest payments, and profits due abroad, all in period $t$. $r^o_t$, $r^p_t$, $r^D_t$, and $r^f_t$ are respectively the dividend yield on ordinary shares, the dividend yield on preference shares, the redemption yield on 20-year debentures, and the capitalisation rate for profits due abroad. The final term inside the brackets represents the value of
foreign owned capital located in the UK. \( \theta \) represents an adjustment factor for income earned by UK companies from assets located abroad and from financial assets.

Data constraints create further difficulties in the construction of \( V_t \). We consider each of these difficulties in turn. Attention is first turned to the value of dividend payments. Published data on the value of dividend payments is not separated by type of share. A series on the value of dividend payments on both ordinary shares and preference shares has been published since 1955. A series measuring the value of dividend payments on ordinary shares was published until 1988. With these two series one could impute the value of dividend payments on preference shares up to 1988. However, there is insufficient published data to calculate the value of dividend payments on preference shares since 1988.

Dividend payments on preference shares have always accounted for only a small proportion of total dividend payments. Given this, the first two terms in equation 4.20 were combined. The capitalisation rate for total dividend payments is taken as the dividend yield on the FT-All Share index. Although the All Share index comprises of only ordinary shares, the associated dividend yield is a good proxy for the combined yield on ordinary and preference shares since the vast majority of shares are ordinary shares.

A series for the value of dividend payments on ordinary and preference shares \( (DPO_t + DPP_t) \) is published back to 1963q1 in ETAS 1996 (Table 1.11). Data for this series back to 1955q1 is published in ETAS 1994 (Table 1.11). The reorganisation of the data in this table in the 1997 edition of ETAS has resulted in the series being combined with the value of interest payments. Thus, post 1995q2 data for dividend payments has been obtained from the ONS Database (the identifying code for this series is CIKB).

The capitalisation rate, \( r^D \), measured by the dividend yield on the FT-All Share Index, is available on a monthly basis from Financial Statistics (June 1996, Table 7.1G, p. 120) and on a quarterly basis on the ONS Database (with the identifying code AJMD). This index combines the FT-Index of 500 ordinary industrial shares and an index of financial companies shares. However, the All Share Index was not published prior to 1962q2 and, as such, there is no associated dividend yield. The broadest
dividend yield series available prior to 1962 is that corresponding to the FT Index of 180 Ordinary Industrial Shares. Monthly estimates of the dividend yield on this index are published (for the period 1955 to 1963) in the *Annual Abstract of Statistics 1965* (Table 351, p. 300). Quarterly estimates are obtained by taking arithmetic averages of the monthly estimates in each quarter. Thus, $r_O$, is obtained by splicing the dividend yield on the FT-All Share Index (available from 1962q2) with the dividend yield on the FT Index of 180 Ordinary Industrial Shares (1955q1 to 1962q2). Combining the value of dividend payments and the capitalisation rate gives a measure of the quarterly dividend flow, $(DPO_t + DPP_t)/r_O$, which was annualised using a four quarter sum.

The series for debt interest payments was slightly more difficult to construct due to a change in the way companies current account data is compiled. Data for this series is available for the period 1963q1 to 1995q2 and can be found in *ETAS 1996* (Table 1.11). Data for the period 1955q1 to 1963q1 can be found in *ETAS 1994* (Table 1.11). In the *ETAS*, the current account data for companies and financial institutions presents data on interest payments in a column referred to as ‘other’ (the definition of which is given as dividend and interest etc. minus payments of dividends). This series has the identifying code CIDV. For the post 1995q2 period, this series has to be constructed since dividends and interest payments data is now collected on a different basis. It is constructed by subtracting series for dividend payments, profits due abroad, UK taxes on income, and unallocated income from total income. Since the series for total income is also affected by the way in which dividend and interest payment data is compiled, this series is also constructed post 1995q2. The data for total income in *ETAS 1996* is spliced with the data for the same series in *ETAS 1997*.

The associated rate of capitalisation, $r_D^P$, is the redemption yield on 20-year debentures. This is also less than ideally measured. (It is noteworthy that debentures form a significant component of interest payments). This redemption yield, which is based on 15 redeemable debentures weighted to give an average maturity of 20 years, was not compiled prior to 1965. However, a flat yield, based on 15 undated debentures, was published in *Financial Statistics* up until the end of 1962. This data is presented in a monthly format from 1960, but is only available on an annual basis between 1955 and 1960. Arithmetic averages of monthly estimates are used to construct quarterly estimates for the period 1960q1 to 1962q4. Quarterly estimates of
the flat yield are generated for the period 1955q1 and 1959q4 by linearly interpolating the annual averages. This results in a quarterly measure of the flat yield on debentures from 1955q1 to 1962q4. The redemption yield data is published in *Financial Statistics* on a monthly basis from 1965q1 to 1994q4, at which time it ceased to be compiled. There is a small discontinuity in 1981q1. Arithmetic averages of these monthly estimates are taken to give quarterly estimates and the discontinuity in 1981q1 was overcome by splicing the data. In order to get a complete run of quarterly data from 1955q1 to 1994q4, the missing quarterly estimates for the period 1963q1 to 1964q4 are imputed. This is done by linearly interpolating between 1962q4 (the last estimate in the flat yield series) and 1965q1 (the first estimate in the redemption yield series). Data from 1995q1 onwards is derived by splicing the debenture yield data up to 1994q4 with the redemption yield on short-dated five year British Government Stock (published in *Financial Statistics* or available on the *ONS Database* with the identifying code AJLV). The debenture and loan stock interest data and the capitalisation rate are then combined to give the quarterly interest flow, $DIP_t/r^p_t$, which are then annualised using a four quarter sum.

The final term inside the brackets of equation 4.20 represents the value of foreign owned capital located in the UK. It is proxied by capitalising the value of profits earned in the UK by foreign firms. A series for ‘profits due abroad’ is published in *ETAS 1996* (Table 1.11). This table contains quarterly data on ‘profits due abroad’ back to 1963, but in previous editions of *ETAS* (e.g. 1994) such data is published back to 1960 with annual estimates going as far back as 1951. Quarterly data on this series from 1960 is also available from the *ONS Database* (with the identifying code CIBU). For the period prior to 1960, quarterly estimates were imputed by interpolation ensuring that the quarterly estimates in any year sum to the annual figure.

The capitalisation rate adopted for profits due abroad, $r^d_t$, is that used by Oulton (1981). Here, profits due abroad are capitalised at the same rate as gross after-tax profits accruing to UK companies are capitalised. That is

$$r^d_t = \frac{GP_t - PDA_t}{\left[ \frac{DPO_t}{r^d_t} + \frac{DPP_t}{r^d_t} + \frac{DIP_t}{r^d_t} \right]} \quad (4.21)$$

where $GP_t$ denotes gross profits after tax, i.e. total income net of UK taxes on income.
Although this rate of capitalisation is not ideal, the fact that profits due abroad constitute only a small proportion of corporate income implies that any error in the value of earnings from foreign owned capital located in the UK, arising from an inappropriate choice of capitalisation rate, will be small.

Gross after tax profits are defined as gross trading profits before stock appreciation plus rent and non-traded income plus income from abroad minus UK taxes on income. A series for gross trading profits before stock appreciation is published in *ETAS 1996* (Table 1.11). *ETAS 1994* contains quarterly estimates of this series back to 1955q1, whereas later editions publish data only as far back as 1960 or 1963. The data is also available back to 1960 on the *ONS Database* (identifying code CIAC). Rent and non-traded income is published from 1955q1 to 1995q2 in *ETAS*. Data on rent and non-traded income for companies and financial institutions has not been published since 1995q2. (This is partly a result of a reorganisation of Table 1.11 of *ETAS*, following a change in the way data on dividend and interest payments are compiled). However, a proxy for rent and non-traded income can be gained by subtracting gross trading profits before stock appreciation (identifying code CIAC) and income from abroad (CIAL) from total income (CIDB). The resulting quarterly estimates for rent and non-traded income are much larger than published estimates of the series in *ETAS*. This is due to the fact that the old method of compiling data on payments of dividends and interest underestimated actual payments. The post 1995q2 data generated by subtracting gross trading profits and income from abroad from total income is spliced with the pre-1995q2 published estimates of rent and non-traded income. Income from abroad is available on a quarterly basis back to 1960q1 in *ETAS* and on the *ONS Database*. Annual estimates of the series are published in *ETAS* back to 1946 (in *ETAS 1994*, for example). Quarterly estimates for income from abroad for the period 1955 to 1960 are also obtained by linear interpolation.

Data for profits due abroad, $PDA_n$, is available on a quarterly basis back to 1960 from *ETAS 1994* (Table 1.11) and from the *ONS Database* (identifying code CIBU). *ETAS* contains annual estimates for this series back to 1951 and these are used to derive quarterly estimates by linear interpolation.

The denominator of equation 4.21 represents the capitalised value of dividend and interest payments. The components of the denominator of equation 4.21 have been
derived above. The resulting measure of $r^i_t$ should only be regarded as a crude approximation to the appropriate rate of capitalisation for profits due abroad. As such, $PDA_t/r^i_t$ will be less than ideally measured. However, the fact that the capitalised value of profits due abroad constitute only a small fraction of $V_t$ implies that any error in $V_t$ that follows from these data deficiencies is unlikely to be of first order importance.

$\theta$ (the adjustment necessary to exclude income from assets located abroad and income from financial assets from the numerator of $q$) is defined by Oulton (1981) as the ratio of domestic income from non-financial assets to total income. Domestic income on non-financial assets is defined here as gross trading profits plus rent. Both of these series are given in *ETAS 1997* (Table 1.12); the respective identifiers are given as CIAC and CAQH. Total income is given in *ETAS 1996* (Table 1.11) and has the identifying code CIDB. Note that the post 1995q2 data for this series is constructed as described above. The necessary data can also be found on the *ONS Database*. Data (from both sources) for gross trading profits and rent are available on a quarterly basis back to 1955q1. Annual estimates for total income for the period 1955 to 1960 are taken from *ETAS 1996* (Table 1.11) and these are used to generate quarterly estimates by interpolation.

Substituting these variables into equation 4.20, yields a measure of the stock market value of firms, $V_t$. The series is plotted (in constant prices) in Figure 4.14. Notice that the real stock market value of companies and financial institutions fell quite sharply between 1972 and 1976. Since 1976, the stock market value of firms has increased rapidly.

4.2.7.3.3 Inventories, $Z_t$

The final variable necessary to derive a tax adjusted measure of $Q$ is that of inventories, $Z_t$. Recall from the discussion in Section 4.2.7.2, a measure of the value of stocks and work in progress is necessary in order to resolve the inconsistency in the definitions of the numerator and the denominator of $q$. The numerator, the market value of the firm $V_t$, includes the value of inventories, whereas the denominator
measures only the value of fixed capital assets. This inconsistency can be resolved by subtracting the value of inventories from the numerator.

The nominal stock of inventories, $Z_n$, has been estimated from a benchmark figure, the ‘book value of stocks and work in progress’ held by companies and financial institutions at the end of 1996, earlier periods being derived by applying the series known as the ‘increase in the book value of stocks and work in progress’. The end of 1996 value for companies and financial institutions’ book value of stocks and work in progress is published in the *Blue Book 1997* (see notes to Table 15.4). Quarterly estimates of the increase in the book value of ICCs and FCIs’ stocks and work in progress are available on the *ONS Database* (the respective identifying codes are AAAT and AAAP).

### 4.2.7.4 A Tax-Adjusted Measure of $Q_t$

The derivation of variables $B_t$, $Z_t$, and $V_t$ in the preceding subsections enables a tax adjusted measure of $Q_t$ to be constructed in accordance with equation 4.16. The resulting measure of $Q_t$ is plotted in Figure 4.15. Since these estimates may be of
some general interest and since they are somewhat tedious to calculate, they are presented in Table 4.1. A marked feature of Figure 4.15 is the sharp fall in \( Q \) from the beginning of 1973. This reflects the fall in the market value of the corporate sector due to rising inflation and interest rates, and low profit and investment levels. Having reached a minimum in the middle of 1975, \( Q \) rises slowly but remains below zero until 1987.

It is useful to compare the measure of \( Q \) described above with those derived by other authors. Although no estimates of \( Q \) for UK companies and financial institutions have been published, estimates do exist for ICCs. Since ICCs represent a significant component of the UK corporate sector, it is reasonable to expect this measure of \( Q \) for companies and financial institutions to be correlated with those measures derived for ICCs. Porteba and Summers (1983), for example, present annual estimates of tax-adjusted \( Q \) for UK ICCs for the period 1948-80. Their measure of \( Q \) is constructed in a similar way to that used here for companies and financial institutions. The authors analyse the effects of alternative forms of dividend taxation on corporate investment. As such, they present two measures of tax-adjusted \( Q \), each conforming to a hypothesis for dividend taxation. It is not particularly useful to compare these tax-adjusted measures of \( Q \) with the measure derived here for UK companies and financial institutions since they have been constructed to test particular hypotheses regarding the effects of each view of dividend taxation on corporate investment decisions. However, they also present estimates of the valuation ratio before adjusting for taxation for the same period. This can be compared with our measure of \( Q \).

Figure 4.16 plots the annual pre-tax valuation ratio of Porteba and Summers and annual averages of the pre-tax measure of \( Q \) derived above. The two series track one another closely. The series derived here is generally higher than that derived by Porteba and Summers (1983). This is due to the fact that the former series includes FCIs, whereas the latter represents a measure of \( Q \) for ICCs alone.

4.2.7.5 Limitations of Measured \( Q_t \)

The first point to note is that \( Q \) has been derived for companies and financial institutions. This sector comprises privately controlled corporate enterprises organised
Figure 4.15: A tax-adjusted measure of $Q$

Figure 4.16: Annualised pre-tax $Q$ and Porteba and Summers' $Q$

annual averages of $Q$

Porteba and Summers' valuation ratio
for profit and resident in the UK. According to the *United Kingdom National Accounts: Sources and Methods 1985* (see CSO (1985), p. 87), the main constituent is 4000-5000 public limited companies. Much greater in number, but smaller in total net assets, are three-quarters of a million private companies many of which are subsidiaries of public companies or other private companies. However, the company sector does not generally include unincorporated businesses, sole traders or partnerships which typically belong to the personal sector and therefore it does not embody all firms in the economy. Since these firms contribute only a small amount to total company profits, any measure of $Q$ for the ‘companies and financial institutions’ will be representative of an ‘all firms’ $Q$.

The second problem is more substantial. The purpose of this thesis is to model new private industrial and commercial construction output. The investment series of interest is investment in industrial and commercial buildings. Therefore, a Tobin’s $q$ model of investment should therefore contain a $Q$ variable that measures the ratio of the market value of firms’ industrial and commercial buildings relative to the replacement cost of these assets. The $Q$ variable derived above measures the ratio of the market value of firms’ total fixed capital assets to the replacement cost of these assets. Therefore, this series is a suitable variable in a model of aggregate investment, but is not ideal in a model of investment in industrial and commercial buildings. Unfortunately, there exist no estimates, or means of deriving estimates, for the market value of the firms’ industrial and commercial buildings. Thus, the best we can do is to construct a $Q$ variable for investment assets in general. Of course, if firms held an optimal portfolio of assets, one would expect $Q$ to be the same for different types of asset.

It should be highlighted that the capital stock variable used in the denominator of $Q$ must be the stock of all fixed assets held by companies and financial institutions and not the stock of industrial and commercial buildings held by companies and financial institutions derived in Section 4.2.5. The data on the value of all fixed assets held by companies and financial institutions is available in the *Blue Book 1997* (Table 14.7). The stock of all fixed capital held by ICCs has the identifying code EXHK and the stock of all fixed capital held by FCIs has the identifying code EXHL. Both of these two series are also available on the *ONS Database*. The sum of these two series gives
the stock of fixed capital held by companies and financial institutions. The data on these series is not available on a quarterly basis. Hence, quarterly estimates for the stock of all fixed capital held by companies and financial institutions is derived by linearly interpolating annual estimates. Although not ideal, the resulting series was checked against an alternative series constructed using the GDFCF data and the perpetual inventory method. Using a starting value in the perpetual inventory method (set equal to the published estimate for the stock of all fixed capital held by companies and financial institutions at the end of 1954) and a depreciation rate of 0.015% per quarter, the resulting series was almost identical to the quarterly series interpolated from the annual *Blue Book* estimates.

Similarly, the depreciation rate used in the construction of $Q_t$ will not be equal to the rate used to construct the series for the stock of industrial and commercial buildings described in Section 4.2.5. Since $Q_t$ is the ratio of the market value of all fixed assets held by companies and financial institutions to the replacement cost of those assets, the depreciation rate used in its construction must represent an average rate at which all assets depreciate. This is assumed to be 0.015% per quarter; the rate necessary for the measure of capital stock derived using the GDFCF data and the perpetual inventory method, and described in the previous paragraph, to approximate most closely the measure derived by interpolating the annual *Blue Book* estimates.

### 4.3 Definitions, Sources and Limitations of Subsidiary Data

In this section we consider a number of other economic time series thought to be of value in forecasting new private industrial and commercial construction output. These series are not consistent with a strict interpretation of the four dominant theories of investment, although a significant role for some of these variables has been found in the empirical investment literature, a review of which was provided in Chapter 2. In addition to the variables found to affect investment in the empirical literature, we consider a number of variables thought to be (leading) indicators of construction activity. Whilst the variables discussed in Section 4.2 will be useful to the estimation of models consistent with the dominant theories of investment, the
variables considered in this section will be useful to the development of atheoretical VAR models.

As with the data collected in Section 4.2, data for the series discussed below is collected on a quarterly seasonally unadjusted basis. The availability of data is problematic. For many series, data for the early part of the sample period does not exist in any form. Data for each series is collected back as far as is reasonably possible. We begin by discussing a number of construction industry aggregates and subsequently consider more general indicators of activity.

4.3.1 Indicators of Construction Industry Activity

In this section we consider a number of direct measures of conditions in the construction sector. These variables are thought to be leading indicators of construction activity. However, they can not be considered determinants of new private industrial and commercial construction output. For many of these series, data is not available back to 1955q1.

4.3.1.1 Private Industrial and Commercial Construction New Orders, $O$,

This series relates to contracts awarded to private contractors for new industrial and commercial construction work (excluding subcontracts to avoid double counting), including extensions to existing contracts and construction work in 'package deals'. Also included is speculative work, undertaken on the initiative of the firm, where no contract or order is awarded. New construction work includes extensions, major alterations (that is improvements), site preparation and demolition. It excludes site value and architects or consultants fees.

This definition of new orders is closely related to that of construction output. The latter is defined in Section 4.2.1.1 as the amount chargeable to customers for industrial and commercial building and civil engineering work in the relevant period. When a contractor is awarded a new order for industrial and commercial buildings, it does not receive an immediate payment for the order. Instead, a series of payments are made over the period it takes to complete the work. The size of the payment accords
to the value of the work done in that period. These payments are registered in the construction output data. If all orders were completed to budget, one could think of the output data as being a distributed lag relationship of the orders data. However, a small proportion of new orders are not completed and a somewhat larger proportion exceed budget. These factors tend to distort the relationship between construction output and orders, but the correlation between the two series remains strong.

Like the output data, orders data is published by the DoE in HCS (see, for example, Tables 2.6 and 2.7 in the March 1997, part 2, issue). The orders data is also disaggregated by type of work in the same way as the output data, that is, new orders are disaggregated in to housing, infrastructure, other public work, private industrial and private commercial work. Given our desire to forecast private industrial and commercial output, we are interested in private industrial and commercial new orders. Thus, repair and maintenance, housing, infrastructure and other public work can be disregarded. As with the output data, the change in the way the infrastructure data is presented from 1980 introduces a discontinuity in the data. The problem is overcome in the same way as for the output data and is described in Section 4.2.1.2 above.

New orders data was not collected prior to 1958q1 and so our data on this series begins at this date. Estimates are collected in current price terms and are revalued in 1990 prices using the tender price index, TPI, described in Section 4.3.1.2. An output price deflator is unsuitable for deflating new orders, since it attempts to measure changes in the price level for currently executed work rather than tendering levels for work yet to be started.

Whilst many of the limitations of the output series (discussed in Section 4.2.1.3) apply to the new orders series, they are relatively unimportant. We are interested in the new orders series since it is likely to be a good, although imperfect, indicator of construction output. Given that the coverage and basis for collecting new orders data is consistent with those of the output data, any limitations specific to the new orders series or shared with the output series, are unlikely to be of first order importance. A plot of real private industrial and commercial construction new orders over time is presented in Figure 4.1. Notice that the series clearly and consistently leads the real output series.
Broadly speaking, a tender price is defined as the price for which a builder offers to erect a building that the client has to pay. The level of individual tender prices may be assessed, in order to judge its competitiveness, by indexing the tender in some way and comparing it with a benchmark given by one of the many published general tender price indices. Tender price indices are generally based on a sample of tenders in each time period. Each of these tenders is indexed, that is to say, the tender is revalued (in whole or in part) using a base schedule of rates and the current value is expressed as a percentage of the rebased value. This gives an individual 'project index'. The average (either arithmetic or geometric mean or median) of the project indices constitutes the general index of tender prices. The lower an individual project index is relative to a general index of tender prices, the more competitive it may be said to be. Sharp increases in the general price index might be indicative of contractors' increasing workloads and may therefore provide an indication of the level of future construction output. It should be noted that other factors, unrelated to contractors' workload, could result in accelerating tender prices.

Just as the output price index is the appropriate index for revaluing current output into constant 1990 prices, the general index of tender prices is the appropriate index with which to deflate current new orders. A variety of tender price indices, differing in coverage, are published by a number of institutions. Many of the indices available are published on an historical basis in Flemming and Tysoe (1991). One of the first institutions to publish a general index of tender price indices on a regular basis was Building Cost Information Services, BCIS. The general index, which is available quarterly, has been published since 1974q1. However, more recently, they have begun to publish indices on the level of tender prices by sector, by building type and by type of contract.

Given that our new orders series discussed in Section 4.3.1.1 covers new private industrial and commercial orders only, the appropriate index would be an index of private industrial and commercial tender prices. Indices with such specific coverage are less widely published. BCIS do publish an index of private sector, private industrial and private commercial tender prices, however, these were not published prior to 1984q1. An all-in tender price is published by BCIS and this is available back
to 1974q1 (and is given in Flemming and Tysoe (1991)). Prior to 1974q1 few indices of any kind exist. However, BCIS in collaboration with the DoE have recently published an historic index of public sector tender prices back to the mid-1950s (see Packham (1993)). This series excludes all private sector tenders. Unfortunately, this series is likely to be highly correlated with an all-in tender price index prior to 1974q1. Thus, in order to derive an all-in index of tender prices back to 1955q1, the Packham series is spliced with the BCIS All-In Tender Price Index to obtain pre-1974q1 data. This series, although not ideal since it is not an index of private sector tender prices, should provide a general indication of trends in the tender prices and, moreover, ought to be adequate for the purposes of deflating the current new orders series from 1958q1. The resulting series is plotted in Figure 4.17.

4.3.1.3 A Market Conditions Index, MCI,

The market conditions index combines data on tender prices and building costs to yield a basic measure of the profitability of investment in buildings. An increase in tender prices over building costs is assumed to indicate an increase in profit margins, which will eventually result in an increase in investment. As such, the MCI is related to the Q variable discussed in Section 4.2. Recall, marginal \( q \) is defined as the ratio of the market valuation of an additional unit of capital to the current replacement cost of that unit. If we make the rather strong assumption that the market value of a planned building, at the time the tender for that planned building is placed, is given by the tender price, then a very unsophisticated proxy for marginal \( q \) is given by the ratio of tender prices to building costs.

The tender price index used to construct this market conditions index is described in the previous subsection. The necessary building cost index should describe the change in the cost of private industrial and commercial construction to the contractor. However, given that such a specific index does not exist and that the chosen tender price index is an all-in index, we have had to settle for an all-in general building cost index. Again, a number institutions publish an index of general building costs. However, few of these series have been published regularly since the mid-1950. One of the longest complete quarterly series available is Spon’s Building Cost Index. This
Figure 4.17: Indices of building costs and tender prices

(1990=100)

Figure 4.18: A market conditions index
series has been published since 1966q1 and is presented in Flemming and Tysoe (1991) and Spon’s Architects’ and Builders’ Price Book 1993 (see Davis et al (1993)). Prior to 1966, a number of annual indices of building costs are available, but few of these detail trends since the 1950s. Flemming and Tysoe (1991) present a series called the Venning Hope Cost of Building Index, which provides a complete run of annual data from 1914 to 1975. This series is linearly interpolated prior to 1966 to give quarterly estimates and these are spliced with Spon’s post 1966 quarterly index. The resulting building cost index (rebased to 1990=100) is plotted in Figure 4.17 for the period from 1955q1 to 1996q4.

The obvious way to combine the tender price and building cost indices to obtain a market conditions index is to divide tender prices by building costs. However, the upward trend in the building costs index is much stronger than the trend in tender prices. This is due, in part, to the fact that the tender price index makes some allowance for market conditions, whereas the building cost index makes no such allowance. For this reason, dividing the former by the latter will give the impression of continually declining conditions in the market. An alternative means by which the two series might be combined is as follows. Take logs of both series and regress tender prices on a constant, a trend (to account for the fact that market conditions are to some extent already taken into account in the TPI) and the building cost index. The residuals from this regression give the market conditions index. It is plotted in Figure 4.18. Notice that strong positive residuals correspond to periods of vibrant activity in the construction sector. It should be noted that this is an index of conditions in the construction sector as a whole: both the TPI and BCI are general or all-in indices. It is not necessarily indicative of conditions in private industrial and commercial sectors, although it must be said that these sectors do contribute much to the sector as a whole.

4.3.1.4 Construction Employment as a Share of Total Employment, RE,

Construction employment as a share in total employment, RE, other things being equal, provides a crude measure of the relative prosperity of the construction sector. In other words, other things being equal, an increase in RE is likely to lead to a future
increase in construction activity/output relative to total output. To this extent, the series may be useful in forecasting construction output.

In order to construct this measure we need two series, namely construction employment and total employment. In this subsection we discuss the construction employment variable. We also discuss how a measure for relative employment can be derived, given series for construction and total employment. A discussion of the total employment variable is reserved until Section 4.3.2.2.

Construction employment data is compiled by the DoE (and published in HCS) and by the Department for Education and Employment (DfEE, published in the Employment Gazette). Although a large component of construction employment is self employment, a lack of data prior to 1975 necessitates that we focus solely on employees in employment. Both Departments publish a quarterly series for employees in employment. Due to differing compilation methods, the two series are not directly comparable. Furthermore, due to a number of changes in industry definition (in 1958, 1968, 1980 and 1992), the coverage of both series has changed through time leading to discontinuities in each series. These are substantial difficulties that must be overcome to obtain a continuous time series.

The DfEE have published a quarterly series for employees in employment back to 1978q2 in which the discontinuities, caused by changes in industry definition (in 1980 and 1992), have been corrected for. This series is published in the Employment Gazette and is available from the ONS Database using identifying code LMAY. Quarterly estimates for the period prior to 1978q2 have been obtained from various issues HCS back to 1964q1. The discontinuity in 1968, caused by a change in industry definition, is corrected by splicing. The resulting series is then spliced with the post 1978q2 DfEE data. Data for this series has not been collected prior to 1964q1. This is because estimates are likely to be unreliable and are based on a 1958 definition of the industry which is substantially different from the 1968 definition. Therefore, it is unlikely that the relationship between the pre and post 1964 series can be applied retrospectively to the whole of the earlier series. The resulting series, plotted in Figure 4.19, indicates a steady decline in construction employment over the period. This series and the series for total employment, to be discussed in Section 4.3.2.2, are used to construct the measure of \( RE \).
Due to differences in productivity between the construction sector and the economy as a whole, it is inappropriate to simply divide construction employment by total employment in the economy to obtain a consistent series for employment in the construction sector relative to the whole economy. We therefore use a similar approach to that used to develop the MCI series described above. First, both series are logged. Then construction employment is regressed on a trend and total employment. The inclusion of a constant is inappropriate since intuition suggests that the relationship needs to be forced through the origin. The residuals from this regression equation provide our measure of $RE$. The series is plotted in Figure 4.20 for the period from 1964q1 to 1996q4.

4.3.1.5 An Index of Real Wages in the Construction Sector, $W_t$

Wage rates are likely to be correlated with current activity or workload and, therefore, future output in the construction sector. In periods of heavy workload, contractors will need to offer higher wages to attract new employees. Wage rises are likely to be especially acute in times of skill shortages. Moreover, contractors will need to offer more overtime to existing workers in order to meet tight completion dates. Since current wage rates are likely to be correlated with future construction output, wage rate data is likely to contain some information useful in forecasting construction output.

Wages are defined here to include overtime and bonus payments. Under this definition, wages are likely to be more sensitive to changes in construction activity. We are therefore interested in a measure of actual earnings rather than basic wage rates. From 1963, a small scale survey has been conducted to obtain monthly data on average earnings in the construction industry in Great Britain. Details of the methodology, including retrospective revisions, are given in the Ministry of Labour Gazette (March 1967) and the Department and Employment Gazette (April 1976). The monthly index covers all workers, manual and non-manual, full and part-time, of both sexes without distinction. Until 1966, however, separate indices are published for weekly-paid and monthly-paid employees. The results are currently published in the Employment Gazette. Current price quarterly indices are published in the Monthly...
4.3.2 More General Indicators of Activity

Here we consider a number of indicators of economic activity. To the extent that the general economic climate affects the demand for construction services, indicators of economic activity will contain information that may be exploited for forecasting new private industrial and commercial construction output. Some of the variables discussed below are known to be leading indicators of the general economic cycle.

4.3.2.1 Real Gross Domestic Product, $Y_t^A$

According to the accelerator model of investment behaviour, investment undertaken by private sector clients is a function of private sector output. A series for private sector output is described in Section 4.3.3. In order to avoid a spurious correlation between private sector output and investment (as measured by construction output), it was necessary to deduct the value of the latter from the former. Such an approach has been suggested by Gordon and Veitch (1986). In this section we define an aggregate measure of output. Aggregate output, $Y_t^A$, is likely to provide a good indicator of economic conditions. It may also provide a proxy for confidence in the economy and, to the extent that investment is sensitive to confidence, aggregate output may contain useful information for forecasting construction output.

A measure of real GDP must be constructed, since seasonally unadjusted estimates are no longer published. However, such a measure is easily constructed and the method of construction is discussed in the development of the implicit price deflator for private sector output in Section 4.2.4. In short, it is most easily constructed using the expenditure based measure of GDP. Real GDP at market prices is calculated as real total final expenditure less the real value of imports of goods and services. Both series are published in ETAS (see Table 1.3 of the 1996 edition, for example) and on the

Digest of Statistics, Housing and Construction Statistics and on the ONS Database (under the identifying code DHNE). The series is revalued to constant prices using the GDP implicit price deflator. This deflator is defined and discussed in Section 4.2.4. The resulting series is plotted in Figure 4.21 for the period from 1966q1 to 1996q4.
Figure 4.21: An index of real wages in the construction sector

Figure 4.22: GDP at market prices
(constant £ million)
ONS Database (under the respective identifying codes DJDA and DJCY). The value of construction output is deducted to avoid the problem of spurious correlation and the resulting measure, $\gamma^a$, is plotted in Figure 4.22 for the period from 1955q1 to 1996q4.

4.3.2.2 Total Employment, $E$

Assuming that the optimal capital-labour ratio is constant, an increase in the quantity of labour employed by a profit maximising firm must be matched by a proportionate increase in capital (plant, machinery and buildings). Moreover, to the extent that this additional labour must be accommodated, an increase in building activity is likely to result. As such, increases in total employment, $E$, are likely to be correlated with future increases in construction output. In this sense, the level of employment could be seen as a leading indicator of investment in industrial and commercial buildings.

Employment and unemployment data are notorious for discontinuities caused by changes in government definitions. However, a seasonally unadjusted measure of aggregate employment can be relatively easily obtained from government statistics. The DfEE currently publish two such series, workforce in employment and employees in employment, on a quarterly basis back to 1950. The main difference between the two series is self-employment which grew steadily throughout the 1980s. Given that the self employed require premises from which to operate businesses, we chose to work with the more aggregate measure. An unadjusted series measuring the absolute size of the workforce in employment is published in ETAS (see Table 3.2 in the 1996 edition, for example) and on the ONS Database (under identifying code DYDA). The series is plotted in Figure 4.23 for the period from 1955q1 to 1996q4.

This series is used to construct the measure of construction employment relative to total employment, $RE$, which is described in Section 4.3.1.4. It is noteworthy that there is an inconsistency between the two series caused by the fact that the series for construction employment measures employees in employment, whereas the series for total employment includes the self employed. Ideally, the measure of construction employment would have included self employment, but such data does not exist for the early part of the sample period.
Figure 4.23: Total employment

Figure 4.24: ICCs' liquidity ratio
4.3.2.3 A Measure of Corporate Liquidity, \( LR \),

It is often argued that the reason that investment did not take off when the real user cost of capital was negative in the mid-1970s was the strong adverse effect from company liquidity (see Kelly and Owen (1985)). Implicitly, of course, this recognises that capital markets are imperfect. Certainly, the exclusion of corporate liquidity may go some way to explaining the poor performance of neoclassical models through the 1970s. By including such a variable, the neoclassical model's ability to track movements in investment during the 1970s may be enhanced. This approach has been suggested by Kelly and Owen (1985). Although this approach is inconsistent with Jorgenson's model, which relies heavily on the assumption of perfect capital markets, there are no theoretical objections to including the variable in a VAR model.

Obtaining a measure of corporate liquidity is not easy. Ideally, we need a measure of the liquidity of private sector firms. However, liquidity data on financial companies is sparse. Several series exist for ICCs, but these do not span the sample period of 1955q1 to 1996q4. Since 1990 a liquidity ratio for ICCs (defined as ICCs liquid assets over liabilities) has been published in \( \text{Financial Statistics} \). Prior to this, a series was published on the liquidity positions of large ICCs. This also appears in \( \text{Financial Statistics} \). Unfortunately, this series is unlikely to be a good proxy for the liquidity of all ICCs, since large ICCs are less likely to be financially constrained. In the \( \text{Handbook of Financial Statistics 1997} \) (see ONS (1997)), ICCs liquid assets are defined to include bank and building society deposits, holdings of gilts, treasury bills, bonds, notes and coins, commercial paper, overseas securities, and other financial assets that can be realised within twelve months. Liabilities are defined as all lending from UK banks plus issues of commercial paper and other financial liabilities that might have to be repaid within 12 months. Given these definitions of ICCs liquid assets and liabilities, it was possible to derive a measure of the liquidity ratio prior to 1990q1 using ICCs balance sheet data published in \( \text{Financial Statistics} \). Prior to 1968, quarterly balance sheet data for ICCs was not published in \( \text{Financial Statistics} \). Between 1960 and 1968, annual balance sheet data is given in the \( \text{Blue Book} \) and this is used to obtain annual series for ICCs liquid assets and liabilities. Dividing assets by liabilities gives an annual series for ICCs liquidity ratio between 1960 and 1968. Quarterly estimates are obtained by linear interpolation. It is not possible to derive a
series for ICCs' liquidity prior to 1960 due to a lack of balance sheet data. The resulting series is plotted in Figure 4.24 for the period from 1960q1 to 1996q4.

4.3.2.4 Real Interest Rates, $RIR_i$

The measure of the user cost of capital developed in Section 4.2.6 above measures the cost of hiring one unit of capital for one period. It is comprised of the interest cost of the capital, the depreciation cost and a term for capital gains or losses during the period. In deriving this measure, the capital gains term was scaled with a coefficient of 0.2 in order that the real user cost of capital remained positive throughout the sample period. Recall, a negative real user cost of capital implies that investment should be infinite and this possibility undermines the neoclassical model. This adaptation, suggested by Nickel (1978) and implemented above, and by Jenkinson (1981) for example, is rather ad hoc.

For the less theoretically rigorous models to be developed in the following chapter, an alternative, and more simplistic, measure of the cost of capital is provided by the real interest rate. This is constructed as the nominal interest rate minus the expected rate of general price inflation. The nominal interest rate series used here is the same as that used in the development of the user cost of capital variable. For the period since 1963, this is given by the gross redemption yield on short dated five year British government stock. Details of the data sources and construction of this series prior to 1963 is given in Section 4.2.6.2. Inflation is calculated as the percentage change in the GDP deflator. The GDP deflator is calculated by dividing nominal GDP by real GDP. The construction of the real GDP variable is discussed in Section 4.3.2.1. Nominal GDP is constructed in the same way: nominal total final expenditure less the nominal value of imports of goods and services. Both series are published in ETAS (see Table 1.3 of the 1996 edition, for example) and on the ONS Database (under the respective identifying codes DJAK and DJAG). The expected rate of inflation is given by the predicted values from an ARIMA model of inflation. An ARMA(5,0) fits the data well and satisfies the diagnostics. The resulting real interest rate series is plotted in Figure 4.25 for the period from 1955q1 to 1996q4.
4.3.2.5 Capacity Utilisation, \( CU \),

Capacity utilisation has been used to explain investment demand in a number of studies in the literature (see Chenery (1952) for an example of how capacity utilisation has been used in an accelerator type model, and see Engle and Foley (1975) and von Furstenburg (1977) for examples of capacity utilisation variables in the \( q \) model). Recall, the accelerator states that investment is related to changes in output. This relationship depends on the rather strong assumption that the firm is already operating at full capacity. If firms are not already operating at full capacity they can respond to an increase in output by using existing capacity more intensively. Thus, the inclusion of a capacity utilisation variable may improve the performance of an accelerator type model. *Ceteris paribus*, the pressure on firms to increase capacity will be higher in periods when firms have little excess capacity. In this sense, one might observe increased investment in buildings and, therefore, increased new private industrial and commercial construction output, in periods when capacity utilisation is high. Therefore, capacity utilisation may be a useful indicator of future construction output.

One of the most well known measures of capacity utilisation is published by the CBI in its Industrial Trends Survey. This is a qualitative survey of member firms of the CBI in manufacturing. This variable is constructed from the responses of firms to a particular question in the survey which asks whether the firm is operating below capacity. The series is presented as the proportion of firms operating below capacity. This variable is available in an unadjusted form back to April 1958. It is this measure of capacity utilisation that is used in this work. The timing of the publishing of the series is worthy of comment. There is a time lag between firms’ completion of the survey and its publication. Moreover, the survey is published at the beginning of the each quarter, whereas most other economic variables are measured at the end of the quarter. Thus, the survey provides information on economic conditions faced by firms in the previous quarter. Accordingly, the CBI measure of capacity utilisation variable is lagged by one quarter in this work and then treated as a contemporaneous variable. The series is plotted in Figure 4.26 for the period from 1958q1 to 1996q4.

It is noteworthy that the measure of capacity utilisation used here is a component in the Office of National Statistics' cyclical indicator of the UK economy. Details of the methodology of constructing these cyclical indicators is given in O'Dea (1975) and
Given that the series is a component in the UK cyclical indicators, it may provide an indication of future trends in aggregate output and construction output.

4.3.2.6 A Measure of Business Optimism, $BO_t$

Optimism in the business community surrounding current and future conditions in the economic environment is frequently cited as having an important effect on investment (for example, see Driver and Moreton (1991)). The reason for this is clear. Faced with the choice of investing now or delaying the investment, a firm must consider future economic conditions. If the business community is uncertain about the future climate, then investment projects will be postponed or cancelled (this idea is formalised in the literature on investment irreversibility discussed in Section 2.7.3). Thus, other things being equal, an increase in business optimism will result in an increase in investment.

The measurement of something as intangible as business optimism is not without its difficulties. However, a useful source of information is provided by surveys of the business community. Since 1959, the CBI Industrial Trends Survey has inquired as to whether firms feel more or less optimistic about the economic climate than they did four months previous. The balance of business optimism, i.e. the proportion feeling more optimistic minus the proportion feeling less optimistic, is the most frequently used measure of industry optimism. A complete run of quarterly seasonally unadjusted data is available from 1959q1 to 1996q4. As with the capacity utilisation variable, we have lagged the series by one quarter to allow for the fact that the survey is published (with a lag) at the beginning of each quarter. The resulting series is plotted in Figure 4.27 for the period from 1958q4 to 1996q1.

It is noteworthy that this series consistently leads the aggregate economic cycle. Indeed, the series is one of five series comprising the Office of National Statistics' Longer Leading Index. As such, the cycles in business optimism are likely to lead cycles in aggregate investment and the series may therefore be of some value as a leading indicator of construction output in the VAR models to be estimated in the following chapter.
Figure 4.27: Change in business optimism

(percentage balance)

Figure 4.28: Investment intentions

(percentage of firms expecting to authorise more minus percentage expecting to authorise less)
4.3.2.7 Firms' Investment Intentions, \( I_t \)

The CBI Industrial Trends Survey also asks firms whether they expect to authorise more, the same or less capital expenditure in the following twelve months than in the previous twelve months. Building expenditure is distinguished from expenditure on plant and machinery. The resulting data is summarised in a table in ET 'Indicators of Fixed Investment'. The series measuring expected capital expenditure on buildings is, as one would expect, highly correlated with future construction new orders. To this extent, this variable is a leading indicator of construction output. A complete run of quarterly seasonally unadjusted data is available from 1959q1 to 1996q4. As with the capacity utilisation and business optimism variables, we have lagged the series by one quarter to allow for the fact that the survey is published (with a lag) at the beginning of each quarter. The resulting series is plotted in Figure 4.28 for the period from 1958q4 to 1996q1.

4.4 Preliminary Data Analysis

In this section we consider the statistical properties of the data described in Section 4.2 and 4.3. In particular, we will examine the data for non-stationarity. We introduced the concepts of stationarity and integration in Chapter 3 (see Section 3.3) and outlined the importance of these concepts in econometric work. Moreover, we described a number of procedures that can be used to test for the presence of non-stationarities in time series data. We now apply these procedures to the data described in Sections 4.2 and 4.3.

4.4.1 The Order of Integration Determined for Core Data

The HEGY test, the Augmented Dickey-Fuller test and the Phillips-Perron test are applied to the data suggested by the four dominant theories of investment behaviour. These series were discussed in Sections 4.2. Since the \( Q \) model posits a relationship between the rate of investment and \( Q \), we also apply these tests to a variable measuring the rate of net investment, \( \Delta K/K \). These tests were also applied to the data considered in Section 4.3 and the results of that analysis are presented in Section
4.4.2. In the cases where the order of integration is not clearly determined by these tests, the autocorrelation function and spectral density are examined. We proceed with the HEGY test for a seasonal unit root.

4.4.1.1 Results of Seasonal Unit Root Tests

The results of the HEGY test for seasonal unit roots are summarised in Table 4.2. As noted above, the HEGY procedure enables one to test for simple and seasonal unit roots simultaneously. Before we discuss the evidence regarding the presence of seasonal unit roots it is worth noting that, with the exception of \( c/p \), the real user cost of capital, the null hypothesis of a simple unit root could not be rejected for any of the series tested. It is interesting that the HEGY test should suggest that the real user cost of capital should be stationary. Casual inspection of Figure 4.12 indicates that both the mean and variance of the real user cost of capital (with capital gains) increase over time. We examine this series more closely in Section 4.4.1.2. It has been noted above that the HEGY test has low power. However, this is not a good reason for ignoring the results of the HEGY test altogether. If the HEGY test rejects the null, we may have 90% confidence in the test results for any given series. On the other hand, here we have applied the test to eight series. Thus, the overall size of the test for the data is \( 1 - 0.98 = 0.57 \). In other words, we have a 57% probability that a valid null has been rejected. For this reason, the ADF and PP tests are used in addition to the HEGY test. Therefore, that we also use the ADF and PP tests is not a result of the low power of the HEGY test, but rather an issue of repeated testing.

The results of the HEGY tests reveal little evidence in support of seasonal unit roots. There is no evidence of a seasonal unit root at the annual frequency in any of the series. The null hypothesis of a seasonal unit root at the annual cycle (or, in terms of equation 3.8 in Chapter 3, that \( \delta_3 = \delta_4 = 0 \)) is rejected in tests on all series (although only marginally for real private sector output, \( Y \)). With the exception of the price of private sector output, \( p \), there is also no evidence of seasonal unit roots at the biannual frequency: the null hypothesis of a seasonal unit root at the biannual frequency (or, in terms of equation 3.8, that \( \delta_2 = 0 \)) is rejected for all series, except \( p \). For \( p \), the null hypothesis can not be rejected at the 10% level. Thus, on the basis of these test results.

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it is possible to conclude that $p$ contains a stochastic seasonal component whilst any seasonal component that may exist in the other series is deterministic which can, of course, be modelled using seasonal dummies.

Table 4.2: HEGY tests for seasonal unit roots in the core data

<table>
<thead>
<tr>
<th>Series</th>
<th>Frequency</th>
<th>Zero$^1$</th>
<th>Biannual$^1$</th>
<th>Annual$^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta K(1)$</td>
<td>10% CV</td>
<td>-2.62</td>
<td>-2.59</td>
<td>5.56</td>
</tr>
<tr>
<td>$K(2)$</td>
<td>-2.11</td>
<td>20.92</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta K/K(2)$</td>
<td>-1.27</td>
<td>32.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y(1)$</td>
<td>1.19</td>
<td>34.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z(2)$</td>
<td>0.18</td>
<td>5.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p(6)$</td>
<td>-4.34</td>
<td>37.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q(2)$</td>
<td>-2.18</td>
<td>7.71</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c/p(2)$</td>
<td>-7.57</td>
<td>73.66</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. These test results are derived from the model given in equation 3.8.
2. The lag length (in parenthesis) on the HEGY regression is determined using the minimum AIC criterion up to a maximum of nine lags.

We employ a test size of 10% rather than the conventional 5%, for the reasons outlined in Section 3.3.1. The failure to reject the null of a seasonal unit root in $p$ at the biannual frequency may be due to the HEGY tests' low power. In order to shed some light on this possibility, spectral techniques are employed. In the discussion of spectral techniques in Section 3.3.2.2, two lines of enquiry were suggested. First, it was noted that if there is a seasonal unit root in a time series, one would expect to find large spikes in the estimated spectral density at one or more of the seasonal frequencies. However, as we discussed, these spikes are open to misinterpretation since they may also indicate a strong regular, or deterministic, cycle. It was suggested, therefore, that deterministic seasonality be removed prior to estimation of the spectral density. If the power at the seasonal frequencies is large once the deterministic seasonality has been removed, this may be taken as evidence of seasonal non-stationarity. Second, if the nature of the seasonal pattern is gradually changing, spectral techniques can be of use in identifying these changes. This can be examined, as discussed in Section 3.3.2.2, by dividing the sample period for a series into two sub-periods and estimating a spectral density for each sub-period. If the size of spikes at the seasonal frequencies (relative to other frequencies) differs in the two
sub-samples, then there is evidence that the importance of the seasonal cycle has changed.

We apply these procedures to $p$. Deterministic seasonality is removed from $p$ by regressing the series on four seasonal dummies. Spectral analysis is performed on the regression’s residuals. Figure 4.29 shows the estimated spectral density of these residuals. Given that the deterministic seasonality has been removed, high power at the seasonal frequencies would provide evidence of stochastic seasonality, thereby supporting the results from the HEGY tests. One can immediately observe, however, that the spectral density displays lower power at the seasonal frequencies relative to other frequencies. Thus, on this basis, one may conclude that there is no evidence of stochastic seasonality in $p$.

We have forty two years of data for $p$. Spectral densities are estimated for the periods 1955q1-1975q4 and 1976q1-1996q4. Figure 4.30 shows the (scaled) spectral densities (plotted on a log scale) for $p$. It is noteworthy that the spectral densities do not indicate a significant spike at either the annual ($\pi/2$) or biannual ($\pi$) frequency. On this basis, one might conclude that there is no evidence of a seasonal cycle (of any kind) and certainly no evidence supporting the hypothesis of a seasonal unit root (at either frequency). It is the biannual frequency ($\pi$) that is of most interest here, since it is at this frequency that the HEGY test suggests non-stationarity. Since the relative importance of the biannual frequency is constant across time periods, it may be concluded that there is no evidence of stochastic seasonality in $p$. A noteworthy feature of Figure 4.30 is that middle and high frequencies are relatively more important in the second period.

In summary, the spectral analysis suggests that the failure of the HEGY test to reject the null hypothesis of a seasonal unit root at the biannual frequency is likely to be due to the test’s low power. Thus, on the basis of the HEGY tests and the spectral analysis, it is assumed in further analysis that none of the series tested in this section contain seasonal unit roots. It is noteworthy that these results are broadly consistent with the results of tests for seasonal unit roots in UK macro data in the sense that evidence supporting the presence of seasonal unit roots is conspicuously scarce.
Figure 4.29: The spectral density of $p$ (deterministic seasonality removed)

Figure 4.30: The spectral densities for $p$
4.4.1.2 Results of Simple Unit Root Tests

Prior to applying these tests, all series are regressed on quarterly dummies. For those series for which the quarterly dummies are significant, the ADF and PP tests are performed on the residuals from these regressions. Where the seasonal dummies are not significant, the ADF and PP tests are performed on the actual series. This procedure for applying traditional unit root tests to seasonal data has been justified by Osbourne et al. (1988), Dickey, Hasza and Fuller (1984) and Dickey, Bell and Miller (1986).

The results of the simple unit root tests are presented in Tables 4.3 to 4.6 for the data described in Section 4.2. Attention is turned first to the ADF test results. Following the procedure outlined by Dickey and Pantula (1987), the ADF test is applied to the first differences of the data and then to the levels. Thus, rather than testing the null hypothesis that a series is I(1) against an I(0) alternative, we begin by testing the null hypothesis that a series is I(2) against the alternative that the series is integrated of an order less than I(2). The null hypothesis of a simple unit root in the first differenced data is not rejected for $K$ using the general (i.e. with trend) ADF regression (see Table 4.3).

As discussed in Section 3.3.1.1, it is important to check the appropriateness of the trend term in the general ADF regression. If the trend is inappropriate in the ADF regression then the model will be over parameterised. Test statistics generated from an over parameterised model are, in general, lower in power than the same test statistics generated from a correctly specified model. Thus, it is conceivable that a null hypothesis not rejected by tests generated using an over parameterised model may be rejected if the model was correctly specified. Therefore, when a null hypothesis cannot be rejected, it is important to determine whether or not the model is correctly specified. Failure to reject the null might be a consequence of the reduced test power concomitant with over parameterisation. In the case of the general ADF regression, this amounts to testing the appropriateness of the trend term. The joint test of $a_1=a_2=0$ (in terms of equation 3.5) provides such a check. If this test is not rejected, then we can be confident that the trend term is inappropriate. Under these circumstances, the appropriate procedure is to estimate the ADF regression without the trend term. Note that if the null is rejected there is no need to test whether the model is over
parameterised. Over parameterisation simply reduces the probability of correctly rejecting a false null. If the null is rejected in an over parameterised model, then it should be rejected more strongly in a correctly specified model. In terms of the capital stock variable $K$, the joint test $\alpha_1=\alpha_2=0$ can not be rejected and so one can conclude that the general ADF regression is an inappropriate model for testing a unit root in these series. However, when the regression is re-estimated without the trend, the null hypothesis of a unit root in the first differenced data is still not rejected. Economic data is rarely I(3) and so we do not test such a hypothesis for $K$. Given that capital stock is I(2), we would expect net investment to be I(1) since net investment is defined as the first change in capital stock.

For other series tested, the I(2) null is rejected (although only marginally in the case of $p$). Since the null ($\alpha_0=0$ in terms of equation 3.5) is rejected using the general ADF regression, there is no need to check the appropriateness of the trend term. Thus, only $K$ is I(2). For the other series, the ADF test is applied to the levels data.

In testing for a unit root in the levels data, the null hypothesis is that the series is I(1). Rejection of this hypothesis implies that the series is I(0). Test statistics generated using the general ADF model (i.e. that including a trend) indicate that the null hypothesis (that $\alpha_1=0$) can not be rejected for $\Delta K$, $\Delta K/K$, $Y$, $z$, $p$, and $Q$ (see Table 4.4). However, before concluding that these series are I(1), it is useful to check the appropriateness of the trend term in the ADF model. For all of these series, the joint hypothesis that $\alpha_1=\alpha_2=0$ can not be rejected. Therefore, the ADF regressions are re-estimated without the trend. The resulting test statistics still suggest that the null hypothesis of a unit root can not be rejected. Thus, on the basis of the ADF tests on levels data, we can conclude that $\Delta K$, $\Delta K/K$, $Y$, $z$, $p$, and $Q$ are non-stationary. More precisely, since the I(2) null has already been rejected, the ADF tests suggest that these series are I(1).

The null hypothesis of a simple unit root in the levels data is rejected, however, for $c/p$, albeit marginally. Recall, the HEGY test also found against a unit root for this series. Given that the null hypothesis is rejected using the general model, there is no need to check the appropriateness of the trend term in the ADF regression for this
series. The null is rejected, despite the possibility of over parameterisation and reduced test power.

Thus, on the basis of these ADF tests, one may conclude that $K$ contains two unit roots, $c/p$ does not contain any unit roots and the remaining series contain a single simple unit root. In other words, $K$ is I(2), $\Delta K$, $\Delta K/K$, $Y$, $z$, $p$, and $Q$ are I(1) and $c/p$ is I(0).

Table 4.3: ADF tests for simple unit roots in the core data (first differences)

<table>
<thead>
<tr>
<th>Series</th>
<th>Test</th>
<th>$\alpha_1=0$</th>
<th>$\alpha_1=\alpha_2=0$</th>
<th>$\alpha_1=0$ (with $\alpha_2=0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta K$</td>
<td>10% CV</td>
<td>-3.13</td>
<td>5.34</td>
<td>-2.57</td>
</tr>
<tr>
<td>$K$</td>
<td>-3.82</td>
<td>7.32</td>
<td>-3.84</td>
<td></td>
</tr>
<tr>
<td>$\Delta K/K$</td>
<td>-2.66</td>
<td>4.61</td>
<td>-1.96</td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>-5.07</td>
<td>13.23</td>
<td>-5.13</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>-4.43</td>
<td>9.83</td>
<td>-4.13</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>-5.83</td>
<td>17.07</td>
<td>-5.83</td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>-3.34</td>
<td>5.59</td>
<td>-2.47</td>
<td></td>
</tr>
<tr>
<td>$c/p$</td>
<td>-5.49</td>
<td>15.06</td>
<td>-5.49</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.4: ADF tests for simple unit roots in the core data (levels)

<table>
<thead>
<tr>
<th>Series</th>
<th>Test</th>
<th>$\alpha_1=0$</th>
<th>$\alpha_1=\alpha_2=0$</th>
<th>$\alpha_1=0$ (with $\alpha_2=0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta K$</td>
<td>10% CV</td>
<td>-3.13</td>
<td>5.34</td>
<td>-2.57</td>
</tr>
<tr>
<td>$K$</td>
<td>-2.06</td>
<td>4.68</td>
<td>-1.91</td>
<td></td>
</tr>
<tr>
<td>$\Delta K/K$</td>
<td>-0.18</td>
<td>2.63</td>
<td>2.23</td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>-1.84</td>
<td>1.83</td>
<td>-1.43</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>-1.27</td>
<td>2.02</td>
<td>1.16</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>-1.86</td>
<td>1.89</td>
<td>-0.03</td>
<td></td>
</tr>
<tr>
<td>$Q$</td>
<td>-2.02</td>
<td>4.57</td>
<td>1.27</td>
<td></td>
</tr>
<tr>
<td>$c/p$</td>
<td>-1.92</td>
<td>1.9</td>
<td>-1.93</td>
<td></td>
</tr>
</tbody>
</table>

Notes
1. Test for a simple unit root using the model given in equation 3.5 on first differenced data.
2. Test for a simple unit root using the model given in equation 3.5 on first differenced data but with $\alpha_2$ constrained to zero (i.e. the no trend case).
3. The lag length (in parenthesis) is determined using the minimum AIC criterion up to a maximum of nine lags.
These results are largely confirmed by the results of the PP tests presented in Table 4.5 (for first differenced data) and Table 4.6 (for levels data). The testing procedure is identical to that followed for the ADF tests. Performing the PP tests on the first differenced data (see Table 4.5) reveals that only $K$ is (at least) $I(2)$. Tests for a simple unit root in the levels data, derived from the model including a trend term, clearly suggest that all series are non-stationary (even $c/p$ is found to be non-stationary here). However, the joint tests that $\alpha_1=\alpha_2=0$ in equation 3.5 can not be rejected, except in the case of $p$. That is to say, for all series other than $p$, the model used to generate the test statistics is over parameterised when the trend is included. When the models are re-estimated without a trend, the null hypothesis of a simple unit root in these series is still not rejected. Thus, the PP tests for a simple unit root suggest that all series tested here are non-stationary: $K$ is found to be $I(2)$, whilst the remaining series are found to contain a single simple unit root and are therefore $I(1)$.

With the exception of the series $c/p$, the ADF and PP tests for simple unit roots agree as to each series' order of integration. $\Delta K$, $\Delta K/K$, $Y$, $z$, $p$, and $Q$ have been found to be $I(1)$ and, therefore, require first differencing for stationarity. $K$ has been found to be $I(2)$. The ADF test (and the HEGY test) suggest that $c/p$ is stationary, whereas the PP test presents clear evidence to the contrary. Visual inspection of the time series (see Figure 4.12) indicates that the series is non-stationary since the mean and variance of the series appear to increase over time. Also apparent in Figure 4.12 are the large spikes in the series in the mid-1970's and early 1980's. These outlying observations are unlikely to be generated by an autoregressive process and, therefore, the use of an ADF (or HEGY) test (which assumes that the series to be tested is generated by an autoregressive process) may not be appropriate. Also, common sense suggests that the dominance of these outlying observations (or, more precisely, the mean reversion about the observations) biases the ADF test to find in favour of stationarity. For these reasons an investigator might place more weight on the results from the PP test results. To establish the appropriate order of integration of $c/p$, the ACF and spectrum are examined. These are presented in Figure 4.31 and Figure 4.32 respectively. If $c/p$ is non-stationary one would expect the ACF to decay slowly and the spectrum to contain a large peak at the zero frequency relative to other frequencies (this peak would be infinite in a theoretical power spectrum). The evidence in Figures 4.31 and
Figure 4.31: The ACF for the real user cost of capital (levels data)

Figure 4.32: The spectral density for the real user cost of capital
4.32 certainly suggests that \( c/p \) is non-stationary. Visual Inspection of the ACF for the first differences of \( c/p \) (see Figure 4.33) supports the view that this series is integrated of an order no greater than one. Therefore, it is assumed that \( c/p \) is also I(1).

| Table 4.5: PP tests for simple unit roots in the core data (first differences) |
|---------------------------------|----------|----------|------------------|
| Series  | Test  | \( \alpha_1=0 \) | \( \alpha_1=\alpha_2=0 \) | \( \alpha_1=0 \) (with \( \alpha_2=0 \)) |
| \( \Delta K(1) \) | -3.13 | 5.34 | -2.57 |
| \( K(1) \) | -2.04 | 2.13 | -1.55 |
| \( \Delta K/K(1) \) | -10.52 | 55.38 | -10.54 |
| \( Y(1) \) | -14.73 | 108.64 | -14.54 |
| \( z(1) \) | -7.58 | 28.68 | -7.56 |
| \( p(1) \) | -15.38 | 118.29 | -13.98 |
| \( Q(1) \) | -22.8 | 259.6 | -22.87 |
| \( c/p(1) \) | -8.55 | 36.57 | -8.57 |

Notes:
1. Test for a simple unit root using the model given in equation 3.5 on first differenced data.
2. Test for a simple unit root using the model given in equation 3.5 on first differences data but with \( \alpha_2 \) constrained to zero (i.e. the no trend case).
3. The lag length (in parenthesis) is determined according to the highest significant lag (up to \( T \), which is lag 12 given that \( T=165 \)) from either the ACF or the PACF of the first differenced series.

| Table 4.6: PP tests for simple unit roots in the core data (levels) |
|---------------------------------|----------|----------|------------------|
| Series  | Test  | \( \alpha_1=0 \) | \( \alpha_1=\alpha_2=0 \) | \( \alpha_1=0 \) (with \( \alpha_2=0 \)) |
| \( \Delta K(1) \) | -3.13 | 5.34 | -2.57 |
| \( K(1) \) | -1.75 | 1.59 | -1.39 |
| \( \Delta K/K(1) \) | -1.26 | 61.25 | 11.1 |
| \( Y(1) \) | -2.12 | 2.49 | -0.62 |
| \( z(1) \) | -1.64 | 2.8 | 1.25 |
| \( p(1) \) | -1.79 | 2.25 | 0.47 |
| \( Q(1) \) | -1.98 | 11.2 | 3.28 |
| \( c/p(1) \) | -2.79 | 3.99 | -2.13 |

Notes:
1. Test for a simple unit root using the model given in equation 3.5.
2. Test for a simple unit root using the model given in equation 3.5 but with \( \alpha_2 \) constrained to zero (i.e. the no trend case).
3. The lag length (in parenthesis) is determined according to the highest significant lag (up to \( T \), which is lag 12 given that \( T=165 \)) from either the ACF or the PACF of the first differenced series.
4.4.2 The Order of Integration Determined for Subsidiary Data

The HEGY test, the Augmented Dickey-Fuller test and the Phillips-Perron test are now applied to the subsidiary data discussed in Section 4.3. We proceed with the seasonal unit root tests.

4.4.2.1 Results of Seasonal Unit Root Tests

The results of the seasonal unit root tests on the subsidiary data are given in Table 4.7. Where possible, the data is logged prior to testing. These series are prefixed with an ‘L’. We begin by commenting on the results relating to the zero frequency. The test results suggest that the series for employment, capacity utilisation, investment intentions, real interest rate, and business optimism, respectively denoted \( LE, LCU, II, RIR, \) and \( BO \), appear to \( \text{I}(0) \). In the case of \( LE \) this finding is surprising: the plot of the
series over time, given in Figure 4.23, displays an upward trend and long periods of deviation from this trend. These results are checked in Section 4.4.2.2.

### Table 4.7: HEGY tests for seasonal unit roots in the subsidiary data

<table>
<thead>
<tr>
<th>Series</th>
<th>Frequency</th>
<th>Zero(^1)</th>
<th>Biannual(^1)</th>
<th>Annual(^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO(1)</td>
<td>10% CV</td>
<td>-2.62</td>
<td>-2.59</td>
<td>5.56</td>
</tr>
<tr>
<td>LY(8)</td>
<td>-1.83</td>
<td>-4.55</td>
<td>40.91</td>
<td></td>
</tr>
<tr>
<td>LE(1)</td>
<td>-1.41</td>
<td>-2.64</td>
<td>6.58</td>
<td></td>
</tr>
<tr>
<td>LTP(1)</td>
<td>-2.9</td>
<td>-4.25</td>
<td>29.11</td>
<td></td>
</tr>
<tr>
<td>LW(6)</td>
<td>-0.62</td>
<td>-6.99</td>
<td>48.49</td>
<td></td>
</tr>
<tr>
<td>L(7)</td>
<td>-0.12</td>
<td>-3.48</td>
<td>11.63</td>
<td></td>
</tr>
<tr>
<td>LC(1)</td>
<td>-2.61</td>
<td>-2.97</td>
<td>12.92</td>
<td></td>
</tr>
<tr>
<td>L(1)</td>
<td>-3.95</td>
<td>-7.24</td>
<td>86.62</td>
<td></td>
</tr>
<tr>
<td>RIR(1)</td>
<td>-2.91</td>
<td>-4.72</td>
<td>56.76</td>
<td></td>
</tr>
<tr>
<td>CI(2)</td>
<td>-2.97</td>
<td>-8.71</td>
<td>80.68</td>
<td></td>
</tr>
<tr>
<td>BO(1)</td>
<td>-4.65</td>
<td>-5.51</td>
<td>37.22</td>
<td></td>
</tr>
<tr>
<td>RR(1)</td>
<td>-2.24</td>
<td>-2.83</td>
<td>8.45</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. These test results are derived from the model given in equation 3.8.
2. The lag length (in parenthesis) on the HEGY regression is determined using the minimum AIC criterion up to a maximum of nine lags. Variables prefixed with \(L\) are logged.

The results of tests for the presence of a unit root at the seasonal frequencies are clear cut: none of the series appear to contain a seasonal unit root at either the bi-annual or annual frequencies. We therefore conclude that the seasonality in these series (should it exist) will be adequately captured by deterministic seasonal dummies.

#### 4.4.2.2 Results of Simple Unit Root Tests

As with the series considered in Section 4.4.1.2, the series to be tested for a simple unit root in this section are regressed on seasonal dummies (after taking logs). For those series for which the seasonal dummies were significant, the ADF and PP tests are performed on the residuals from these regressions. Where the seasonal dummies are not significant, the ADF and PP tests are performed on the actual series. It is noteworthy that, where possible, the data is logged prior to testing. The results of the simple unit root tests are given in Tables 4.8 to 4.11. The testing procedure is identical to that described in Section 3.3.1.1 and used in Section 4.4.1.2. We start by testing for a unit root in the first differenced data, as suggested by Dickey and Pantula (1987).
As is clear from Table 4.8, the ADF test rejects the null hypothesis of a unit root in the first differenced data in every case. Thus, we can conclude that none of the series are I(2). When the ADF test is applied to the levels data, the null hypothesis (of I(1) vs. I(0)) can not be rejected for any series except capacity utilisation, LCU, investment intentions, II, and business optimism, BO (see Table 4.9). Thus, on the basis of the ADF tests, we conclude that, with the exception of LCU, II, and BO, all variables are I(1). Capacity utilisation, investment intentions, and business optimism are taken to be I(0). Notice that the trend term is found to be inappropriate in many of the ADF regressions. However, in no case are the initial findings overturned when the ADF model is appropriately specified without the trend term. It is also noteworthy that the employment variable is found to be I(1) here, in contrast to the results of the zero frequency HEGY test.

Table 4.8: ADF tests for simple unit roots in the subsidiary data (first differences)

<table>
<thead>
<tr>
<th>Series</th>
<th>Test</th>
<th>$\alpha_1=0^1$</th>
<th>$\alpha_1=\alpha_2=0^1$</th>
<th>$\alpha_1=0$ (with $\alpha_2=0)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LO(1)$</td>
<td>10% CV</td>
<td>-11.42</td>
<td>65.16</td>
<td>-11.43</td>
</tr>
<tr>
<td>$LY(8)$</td>
<td>-3.77</td>
<td>7.11</td>
<td>-3.73</td>
<td></td>
</tr>
<tr>
<td>$LE(7)$</td>
<td>-4.46</td>
<td>9.95</td>
<td>-4.46</td>
<td></td>
</tr>
<tr>
<td>$LTPI(3)$</td>
<td>-6.76</td>
<td>22.86</td>
<td>-6.74</td>
<td></td>
</tr>
<tr>
<td>$LW(5)$</td>
<td>-3.73</td>
<td>6.99</td>
<td>-3.61</td>
<td></td>
</tr>
<tr>
<td>$LLR(1)$</td>
<td>-10.97</td>
<td>60.19</td>
<td>-10.95</td>
<td></td>
</tr>
<tr>
<td>$LCU(4)$</td>
<td>-10.08</td>
<td>50.82</td>
<td>-10.11</td>
<td></td>
</tr>
<tr>
<td>$II(1)$</td>
<td>-7.01</td>
<td>24.56</td>
<td>-7.03</td>
<td></td>
</tr>
<tr>
<td>$RIR(4)$</td>
<td>-7.92</td>
<td>31.41</td>
<td>-7.95</td>
<td></td>
</tr>
<tr>
<td>$MC(4)$</td>
<td>-4.36</td>
<td>9.51</td>
<td>-4.36</td>
<td></td>
</tr>
<tr>
<td>$BO(3)$</td>
<td>-7.12</td>
<td>25.43</td>
<td>-7.16</td>
<td></td>
</tr>
<tr>
<td>$RE(6)$</td>
<td>-4.64</td>
<td>10.81</td>
<td>-4.6</td>
<td></td>
</tr>
</tbody>
</table>

Notes
1. Test for a simple unit root using the model given in equation 3.5 on first differenced data.
2. Test for a simple unit root using the model given in equation 3.5 on first differenced data, but with $\alpha_1$ constrained to zero (i.e. the no trend case).
3. The lag length (in parenthesis) is determined using the minimum AIC criterion up to a maximum of nine lags. Variables prefixed with an $L$ are logged.

On the whole, the results of the PP tests support those of the ADF tests. When the test is applied to the first differenced data the null of a unit root is clearly rejected in every case (see Table 4.10). In the levels data, there are a number of results that merit individual comment. Firstly, the PP test rejects the null of a unit root in the levels data for real GDP. $L^{I'}$. This is contrary to the bulk of evidence found by other researchers.
in this area. Casual inspection of Figure 4.22 reveals that the series can not possibly be stationary and we therefore dismiss this result as implausible. Secondly, the results concerning the liquidity variable, $LLR$, are not conclusive: the series is on the I(0)/I(1) borderline. However, the time series plot given in Figure 4.24 suggests that $LLR$ is non-stationary. Given this, and the fact that the ADF test suggests non-stationarity, it is taken to be I(1) here. Finally, the capacity utilisation variable appears to be I(1) when the trend is included in the ADF regression. This result contradicts the result of the ADF test given in Table 4.9. However, when the model given by equation 3.5 is reestimated without the trend term, the null hypothesis in the PP test is rejected.

Table 4.9: ADF tests for simple unit roots in the subsidiary data (levels)

<table>
<thead>
<tr>
<th>Series</th>
<th>Test</th>
<th>$\alpha_1=0^1$</th>
<th>$\alpha_1=\alpha_2=0^1$</th>
<th>$\alpha_1=0$ (with $\alpha_2=0$)$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO(1)</td>
<td>10% CV</td>
<td>-2.93</td>
<td>4.4</td>
<td>-2.03</td>
</tr>
<tr>
<td>LY(8)</td>
<td>-2.76</td>
<td>3.98</td>
<td></td>
<td>-0.92</td>
</tr>
<tr>
<td>LE(7)</td>
<td>-3.11</td>
<td>4.9</td>
<td></td>
<td>-1.96</td>
</tr>
<tr>
<td>LTP(3)</td>
<td>-1.73</td>
<td>1.52</td>
<td></td>
<td>-1.73</td>
</tr>
<tr>
<td>LW(5)</td>
<td>-1.87</td>
<td>2.25</td>
<td></td>
<td>-0.41</td>
</tr>
<tr>
<td>LLR(1)</td>
<td>-3.07</td>
<td>5.74</td>
<td></td>
<td>-3.09</td>
</tr>
<tr>
<td>LCU(4)</td>
<td>-3.86</td>
<td>7.51</td>
<td></td>
<td>-3.89</td>
</tr>
<tr>
<td>II(4)</td>
<td>-5.67</td>
<td>16.16</td>
<td></td>
<td>-5.69</td>
</tr>
<tr>
<td>RIR(4)</td>
<td>-3.06</td>
<td>5.92</td>
<td></td>
<td>-3.04</td>
</tr>
<tr>
<td>MCI(4)</td>
<td>-2.08</td>
<td>2.24</td>
<td></td>
<td>-2.09</td>
</tr>
<tr>
<td>BO(3)</td>
<td>-4.78</td>
<td>11.44</td>
<td></td>
<td>-4.81</td>
</tr>
<tr>
<td>RE(6)</td>
<td>-1.79</td>
<td>1.86</td>
<td></td>
<td>-1.89</td>
</tr>
</tbody>
</table>

Notes
1. Test for a simple unit root using the model given in equation 3.5.
2. Test for a simple unit root using the model given in equation 3.5 but with $\alpha_2$ constrained to zero (ie the no trend case).
3. The lag length (in parenthesis) is determined using the minimum AIC criterion up to a maximum of nine lags. Variables prefixed with an $L$ are logged.

In summary, the results of the unit root tests reveal that all but three of the subsidiary series are I(1). Capacity utilisation, $LCU$, investment intentions, $II$, and business optimism, $BO$, are found to be stationary. This finding is consistent with the evidence in the time series plots: each variable displays a constant mean and strong mean reverting behaviour (see Figure 4.26, 4.28 and 4.27 for capacity utilisation, investment intentions, and business optimism respectively). Indeed, these series were expected to be I(0) a priori, since they are bounded from above and below.
Table 4.10: PP tests for simple unit roots in the subsidiary data (first differences)

<table>
<thead>
<tr>
<th>Series</th>
<th>Test</th>
<th>$\alpha_i=0$</th>
<th>$\alpha_i=\alpha_2=0$</th>
<th>$\alpha_i=0$ (with $\alpha_2=0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO(1)</td>
<td>10% CV</td>
<td>-3.13</td>
<td>5.34</td>
<td>-2.57</td>
</tr>
<tr>
<td>LY(1)</td>
<td>-18.28</td>
<td>167.08</td>
<td>-18.32</td>
<td></td>
</tr>
<tr>
<td>LE(1)</td>
<td>-18.67</td>
<td>174.06</td>
<td>-18.72</td>
<td></td>
</tr>
<tr>
<td>LTP(1)</td>
<td>-10.81</td>
<td>58.39</td>
<td>-10.82</td>
<td></td>
</tr>
<tr>
<td>LW(1)</td>
<td>-11.12</td>
<td>61.86</td>
<td>-11.12</td>
<td></td>
</tr>
<tr>
<td>LLR(1)</td>
<td>-9.85</td>
<td>48.52</td>
<td>9.78</td>
<td></td>
</tr>
<tr>
<td>LCU(1)</td>
<td>-10.97</td>
<td>60.19</td>
<td>-10.95</td>
<td></td>
</tr>
<tr>
<td>II(1)</td>
<td>-10.48</td>
<td>54.95</td>
<td>-10.51</td>
<td></td>
</tr>
<tr>
<td>RIR(1)</td>
<td>-9.63</td>
<td>46.39</td>
<td>-9.67</td>
<td></td>
</tr>
<tr>
<td>MC(1)</td>
<td>-10.07</td>
<td>50.67</td>
<td>-10.09</td>
<td></td>
</tr>
<tr>
<td>BO(1)</td>
<td>-11.31</td>
<td>63.95</td>
<td>-11.29</td>
<td></td>
</tr>
<tr>
<td>RE(1)</td>
<td>-12.89</td>
<td>83.15</td>
<td>-12.92</td>
<td></td>
</tr>
</tbody>
</table>

Notes
1. Test for a simple unit root using the model given in equation 3.5 on first differenced data.
2. Test for a simple unit root using the model given in equation 3.5 on first differences data but with $\alpha_2$ constrained to zero (i.e. the no trend case).
3. The lag length (in parenthesis) is determined according to the highest significant lag (up to $T^*$), from either the ACF or the PACF of the first differenced series. Note that $T$ varies across series. Variables prefixed with an $L$ are logged.

Table 4.11: PP tests for simple unit roots in the subsidiary data (levels)

<table>
<thead>
<tr>
<th>Series</th>
<th>Test</th>
<th>$\alpha_i=0$</th>
<th>$\alpha_i=\alpha_2=0$</th>
<th>$\alpha_i=0$ (with $\alpha_2=0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO(1)</td>
<td>10% CV</td>
<td>-3.13</td>
<td>5.34</td>
<td>-2.57</td>
</tr>
<tr>
<td>LY(1)</td>
<td>-2.65</td>
<td>6.67</td>
<td>-2.62</td>
<td></td>
</tr>
<tr>
<td>LE(1)</td>
<td>-6.19</td>
<td>19.15</td>
<td>-0.88</td>
<td></td>
</tr>
<tr>
<td>LTP(1)</td>
<td>-1.86</td>
<td>1.83</td>
<td>-1.19</td>
<td></td>
</tr>
<tr>
<td>LW(1)</td>
<td>-0.59</td>
<td>0.41</td>
<td>-0.79</td>
<td></td>
</tr>
<tr>
<td>LLR(1)</td>
<td>-1.07</td>
<td>1.24</td>
<td>-0.23</td>
<td></td>
</tr>
<tr>
<td>LCU(1)</td>
<td>-3.13</td>
<td>5.35</td>
<td>-3.06</td>
<td></td>
</tr>
<tr>
<td>II(1)</td>
<td>-2.67</td>
<td>3.61</td>
<td>-2.69</td>
<td></td>
</tr>
<tr>
<td>RIR(1)</td>
<td>-3.64</td>
<td>6.66</td>
<td>-3.64</td>
<td></td>
</tr>
<tr>
<td>MC(1)</td>
<td>-3.03</td>
<td>5.53</td>
<td>-3.08</td>
<td></td>
</tr>
<tr>
<td>BO(1)</td>
<td>-0.88</td>
<td>0.71</td>
<td>-1.08</td>
<td></td>
</tr>
<tr>
<td>RE(1)</td>
<td>-4.58</td>
<td>10.64</td>
<td>-4.61</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.46</td>
<td>1.22</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Notes
1. Test for a simple unit root using the model given in equation 3.5.
2. Test for a simple unit root using the model given in equation 3.5 but with $\alpha_2$ constrained to zero (i.e. the no trend case).
3. The lag length (in parenthesis) is determined according to the highest significant lag (up to $T^*$) from either the ACF or the PACF of the first differenced series. Note that $T$ varies across series. Variables prefixed with an $L$ are logged.
4.4.3 Statistical Analysis: A Summary

The order of integration of several economic time series has been tested. The procedure proposed by Hylleburg et al (1990) was used to test for the presence of seasonal unit roots in the data. Spectral techniques were used to complement this test. It was concluded that none of the series tested contained seasonal unit roots. The procedures suggested by Dickey and Fuller (1979) and Phillips and Perron (1988) were used to test for simple unit roots. Spectral techniques and the ACF were used in the event that these tests provided conflicting evidence as to a series' order of integration. All series tested in Section 4.4.1 were found to be non-stationary: $K$ was found to have two unit roots, whilst $\Delta K$, $\Delta K/K$, $Y$, $z$, $p$, $Q$ and $c/p$ were found to contain just one simple unit root. As such, care needs to be taken when estimating regressions with these series so as to avoid the spurious regression problem described by Granger and Newbold (1974). These series can be made stationary by at most two applications of the first difference operator. Of the series tested in Section 4.4.2 all but three series were found to be non-stationary: $LO$, $LY^a$, $LE$, $LTPI$, $LW$, $LLR$, $RIR$, $MCI$, and $RE$ were found to be I(1). The other variables tested in Section 4.4.2, namely $LCU$, $II$ and $BO$, were found to be stationary.

It is noteworthy that in Section 4.4.1 the tests were conducted over the sample period of 1955q1 to 1996q4. Pre-1963 data for these variables is somewhat less reliable than the post-1963 data. This is a result of the fact that some of the data necessary to construct these variables was not compiled in an appropriate, or continuous, form prior to 1963. As discussed in Section 4.2, a number of rather arbitrary techniques, such as interpolation and splicing, were employed in an attempt to maximise the length of the sample period, thereby maximising the degrees of freedom. These techniques result in a less than perfect time series for the early part of the sample period. Indeed, it may be the case that these techniques actually introduce some non-stationarity into the data. With this in mind, the unit root tests of Section 4.4.1 were repeated for the period using the truncated sample from 1963q1 and 1996q4. There were no significant differences in the test results over the two sample periods. The test results for the period 1963q1 to 1996q4 are not reported here.

It is also noteworthy that the test statistics reported in Section 4.4.1 are derived from the actual data rather than the logarithms of the data. This is a result of the fact that
investment theories posit a linear relationship between actual investment and its determinants. Indeed, the vast majority of empirical studies of investment, considered in Chapter 2, estimate investment functions with actual data rather than the logged data. In order to be consistent with this body of literature, the initial models (i.e. those corresponding to the dominant theories of investment) estimated in the following chapter are estimated with actual rather than logged data. For variables that typically increase over time, such as GDP for example, one typically observes higher absolute variance at the end of the sample period than the beginning. However, the variance of the series as a percentage of its actual value in any given period tends to be constant. This is one of the arguments in favour of conducting analysis with logged data. In using actual data, increasing absolute variance may be mistakenly identified as evidence of non-stationarity in unit root tests. In order to establish whether the results of tests for non-stationarity conducted in Section 4.4.1 are unique to the actual data, we also perform all of the tests used above on the logged data. For every variable, the order of integration of the logged data was found to be the same as that determined above. The test results obtained using logged data are not reported here. Note that the data described in Section 4.3 and tested in 4.4.2 is logged. The VAR models to be estimated in the following chapter contain variables that are, where possible, logged.

The results of some of the unit root tests are worthy of comment. $K$, the stock of privately owned industrial and commercial buildings, has been found to be $I(2)$. This implies that private sector net investment in industrial and commercial buildings, $\Delta K$, should be $I(1)$ and indeed, this is confirmed by the ADF and PP tests. However, this finding is at odds with much of the empirical literature. Consider, for example, the accelerator model of investment. This suggests that net investment is some function of changes in output. Given that output is known to be $I(1)$, changes in output are $I(0)$. Thus, the accelerator model suggests that an $I(1)$ variable, net investment, is explained by current and lagged values of an $I(0)$ variable, changes in output. Clearly, such a relationship is unbalanced since an $I(0)$ variable is being used to explain an $I(1)$ variable. Such a strategy will eventually fail as the two variables diverge by ever larger amounts. Thus, assuming that output really is $I(1)$, for the accelerator model to be a credible theory of investment, investment must be $I(0)$. This is the implicit assumption in much of the empirical work on the accelerator model although, since
most of the work on the accelerator was conducted prior to the realisation of the
importance of non-stationary data in econometric work, it is never actually tested.
Since the series used to measure net investment has been found to be I(1), this
assumption, and therefore the framework of the accelerator model, is clearly invalid
for our investment problem. Whether or not the accelerator is a valid framework for
analysing more general physical investment problems depends on whether the
investment series being considered is stationary. Evidence in Engle and Yoo (1991)
and in Stock and Watson (1988) suggests that it is not.

That $K$ has been found to be I(2) also has implications for Jorgenson’s neoclassical
model. Recall from equation 2.28, desired or optimal capital stock, with
Cobb-Douglas technology, is determined by the ratio of real output to the real user
cost of capital. Both of these variables have been found to be I(1). This suggests that
the optimal capital stock is not I(1). Thus, whilst $K$ is I(2), $K^*$ is I(1). This result is
rather bizarre since it implies that actual and desired capital stock will diverge by ever
larger amounts. Moreover, since the Jorgenson model posits a relationship between
net investment, $\Delta K$, which is I(1), and changes in the desired capital stock, which are
I(0), it, like the accelerator, gives rise to an unbalanced regression. Empirical work
with the Jorgenson model with Cobb-Douglas technology, like work with the
accelerator, implicitly assumes that investment is I(0). In this case, capital stock will
be I(1) and of the same order of integration as the desired capital stock with
Cobb-Douglas technology. Again, however, much of the work on the neoclassical
model predates realisation of the importance of non-stationarity in econometrics.
Therefore, this implicit assumption is never tested. That $K$ and $\Delta K$ have been found to
be I(2) and I(1) respectively in this work suggests that the Jorgenson model is also an
inappropriate framework for analysing our specific investment problem.

Of course, there exists the possibility that $K$ is not in fact I(2). Moreover, $K$ may not
measure the stock of privately owned industrial and commercial buildings accurately.
Thus, before we reject the accelerator and neoclassical models as credible frameworks
within which to analyse private sector net investment in industrial and commercial
building, we must examine these possibilities more closely. To this end, we perform
unit root tests on a number of other related measures of capital stock and investment.
In particular, we consider private sector GDFCF in other buildings and works, the
stock of other buildings and works held by the private sector, as measured by $K^{BB}$ (and described in Section 4.2.5.2), and the stock of privately owned industrial and commercial buildings, as measured by $K^a$ (and described in Section 4.2.5.3.1). Recall from the discussion in Section 4.2.1.3, private sector GDFCF in buildings and works measures private sector net investment in non-residential buildings and infrastructure. The series is published in the *Blue Book 1997* (Table 13.1). Although there are a number of differences between this measure of investment and $\Delta K$ (some of which are discussed in Section 4.2.1.3), evidence suggesting that this series is I(1) would support the finding that $\Delta K$ is I(1). This would also be indirect evidence in support of $K$ being I(2). More direct support of $K$ being I(2) would be provided if the stock of other buildings and works held by the private sector, $K^{BB}$, and the stock of privately owned industrial and commercial buildings, as measured by $K^a$, were found to be I(2). With this in mind, we apply the ADF and PP tests to these three series. Given that quarterly data on the stock of other buildings and works held by the private sector, $K^{BB}$, do not exist, the tests are applied to annual data. The results of these unit root tests are presented in Table 4.12 along with the results of tests on annualised estimates of $K$.

Table 4.12 provides clear evidence in support of the finding that $K$ is I(2). Moreover, they provide no evidence to suggest that $K$ is an inadequate measure of the stock of privately owned industrial and commercial buildings. We therefore reject the accelerator and neoclassical models as appropriate frameworks within which to analyse our investment problem. However, in Chapter 5 we examine how the regression equations suggested by these theories can be modified to give econometrically sound specifications.

Finally, we note that if $K$ is I(2) but $Y$ is I(1), the capital-output ratio will be non-stationary (either I(1) or I(2)). The capital-output ratio (calculated as the stock of privately owned industrial and commercial buildings, $K$, over private sector output, $Y$) is plotted in Figure 4.34 for the period from 1955 to 1996. The non-stationarity in the capital output-ratio is clearly apparent. The capital-output ratio rapidly increased between 1955 and 1975 from 0.35 to 0.8. Since 1975, the ratio appears to deviate about a mean of 0.83. Of course, this apparent change of behaviour in the capital-output ratio around 1975 might be indicative of a structural break in the capital
stock series. This may explain why $K$ has been found to be I(2). However, we do not check for a structural break in $K$ in this work.

Table 4.12: ADF and PP tests on alternative investment and capital stock series

<table>
<thead>
<tr>
<th>Series</th>
<th>Test</th>
<th>$\alpha_1=0^1$</th>
<th>$\alpha_1=\alpha_2=0^1$</th>
<th>$\alpha_1=0$ (with $\alpha_2=0^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta GDCF(4)$</td>
<td>ADF$^3$</td>
<td>-3.68</td>
<td>7.02</td>
<td>-3.05</td>
</tr>
<tr>
<td>$GDCF(1)$</td>
<td>ADF$^3$</td>
<td>-2.24</td>
<td>3.09</td>
<td>-0.28</td>
</tr>
<tr>
<td>$\Delta GDCF(1)$</td>
<td>PP$^4$</td>
<td>-3.71</td>
<td>5.16</td>
<td>-3.06</td>
</tr>
<tr>
<td>$GDCF(1)$</td>
<td>PP$^4$</td>
<td>-1.52</td>
<td>1.52</td>
<td>-0.08</td>
</tr>
<tr>
<td>$\Delta K(6)$</td>
<td>ADF$^3$</td>
<td>-1.19</td>
<td>0.97</td>
<td>-0.13</td>
</tr>
<tr>
<td>$K(2)$</td>
<td>ADF$^3$</td>
<td>0.31</td>
<td>2.92</td>
<td>2.45</td>
</tr>
<tr>
<td>$\Delta K(1)$</td>
<td>PP$^4$</td>
<td>-2.06</td>
<td>2.18</td>
<td>-1.53</td>
</tr>
<tr>
<td>$K(1)$</td>
<td>PP$^4$</td>
<td>0.28</td>
<td>14.3</td>
<td>5.31</td>
</tr>
<tr>
<td>$\Delta K^a(1)$</td>
<td>ADF$^3$</td>
<td>-2.79</td>
<td>3.92</td>
<td>-2.16</td>
</tr>
<tr>
<td>$K^a(2)$</td>
<td>ADF$^3$</td>
<td>-0.17</td>
<td>1.43</td>
<td>1.66</td>
</tr>
<tr>
<td>$\Delta K^a(1)$</td>
<td>PP$^4$</td>
<td>-1.85</td>
<td>1.74</td>
<td>-1.49</td>
</tr>
<tr>
<td>$K^a(1)$</td>
<td>PP$^4$</td>
<td>0.48</td>
<td>5.37</td>
<td>-3.29</td>
</tr>
<tr>
<td>$\Delta K^b(2)$</td>
<td>ADF$^3$</td>
<td>-2.39</td>
<td>3.18</td>
<td>-1.28</td>
</tr>
<tr>
<td>$K^b(0)$</td>
<td>ADF$^3$</td>
<td>1.06</td>
<td>6.86</td>
<td>3.75</td>
</tr>
<tr>
<td>$\Delta K^b(1)$</td>
<td>PP$^4$</td>
<td>-2.57</td>
<td>16.1</td>
<td>-1.96</td>
</tr>
</tbody>
</table>

Notes
1. Test for a simple unit root using the model given in equation 3.5.
2. Test for a simple unit root using the model given in equation 3.5 but with $\alpha_2$ constrained to zero (i.e. the no trend case).
3. The lag length (in parenthesis) is determined using the AIC criterion (up to a maximum of six).
4. The lag length (in parenthesis) is determined according to the highest significant lag (up to $T^*$) from either the ACF or the PACF of the first differenced series. Note that $T$ varies across series.

4.5 Concluding Remarks on the Data

In this chapter we have carefully constructed a number of data series some of which are thought to be determinants of investment behaviour and some indicators of construction output. Section 4.2 is concerned with the development of variables that are consistent with a rigorous interpretation of the accelerator, neoclassical and $Q$ theories of investment. In particular, considerable effort has been spent developing measures of capital stock, the user cost of capital, and $Q$. These measures have, where possible, been tailored to the precise nature of the capital under consideration in this work, namely industrial and commercial building. Such narrowly defined constructs have not existed previously.
The variables developed in Section 4.3 are not consistent with any particular theory of investment. However, many of these variables have been suggested as important determinants of investment in the empirical literature and others were included on the grounds that they are thought to be indicators of investment or construction output and are therefore useful variables in such a forecasting model.

In Section 4.4 the statistical properties of the series are examined. Unit root tests provide the main tool of analysis. A variety of unit root tests are employed and, in cases where the order of integration of a variable is not unequivocally determined by these tests, we employ the autocorrelation function and the spectrum (discussed in Chapter 3) as tools for discrimination. The spectrum is rarely used in modern analysis due to the fact that it requires long runs of data in order to identify cycles in the data. We are fortunate to have a relatively (but not ideally) long sample period of 168 quarterly observations for many variables. There is no evidence to suggest that any of the series are seasonally integrated. In the following chapters we therefore model seasonality using deterministic seasonal dummies. Most of the variables tested appear to contain one simple unit root, although capital stock appears to be 1(2) and measures of capacity utilisation, investment intentions, and business optimism appear to be
stationary. It was noted in Section 4.2.5.5 that the preferred measure of capital stock is $K'$ (denoted $K$) on the grounds that first changes in this series are identically equal to the net investment series, $\Delta K$. The results of the unit root tests on $K$, $\Delta K$, and a number of other measures of investment and capital stock do nothing to undermine this view and so it is with this series that we work in the following chapters.

We conclude this chapter by examining the time series plots of $\Delta K$ and the determinants of investment suggested by theory. Attention is focused on the eight time series plots given in Figures 4.35 to Figures 4.42. The relationship between real investment and (private sector) output is given in Figure 4.35. Notice the correlation between the two series. Whilst both series have an increasing trend over the period, output has increased much more than investment. However, increases (decreases) in investment tend to occur at the same time as increases (decreases) in the rate of output as the accelerator predicts. This relationship is not easy to see in time series plots of investment and first changes in output, due to the dominance of the strong seasonal pattern. However, the relationship is clear when investment is plotted with annual changes in output as in Figure 4.36. Recall, net investment has been found to be I(1) and changes in output are I(0) suggesting that output and investment will diverge by ever larger amounts.

The relationship between investment and user cost is plotted in Figure 4.37. We plot the negative of the user cost series so that the relationship between the two series can be seen more clearly. (Note that increases in user cost should correspond to lower investment, *ceteris paribus*). There is obviously some correlation between the two series but the relationship is far from strong. For example, the low levels of user cost in the early to mid 1970s do not lead to an increase in investment. As suggested above, this may be because firms faced liquidity constraints at this time. From 1975 onwards, the relationship between the two series is relatively strong, although investment rose disproportionately in the late 1980s. As is clear from Figure 4.38, the relationship between investment and the first changes in (transposed) user cost is difficult to interpret.

Of course, the neoclassical model posits a relationship between investment and changes in the optimal capital stock, where the optimal capital stock is measured as real output over the real user cost of capital with Cobb-Douglas technology. However,
Figure 4.35: Net investment and non-construction output

(constant £ million)

output (RHS)

investment (LHS)

Figure 4.36: Net investment and the change in output

(constant £ million)

changes in output (RHS)

investment (LHS)
Figure 4.37: Net investment and (negative) user cost

(constant £ million)

user cost (RHS)

investment (LHS)

1955q1 1960q1 1965q1 1970q1 1975q1 1980q1 1985q1 1990q1 1995q1

Figure 4.38: Net investment and the change in user cost

(constant £ million)

change in user cost (RHS)

investment (LHS)

1955q1 1960q1 1965q1 1970q1 1975q1 1980q1 1985q1 1990q1 1995q1
Figure 4.39: Net investment and optimal capital stock

Figure 4.40: Net investment and optimal capital stock
Figure 4.41: Net investment and the change in optimal capital stock

(change in $Y/c$ (RHS))

Figure 4.42: The rate of net investment and tax-adjusted $Q$

tax-adjusted $Q$ (RHS)

rate of investment (LHS)
in the statistical analysis of these series, we found investment to be I(1) and both output and user cost to be I(1). This implies that changes in desired capital stock are I(0). The relationship between investment and optimal capital stock is shown on Figure 4.39. The large spike in the series occurs at 1973q4. It should be noted that the value for this series in 1974q1 and 1974q2 exceeds 3.5 and 12 million respectively, and have been taken out of the series in order that some appreciation for the relationship between the two series can be gained from the time series plot. The very high values of optimal capital stock are a result of the fact that the series measuring the user cost of capital approaches zero in 1974. In fact, the user cost of capital is highly negative at this time, due to the prevailing negative real interest rate and so the capital gains term is scaled, as discussed in Section 4.2.6. This is the solution suggested by Nickel (1978). Again, there appears to be some correlation between the series, the main anomaly between the two series occurring in the early to mid 1970s.

Because a negative user cost of capital implies an infinite rate of investment in the neoclassical model, capital gains are often excluded. This is sometimes justified on the grounds that in the 1970s, the user cost measured inclusive of capital gains provides a poor proxy for the \textit{ex ante} cost of capital. The relationship between investment and optimal capital stock, when capital gains are excluded from the user cost term, is plotted in Figure 4.40. Clearly, the correlation between the two series improves when capital gains are excluded. The inclusion of this graph is intended to be purely illustrative however, since the exclusion of capital gains is not justifiable from a theoretical perspective.

Figures 4.39 and 4.40 indicate some correlation between $\Delta K$ and desired capital stock, but the neoclassical model posits a relationship between investment and changes in the optimal capital stock. As noted above, we believe the first changes in desired capital stock is a stationary series and therefore investment and desired capital stock will diverge by ever larger amounts. Indeed, any relationship between investment and changes in desired capital stock in time series plots is difficult to identify, even if the observations in the mid 1970s are removed (see Figure 4.41 which plots $\Delta K$ against annual changes in desired capital stock).

The relationship between the rate of net investment, $\Delta K/K$ and $Q$, is plotted in Figure 4.42. There is a reasonably strong correlation between the two series, but this
relationship is perhaps not as strong as one would expect given that theory states that contemporaneous $Q$ alone should provide a sufficient explanation of investment. The first point to note is that $Q$ appears to lead the rate of investment by between 6 and 10 quarters. The lead time, however, does not appear to remain constant throughout the sample period. It is also noteworthy that for the first five years of the sample period it is difficult to identify a positive relationship between investment and $Q$. This lack of relationship, of course, may be a result of the imperfect procedures adopted to obtain a series for $Q$ prior to 1960. In any case, from the evidence in Figure 4.42, it is unlikely that the theoretical implications of the $Q$ model will be upheld by the empirical analysis of the following chapter.

The relationship between $\Delta K$ and its determinants is examined more rigorously in the following chapter. Chapter 5 begins by estimating forms of accelerator, neoclassical, putty-clay and $Q$ models of investment. Later in the Chapter 5, we move away from the models suggested by investment theory as we develop a number of VAR models.
5.1 Introduction

In this chapter a number of models of private industrial and commercial construction output are estimated. An assessment of the forecasting performance of these models is conducted in Chapter 6. We begin, in Section 5.2, with the estimation of models consistent with the four dominant theories of investment. The theoretical details of these models were outlined in Chapter 2. Models of this type have been estimated many times in the literature and a review of many of the studies was given in Chapter 2. In Section 5.2 we attempt to estimate the accelerator, neoclassical (pure and putty-clay) and $Q$ models (for the sample period 1955q1 to 1996q4) in a form consistent with the theory of investment. We can compare the results from these models with those of previous studies. Given that the empirical literature is very large, we choose to compare results with just five studies, each of which is representative of the literature of the time. These studies consider construction investment explicitly and moreover, each aims to systematically compare the within and out of sample performance of the four dominant models. The results of unit root tests conducted in Section 4.4 however, suggest that theoretically consistent estimation may be econometrically invalid. Therefore, the traditional models of investment are respecified and an empirical comparison of the within sample properties of these models is provided. The models are also tested for structural stability. An assessment of their forecasting performance is reserved until Chapter 6.

In Section 5.3 we attempt to model private industrial and commercial construction output as an ARIMA process. A combination of the traditional Box-Jenkins approach and a more modern rule-based approach is used to develop model portfolios. Diagnostic checking provides the means of discriminating between alternative specifications within a portfolio. The purpose of developing these models is to use the resulting forecast as a benchmark forecast against which forecasts generated using other models can be compared. Such comparisons are made in Chapter 6.

A problem with the models estimated in Section 5.2 is that identification of structural parameters may be extremely difficult. A relatively non-structural approach to
modelling private industrial and commercial construction output is adopted in Section 5.4. The basic idea was first suggested by Sims (1980) who treats each variable in the system as endogenous and regresses current values on their own lags and those of all other variables in the system. One of the problems with this approach is that the endogenous variables have to be I(0) and, given that many economic variables are I(1), this often results in a loss of long-run information. Only a few authors have applied this approach to investment spending (see Gordon and Veitch (1986), McMillan (1985) and Funke (1989)) but none of these studies use the models to forecast investment. Johansen (1988) develops an estimation procedure for a system of equations which, in some cases, allows long-run information to be retained. This procedure is described in Section 3.4.4 and is used to estimate a number of models in Section 5.4. We focus on determining the appropriate lag length for each model and the number of cointegrating relationships. Interpretation of the resulting specifications is known to be difficult or, in cases where more than one cointegrating relationship exists, impossible unless the resulting system subsumes a structural model (see Wickens (1993)). The response of some researchers faced with multiple cointegrating relationships is to select that relationship which makes most economic sense. An alternative is to partition variables into endogenous and weakly exogenous variables by testing certain restrictions on the system (see Section 3.4.4 for a fuller description of the testing procedure). This may allow the estimation of a single equation rather than the system as a whole. When the models are developed with forecasting in mind, however, they must necessarily be estimated as a system and moreover, the theoretical implications of the models are largely irrelevant. As such, no attempt is made at interpreting the resulting equations. We begin Section 5.4 by using the Johansen procedure to estimate forms of the four dominant theories of investment. These basic systems are then augmented in ways suggested by the literature and then re-estimated. In the last part of Section 5.4, we develop a set of models that make use of construction specific data and another set that use more aggregate data. Although most of the variables in these systems are chosen because they are believed to be leading indicators of construction output, investment or economic activity in general, none of the systems are consistent with theory. All of the models estimated go forward to the forecasting contest in Chapter 6. It is noteworthy that the sample period used in
Section 5.4 varies across models, and is typically shorter than for the models estimated in Section 5.2. This is due to a lack of data.

5.2 The Traditional Models of Investment

In this section the four traditional models of investment are estimated. Models of this kind have been estimated by a large number of researchers in the past. Many of these researchers however, did not have the benefit of the sophisticated econometric techniques that are available today and therefore, some of these studies can be regarded as somewhat simplistic by today’s standards. Where possible attempts are made to compare the results of this work with that published elsewhere in the literature. Given that the empirical investment literature is vast, comparisons are concentrated on five papers which have already received attention in the empirical literature review contained within Chapter 2 (see Section 2.7 in particular). These papers (by Bischoff (1971b), Clark (1979), Jenkinson (1981), Kopcke (1985) and Bernanke et al (1988)) are distinctive in that their main objective is to provide a comparison of the empirical performance of the four traditional models of investment. These papers are also thought to be representative of the literature at the time they were written. Other empirical studies are referred to when the need to widen comparisons arises.

5.2.1 Estimation Issues and the Traditional Investment Models

In the accelerator, neoclassical and putty-clay models, investment is related to its determinants through some distributed lag relation. There are a number of problems associated with the estimation of distributed lag models and these are outlined in Section 3.2. The estimation issues specific to each model are discussed below.

5.2.1.1 The Accelerator Model

In Section 2.2.1 (equation 2.9) it was shown that according to the accelerator model, real gross investment, $I_t$, is related to changes in real output, $\Delta Y_t$, according to the specification
\[ I_t = yv \sum_{i=0}^{k} (1 - v)^i \Delta Y_i + \delta K_{t-1} \]  \hspace{1cm} (5.1)

where \( y \) is fixed capital-output ratio, \( v \) is a partial adjustment parameter reflecting the fact that adjustment of actual capital stock to its desired level is not immediate as in the simple accelerator. Equations of this kind have been frequently estimated in the literature (see Section 2.2.4). Subtracting depreciation and appending a constant, \( \kappa \), seasonal dummies, \( s_j \) (where \( j = 1 \ldots 3 \)), and an error term, \( u_t \), gives an equation for real net investment, \( \Delta K_t \), as

\[ \Delta K_t = \kappa + \sum_{j=1}^{3} s_j + \sum_{i=0}^{k} \beta_i \Delta Y_{t-i} + u_t \]  \hspace{1cm} (5.2)

Researchers have encountered a number of problems in estimating such relationships. For example, the residuals are often heteroskedastic. Many researchers do not bother testing for heteroskedasticity but instead assume it to be present \( a \ priori \) (see Clark (1979), Jenkinson (1981), Bernanke et al. (1988), and Bischoff (1971b) for examples). The typical approach to this problem is to deflate all variables in the equation (including the constant and seasonal dummies). Many US studies, such as Bischoff (1971b), Bernanke et al. (1988) and Clark (1979), use the US Council of Economic Advisers’ estimates of ‘potential output’ as a deflator. For the UK, Jenkinson (1981) deflates variables by last period’s capital stock. Division of variables by potential output or lagged capital stock is presumably based on an assumption that the error variance rises in proportion to potential output or lagged capital stock. Since none of these studies explore the nature of the heteroskedasticity such a deflation is somewhat arbitrary. We choose instead to test for heteroskedasticity.

Another problem encountered by researchers estimating equation 5.2 is that of serially correlated residuals. Estimated first order serial correlation coefficients are typically very high. For example, Bernanke et al. (1988) find a serial correlation coefficient of 0.972 on their accelerator model for investment in structures. Kopcke (1985) presents an estimate of 0.97, Clark’s (1979) estimate is 0.90, Bischoff’s (1971b) estimate is 0.849, while for aggregate UK investment, Jenkinson (1981) derives an estimate of 0.77. In each of these studies, equations are reestimated using the Cochrane-Orcutt procedure to correct for the serial correlation (see Cochrane and Orcutt (1949)). Only
Bernanke et al (1988) make any attempt to determine the source of the serial correlation. They note that the high degree of serial correlation in the untransformed residuals suggests that one should examine the validity of the first order serial correlation correction. Applying a likelihood ratio test of common factor restrictions, as suggested by Hendry and Mizon (1978), Bernanke et al (p. 310) find ‘disturbing evidence’ against the first order serial correlation process. Thus, they conclude that the serially correlated errors are a result of dynamic misspecification.

That equation 5.2 is misspecified is obvious from the results of the unit root tests. Recall from Section 4.4, $\Delta K_t$ is a non-stationary, I(1) process, whereas $\Delta Y_t$ is a stationary process. As a result equation 5.2 is said to be ‘unbalanced’; an I(0) variable is being used to explain an I(1) variable. Such a strategy will eventually fail as the two variables must diverge by ever-larger amounts. Thus, the equation does not make sense. The problems of applying standard distribution theory to unbalanced regressions was first discussed by Mankiw and Shapiro (1985, 1986).

In the studies listed above as finding evidence of first order serial correlation, the order of integration is not tested. This is probably due to the fact that the importance of testing for a unit root was not fully realised when these papers were written. This is certainly true of the early papers. Without the test results we can not say with certainty that the accelerator equations in these papers are unbalanced, but such an assumption would not be unreasonable given the very high first order serial correlation coefficients found. If investment in these studies is I(1) whilst first changes in output are I(0), then a Cochrane-Orcutt correction is inappropriate since the serial correlation indicates model misspecification. It is clear from their comment on the results of a test for a common factor restriction that Bernanke et al are aware of this, however, they make no attempt to reestimate the model appropriately. This said, an appropriately specified model would undermine the accelerator hypothesis. Recall, the accelerator theory states that the level of investment depends entirely on (current and lagged) changes in output. Given that investment is an I(1) process and changes in output are I(0), econometric theory says that there can not be a relationship between these two variables in the way that the accelerator theory of investment suggests. Thus, the results of the preliminary econometric analysis directly contradict the investment theory.
Although, on the basis of the unit root tests, the estimation of equation 5.2 is plainly wrong, it is estimated here in order to provide an initial comparison with some of the other studies discussed above. As in Bernanke et al, common factor analysis is adopted in order determine the source of the serial correlation. Given that investment is I(1) and the change in output is I(0) the resulting model misspecification should be identified by a rejection of the common factor restrictions. Since with serially correlated errors traditional tests of significance are invalid (in addition to the estimates being inefficient) the lag length $k$ in equation 5.2 is determined (using the AIC as described in Section 5.2.1) after the Cochrane-Orcutt correction.

Equation 5.2 is then respecified to give a balanced regression. One might be tempted to take first differences of both sides of equation 5.2 such that

$$\Delta^2 K_t = \sum_{j=1}^{3} s_j + \sum_{t=0}^{k} \beta_t \Delta^2 Y_{t-t} + u_t$$ (5.3)

where $\Delta^2 K$ denotes first differences of net investment (or second differences in capital stock) and $\Delta^2 Y$ denotes second differences of output. However, such an approach is inappropriate since, although the equation will be balanced in the sense that both sides of equation 5.3 are I(0), $\Delta^2 Y$ is overdifferenced. Moreover, estimates in equation 5.3 may be biased.

An alternative means of obtaining a balanced regression is to append the right hand side of equation 5.2 with a lagged dependant variable. Since $\Delta K_t$ is I(1), $\Delta K_{t-1}$ will also be I(1). Moreover, $\Delta K_t$ and $\Delta K_{t-1}$ are likely to be cointegrated, that is to say a linear combination of these variables will be I(0) (the concept of cointegration in a single equation framework is discussed in Section 3.4). Thus, the equation will be balanced. If the common factor restrictions on equation 5.2 are rejected, thereby suggesting dynamic misspecification, one would hope that appending a lagged dependant variable to it may be sufficient to correct misspecification. Thus,

$$\Delta K_t = \kappa + \sum_{j=1}^{3} s_j + \sum_{t=0}^{k} \beta_t \Delta Y_{t-t} + b \Delta K_{t-1} + u_t$$ (5.4)

gives an econometrically valid equation. Note that equation 5.4 subsumes equation 5.3. The lag length $k$ in equation 5.4 can be determined according to the AIC.
(assuming residuals are well-behaved) and the coefficients $\beta$ can be constrained to lie on an Almon polynomial.

In summary, two versions of the accelerator model are to be estimated. The version of the accelerator typically estimated in the literature and given in equation 5.2 is not consistent with the results of unit root tests given in Section 4.4. According to the results of these tests, investment can not be related to changes in output in the way that the accelerator of equation 5.2 suggests: investment is a non-stationary $I(1)$ process and the first change in output is an $I(0)$ process. That is, an $I(0)$ process can not possibly explain an $I(1)$ process. The very high first order serial correlation coefficients found in other studies would suggest that the estimated equations are also unbalanced and therefore misspecified. Although equation 5.2 is misspecified, it is estimated here with a Cochrane-Orcutt correction for serial correlation in order to facilitate comparisons with these other studies. However, equation 5.2 is also respecified to obtain a balanced regression and this is given in equation 5.4. It is this version of the model that is used in subsequent analysis. The estimation results for the misspecified accelerator given by equation 5.2, and its respecification given by equation 5.4, are presented and described in Section 5.2.2.1.

5.2.1.2 The Neoclassical Model

In Section 2.3.3 it was shown that Jorgenson, assuming Cobb-Douglas technology, derived a model of investment in which firms' investment behaviour was affected by changes in relative prices in addition to changes in output. Jorgenson specified a model, given by equation 2.32, in which real gross investment was related to changes in output and user cost as

$$I_t = \kappa + \sum_{i=0}^{k} \phi_i \Delta \left( \frac{P_t}{C_t} \right) + \delta K_{t-1}$$

(5.5)

where $p$ is the price of output and $c$ is the nominal user cost of capital. Subtracting depreciation and appending seasonal dummies, $s_j$ (where $j=1...3$), and an error term, $\epsilon$, gives an equation for real net investment as
\[ \Delta K_t = \kappa + \sum_{j=0}^{3} s_j + \sum_{i=0}^{k} \beta_i \Delta \left( \frac{pY}{c} \right)_{t-i} + u_t \]  

(5.6)

Like equation 5.2 for the accelerator model, there are a number of problems associated with the estimation of this model. In addition to problems associated with estimating distributed lag models, residuals from equations of the form of 5.6 have typically been heteroskedastic and serially correlated. Further, like the accelerator model of equation 5.2, equation 5.6 is unbalanced since investment is \( I(1) \) and the right hand side variables have been found to be \( I(0) \) in the analysis conducted in Section 4.4. This is likely to be true of other neoclassical investment equations in the literature since many have found first order serial correlation coefficients in excess of 0.95 (see Bernanke et al. (1988), and Kopcke (1985) for examples). Although equation 5.6 is misspecified it is estimated here in order to compare results (many of which, it is accepted, are invalid) with those found in these other studies. Equation 5.6 is then respecified to obtain a balanced regression. If one were to take first differences of both sides of equation 5.6 such that

\[ \Delta^2 K_t = \sum_{j=0}^{3} s_j + \sum_{i=0}^{k} \beta_i \Delta^2 \left( \frac{pY}{c} \right)_{t-i} + u_t \]  

(5.7)

the equation would be balanced in the sense that both sides of equation 5.7 are \( I(0) \). However, the right hand side is overdifferenced and the estimates from such an equation may be biased. Alternatively, we can append the right hand side of equation 5.6 with a lagged dependent variable such that

\[ \Delta K_t = \kappa + \sum_{j=0}^{3} s_j + \sum_{i=0}^{k} \beta_i \Delta \left( \frac{pY}{c} \right)_{t-i} + b \Delta K_{t-1} + u_t \]  

(5.8)

The lag length \( k \) on equations 5.8 and 5.6 is determined according to the AIC criterion (after correcting \( u_t \) for any remaining serial correlation and testing for heteroskedasticity) and the potential problem of multicollinearity is overcome by forcing the estimated coefficient to lie on an Almon polynomial.
5.2.1.3 The Putty-Clay Model

Bischoff’s (1968, 1971a) amendment to the pure neoclassical model was described in Section 2.4. The amendment arises from the empirical observation that most modifications to the capital-labour ratio are made to new capital since existing capital stock cannot be remoulded in response to fluctuations in relative prices. Therefore, the effects of changes in the user cost of capital can only be realized as older existing capital stock is retired or as total capacity is increased. By contrast, an unexpected increase in output requires an immediate increase in capital stock, having the same capital-labour ratio as existing capital, and its installation will not be impeded by the need to retire existing capital first. Thus, the response of investment to an increase in output will be more rapid than the response to a decrease in the user cost of capital.

Under these circumstances the distributed lag of investment on changes in output will have a different shape from the distributed lag of investment on changes in the relative price of inputs. To account for this possibility, Bischoff (1968, 1971a) formulates the specification for real gross investment given in equation 2.34, and rewritten below.

\[ I_t = \kappa + \sum_{i=0}^{k} \beta_{1i} \left( \frac{p_{t-1-i} Y_{t-i}}{c_{t-i-1}} \right) + \sum_{i=0}^{k} \beta_{2i} \left( \frac{p_{t-1-i} Y_{t-i}}{c_{t-i-1}} \right) + \delta K_{t-1} \]  

(5.9)

Subtracting depreciation and appending seasonal dummies, \( s_j \) (where \( j=1...3 \)), and an error term, \( u_t \), gives an equation for real net investment,

\[ \Delta K_t = \kappa + \sum_{j=1}^{3} s_j + \sum_{i=0}^{k} \beta_{1i} \left( \frac{p_{t-1-i} Y_{t-i}}{c_{t-i-1}} \right) + \sum_{i=0}^{k} \beta_{2i} \left( \frac{p_{t-1-i} Y_{t-i}}{c_{t-i-1}} \right) + u_t \]  

(5.10)

Again there are problems in estimating equation 5.10. These problems relate to multicollinearity, heteroskedasticity and serial correlation. They are solved in the same way they were solved in the accelerator and neoclassical models. However, unlike the accelerator and neoclassical models of investment given in equations 5.2 and 5.6, respectively, this equation is not unbalanced. Net investment on the left hand side of equation 5.10 is I(1), as are real output and the real user cost of capital. In the pure version of the neoclassical model, with Cobb-Douglas technology, net investment is a function of changes in output and the user cost, whereas in equation 5.10, net investment is a function of the levels of output and the user cost. Although equation 5.10 is balanced it cannot be estimated as it stands unless \( \Delta K_t \) is cointegrated.
with the right hand side variables. If this is the case, then the long-run information contained in equation 5.10 can be retained. Otherwise, equation 5.10 must be transformed in some way so as to rule out the possibility of estimating a ‘nonsense regression’ (in the words of Yule (1926)), or a ‘spurious regression’ (in the terminology of Granger and Newbold (1974)), discussed in Section 3.4. By taking first differences of both sides of equation 5.10 the non-stationary variables become stationary, the equation remains balanced and the nonsense or spurious regression is ruled out. Thus, the appropriate way to estimate equation 5.10 if its right and left hand side variables are not cointegrated is as

\[ \Delta^2 K_t = \sum_{j=1}^{3} s_j + \sum_{i=0}^{k} \beta_{1i} \Delta \left( \frac{p_{t-i} Y_{t-i}}{c_{t-i}} \right) + \sum_{i=0}^{k} \beta_{2i} \Delta \left( \frac{p_{t-i} Y_{t-i-1}}{c_{t-i}} \right) + u_t \]  

(5.11)

Unfortunately, the long-run information in equation 5.10 is lost under such a transformation.

In order to maintain consistency with neoclassical and accelerator models and the models estimated elsewhere in the literature, this is estimated as

\[ \Delta K_t = \sum_{j=1}^{3} s_j + \sum_{i=0}^{k} \beta_{1i} \Delta \left( \frac{p_{t-i} Y_{t-i}}{c_{t-i}} \right) + \sum_{i=0}^{k} \beta_{2i} \Delta \left( \frac{p_{t-i} Y_{t-i-1}}{c_{t-i}} \right) + \Delta K_{t-1} + u_t \]  

(5.12)

using restricted least squares to constrain the coefficient on the lagged capital stock term to unity. The lag length \( k \) (which is taken to be the same for both distributed lag terms) is determined according to the AIC criterion after any remaining residual autocorrelation has been corrected for. The potential problem of multicollinearity is overcome by forcing the estimated coefficients to lie on an Almon polynomial. The validity of endpoint conditions are tested. Here, the presence of heteroskedasticity (and form, if any is found to exist) is also tested for. The estimation results associated with equations 5.10 and 5.12 are presented in Section 5.2.2.3.

5.2.1.4 The \( Q \) Model

A \( Q \) model of investment behaviour was derived in Section 2.6. With an appropriately defined adjustment cost function, it can be shown that the relationship between the rate of (gross) investment and \( Q \) is given by
where $J_3$ is the adjustment cost parameter, $u_t$ is an error term (which is synonymous with a technology shock and follows explicitly from theory). $Q_t$ is defined as

$$Q_t = (q_t - 1) \frac{z_t}{p_t}$$  \hspace{1cm} (5.14)$$

where $q_t$, known as marginal $q$, measures the ratio of the shadow price to the replacement cost of an extra unit of (uninstalled) capital, and $z_t$ is the price of investment goods. Under certain conditions (derived by Hayashi (1982) and discussed in Section 2.6), marginal $q$ can be proxied by average $q$ (the shadow price of existing or installed capital to the replacement cost of that installed capital). If it is further assumed that the stock market is strongly efficient, in the sense that the observed valuation of a firm equals the present discounted value of its future profits, then the shadow price of a firm’s existing capital stock can be measured by the stock market’s valuation of that capital.

Notice that equation 5.13 expresses gross investment (or more specifically the rate of gross investment) as a function of $Q$. This is a result of the fact that the adjustment cost function is specified in terms of gross investment. An equation relating net investment to $Q$ can be derived simply by specifying the adjustment cost function in terms of net investment. Choosing such an adjustment cost function appropriately one can obtain the relationship

$$\frac{\Delta K_t}{K_t} = \kappa + \frac{1}{\beta} Q_t + u_t$$  \hspace{1cm} (5.15)$$

Notice that this expression implies that net investment is related to $Q$ in the same way as gross investment. (That the rate of net investment can be explained by $Q$ alone has been shown by Motahar (1992)). Appending equation 5.15 with seasonal dummies yields

$$\frac{\Delta K_t}{K_t} = \kappa + \sum_{j=1}^{3} s_j + \frac{1}{\beta} Q_t + u_t$$  \hspace{1cm} (5.16)$$

This is the $Q$ equation to be estimated in this work. Note that unlike the previous
models, net investment is not a distributed lag of the independent variable: according to the theory only contemporaneous \( Q \) matters.

It is noteworthy that both \( \Delta K_t/K_t \) and \( Q_t \) are I(1) variables. Since \( Q \) theory posits a relationship between these series one should expect to find a cointegrating relationship between them. If \( \Delta K_t/K_t \) and \( Q_t \) are not cointegrated but are I(1) then the spurious regression problem can arise. One way to avoid the spurious regression problem is to difference the data prior to estimation so that both series are stationary. Thus, if \( \Delta K_t/K_t \) and \( Q_t \) are not cointegrated then a possible respecification is

\[
\Delta \left( \frac{\Delta K_t}{K_t} \right) = \sum_{j=1}^{3} s_j + \nu \Delta Q_t + u_t
\]  

(5.17)

Of course, if \( \Delta K_t/K_t \) and \( Q_t \) are not cointegrated, the implications for \( Q \) theory are disturbing.

Typically, \( Q \) equations suffer from a high degree of serial correlation; first order serial correlation coefficient commonly exceed 0.95 (see Bernanke et al (1988) and Clark (1979) for examples). Therefore, estimates generated from equation 5.17 are unlikely to differ markedly from those generated from equation 5.16. Indeed, Blough (1992) shows that a Cochrane-Orcutt correction for first order serial correlation of the residuals (i.e. generalised least squares) is asymptotically equivalent to the first differenced regression. Nevertheless, estimates from both equations 5.16 and 5.17 are presented and described in Section 5.2.2.4.

Although theory posits a relationship between investment and contemporaneous \( Q \), many authors estimate a distributed lag relation. In particular, all the studies comparing the empirical performance of alternate investment models referenced in this chapter (namely Bischoff (1971b), Clark (1979), Jenkinson (1981), Kopcke (1985) and Bernanke et al (1988)) model investment as a distributed lag relation of \( Q \).

In order to make some comparisons with these \( Q \) models, the first results to be presented in Section 5.2.2.4 are those generated from a model in which investment is related to \( Q \) through a distributed lag. However, it is realised that such an approach is theoretically invalid. The theoretically consistent \( Q \) model given in equation 5.16 is then estimated. Cointegration analysis is used to assess whether any long-run relationship between net investment and \( Q \) exists. If \( \Delta K_t \) and \( Q_t \) are not cointegrated.
equation 5.16 is respecified to yield an econometrically valid equation. This is given by equation 5.17.

5.2.2 The Traditional Investment Models: Estimation Results

In this section we present the results of estimation of the models developed in Section 5.2.1. The net investment variable is given by new private industrial and commercial construction output and the models are estimated for the sample period 1955q1 to 1996q4.

5.2.2.1 The Accelerator Models

The results from estimating the accelerator models of investment are given in Table 5.1. As noted in Sections 3.2 and 5.2.1.1, the lag length $k$ on the distributed lag term is determined using the AIC. The AIC relies on well behaved residuals, but when equation 5.2 is estimated there is, in common with the studies of Bischoff (1971b), Clark (1979), Jenkinson (1981), Kopcke (1985) and Bernanke et al (1988), evidence of strong first order serial correlation, regardless of the lag length specified. As such, a Cochrane-Orcutt correction for first order serial correlation is made to equation 5.2 and this is sufficient to ensure well behaved residuals. The AIC is minimised when $k=33$, implying that it takes about eight years for the full impact of a change in output to be realised as actual investment. This seems rather long and is certainly a greater lag length than those used in other studies. However, in many of these other studies it is not clear how $k$ is determined. Bischoff (1971b) for example, takes $k=23$ for his structures equation and he states (on p. 18) that it is "determined by experimentation". In many of the studies the sample period could not support a lag of eight years. Given that $k=33$ is somewhat long, the lag length is also determined using Frost's (1975) adaptation to the $\bar{R}^2$ criterion suggested by Schmidt and Waud (1973) which is discussed in Section 3.2. This yielded an adjustment time of exactly the same length. Of course, it is not unknown for very large construction projects to take eight or more years and so such a lag length should not be regarded as totally implausible. In addition, the shape of the (unrestricted) lag distribution when $k=33$ is plausible. All
<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation 5.2</th>
<th>Equation 5.4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient ( \times 10^2 )</td>
<td>Standard Error ( \times 10^2 )</td>
</tr>
<tr>
<td>( \Delta Y )</td>
<td>0.74</td>
<td>0.44</td>
</tr>
<tr>
<td>( \Delta Y_{t-1} )</td>
<td>0.83</td>
<td>0.64</td>
</tr>
<tr>
<td>( \Delta Y_{t-2} )</td>
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<td>( \Delta Y_{t-3} )</td>
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<td>( \Delta Y_{t-4} )</td>
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<td>( \Delta Y_{t-5} )</td>
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<td>( \Delta Y_{t-6} )</td>
<td>7.51</td>
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</tr>
<tr>
<td>( \Delta Y_{t-12} )</td>
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<td>0.87</td>
</tr>
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<td>( \Delta Y_{t-13} )</td>
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<td>0.88</td>
</tr>
<tr>
<td>( \Delta Y_{t-14} )</td>
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</tr>
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<td>( \Delta Y_{t-15} )</td>
<td>8.79</td>
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<tr>
<td>( \Delta Y_{t-16} )</td>
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</tr>
<tr>
<td>( \Delta Y_{t-17} )</td>
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<td>0.93</td>
</tr>
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<td>( \Delta Y_{t-18} )</td>
<td>8.53</td>
<td>0.94</td>
</tr>
<tr>
<td>( \Delta Y_{t-19} )</td>
<td>8.48</td>
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<tr>
<td>( \Delta Y_{t-20} )</td>
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</tr>
<tr>
<td>( \Delta Y_{t-21} )</td>
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</tr>
<tr>
<td>( \Delta Y_{t-22} )</td>
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<td>0.93</td>
</tr>
<tr>
<td>( \Delta Y_{t-24} )</td>
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<td>0.93</td>
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<tr>
<td>( \Delta Y_{t-27} )</td>
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<td>( \Delta Y_{t-28} )</td>
<td>5.85</td>
<td>0.95</td>
</tr>
<tr>
<td>( \Delta Y_{t-29} )</td>
<td>5.26</td>
<td>0.93</td>
</tr>
<tr>
<td>( \Delta Y_{t-30} )</td>
<td>4.59</td>
<td>0.90</td>
</tr>
<tr>
<td>( \Delta Y_{t-31} )</td>
<td>3.78</td>
<td>0.87</td>
</tr>
<tr>
<td>( \Delta Y_{t-32} )</td>
<td>2.71</td>
<td>0.75</td>
</tr>
<tr>
<td>( \Delta Y_{t-33} )</td>
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<td>0.52</td>
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<tr>
<td>s1</td>
<td>-72.19</td>
<td>23.24</td>
</tr>
<tr>
<td>s2</td>
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</tr>
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<td>19.62</td>
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<td>constant</td>
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<td>Rho3</td>
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<td>DW3</td>
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<td>SSE3</td>
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<td>87.73</td>
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<td>R2</td>
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<td>0.977</td>
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<tr>
<td>AR2</td>
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<td>0.976</td>
</tr>
<tr>
<td>DH4</td>
<td>1.31</td>
<td>0.50</td>
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<tr>
<td>LM(5)5</td>
<td>3.62</td>
<td>8.53</td>
</tr>
<tr>
<td>BGP6</td>
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<tr>
<td>CF7</td>
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<td>2.11</td>
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<tr>
<td>PS8</td>
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<td>1.56</td>
</tr>
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<td>Almon9</td>
<td>7.00</td>
<td>(3)</td>
</tr>
</tbody>
</table>

Notes:
1. The sample period is 1955q1 to 1996q4.
2. After serial correlation correction and imposition of Almon restrictions.
3. Before correction for serial correlation and imposition of Almon restrictions.
4. Durbin's \( h \)-statistic follows a normal distribution.
5. LM statistic for serial correlation up to the fifth order (after first order correction, if necessary). This is distributed \( \chi^2 \) with 5 degrees of freedom. The 5% critical value is 11.1.
6. The Breusch, Pagan and Godfrey test for heteroskedasticity follows a \( \chi^2 \) distribution with \( N-1 \) degrees of freedom where \( N \) is the number of estimated parameters. The 5% critical value with 38 degrees of freedom is 53.6.
7. CF denotes a Wald test for a common factor (see Hendry and Mizon (1978)). The test statistic is distributed \( \chi^2 \) with one degree of freedom. The 5% critical value is 3.84.
8. The Chow test for predictive stability follows an \( F \)-distribution with \( T-2 \) and \( \nu_{TA1}' \) degrees of freedom. The 5% critical value is 1.86.
9. This denotes the degree of Almon polynomial fitted. The number in parenthesis denotes the endpoint restriction imposed: (0) implies no endpoint restrictions; (1) implies \( \beta_i = 0 \); (2) implies \( \beta_i = 0 \); (3) implies the imposition of both restrictions.
coefficients are positive, the implied lag structure is relatively smooth and stable, and the peak reaction occurs after 10 quarters, decreasing gradually thereafter.

The residuals when $k=33$ are well behaved once the errors are corrected for first order serial correlation. Notice in Table 5.1, the high value of the first order serial correlation coefficient of 0.85 and the very low Durbin-Watson statistic (see Durbin and Watson (1951)) in the uncorrected equation. Both of these are similar to those found in other studies of construction investment.

An LM test for higher order serial correlation is used to test for serial correlation in the residuals after the Cochrane-Orcutt correction. Here the LM statistic given in Table 5.1 jointly tests for serial correlation up to the fifth degree. The test statistic is 3.62 and is distributed $\chi^2$ with 5 degrees of freedom. Since the test statistic is less than the 5% critical value of 11.1, the null hypothesis of no higher order serial correlation is comfortably accepted. The Durbin’s $h$ statistic (see Durbin (1970)), which is normally distributed, is 1.31. This also suggests an absence of higher order serial correlation. We have no evidence to suggest that the residuals are heteroskedastic: the Breusch-Pagan-Godfrey test, due to Breusch and Pagan (1979) and Godfrey (1978) and described in Judge et al (1985) yields a test statistic of 45.2. The test has a $\chi^2$ distribution (with $N-1$ degrees of freedom, where $N$ is the number of explanatory variables). Since the test statistic is less than the 5% critical value of 52.2 (with 37 degrees of freedom) we can not reject the null hypothesis of homoskedastic residuals at this level of significance.

The degree of Almon polynomial is determined in accordance with the method suggested by Anderson (1966) which is described in Section 3.2. As such equation 5.2 is estimated with a seventh order polynomial. Endpoint restrictions $\beta_1=0$ and $\beta_{k+1}=0$ are rejected by an $F$-test and are therefore not imposed on the distributed lag.

Although the final model can explain over 98% of the variation in net investment the high degree of serial correlation in the untransformed residuals suggests that one should examine the validity of the first order serial correlation correction. The test suggested by Hendry and Mizon (1978) provides disturbing evidence against the first order serial correlation process: the Wald test, which is asymptotically distributed $\chi^2$ with one degree of freedom, gives a statistic of 11.2 which exceeds the critical value.
of 3.84. Of course, this particular result merely confirms what is obvious from the unit root tests: the dynamics of equation 5.2 are misspecified.

In Section 5.2.1.1 equation 5.2 was respecified to yield the balanced regression. A lagged net investment term is appended to the right hand side of the accelerator relation as in equation 5.4. Since net investment and lagged net investment are both I(1) equation 5.4 is balanced. Moreover, since these variables are cointegrated, the spurious regression problem is avoided. The lag length \( k \), determined by the AIC, was found to be 20 quarters, lower than for the accelerator given in equation 5.4 and more in line with the lag length used in other studies (albeit with versions of equation 5.2).

There is no evidence of first or higher order serial correlation in this equation: the first order serial correlation coefficient is low and insignificant, the Durbin-Watson statistic (although not strictly valid in models with lagged dependent variables) is close to 2 and the LM statistic is also substantially below the critical value of 11.1. Also, we have no evidence to suggest that the residuals are heteroskedastic. The coefficients on the current and lagged (first differences in) output terms (before Almon restrictions are imposed) are plausible; the implied lag structure is relatively smooth and stable, all coefficients are positive and the peak reaction occurs after 6 quarters, decreasing thereafter. The procedure suggested by Anderson (1966) suggests a third degree polynomial. The restrictions that \( \beta_1 = 0 \) and \( \beta_{k+1} = 0 \) are accepted at the 5% level of significance. Notice that the coefficient on the lagged investment term is 0.94 and highly significant. A coefficient close to unity is indicative of a unit root in net investment and suggests that Cochrane-Orcutt correction for first order serial correlation in the residuals of equation 5.2 is highly inappropriate. The model explains 98% of the variation in net investment. Chow’s test for predictive stability over the period 1994q1 to 1996q4 reveals that the prediction error has a zero mean. The test statistic, which has an \( F \)-distribution with 12 and 96 degrees of freedom, is 1.56 (the critical value is 1.86). This is to say, there is no evidence that predictions made with this model, for the period from 1994q1 to 1996q4, are biased. As Rea (1978) points out this does not necessarily imply that the coefficients are stable.
5.2.2.2 The Neoclassical Models

The results of estimating the neoclassical models are given in Table 5.2. The residuals of equation 5.6 appear to be strongly serially correlated for all values of \( k \) (the first-order serial correlation coefficient was typically in excess of 0.95 (not reported in Table 5.2). Similar results are reported by Bischoff (1971b), Clark (1979), Kopcke (1985) and Bernanke et al (1988)). When the equation was corrected for first order serial correlation using the Cochrane-Orcutt procedure, an LM test revealed that significant serial correlation remained for all values of \( k \). Thus, equation 5.6 was corrected for first and second order serial correlation. This was sufficient to remove much of the serial correlation at most lag lengths. The AIC is minimised when \( k=29 \), implying that it takes about seven years for the full impact of a change in desired capital stock to be realised as actual investment. This lag length is longer than those estimated elsewhere in the literature (Bischoff (1971b), for example, adopts a lag length of 23) and is one year less than the estimated lag on the accelerator model 5.2.

At first sight, this would appear to be evidence against the putty-clay hypothesis. As discussed in Section 5.2.1.3, this hypothesis contends that investment will respond more slowly to changes in the user cost of capital than to changes in output. Thus, investment should respond more slowly to changes in a term that combines output and user cost than to a simple output variable. However, the peak lag in this neoclassical model is 10 quarters behind that on the accelerator equation (10 in the neoclassical model vs 10 for the accelerator) suggesting that the adjustment to changes in relative prices is slower than the adjustment to changes in output as the putty-clay model contends. The shape of the (unrestricted) lag distribution when \( k=29 \) is plausible. All coefficients are positive and the implied lag structure is relatively smooth and stable.

The residuals from this equation are reasonably well behaved, although the LM test for serial correlation up to the fifth degree indicates that some serial correlation remains after the initial corrections. The test statistic is 14.1, which is distributed \( \chi^2 \) with 5 degrees of freedom, has a critical value of 11.1. Given that this equation is misspecified (as discussed in Section 5.2.1.2) and will not be used in subsequent analysis, no attempt has been made to investigate this further. There is no evidence of heteroskedastic residuals; the Breusch-Pagan-Godfrey test yields a test statistic of 8.66 which is distributed \( \chi^2 \) with 33 degrees of freedom. Since the test statistic is less than
Table 5.2: Estimation results for the neoclassical models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation 5.6'</th>
<th>Equation 5.8'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient $\times 10^6$</td>
<td>Standard Error $\times 10^6$</td>
</tr>
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<td>0.788</td>
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</tr>
<tr>
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</tr>
<tr>
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</tr>
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<td>3.011</td>
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<td>$\Delta \rho_{Y/c_6}$</td>
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<td>1.373</td>
</tr>
<tr>
<td>$\Delta \rho_{Y/c_7}$</td>
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</tr>
<tr>
<td>$\Delta \rho_{Y/c_8}$</td>
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</tr>
<tr>
<td>$\Delta \rho_{Y/c_9}$</td>
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<td>4.116</td>
</tr>
<tr>
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<td>4.272</td>
</tr>
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<td>4.392</td>
</tr>
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<td>$\Delta \rho_{Y/c_{12}}$</td>
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<td>4.482</td>
</tr>
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<td>$\Delta \rho_{Y/c_{13}}$</td>
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<td>4.545</td>
</tr>
<tr>
<td>$\Delta \rho_{Y/c_{14}}$</td>
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<td>4.586</td>
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<td>4.608</td>
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<td>4.590</td>
</tr>
<tr>
<td>$\Delta \rho_{Y/c_{18}}$</td>
<td>5.919</td>
<td>4.551</td>
</tr>
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<td>$\Delta \rho_{Y/c_{19}}$</td>
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<td>4.489</td>
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<td>4.401</td>
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<td>4.128</td>
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<td>$\Delta \rho_{Y/c_{23}}$</td>
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<td>3.931</td>
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<td>$\Delta \rho_{Y/c_{24}}$</td>
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<td>3.686</td>
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<td>$s_2$</td>
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<td>$s_3$</td>
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</tr>
<tr>
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<td>435.500</td>
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</tbody>
</table>

Notes:
1. The sample period is 1955q1 to 1996q4.
2. After serial correlation correction and imposition of Almon restrictions.
3. Before correction for serial correlation and imposition of Almon restrictions.
4. Durbin’s $h$ statistic follows a normal distribution.
5. LM statistic for serial correlation up to the fifth order after serial correlation correction. This is distributed $\chi^2$ with 5 degrees of freedom. The 5% critical value is 11.1.
6. The Breusch, Pagan and Godfrey test for heteroskedasticity follows a $\chi^2$ distribution with $V-1$ degrees of freedom where $V$ is the number of estimated parameters. The 5% critical value with 33 degrees of freedom is 47.4.
7. CF denotes a Wald test for a common factor (see Hendry and Mizon (1978)). The test statistic is distributed $\chi^2$ with one degree of freedom. The 5% critical value is 3.84.
8. The Chow test for predictive stability follows an $F$-distribution with $T_2$ and $T_1-V$ degrees of freedom. The 5% critical value is 1.66.
9. This denotes the degree of Almon polynomial fitted. The number in parenthesis denotes the endpoint restriction imposed. (0) implies no endpoint restrictions; (1) implies $\beta_1=0$; (2) implies $\beta_1=0$; (3) implies the imposition of both restrictions.
the 5% critical value of approximately 46.0 we can not reject the null hypothesis of homoskedastic residuals at this level of significance.

The model is estimated with coefficients constrained to lie on a third order polynomial. Endpoint restrictions $\beta_1=0$ and $\beta_{k+1}=0$ are both accepted and are therefore imposed on the distributed lag. The final model can explain over 97% of the variation in investment which is perhaps slightly higher than other studies (see Bischoff (1971b), Clarke (1979), and Bernanke (1988). The Chow test for predictive stability indicates that predictions made with this model will be biased. Many of these statistics are invalid given the serial correlation that remains in the error and the fact that the model is misspecified.

In Section 5.2.1.2 equation 5.6 was respecified to yield a balanced regression. Equation 5.8 relates net investment to a distributed lag of the first differences in desired capital stock (where desired capital stock is defined as real output divided by the real user cost of capital, assuming Cobb-Douglas technology), and a lagged net investment term. Since current and lagged net investment are I(1), the equation is balanced and moreover, since these variables are cointegrated the spurious regression problem is avoided and the residuals from the equation will be stationary. The lag length $k$, determined by the AIC, is found to be 19 quarters, which is lower than for the incorrectly specified version of the neoclassical model given by equation 5.6 and is intuitively more reasonable. Prior to using the AIC, the residuals were corrected for the fourth order serial correlation (apparent in the ACF and PACF) using the Cochrane-Orcutt procedure. Notice from Table 5.2 that the coefficient on $\rho_4$ is significant with a $t$-ratio in excess of three. After this correction, the LM statistic is 2.81 which is substantially below the critical value of 11.1. The residuals appear to be homoskedastic. The coefficients on the lagged (first differences in) desired capital stock (before Almon restrictions are imposed) are not as stable as those on the accelerator, although they are all positive. The model is fitted with a third degree Almon polynomial. Once fitted it is clear that the peak reaction of net investment occurs 13 quarters after a change in desired capital stock. Recall, the peak reaction to a change in output in equation 5.4 occurred at $k=6$. The fact that the peak reaction to changes in desired capital stock occurs later than that to changes in output provides some support for the putty-clay hypothesis. The $\beta_1=0$ and $\beta_{k+1}=0$ restrictions can not
be rejected and are therefore imposed. Notice that the coefficient on the lagged capital stock term is close to unity and highly significant. This is consistent with the finding of a unit root in the net investment series and suggests that a Cochrane-Orcutt correction for first order serial correlation, as applied to equation 5.6, is inappropriate. After the correction for fourth order serial correlation, the model explains over 97% of the variation in net investment. Chow's test for predictive stability over the period from 1994q1 to 1996q4 reveals that the prediction error mean is not significantly different from zero. The test statistic, which has an $F$-distribution with 12 and 104 degrees of freedom, is 1.47 (the critical value is 1.86). This implies that predictions made with this model will not be significantly biased.

5.2.2.3 The Putty-Clay Models

The results of estimating equations 5.10 and 5.12 are shown in Table 5.3. The estimation results of equation 5.10 are discussed first. The maximum value for $k$, the lag length on the distributed lag terms, is limited to 25 given the large number of parameters to be estimated. That is, the AIC is used to determine $k$ up to a maximum of 25. In common with other studies (see Bischoff (1971b), Clark (1979), Kopcke (1985) and Bernanke et al (1988) for examples), the residuals are strongly serially correlated for all values of $k \leq 25$ (the first-order serial correlation coefficient, not reported in Table 5.3, is typically in excess of 0.95). When the equation is corrected for first order serial correlation using the Cochrane-Orcutt procedure, an LM test revealed that significant serial correlation remained for all values of $k$. Examination of the ACF and PACF suggested that third order serial correlation remained. Thus, equation 5.10 is estimated with a correction for first and third order serial correlation. Table 5.3 shows that the first and third order serial correlation coefficients are highly significant (carrying $t$-ratios of 24.79 and -4.41, respectively).

The AIC is minimised with $k=20$, implying that it takes five years for the full impact of a change in desired capital stock to be realised as actual investment. This is somewhat less than the estimated lag on the accelerator and neoclassical models. The shape of the (unrestricted) lag distribution when $k=20$ is worthy of comment. On the first distributed lag, the value of the coefficient increases from close to zero at the
Table 5.3: Estimation results for the putty-clay models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equation 5.10</th>
<th>Equation 5.12</th>
<th>Equation 5.10</th>
<th>Equation 5.12</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient/Standard Error</td>
<td>Coefficient/Standard Error</td>
<td>Coefficient/Standard Error</td>
<td>Coefficient/Standard Error</td>
</tr>
<tr>
<td>$(p_{n+1}Y_{t},C_{t})$</td>
<td>-0.187/0.193</td>
<td>-0.324/0.215</td>
<td>0.193/0.230</td>
<td>0.335/0.221</td>
</tr>
<tr>
<td>$(p_{n+2}Y_{t},C_{t})$</td>
<td>-0.210/0.400</td>
<td>-0.388/0.374</td>
<td>0.218/0.411</td>
<td>0.403/0.384</td>
</tr>
<tr>
<td>$(p_{n+3}Y_{t},C_{t})$</td>
<td>-0.108/0.572</td>
<td>-0.251/0.513</td>
<td>0.115/0.586</td>
<td>0.265/0.525</td>
</tr>
<tr>
<td>$(p_{n+4}Y_{t},C_{t})$</td>
<td>0.084/0.758</td>
<td>0.029/0.655</td>
<td>-0.082/0.774</td>
<td>-0.021/0.669</td>
</tr>
<tr>
<td>$(p_{n+5}Y_{t},C_{t})$</td>
<td>0.337/0.958</td>
<td>0.404/0.806</td>
<td>-0.340/0.977</td>
<td>-0.404/0.822</td>
</tr>
<tr>
<td>$(p_{n+6}Y_{t},C_{t})$</td>
<td>0.622/1.162</td>
<td>0.830/0.961</td>
<td>-0.631/1.184</td>
<td>-0.840/0.979</td>
</tr>
<tr>
<td>$(p_{n+7}Y_{t},C_{t})$</td>
<td>0.915/1.354</td>
<td>1.267/1.110</td>
<td>-0.931/1.380</td>
<td>-1.288/1.131</td>
</tr>
<tr>
<td>$(p_{n+8}Y_{t},C_{t})$</td>
<td>1.195/1.522</td>
<td>1.684/1.241</td>
<td>-1.218/1.551</td>
<td>-1.715/1.265</td>
</tr>
<tr>
<td>$(p_{n+9}Y_{t},C_{t})$</td>
<td>1.445/1.654</td>
<td>2.051/1.346</td>
<td>-1.474/1.686</td>
<td>-2.092/1.371</td>
</tr>
<tr>
<td>$(p_{n+10}Y_{t},C_{t})$</td>
<td>1.649/1.742</td>
<td>2.348/1.416</td>
<td>-1.684/1.776</td>
<td>-2.396/1.443</td>
</tr>
<tr>
<td>$(p_{n+11}Y_{t},C_{t})$</td>
<td>1.798/1.781</td>
<td>2.557/1.447</td>
<td>-1.838/1.816</td>
<td>-2.612/1.476</td>
</tr>
<tr>
<td>$(p_{n+12}Y_{t},C_{t})$</td>
<td>1.883/1.768</td>
<td>2.669/1.438</td>
<td>-1.926/1.804</td>
<td>-2.728/1.467</td>
</tr>
<tr>
<td>$(p_{n+13}Y_{t},C_{t})$</td>
<td>1.901/1.704</td>
<td>2.678/1.390</td>
<td>-1.946/1.740</td>
<td>-2.737/1.418</td>
</tr>
<tr>
<td>$(p_{n+14}Y_{t},C_{t})$</td>
<td>1.850/1.594</td>
<td>2.583/1.306</td>
<td>-1.896/1.628</td>
<td>-2.642/1.333</td>
</tr>
<tr>
<td>$(p_{n+15}Y_{t},C_{t})$</td>
<td>1.734/1.445</td>
<td>2.392/1.192</td>
<td>-1.778/1.477</td>
<td>-2.447/1.218</td>
</tr>
<tr>
<td>$(p_{n+16}Y_{t},C_{t})$</td>
<td>1.558/1.266</td>
<td>2.115/1.058</td>
<td>-1.599/1.295</td>
<td>-2.165/1.082</td>
</tr>
<tr>
<td>$(p_{n+17}Y_{t},C_{t})$</td>
<td>1.332/1.068</td>
<td>1.770/0.911</td>
<td>-1.368/1.095</td>
<td>-1.812/0.933</td>
</tr>
<tr>
<td>$(p_{n+18}Y_{t},C_{t})$</td>
<td>1.067/0.864</td>
<td>1.379/0.760</td>
<td>-1.098/0.888</td>
<td>-1.412/0.779</td>
</tr>
<tr>
<td>$(p_{n+19}Y_{t},C_{t})$</td>
<td>0.781/0.663</td>
<td>0.969/0.605</td>
<td>-0.804/0.682</td>
<td>-0.993/0.622</td>
</tr>
<tr>
<td>$(p_{n+20}Y_{t},C_{t})$</td>
<td>0.492/0.463</td>
<td>0.576/0.442</td>
<td>-0.508/0.478</td>
<td>-0.591/0.455</td>
</tr>
<tr>
<td>$(p_{n+21}Y_{t},C_{t})$</td>
<td>0.223/0.253</td>
<td>0.238/0.250</td>
<td>-0.231/0.261</td>
<td>-0.244/0.258</td>
</tr>
<tr>
<td>constant</td>
<td>-60.01/17.53</td>
<td>53.10/17.26</td>
<td>2.728/1.467</td>
<td>2.737/1.418</td>
</tr>
</tbody>
</table>

Notes:
1. The sample period is 1955q1 to 1996q4.
2. After serial correlation correction and imposition of Almon restrictions.
3. Before correction for serial correlation and imposition of Almon restrictions.
4. Durbin's h statistic follows a normal distribution.
5. LM statistic for serial correlation up to the fifth order after serial correlation correction. This is distributed $\chi^2$ with 5 degrees of freedom. The 5% critical value is 11.1.
6. The Breusch, Pagan and Godfrey test for heteroskedasticity follows a $\chi^2$ distribution with $N-1$ degrees of freedom where $N$ is the number of estimated parameters. The 5% critical value with 45 degrees of freedom is 61.7.
7. CF denotes a Wald test for a common factor (see Hendry and Mizon (1978)). The test statistic is distributed $\chi^2$ with one degree of freedom. The 5% critical value is 3.84.
8. The Chow test for predictive stability follows an $F$-distribution with $T_2$ and $T_1-N$ degrees of freedom. The 5% critical value is 1.86.
9. This denotes the degree of Almon polynomial fitted. The number in parenthesis denotes the endpoint restriction imposed: (0) implies no endpoint restrictions; (1) implies $\beta_1=0$; (2) implies $\beta_{n+1}=0$; (3) implies the imposition of both restrictions.

zero lag to a maximum at lag 11. The value of the coefficients fall thereafter to a minimum (which is negative) at lag 18 and, rather oddly, rise again in lags 19 and 20. The opposite pattern is apparent for the coefficients on the second distributed lag.
term. When the coefficients are constrained to lie on a fourth degree Almon polynomial (with endpoint constraints imposed), this odd behaviour at longer lags is eliminated. However, this is achieved at the expense of introducing odd behaviour in the early lags. On the first distributed lag term, the first three lags are negative implying that a favourable movement in investment determinants actually results in decreased investment in the three subsequent quarters. It is noteworthy that the Almon and endpoint restrictions on the distributed lags can not be rejected by the data.

The residuals from this equation are well behaved. The null hypothesis of no serial correlation (up to the fifth degree) is not rejected: the LM statistic of 8.47 is well below the $\chi^2(5)$ 5% critical value of 11.1. Also, the residuals appear to be homoskedastic: the Breusch-Pagan-Godfrey test yields a test statistic of 29.44 which is comfortably less than the $\chi^2(45)$ 5% critical value of 61.6. The final model explains over 97% of the variation in investment. The Chow test for predictive stability indicates that predictions made with this model are likely to be biased: the $F$-statistic of 2.29 with 12 and 98 degrees of freedom is greater than the 5% critical value of 1.85 (although less than the 1% critical value of 2.36).

As discussed in Section 5.2.1.3, the problem of spurious regression may apply to equation 5.10 if $\Delta K_i$ and the right-hand side variables are not cointegrated. As such, the test results discussed above may be meaningless. We now apply the test for cointegration described in Section 3.4.2. The results of this cointegration test are presented in Table 5.4.

The lag length $k$ in equation 3.24 is determined according to the minimum AIC (these are given in parenthesis in Table 5.4). $\Delta K_i$ is taken as the regressand in the cointegrating regressions. As with the unit root tests conducted in Section 4.4, 10% critical values are quoted here due to the low power of Dickey-Fuller type tests.

None of the test results given in Table 5.4 suggest that the residuals from the cointegrating regression are stationary. We therefore conclude that there is no evidence of a long run relationship between the variables on the right and left hand sides of equation 5.10. Moreover, this suggests that estimating such an equation may give rise to spurious results.
Table 5.4: Tests for cointegration: results for the putty-clay model

<table>
<thead>
<tr>
<th>∆K, on:</th>
<th>Equation 3.22</th>
<th>Equation 3.23</th>
</tr>
</thead>
<tbody>
<tr>
<td>p_{t+1}Y_{t+1}/c_{t+1}</td>
<td>-1.74(12) -3.04</td>
<td>-2.55(12) -3.5</td>
</tr>
<tr>
<td>p_{t+1}Y_{t+1}/c_{t+1}</td>
<td>-1.74(12) -3.04</td>
<td>-2.55(12) -3.5</td>
</tr>
<tr>
<td>both</td>
<td>-1.77(11) -3.45</td>
<td>-2.09(11) -3.84</td>
</tr>
</tbody>
</table>

As discussed in Section 5.2.1.3, the appropriate way to proceed from here is to take first differences of both sides of equation 5.10 so that all variables are I(0). The resulting specification is given in equation 5.12. The results of estimating equation 5.12 are presented in Table 5.3.

The lag length \( k \) (again constrained to be less than or equal to 25 because of the large number of parameters to be estimated) was found to be 20 quarters as determined by the AIC. Prior to using the AIC the residuals are corrected for serial correlation. The ACF and PACF of the residuals from this equation suggest that there is evidence of first and second order serial correlation. The first and second order coefficients are significant with \( t \)-ratios of 2.67 and 2.97, respectively. When the residuals are corrected for this serial correlation, the LM statistic is 6.89 compared with a 5% \( \chi^2(5) \) critical value of 11.1. The residuals also appear to be homoskedastic; the Breusch-Pagan-Godfrey test yields a test statistic of 32.17 against a 5% \( \chi^2(45) \) critical value of 61.6. The distributed lag coefficients (before Almon restrictions are imposed) are plausible; the implied lag structure is relatively smooth and stable, all coefficients are positive and the peak reaction occurs after 12 quarters, decreasing thereafter. Coefficients are constrained to lie on a third degree Almon polynomial. The restrictions \( \beta_1 = 0 \) and \( \beta_{k+1} = 0 \) can not be rejected and are therefore imposed. As with equation 5.10, the imposition of the Almon restrictions results in the first three terms in both of the distributed lags taking perversely signed coefficients, implying that firms' initial response to a favourable change in investment determinants is to reduce net investment.

When the model is estimated in the form of equation 5.12 with the coefficient on the lagged investment term restricted to unity, over 97% of the variation in net investment can be explained. Chow's test for predictive stability over the period from 1994q1 to 1996q4 reveals that the prediction error mean is not significantly different from zero.
The test statistic, which has an $F$-distribution with 12 and 99 degrees of freedom, is 1.77 (the 5% critical value is approximately 1.85). This is to say, predictions made with this model are not likely to be significantly biased.

5.2.2.4 The $Q$ Models

The results of estimating equations 5.16 and 5.17 are shown in Table 5.6 and 5.8, respectively. The results of a distributed lag $Q$ model are discussed first. These are presented in Table 5.5. The AIC suggests a lag length of $k=30$ after a Cochrane-Orcutt correction for first order serial correlation. Notice that the first order serial correlation coefficient of 0.99 is highly significant. The LM statistic indicates that the first order correction is sufficient to remove most of the serial correlation. Such corrections are commonplace in the literature on $Q$ models (see Bischoff (1971b), Clark (1979), Kopcke (1985) and Bernanke et al (1988) for examples).

The lag length is similar to that found on the accelerator and neoclassical models discussed above, but is somewhat longer than that found elsewhere in the literature. For example, Jenkinson (1981) estimates a $Q$ model with 14 lags, Bischoff's (1971b) model contains 13 lags, Clark's (1979) and Kopcke's (1985) models contain 8 lags and the model estimated by Bernanke et al (1988) contains 11. However, it is not clear how the lag length is determined in these studies.

The shape of the (unrestricted) lag structure when $k=30$ is also worthy of comment. Rather than following the expected pattern observed in the other models, the shape of the lag coefficients is rather irregular. Indeed many of the lag coefficients are negative. Bernanke et al (1988), one of the few studies to present unconstrained lag coefficient estimates, also find many negative coefficients in their structures equation. Given this irregular pattern to the lag coefficients it would seem pointless to attempt to constrain the coefficients to lie on the Almon polynomial. Thus, unlike the other models, the coefficient estimates presented in Table 5.5 are unconstrained estimates.

The model satisfies tests for heteroskedasticity and predicative stability and, once corrected for first order serial correlation, the equation explains nearly 99% of the variation in rate of net investment. This is a somewhat higher proportion than that
Table 5.5: Estimation results for a distributed lag \( Q \) model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient(^{\times 10^{3}})</th>
<th>Standard Error</th>
<th>( t )-ratio</th>
<th>Variable</th>
<th>Coefficient(^{\times 10^{3}})</th>
<th>Standard Error</th>
<th>( t )-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>-0.219</td>
<td>0.371</td>
<td>-0.591</td>
<td>( Q_{16} )</td>
<td>-0.366</td>
<td>0.354</td>
<td>-1.034</td>
</tr>
<tr>
<td>( Q_{1} )</td>
<td>0.915</td>
<td>0.382</td>
<td>2.397</td>
<td>( Q_{17} )</td>
<td>0.102</td>
<td>0.336</td>
<td>0.304</td>
</tr>
<tr>
<td>( Q_{2} )</td>
<td>0.103</td>
<td>0.386</td>
<td>0.267</td>
<td>( Q_{18} )</td>
<td>0.809</td>
<td>0.335</td>
<td>2.419</td>
</tr>
<tr>
<td>( Q_{3} )</td>
<td>0.011</td>
<td>0.385</td>
<td>0.028</td>
<td>( Q_{19} )</td>
<td>-0.497</td>
<td>0.333</td>
<td>-1.492</td>
</tr>
<tr>
<td>( Q_{4} )</td>
<td>-0.218</td>
<td>0.387</td>
<td>-0.565</td>
<td>( Q_{20} )</td>
<td>-0.020</td>
<td>0.320</td>
<td>-0.061</td>
</tr>
<tr>
<td>( Q_{5} )</td>
<td>0.824</td>
<td>0.388</td>
<td>2.125</td>
<td>( Q_{21} )</td>
<td>-0.239</td>
<td>0.326</td>
<td>-0.733</td>
</tr>
<tr>
<td>( Q_{6} )</td>
<td>0.685</td>
<td>0.380</td>
<td>1.802</td>
<td>( Q_{22} )</td>
<td>0.573</td>
<td>0.325</td>
<td>1.764</td>
</tr>
<tr>
<td>( Q_{7} )</td>
<td>-0.647</td>
<td>0.378</td>
<td>-1.710</td>
<td>( Q_{23} )</td>
<td>-0.059</td>
<td>0.322</td>
<td>-0.185</td>
</tr>
<tr>
<td>( Q_{8} )</td>
<td>0.266</td>
<td>0.380</td>
<td>0.699</td>
<td>( Q_{24} )</td>
<td>-0.116</td>
<td>0.321</td>
<td>-0.361</td>
</tr>
<tr>
<td>( Q_{9} )</td>
<td>0.265</td>
<td>0.376</td>
<td>0.705</td>
<td>( Q_{25} )</td>
<td>0.431</td>
<td>0.330</td>
<td>1.307</td>
</tr>
<tr>
<td>( Q_{10} )</td>
<td>0.787</td>
<td>0.365</td>
<td>2.157</td>
<td>( Q_{26} )</td>
<td>-0.540</td>
<td>0.326</td>
<td>-1.658</td>
</tr>
<tr>
<td>( Q_{11} )</td>
<td>-0.039</td>
<td>0.367</td>
<td>-0.107</td>
<td>( Q_{27} )</td>
<td>-0.119</td>
<td>0.330</td>
<td>-0.359</td>
</tr>
<tr>
<td>( Q_{12} )</td>
<td>-0.214</td>
<td>0.368</td>
<td>-0.581</td>
<td>( Q_{28} )</td>
<td>0.625</td>
<td>0.336</td>
<td>1.857</td>
</tr>
<tr>
<td>( Q_{13} )</td>
<td>0.699</td>
<td>0.365</td>
<td>1.912</td>
<td>( Q_{29} )</td>
<td>-0.050</td>
<td>0.329</td>
<td>-0.152</td>
</tr>
<tr>
<td>( Q_{14} )</td>
<td>-0.556</td>
<td>0.365</td>
<td>-1.527</td>
<td>( Q_{30} )</td>
<td>-0.062</td>
<td>0.317</td>
<td>-0.195</td>
</tr>
<tr>
<td>( Q_{15} )</td>
<td>0.070</td>
<td>0.340</td>
<td>0.200</td>
<td>( R )</td>
<td>0.990</td>
<td>0.012</td>
<td>83.191</td>
</tr>
<tr>
<td>( s )</td>
<td>-0.543</td>
<td>0.203</td>
<td>-2.675</td>
<td>( \text{DW}^2 )</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s_{2} )</td>
<td>0.647</td>
<td>0.142</td>
<td>4.550</td>
<td>( \text{SSE}^2 )</td>
<td>14.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s_{3} )</td>
<td>0.610</td>
<td>0.191</td>
<td>3.191</td>
<td>( R^2 )</td>
<td>0.986</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{constant} )</td>
<td>-0.146</td>
<td>0.115</td>
<td>-1.275</td>
<td>( \text{ar}^2 )</td>
<td>0.984</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{DH}^4 )</td>
<td>1.31</td>
<td></td>
<td></td>
<td>( \text{LM}(5)^5 )</td>
<td>5.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{BGP}^6 )</td>
<td>26.81</td>
<td></td>
<td></td>
<td>( \text{CF}^7 )</td>
<td>12.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{PS}^8 )</td>
<td>0.55</td>
<td></td>
<td></td>
<td>( \text{Almon}^9 )</td>
<td>n/a</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. The sample period is 1955q1 to 1996q4.
2. After serial correlation correction and imposition of Almon restrictions.
3. Before correction for serial correlation and imposition of Almon restrictions.
4. Durbin's \( h \) statistic follows a normal distribution.
5. LM statistic for serial correlation up to the fifth order after serial correlation correction. This is distributed \( x^2 \) with 5 degrees of freedom. The 5% critical value is 11.1.
6. The Breusch, Pagan and Godfrey test for heteroskedasticity follows a \( x^2 \) distribution with \( N-1 \) degrees of freedom where \( N \) is the number of estimated parameters. The 5% critical value with 34 degrees of freedom is 48.6.
7. CF denotes a Wald test for a common factor (see Hendry and Mizon (1978)). The test statistic is distributed \( x^2 \) with one degree of freedom. The 5% critical value is 3.84.
8. The Chow test for predictive stability follows an \( F \)-distribution with \( T \) and \( T-N \) degrees of freedom. The 5% critical value is 1.86.
9. This denotes the degree of Almon polynomial fitted. The number in parenthesis denotes the endpoint restriction imposed: (0) implies no endpoint restrictions; (1) implies \( \beta_1=0 \); (2) implies \( \beta_{i-1}=0 \); (3) implies the imposition of both restrictions.

As noted in Section 5.2.1.4, modelling the rate of net investment as a distributed lag of \( Q \) is inappropriate since, from a theoretical standpoint, contemporaneous \( Q \) alone is sufficient to explain the rate of investment. Table 5.6 presents the estimation results for such a model (given by equation 5.16).
The first point to note is the very high first order serial correlation coefficient of 0.994. The Durbin-Watson statistic and $R^2$ on the uncorrected equation (not reported here) are 0.02 and 0.11, respectively. Although the coefficient on the uncorrected equation is significant (the coefficient is 0.0021 and has a $t$-ratio of 4.49), these statistics and those presented in Table 5.6 do not suggest that $Q$ can provide a satisfactory explanation of the rate of net investment. However, they are typical of the statistics from properly specified $Q$ models presented elsewhere in the literature. For example, in one of the more successful attempts at modelling equation 5.16, Hayashi (1982) obtains an $R^2$ of 0.46 and a Durbin-Watson statistic of 0.43. Summers (1981) estimates a number of $Q$ models (which take into account various taxation factors) and $R^2$'s range from 0.30 to 0.40. Authors often view the high degree of serial correlation as being symptomatic of an omitted variable. As a result, additional variables such as current and lagged output and lagged dependent variables are often included in $Q$ models. As discussed in Section 2.6, this is wholly inappropriate in the context of this type of model, which require that all assumptions are explicitly accounted for prior to the characterisation of the optimal investment policy. When incorporated into the optimisation problem, the resulting specification bears little resemblance to the estimated equation. Chirinko (1987) estimates a $Q$ model with an output variable and only the latter is significant. Blanchard and Wyplosz (1981)
estimate a $Q$ model with lagged $Q$ and current and lagged output variables; the output variables are significant and their equation has an $R^2$ of 0.57. Abel and Blanchard (1986, p. 250) find that ‘$Q$ is generally a significant explanator of investment, but leaves unexplained a large, serially correlated fraction of investment’. In addition, when output (over capital stock) and profit variables are added to their $Q$ equations both enter significantly. These findings certainly cast doubt on the view that $Q$ alone can explain the rate of investment and thereby undermine $Q$ theory. When current output (over capital stock) is added to equation 5.16 in this study, both $Q$ and the output variable are highly significant. Together, these variables appear to explain 81% of the variation in the rate of net investment, but it must be borne in mind that the Durbin-Watson statistic is still very low at 0.12.

When equation 5.16 is corrected for first order serial correlation, the coefficient on $Q$ decreases to $1.37 \times 10^{-4}$ and has a $t$-ratio of 0.48 (see Table 5.6). Most of the residual serial correlation is accounted for by the first order correction as indicated by the LM statistic of 11.1 (as compared with a 5% critical value of 11.1). The residuals appear to be homoskedastic and the equation comfortably passes the Chow test for predictive stability. This specification of the $Q$ model explains 98.5% of the variation in the rate of net investment.

It was noted in Section 5.2.1.4 that the problem of spurious regressions might arise if the rate of net investment, $\Delta K/K$, and $Q_t$ are not cointegrated. As such, the Dickey-Fuller test for a unit root in the residuals of a cointegrating regression is carried out and the results are presented in Table 5.7. This test for cointegration is discussed in Section 3.2.

<table>
<thead>
<tr>
<th>$\Delta K_t/K_t$ on:</th>
<th>Equation 3.22</th>
<th>Equation 3.22</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_t$</td>
<td>Test Statistic</td>
<td>10% CV</td>
</tr>
<tr>
<td></td>
<td>-1.22(5)</td>
<td>-3.04</td>
</tr>
</tbody>
</table>

The results of the tests for cointegration are not clear cut. Results generated using the cointegrating regression 3.22 (which excludes a time trend) suggests that the residuals contain at least one unit root. Therefore, $\Delta K_t/K_t$ and $Q_t$ can not be cointegrated and estimating the $Q$ model in the form suggested by theory, and given in equation 5.16.
could yield spurious results. These econometric results are inconsistent with, and to
some extent undermine, $Q$ theory. $Q$ theory states that there is a long-run relationship
between the rate of investment and $Q$. The results of the cointegration analysis using
equation 3.22 suggest that no long-run relationship exists. On the other hand, results
generated using equation 3.23 (which includes a trend term) suggests that the
residuals are $I(0)$. Further analysis reveals that these results are sensitive to the lag
length $k$ chosen in equation 3.24. The AIC is minimised with a lag length of $k=4$.
However, for all other lag lengths (between zero and fifteen) the residuals appear to be
$I(1)$. We therefore regard this evidence of cointegration to be rather weak, at best.

If $\Delta K/K_i$ and $Q_i$ are not cointegrated, the spurious regression problem can be ruled out
by taking first differences of the right and left hand side variables to yield equation
5.17. The results from estimating equation 5.17 are given in Table 5.8. Given that the
estimated first order serial correlation coefficient for equation 5.16 was close to unity,
the differences in estimation results between 5.16 and 5.17 (presented in Tables 5.6
and 5.8, respectively) are due largely to the fact that the latter does not include a
constant.

Table 5.8: Estimation results from $Q$ model 5.17

<table>
<thead>
<tr>
<th>Equation 5.17</th>
<th>Coefficient x 10^3</th>
<th>Standard Error</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Q_i$</td>
<td>-0.121</td>
<td>0.271</td>
<td>-0.445</td>
</tr>
<tr>
<td>$s1$</td>
<td>-0.584</td>
<td>0.081</td>
<td>-7.195</td>
</tr>
<tr>
<td>$s2$</td>
<td>0.446</td>
<td>0.079</td>
<td>5.651</td>
</tr>
<tr>
<td>$s3$</td>
<td>0.142</td>
<td>0.079</td>
<td>1.811</td>
</tr>
<tr>
<td>Rho1</td>
<td>0.172</td>
<td>0.077</td>
<td>2.228</td>
</tr>
<tr>
<td>DW</td>
<td>1.650</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSE</td>
<td>4x10^{-5}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.984</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$aR^2$</td>
<td>0.984</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DH</td>
<td>2.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LM(5)</td>
<td>12.69</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BGP</td>
<td>-8.98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CF</td>
<td>1.98</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PS</td>
<td>0.19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

See Notes on Table 5.5.

In the model with no constant the coefficient on $Q$ takes the wrong sign (although it is
insignificant). The LM statistic suggests that some serial correlation remains in the
residuals and examination of the ACF and PACF suggests that this is of first and fourth order varieties. The residuals appear to be homoskedastic and the equation passes the Chow test for predictive stability. When the regression is run with the lagged rate of net investment on the right-hand side (with the coefficient restricted to unity) the equation explains over 98% of the variation in the rate of net investment.

If, as theory predicts, $\Delta K_t/K_t$ and $Q_t$ are indeed cointegrated, then estimation of equation 5.16 will not yield spurious results (in which case the results given in Table 5.6 are valid) and moreover, a model in first differences incorporating an error correction term can be sensibly developed. This has the advantage over simple first differenced models in that long-run information can be retained. If $\Delta K_t/K_t$ and $Q_t$ are cointegrated then the residuals from a regression

$$\frac{\Delta K_t}{K_t} = \kappa + \sum_{j=1}^{3} s_j + \frac{1}{\beta} Q_t + u_t$$

(5.18)

will be stationary. In this case, long-run information can be retained by appending the first difference equation with the estimated residuals from such an equation. This gives

$$\Delta \frac{\Delta K_t}{K_t} = a_0 \Delta Q_t + \Delta \sum_{j=1}^{3} s_j + a_1 \left( \frac{\Delta K_t}{K_t} - \kappa - \sum_{j=1}^{3} s_j - \frac{1}{\beta} Q_t \right)_{t-1} + e_t$$

(5.19)

where the term in brackets is known as the error correction mechanism. The concept of an error correction mechanism was introduced by Davidson et al (1978) and the relationship between cointegration and error correction mechanism has been explored by Engle and Granger (1987). Equation 5.19 is estimated here and the results are presented in Table 5.9.

Neither $Q$ nor the long-run component, ECM, are significant although both have the correct signs. The Durbin-Watson statistic is much improved on equation 5.16 and 5.17 but the equation can only explain 38% of the variation in the rate of net investment. Moreover, an examination of the ACF of the residuals from this equation (not included here) reveals a significant first order serial correlation coefficient. Given that it was not at all clear that $\Delta K_t/K_t$ and $Q_t$ were indeed cointegrated, unit root tests were conducted on the residuals from equation 5.19. These Dickey-Fuller tests are
based on equation 3.24 with $\hat{u}_t$ replaced by $\hat{e}_t$ from equation 5.19. Table 5.10 presents the results of the unit root tests.

Table 5.9: Estimation results from $Q$ model 5.19

<table>
<thead>
<tr>
<th>Equation 5.19</th>
<th>Coefficient x10^3</th>
<th>Standard Error</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Q_t$</td>
<td>0.093</td>
<td>0.286</td>
<td>0.324</td>
</tr>
<tr>
<td>ECM</td>
<td>-12.502</td>
<td>10.520</td>
<td>-1.189</td>
</tr>
<tr>
<td>s1</td>
<td>-0.518</td>
<td>0.071</td>
<td>-7.283</td>
</tr>
<tr>
<td>s2</td>
<td>-0.034</td>
<td>0.078</td>
<td>-0.429</td>
</tr>
<tr>
<td>s3</td>
<td>0.162</td>
<td>0.072</td>
<td>2.259</td>
</tr>
<tr>
<td>Rho1</td>
<td>0.185</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DW</td>
<td>1.624</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSE</td>
<td>$3.85 \times 10^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.381</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$aR^2$</td>
<td>0.365</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

See Notes on Table 5.5.

Table 5.10: Unit roots tests on the residuals of equation 5.19

<table>
<thead>
<tr>
<th>Equation 3.22</th>
<th>Test Statistic</th>
<th>10% CV</th>
<th>Equation 3.23</th>
<th>Test Statistic</th>
<th>10% CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta K_t / K_t$ on: $Q_t$</td>
<td>-5.39(3)</td>
<td>-2.57</td>
<td>-5.38(3)</td>
<td>-3.13</td>
<td></td>
</tr>
</tbody>
</table>

As is clear from Table 5.10, the residuals from equation 5.19 appear to be stationary. The AIC is minimised when $k$ in equation 3.24 is equal to three and the results are not sensitive to the lag length. That is, the residuals appear to be stationary for all $k$ (between zero and twelve). This evidence supports the error correction specification of the $Q$ model. However, given the model’s poor within sample performance, equation 5.19 is unlikely to be useful in any forecasting exercise.

5.2.3 A Comparison of the Within Sample Properties of the Traditional Models

The specifications suggested by a rigorous implementation of the four main theories of investment behaviour have proved to be econometrically invalid. This is due, in part, to the fact that net investment has been found to be $I(1)$. These specifications, given by equations 5.2, 5.6, 5.10, and 5.16 for the accelerator, neoclassical, putty-clay, and $Q$ models respectively, have been estimated in this form in numerous studies. We have estimated these specifications in order to facilitate comparisons with some of these studies. However, we have respecified the models to give econometrically
sensible models and these are given in equations 5.4, 5.8, 5.12 and 5.17 for the accelerator, neoclassical putty-clay and $Q$ models, respectively. It is these versions of the traditional models of investment behaviour that we take into the forecasting contest in Chapter 6.

A summary of the within sample properties is given in Table 5.11. Models 5.4, 5.8 and 5.12 are not directly comparable with model 5.17 due to the different dependent variable and lack of a distributed lag relationship in the latter. It is noteworthy that the high $R^2$'s are largely due to the fact that these models are estimated with the level of real net investment ($\Delta K/K$ in equation 5.17) as the dependent variable and with lagged net investment (lagged $\Delta K/K$ in equation 5.17) as a right hand side variable (with the coefficient constrained to unity in equations 5.12 and 5.17). In the accelerator and neoclassical models, the lagged dependent variable takes a value close to unity suggesting that net investment is I(1) and that the Cochrance-Orcutt correction for first order serial correlation (frequently applied to these models in the empirical investment literature) is inappropriate.

 Whereas the respecified accelerator and $Q$ models (given by equations 5.4 and 5.17 respectively) do not require correction for serial correlation, the neoclassical and putty-clay models (given by equations 5.8 and 5.12 respectively) require a correction for higher order serial correlation (in addition to a first order correction for the latter). The putty-clay model also fails the test for predictive stability and exhibits a rather counterintuitive lag pattern at early lags. Therefore, one would not expect this model to do well in the forecasting exercise. The accelerator and neoclassical models suggest a more reasonable lag pattern. The distributed lag coefficients in the neoclassical model, although all positive, are somewhat erratic prior to the imposition of the Almon polynomial. Both the accelerator and neoclassical models pass the Chow test for predictive stability. Although the $Q$ variable, in itself, explains a relatively small proportion of the variation in the rate of net investment, equation 5.17 has satisfactory residual statistics. It is noteworthy that no evidence of heteroskedasticity was uncovered using the Breusch-Pagan-Godrey test. This is slightly surprising given the tendency in the empirical literature to assume heteroskedasticity a priori.
Table 5.11: Summary of within sample statistics for the traditional investment models

<table>
<thead>
<tr>
<th>Model</th>
<th>Accelerator (5.4)</th>
<th>Neoclassical (5.8)</th>
<th>Putty-Clay (5.12)</th>
<th>Q (5.17)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>ΔKₜ</td>
<td>ΔKₜ</td>
<td>ΔKₜ</td>
<td>ΔKₜ</td>
</tr>
<tr>
<td>Explanatory</td>
<td>ΔY (current &amp; 20</td>
<td>ΔKₜ (current &amp; 19</td>
<td>ΔKₜ/ΔKₜ</td>
<td>Δ(ΔK/K)</td>
</tr>
<tr>
<td>Variables</td>
<td>lags), ΔKₜ &amp; κ</td>
<td>lags), ΔKₜ &amp; κ</td>
<td>(current &amp; 19 lags) &amp; κ</td>
<td>(current &amp; 20 lags)</td>
</tr>
<tr>
<td>Peak Lag</td>
<td>6</td>
<td>13</td>
<td>12</td>
<td>n/a</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.976</td>
<td>0.973</td>
<td>0.972</td>
<td>0.984</td>
</tr>
<tr>
<td>SSE</td>
<td>87.73</td>
<td>92.65</td>
<td>95.58</td>
<td>4×10⁻³</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.09 (0.077)</td>
<td>0.142 (0.09)</td>
<td>0.216 (0.081)</td>
<td>0.172 (0.077)</td>
</tr>
<tr>
<td>LM(5)</td>
<td>8.53</td>
<td>2.81</td>
<td>6.89</td>
<td>12.69</td>
</tr>
<tr>
<td>PS</td>
<td>1.56</td>
<td>1.47</td>
<td>2.23</td>
<td>0.19</td>
</tr>
<tr>
<td>Almon</td>
<td>3(3)</td>
<td>3(3)</td>
<td>4(3)</td>
<td>n/a</td>
</tr>
<tr>
<td>Lag shape</td>
<td>plausible</td>
<td>plausible</td>
<td>odd at ends</td>
<td>n/a</td>
</tr>
</tbody>
</table>

In conclusion we can say that the accelerator, neoclassical, putty-clay and Q models suggested by theory and estimated in the empirical literature have proved econometrically invalid for this type of investment problem. We have respecified these models to generate econometrically valid equations. On balance we expect the accelerator type model of equation 5.4 to do best in the forecasting exercise of Chapter 6. Of the distributed lag models, this provides best within sample fit, unconstrained parameter estimates are plausible and the residual statistics are satisfactory. However, it should be noted that the lagged dependent variable in all these models is the dominant regressor and this indicates that the determinants of net investment suggested by theory are not sufficient to explain net investment in industrial and commercial buildings.

5.3 An ARIMA Model of Private Industrial and Commercial Construction Output

5.3.1 Introduction

In this section an ARIMA model of new private industrial and commercial construction output is developed. This model provides the benchmark against which the forecasting performance of the investment models, developed in Section 5.2, and the VAR models, to be developed in Section 5.4, are to be compared. The process of development is described by Box and Jenkins (1976) as an iterative strategy of 'identification-estimation-diagnostic checking' and is based on a sequence of
procedures, many of which are somewhat informal, which are designed to arrive at the most appropriate specification from a general class of ARIMA\((p,d,q)\) model. The general form of an ARIMA model be written as

\[
a(L)\Delta^d y_t = b(L)e_t
\]  

(5.20)

where \(a(L)\) and \(b(L)\) are polynomials, the respective orders of which are denoted \(p\) and \(q\), \(d\) is the degree of differencing necessary to transform \(y\) into a stationary process, and \(e\) is zero-mean, white noise. It is often argued that the strategy described by Box and Jenkins requires a great deal of expertise and experience. However, recent research has been undertaken on the design of more formal model testing procedures and the use of selection criteria, which enable a more standard set of rules to be followed in model specification. Typically, models are developed by making use of both the traditional Box-Jenkins approach and the rule-based approach. The rules are slightly more complex when the series being modelled contains a strong seasonal component. As is clear from Figure 4.1, new private industrial and commercial construction output does indeed display evidence of a strong seasonal component.

Four model portfolios each containing competing specifications are developed. Alternative portfolios are developed to allow for specifications with actual and logged data and to allow for two degrees of differencing. Within each portfolio, discrimination between alternative competing specifications takes the form of diagnostic checking. Discrimination between the preferred models of each portfolio is achieved through an evaluation of their forecast performance and is discussed in Chapter 6. All the models considered in this section are estimated for the period 1955q1 to 1996q4.

5.3.2 Some Preliminary Considerations

The general problem is that of choosing \(p\) and \(q\) such that the final ARIMA\((p,d,q)\) model is the most appropriate representation of the data within this class of model. However, before one can determine the orders \(p\) and \(q\), one must first undertake some preliminary analysis.
5.3.2.1 Data Transformations: Logarithms versus Actual Data

Many economic time series do not have a constant variance. Therefore, a transformation is often employed to stabilise the variance of the series and, more generally, to induce normality. The family of transformations typically used for this purpose is the Box-Cox power family (see Box and Cox (1964)),

$$x_t^{(i)} = (x_t^i - 1)/\lambda$$

(5.21)

for $\lambda \neq 0$ and $x_t^{(0)} = \ln(x_t)$. The value of the index $\lambda$ can be chosen $a$ priori or estimated along with other parameters. The procedures for joint estimation of parameters are discussed by Nelson and Granger (1979). More often the power transformation is chosen prior to estimation. Among the most popular exploratory methods employed to determine a value of $\lambda$ is the 'range-median' plot restricting attention to a limited range of transformations, such as those indexed by 0, $\frac{1}{2}$ and 1 (see Chapter 4 of Mills (1990) for examples of how this method can be applied). In univariate analysis attention is typically restricted to the indexes of zero and unity which represent log and linear models.

The investment models developed in Section 5.2 are estimated with actual rather than logged data. Whilst it is realised that some of the variables used in those models may not have constant variances, they are not subjected to a logarithmic transformation since investment theory posits a linear relationship between investment and its determinants. Moreover, some of the determinants of investment contain negative values that can not be logged. Visual inspection of the time series plot of new private industrial and commercial construction output, given in Figure 4.1, reveals a steady increase in the series itself and an increase in the variance of, for example, the seasonal changes. This suggests that a logarithmic transformation of the series may be beneficial to the modelling process. To this end, we estimate an ARIMA model with logged data. In order to maintain consistency with the models developed in Section 5.2, an ARIMA model of the actual data (that is, without applying the logarithmic transformation) is also developed.
5.3.2.2 Determining the Degree of Differencing

The next stage is to determine the degree of differencing, \( d \). In general, if a series is non-stationary, \( d \) will be strictly positive, and this will usually be apparent from the plot of the series. The actual value of \( d \), however, can not easily be determined through visual inspection of the time series, and examination of the sample autocorrelation function (defined in Section 3.3.2.1), for various differences will also be required. A slow decay of the ACF may be taken as an indication of non-stationarity and hence of the need for first differencing. If the original series is found to be non-stationary, the first differenced series is then analysed. The procedure is repeated until a stationary series is obtained, although, in practice, it is rare for \( d \) to exceed 2.

Mills (1990) notes that sole reliance on the ACF can occasionally lead to over differencing. Although further differencing of a stationary series will still be stationary, over differencing can increase the complexities of estimation. Anderson (1976) notes that the variance of an over differenced series will always be larger than that of the correctly differenced process, and this in itself can provide a useful means of deciding the appropriate level of differencing: the sample variances will generally fall until a stationary process is found, but will tend to increase on over differencing.

Of course, unit root tests provide a more formal method of determining the degree of differencing. These tests have been discussed at some length in Section 3.3.1. For private industrial and commercial construction output an array of unit root tests have already been performed and the results were presented in Section 4.4. In summary, private industrial and commercial construction output, \( \Delta K \), was found to be integrated of order one, implying that first differencing was sufficient to ensure stationarity. It was briefly noted in Section 4.4.5 that the tests were also performed on the logged data. Although the results were not presented, it was noted that, in general, taking logarithms of the data had no significant effect on the test results. The results of the unit root tests for actual and logged private industrial and commercial construction output are presented (represented for the actual data) in Tables 5.12 to 5.14 for completeness. To recapitulate, both the actual and logged series are integrated of order one and therefore, the first differences of these series are stationary.
Table 5.12: HEGY tests for seasonal unit roots

<table>
<thead>
<tr>
<th>Series²</th>
<th>Frequency</th>
<th>Zero¹</th>
<th>Biannual¹</th>
<th>Annual¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔK(1)</td>
<td></td>
<td>-2.11</td>
<td>-5.17</td>
<td>20.92</td>
</tr>
<tr>
<td>LΔK(1)</td>
<td></td>
<td>-2.25</td>
<td>-6.3</td>
<td>32.87</td>
</tr>
</tbody>
</table>

Notes:
1. These test results are derived from the model given in equation 3.8 above.
2. The lag length (in parenthesis) on the HEGY regression is determined using the minimum AIC criterion up to a maximum of nine lags. L denotes logged data.

Table 5.13: ADF tests for simple unit roots

<table>
<thead>
<tr>
<th>Series³</th>
<th>Test</th>
<th>10% CV</th>
<th>α₁=0¹</th>
<th>α₁=α₂=0¹</th>
<th>α₁=0 (with α₂=0)²</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔΔK(3)</td>
<td></td>
<td>-3.13</td>
<td>5.34</td>
<td>-2.57</td>
<td></td>
</tr>
<tr>
<td>ΔK(4)</td>
<td></td>
<td>-3.82</td>
<td>7.32</td>
<td>-3.84</td>
<td></td>
</tr>
<tr>
<td>ΔLΔK(3)</td>
<td></td>
<td>-2.06</td>
<td>4.68</td>
<td>-1.91</td>
<td></td>
</tr>
<tr>
<td>LΔK(4)</td>
<td></td>
<td>-4.51</td>
<td>10.37</td>
<td>-4.56</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. Test for a simple unit root using the model given in equation 3.5.
2. Test for a simple unit root using the model given in equation 3.5 but with α₂ constrained to zero (i.e. the no trend case).
3. The lag length (in parenthesis) is determined using the minimum AIC criterion up to a maximum of nine lags. L denotes logged data.

Table 5.14: PP tests for simple unit roots

<table>
<thead>
<tr>
<th>Series³</th>
<th>Test</th>
<th>10% CV</th>
<th>α₁=0¹</th>
<th>α₁=α₂=0¹</th>
<th>α₁=0 (with α₂=0)²</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔΔK(1)</td>
<td></td>
<td>-10.48</td>
<td>54.48</td>
<td>-10.46</td>
<td></td>
</tr>
<tr>
<td>ΔK(1)</td>
<td></td>
<td>-1.75</td>
<td>1.59</td>
<td>-1.39</td>
<td></td>
</tr>
<tr>
<td>ΔLΔK(1)</td>
<td></td>
<td>-11.98</td>
<td>71.79</td>
<td>-11.9</td>
<td></td>
</tr>
<tr>
<td>LΔK(1)</td>
<td></td>
<td>-2.79</td>
<td>5.11</td>
<td>-2.94</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. Test for a simple unit root using the model given in equation 3.5.
2. Test for a simple unit root using the model given in equation 3.5 but with α₂ constrained to zero (i.e. the no trend case).
3. The lag length (in parenthesis) is determined according to the highest significant lag (up to T, which is lag 12 given that T=168) from either the ACF or the PACF of the first differenced series.

The sample variances of the levels, first differenced and second differenced actual and logged data, also suggest that first differencing is appropriate for stationarity. On first differencing the actual data the sample variance decreases from 36721 to 1147.8 before rising again to 2315.7 when the data is first differenced twice. Applying first differences to the logged data results in a lower sample variance: it decreases from...
0.0363 to 0.0013. On the application of another first difference, the logged data the sample variance increases to 0.0025.

The evidence presented in the ACFs of the actual and logged private industrial and commercial construction output data are also consistent with the results of the simple unit root tests. For the levels data the ACF is slow to decay indicating non-stationarity (see Figure 5.1 for the ACF of the actual data and Figure 5.2 for the ACF of the logged data). First differencing the actual and logged series produces a very distinctive pattern in the ACFs with very large positive autocorrelations at the seasonal lags (see Figures 5.3 and 5.4 for the first differences of the actual and logged series, respectively). The slow decline of these seasonal autocorrelations may be indicative of seasonal non-stationarity and is typically dealt with by seasonal differencing (after first differencing). When the first differenced series are seasonally differenced the sample autocorrelation functions show much more interpretable patterns (see Figure 5.5 for the actual data, and Figure 5.6 for the logged data). Appropriate forms for the autoregressive and moving average polynomials can now, at least in principle, be obtained by the usual methods of identification. Unfortunately, two problems are typically encountered. First, as Mills (1990) notes, the PACFs are extremely difficult to interpret, so conventional identification is usually based solely on the behaviour of the ACF. Second, since the autoregressive and moving average polynomials must account for the seasonal autocorrelations, at least one of them must be of at least the same order as the seasonal periodicity (i.e. at least fourth order with quarterly data). This can mean that the number of models which need to be considered in model selection procedures can become prohibitively large. This latter problem does not apply to the private industrial and commercial construction output data in this exercise since the ACF of the seasonally differenced actual and logged series (given in Figures 5.5 and 5.6) indicate a maximum order of 4 to one, or both, of the AR of MA polynomials. This implies a maximum of twenty possible specifications (ARMA(0,0) to ARMA(4,4), or combinations thereof) which is not a prohibitively large number.

Although the ACFs suggest that seasonally differencing the first differenced data is appropriate for both the actual and logged series, the suggestion is inconsistent with the results of the seasonal unit root tests presented in Section 4.4 and in Table 5.12.
Figure 5.1: The ACF for private industrial and commercial construction output (actual levels data)

Figure 5.2: The ACF for private sector industrial and commercial construction output (log of levels data)
Figure 5.3: The ACF for private industrial and commercial construction output (actual first differenced data)

Figure 5.4: The ACF for private industrial and commercial construction output (log of first differences)
Figure 5.5: The ACF for private industrial and commercial construction output
(actual data first and seasonally differenced)

Figure 5.6: The ACF for private industrial and commercial construction output
(logged data first and seasonally differenced)
Figure 5.7: The PACF for private industrial and commercial construction output (actual first differenced data)

Figure 5.8: The PACF for private industrial and commercial construction output (log of first differences)
above. The seasonal unit root tests clearly reject the hypothesis of seasonal non-stationarity and therefore suggest that seasonal differencing of the series is inappropriate. This implies the ACFs and PACFs of the first differenced series (given in Figures 5.3 and 5.7 for the actual data and Figures 5.4 and 5.8 for the logged data) should be used to determine the order of the autoregressive and moving average polynomials. Casual inspection of the ACFs and PACFs suggests that interpreting these functions may not be at all straightforward.

Since the results of the unit root tests conflict with the evidence presented in the ACFs, the appropriate degree of differencing can not be determined sensibly at this stage. Therefore, we proceed with two alternative degrees of differencing (simple first differences and seasonally differenced first differences) which will necessarily result in alternative models, discrimination between which must be made on other criteria.

5.3.3 Determining the Orders of the AR and MA Polynomials

Once the order of differencing has been determined the next step in the development of an ARIMA model is to specify the orders of the autoregressive and moving average polynomials. In the traditional Box-Jenkins approach this is done by matching the patterns in the sample autocorrelations and partial autocorrelations with the theoretical patterns of known models. We start by attempting to determine \( p \) and \( q \) using this approach. The order of \( p \) and \( q \) are typically small. Given that models are to be developed for actual and logged data and that the degree of differencing has not as yet been determined unambiguously, there are four starting points under consideration.

5.3.3.1 Interpreting the ACFs

We begin by determining \( p \) and \( q \) for the first and seasonally differenced (actual and logged) data. As noted above, interpreting the PACF when data are seasonally differenced is extremely difficult and therefore, researchers tend to rely solely on the ACF. The ACFs of the first differenced and seasonally differenced first differenced data are given in Figures 5.5 and 5.6 for actual and logged data, respectively. In both figures the only significant autocorrelations are at lag 4. In itself this might be
suggestive of an MA(4) process. However, both figures exhibit something of a damped sine wave, which might indicate an autoregressive process. It is not possible to draw firmer conclusions as to the order of $p$ and $q$ without more formal testing and the use of model selection criteria.

When the private industrial and commercial construction output series (actual or logged) is simply first differenced, determining the order of $p$ and $q$ is equally difficult. The ACFs for the first differences of private industrial and commercial construction output are given in Figures 5.3 and 5.4 for actual and logged data, respectively. For both the actual and logged data there are significant negative autocorrelations at lags 2, 6, 10 etc. and significant positive autocorrelations at lags 4, 8, 12 etc. These latter spikes are picking up the seasonal cycle inherent in the data and suggest that an AR(4) process might be entertained. The PACFs shown in Figures 5.7 and 5.8 for actual and logged data, respectively, are dominated by large partial autocorrelations at lag 4. There is also some evidence of sine wave decay in the PACFs suggesting that the generating process may contain an MA component. Again, it is not easy to draw firmer conclusions as to the appropriate order of $p$ and $q$ without more formal testing and the use of model selection criteria.

5.3.3.2 The Use of Model Selection Criteria

A number of model selection criteria have been developed which were originally designed for autoregressive processes, but which have more recently been extended to the more general class of ARIMA process. Three popular criteria are those posited by Akaike (1974), Schwarz (1978) and Hannan and Quinn (1979). All these criteria are structured in terms of the estimated variance plus a penalty adjustment involving the number of estimated parameters. It is in the extent of the penalty that the criteria differ. A maximum order for $p$ and $q$, say $\bar{p}$ and $\bar{q}$, is selected (perhaps on the basis of the ACF and PACF and bearing in mind the number of degrees of freedom) and the value of the model selection criteria is calculated for all $p$ and $q$ up to $\bar{p}$ and $\bar{q}$. The preferred model, denoted $(p_1,q_1)$, is given by the values of $p$ and $q$ that minimise the model selection criteria. In this work we use the AIC (see Akaike (1974)) to determine the order of the ARIMA process.
On the basis of the ACF of the seasonally differenced first differenced actual and logged private industrial and commercial construction output data, a maximum order for $p$ and $q$ was chosen to be 4. Table 5.15 shows the AIC values for different orders $p$ and $q$ up to the maximum of $p = q = 4$.

Table 5.15: AICs for actual and logged first and seasonally differenced data

(i) Actual data

<table>
<thead>
<tr>
<th>$p$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
</table>

(ii) Logged data

<table>
<thead>
<tr>
<th>$p$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5.842</td>
<td>-5.829</td>
<td>-5.812</td>
<td>-5.813</td>
<td>-6.194</td>
</tr>
<tr>
<td>1</td>
<td>-5.828</td>
<td>-5.929</td>
<td>-5.883</td>
<td>-5.834</td>
<td>-6.187</td>
</tr>
<tr>
<td>2</td>
<td>-5.811</td>
<td>-5.914</td>
<td>-5.894</td>
<td>-5.874</td>
<td>-6.161</td>
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<tr>
<td>3</td>
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<td>-5.909</td>
<td>-5.853</td>
<td>-6.128</td>
<td>-6.168</td>
</tr>
<tr>
<td>4</td>
<td>-5.911</td>
<td>-5.933</td>
<td>-5.946</td>
<td>-6.093</td>
<td>-6.202</td>
</tr>
</tbody>
</table>

Table 5.16: AICs for actual and logged first differenced data

(i) Actual data

<table>
<thead>
<tr>
<th>$p$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>

(ii) Logged data

<table>
<thead>
<tr>
<th>$p$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-5.693</td>
<td>-5.668</td>
<td>-5.682</td>
<td>-5.709</td>
<td>-5.888</td>
<td>-5.868</td>
<td>-5.877</td>
</tr>
<tr>
<td>1</td>
<td>-5.666</td>
<td>-5.658</td>
<td>-5.664</td>
<td>-5.712</td>
<td>-5.864</td>
<td>-5.857</td>
<td>-5.859</td>
</tr>
<tr>
<td>2</td>
<td>-5.721</td>
<td>-5.703</td>
<td>-6.088</td>
<td>-6.053</td>
<td>-6.064</td>
<td>-6.051</td>
<td>-6.102</td>
</tr>
<tr>
<td>3</td>
<td>-5.703</td>
<td>-5.749</td>
<td>-6.053</td>
<td>-6.051</td>
<td>-6.157</td>
<td>-6.031</td>
<td>-6.085</td>
</tr>
<tr>
<td>4</td>
<td>-6.026</td>
<td>-6.018</td>
<td>-6.076</td>
<td>-6.185</td>
<td>-6.197</td>
<td>-6.178</td>
<td>-6.178</td>
</tr>
<tr>
<td>5</td>
<td>-6.017</td>
<td>-6.062</td>
<td>-6.052</td>
<td>-5.975</td>
<td>-6.163</td>
<td>-6.159</td>
<td>-6.164</td>
</tr>
<tr>
<td>6</td>
<td>-6.002</td>
<td>-6.032</td>
<td>-6.005</td>
<td>-6.03</td>
<td>-5.988</td>
<td>-6.007</td>
<td>-6.188</td>
</tr>
</tbody>
</table>
For the actual data, the AIC selects the orders (3,4), i.e. an SARIMA4(3,1,4)(0,1,0) model. For the logged data, the AIC selects orders (4,4), i.e. an SARIMA4(4,1,4)(0,1,0) model. The ACFs and PACFs for the first differenced actual and logged data are shown in Figures 5.3, 5.4, 5.7 and 5.8. $\hat{p}$ and $\hat{q}$ are chosen to be 6. This choice is based partly on the ACFs and PACFs whilst being mindful that $\hat{p}$ and $\hat{q}$ should be sufficiently large for the range of models to contain the true model (which may not necessarily be the same as the orders chosen by the criterion under consideration). Table 5.16 shows, for the first differenced data, the AIC values for different orders $p$ and $q$ up to the maximum of $\hat{p}=\hat{q}=6$. For the actual data, the AIC selects an ARIMA(3,1,4) model. For the logged data, the ARIMA(4,1,4) model minimises the AIC.

5.3.3.3 Developing Model Portfolios

Examination of Table 5.15 reveals that there are a number of models that are ‘close to’ the selected model in terms of their criterion value. Poskitt and Tremayne (1987) introduce the idea of a model portfolio, with models being compared with the selected specification by way of the ‘posterior odds ratio’ defined, using the AIC for example, as

$$
\Psi = \exp\left(-\frac{1}{2}T \{\text{AIC}(p_1,q_1) - \text{AIC}(p,q)\}\right)
$$

(5.22)

Although $\Psi$ has no physical meaning, its value may be used to ‘grade the decisiveness of evidence’ against a particular model. Poskitt and Treymayne (1987) suggest that a value of $\Psi$ less than 10 would not be sufficient to warrant discarding the model in favour of that chosen through the criterion minimising procedure, while any $(p,q)$ satisfying $1<\Psi<10^5$ may be thought of as a close competitor to $(p_1,q_1)$. The set of models closely competing with $(p_1,q_1)$ would then be taken as the ‘model portfolio’.

Taking $10^5$ as an approximate upper bound leads to a model portfolio for first and seasonally differenced actual data which contains four specifications; (0,4), (2,4), (3,4) and (4,4). Any one of these models should provide an adequate description of the series. With logged data the model portfolio consists of just two specifications; (0,4) and (4,4).
Model portfolios for simple first differences of private industrial and commercial construction output can be similarly developed. Recall, for the actual data, the AIC selects the (5,5) model. With an upper bound of $10^{14}$, the posterior odds ratio does not admit any other specification into the model portfolio. This suggests that no other model closely competes with the (5,5) model. For logged data, the AIC selects the (4,4) model. The posterior odds ratio admits the (4,3) and (6,6) specifications to the model portfolio. Therefore, this model portfolio consists of three specifications; (4,3), (5,5) and (6,6) each of which should provide an adequate description of first differenced logged data.

In summary, the use of the AIC, when extended to incorporate a portfolio of competing specifications, has led to a number of alternative models for each of the four data transformations of private industrial and commercial construction output considered here. When the data is first and seasonally differenced we have two portfolios: one for the actual data and one for the logged data. For the actual data, the model portfolio consists of four specifications, the (0,4), (2,4), (3,4), and (4,4) models. With logged data, the model portfolio consists of only two specifications, the (0,4) and (4,4) models. Two more portfolios are developed for the first differenced data. For actual data, the model portfolio contains only one specification, namely the (5,5) model. When the data is logged, the model portfolio contains three specifications, the (4,3), (4,4) and (6,6) models. All of these models should provide an adequate explanation of (actual or logged) private industrial and commercial construction output. Fortunately, the analyst has available further tools with which to discriminate between alternative specifications. These tools fall within the domain of diagnostic testing.

5.3.4 Diagnostic Testing

Box, Hillmer and Tiao (1978) argue that the purpose of model building is to transform a series to a structureless white noise process. Hence, a check on whether a particular model is adequate or not revolves around ascertaining whether the calculated residuals approximate a white noise process. This implies that the mean of the residuals should be close to zero, the variance of the residuals should be approximately constant, and
the autocorrelations of the residuals should be negligible. To check whether the mean of the residuals is zero, the sample mean can be compared with its standard error. A plot of the residuals against time is usually sufficient to check whether the variance is constant.

There are a number of ways of checking whether the residuals are uncorrelated. Firstly, one may calculate the sample autocorrelation of the residuals and these can be compared with their standard errors. However, standard errors are difficult to calculate because they depend on the form of the fitted model, the true parameter value and the lag, \( k \). Therefore, much attention is focused on the development of ‘portmanteau’ tests. The original test, developed by Box and Pierce (1970), has been criticised by Prothero and Wallis (1976) among others, on the grounds that significant values for this statistic are rarely observed and consequently, the statistic is of little use for discriminating between models. An improved test has been developed by Ljung and Box (1978) which is based on a modified version of the Box-Pierce portmanteau statistic. The modified portmanteau test statistic is

\[
Q = T(T + 2) \sum_{k=1}^{\mu} (T - k)^{-1} \hat{\rho}_k^2
\]  

(5.23)

The choice of \( \mu \) is somewhat arbitrary but a rule of thumb suggests choosing \( \mu = T^{1/4} \) (see Poskitt and Tremayne (1981)). If the calculated \( Q \) statistic exceeds the tabulated \( \chi^2 \) value with \( \mu - p - q \) degrees of freedom the adequacy of the fitted model must be questioned. Note that if the model contains a constant, the degrees of freedom are reduced by one. It is this modified portmanteau statistic, sometimes referred to as the Ljung-Box-Pierce statistic, that is used here to check whether the residuals are uncorrelated. The test statistics are given in Table 5.17 along with a selection of other diagnostics.

The model portfolio for first and seasonally differenced actual private industrial and commercial construction output contained four specifications. For each of the four models, the mean of the residuals is insignificantly different from zero. Time series plots of the residuals (not presented here) suggest that the residual variances tend to increase very slightly over time. The principal difference between the four models is in the degree of residual autocorrelation. The Ljung-Box-Pierce test reveals that the
(2,4) model provides an inadequate description of the data (the hypothesis of no residual serial correlation is rejected since the test statistic of 14.74 with 6 degrees of freedom is greater than the corresponding 5% critical value of 12.6). In any case, there is evidence that this model is over parameterised: the MA(1), MA(2) and MA(3) parameters are insignificant (see Table 5.18). Although, the (3,4) and (4,4) specifications generate test statistics that suggest that the null hypothesis of no residual serial correlation can not be rejected at the 5% level (the respective test statistics are 10.12, which must be compared with the $\chi^2_3$ critical value of 11.1, and 8.81, which must be compared with the $\chi^2_3$ critical value of 9.49), the models are still ruled out as possible descriptions of the data. Recall, $m$ is chosen arbitrarily (using the rule of thumb $\mu=T/2$). The degrees of freedom (calculated as $\mu-p-q-1$) associated with these statistics are obviously sensitive to the choice of $\mu$. For the (3,4) and (4,4) specifications, the Ljung-Box-Pierce test statistics for values of $\mu$ close to but not equal to 13 ($T_4=168/4$) suggest substantial residual serial correlation.

As with the (2,4) specification, the (3,4) and (4,4) specifications also appear to be over parameterised (see Table 5.18). This leaves the (0,4) specification. The Ljung-Box-Pierce statistics suggest that the model provides an adequate description of the data (the statistic of 4.47 is well below the $\chi^2_8$ critical value of 15.5). Moreover, the residual ACF and PACF exhibit no significant autocorrelations and all of the MA parameters are significant at the 5% level. The specification can explain 30.47% of the variation in the first and seasonally differenced data. Having rejected the (2,4), (3,4) and (4,4) specifications as possible descriptions of the data, the model portfolio is reduced to a single preferred model, namely the (0,4) specification.

The model portfolio for first and seasonally differenced logged private industrial and commercial construction output contained two specifications. The mean of the residuals from both models are insignificantly different from zero and the residual variances are constant. Although the residuals from the (4,4) specification appear to be free from serial correlation, the model is rejected as a satisfactory description of the data on the grounds that the model is over parameterised: the AR parameters are insignificant. It is noteworthy that the moving average component is noninvertible. The (0,4) specification provides an adequate explanation of the data. The Ljung-Box-Pierce statistic is 10.98 (the $\chi^2_8$ critical value is 15.5) and the ACF and

343
PACF do not contain significant autocorrelations. Three of the four MA parameters are strongly significant and the specification can explain 33.54% of the variation in the first and seasonally differenced series (see Table 5.18).

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Specification</th>
<th>Residual Mean</th>
<th>Residual Variance</th>
<th>Q(df)</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta \Delta \Delta K)</td>
<td>(0,4)</td>
<td>-0.11981</td>
<td>7454.9</td>
<td>13.87(8)</td>
<td>0.3047</td>
</tr>
<tr>
<td></td>
<td>(2,4)</td>
<td>3.08730</td>
<td>7130.0</td>
<td>14.74(6)</td>
<td>0.3241</td>
</tr>
<tr>
<td></td>
<td>(3,4)</td>
<td>2.90050</td>
<td>6980.2</td>
<td>10.12(5)</td>
<td>0.3340</td>
</tr>
<tr>
<td></td>
<td>(4,4)</td>
<td>2.82610</td>
<td>6916.6</td>
<td>8.81(4)</td>
<td>0.3356</td>
</tr>
<tr>
<td>(\Delta \Delta \Delta L\Delta K)</td>
<td>(0,4)</td>
<td>-0.00042</td>
<td>0.00182</td>
<td>10.98(8)</td>
<td>0.3354</td>
</tr>
<tr>
<td></td>
<td>(4,4)</td>
<td>0.00022</td>
<td>0.00167</td>
<td>5.10(4)</td>
<td>0.3667</td>
</tr>
<tr>
<td>(\Delta \Delta K)</td>
<td>(5,5)</td>
<td>0.51939</td>
<td>6359.1</td>
<td>7.72(2)</td>
<td>0.4992</td>
</tr>
<tr>
<td>(\Delta L\Delta K)</td>
<td>(4,3)</td>
<td>-0.00015</td>
<td>0.00178</td>
<td>12.83(5)</td>
<td>0.4396</td>
</tr>
<tr>
<td></td>
<td>(4,4)</td>
<td>-0.00054</td>
<td>0.00173</td>
<td>9.35(4)</td>
<td>0.4530</td>
</tr>
<tr>
<td></td>
<td>(6,6)</td>
<td>-0.00045</td>
<td>0.00162</td>
<td>8.32(1)</td>
<td>0.4696</td>
</tr>
</tbody>
</table>

The number of specifications in the portfolios corresponding to first differenced private industrial and commercial construction output can also be reduced. In the case of the actual data, the model portfolio consisted of only one specification, namely the (5,5) specification. The mean of the residuals from this specification are insignificantly different from zero. However, the variance of the series appears to increase slightly over time and, despite the lack of significant autocorrelations in the ACF and PACF of the residuals, the Ljung-Box-Pierce statistic rejects the null
hypothesis of uncorrelated residuals (the statistic is 7.72 and the $\chi^2$ critical value is 5.99). The lack of significant autocorrelations makes it difficult to see how this model could be respecified. It is noteworthy that for alternative values of $\mu$, the modified portmanteau statistic suggests an absence of serial correlation. Since the model portfolio contains no competing models, the (5,5) specification is retained at this stage.

The model portfolio for the first changes in the logged private industrial and commercial construction output series contains three specifications. Although the mean of the residuals is insignificantly different from zero and the residual variance appears to be constant for the (4,3) and (6,6) specifications, these models are rejected as adequate explanations of the series due to their poor residual autocorrelation properties (the respective Ljung-Box-Pierce statistics are 12.83, which should be compared with $\chi^2$ critical value of 11.1, and 8.32, which should be compared with a $\chi^2$ critical value of 3.84). The third specification, the (4,4) also displays a zero mean and constant variance and (marginally) passes the Ljung-Box-Pierce test for uncorrelated errors (see Table 5.17). The ACF and PACF of the residuals (not shown here) exhibit no significant autocorrelations and the model is able to explain 45.3% of the variation in the first differenced series.

The diagnostic checking stage of the ARIMA modelling process has eliminated all except one specification from each of the four model portfolios. The preferred specification for the first and seasonally differenced actual private industrial and commercial construction output series is the (0,4) specification, i.e. a SARIMA$_4(0,1,4)(0,1,0)$. The same specification is preferred with the logged data. With simple first differences of the actual private industrial and commercial construction output data, a (5,5) specification is preferred (albeit by default and with some evidence of residual serial correlation), i.e. an ARIMA$(5,1,5)$. A (4,4) specification, i.e. an ARIMA$(4,1,4)$, is preferred when the data is logged. On the whole, the specifications using logged data are preferred to specifications using actual data on the grounds that the residual variances generated by the latter models tend to increase over time. This, of course, is a common problem with economic data. There is not much to choose between the two logged models (the SARIMA$_4(0,1,4)(0,1,0)$ specification and the ARIMA$(4,1,4)$ specification). Discrimination between these
models must be governed by beliefs regarding the presence of seasonal non-stationarity in the private industrial and commercial construction output series. Recall, the results of the seasonal unit root tests presented in Table 5.12 suggest that there is no evidence of seasonal non-stationarity in the private industrial and commercial construction output series. However, the slowly declining spikes at seasonal lags in the ACF of the first differenced data provide evidence to the contrary. Given that the autocorrelations at the seasonal frequencies slowly decline from about 0.54 rather than from a number closer to unity, the evidence in favour of seasonal non-stationarity provided by the ACF is perhaps weaker than the evidence against seasonal non-stationarity provided by the unit root tests. For this reason, one is led to the conclusion of no seasonal non-stationarity. In this case, simple first differences are sufficient to ensure stationarity, and thus the ARIMA(4,1,4) specification provides a rather more satisfactory description of the data than the SARIMA(0,1,4)(0,1,0) specification.

5.3.5 Concluding Remarks on the ARIMA Model Estimation

A number of ARIMA models of private industrial and commercial construction output have been developed by combining the traditional approach to modelling (that is, the matching of patterns in the ACF and PACF with the theoretical patterns of known models) and a more rule-based approach. When using economic data it is always wise to consider taking logarithms to stabilise the variance of the series. Therefore, some of the ARIMA models developed in this section have been estimated with logged data. However, the models in Section 5.2 were developed without transforming data into logs. To be consistent, other models developed in this section have been estimated without logging the data first.

Since the unit root tests and the ACF of the first differenced (actual and logged) series provide conflicting evidence as to the degree of differencing necessary to achieve stationarity, models were developed for two degrees of differencing. The unit root tests suggest that first differencing was sufficient for stationarity, whilst evidence in the ACF of the first differenced data suggested that the series was seasonally non-stationary and therefore required additional seasonal differencing. Therefore,
model building proceeded with four different transformations of the private industrial and commercial construction output series.

Tentative guesses at possible orders of integration were made by examining the ACFs and the PACFs. By using the AIC and the posterior odds ratio, a model portfolio for each of the transformations of private industrial and commercial construction output was developed. By checking the diagnostics of each of the specifications, the number of competing models within a portfolio was reduced. In fact, after checking the diagnostics it was possible to rule out all but one of the specifications from each of the model portfolios, to give a single preferred model for each data transformation. When the actual private industrial and commercial construction output series was first and seasonally differenced the preferred model was an MA(4) process. Applying the same filters to the logged data also yielded an MA(4) model. When the data was simply first differenced the preferred models for actual and logged data were respectively the (5,5) and the (4,4) specifications. Given that the variances of the residuals from models developed with the actual data were not stable through time, preference was given to the models developed with logged data. Discrimination between the two logged models must be based on beliefs regarding the nature of the non-stationarity in the series. More formal discrimination based on the relative forecast performance of each model is conducted in Chapter 6.

5.4 Estimation of VAR Models

5.4.1 Introduction

In this section a number of models of private industrial and commercial construction output are estimated using the procedure posited by Johansen (1988), a full description of which is provided in Section 3.4.4. We begin by using this procedure to estimate four VAR models that are broadly consistent with the four theories of investment behaviour described in Chapter 2, namely the accelerator model, the neoclassical model, the putty-clay model and the $Q$ model. After some discussion of the properties of these models, four further specifications are estimated in Section
5.4.3 which do not exactly correspond to any particular theory, but are intuitively sensible.

The first is an adaptation of the accelerator model which makes use of capacity utilisation data. The basic accelerator suggests that an increase in the rate of change of output results in net investment. This, of course, suggests that firms are operating at full capacity and must therefore invest in order to meet the change in demand. In reality of course, firms do not operate at full capacity all of the time. Including a capacity utilisation variable in the accelerator type system results in a more realistic model. The second and third specifications estimated in Section 5.4.3 are modifications to the neoclassical models (pure and putty-clay, respectively) in which the assumption of perfect capital markets is relaxed. The final specification considered here evolves from the fact that, in many studies, an output variable is a significant regressor in $Q$ type investment equations. Therefore, the basic $Q$ type VAR is augmented with an output variable.

The fact that VARs are regarded as an atheoretical approach to modelling (see Sims (1980) for a discussion), provides the analyst with the opportunity to develop models that are not strictly consistent with any one single theoretical construct. This is particularly useful when the model is ultimately to be used for forecasting. It is often possible to develop a model, which is not strictly consistent with theory, but which performs significantly better than its theoretically consistent counterpart in forecasting contests. With this in mind, we develop a number of models which contain variables that are not suggested by a rigorous derivation of the four dominant theories of investment, but that may contain valuable forecasting information. These atheoretical models are developed in Section 5.4.4.

5.4.2 VAR Specifications Suggested by Investment Theory

In this section the four theories of investment are modelled in VAR form. Consider the VAR presented in equation 3.29 in Section 3.4.4 which is rewritten below for convenience.

\[
X_t = \Pi_1 X_{t-1} + \ldots + \Pi_k X_{t-k} + \mu + \phi D_t + U_t
\]  
(5.24)

$\mu$ denotes a constant and $D_t$ is a vector of centred seasonal dummies. The variables
appearing in vector $X_t$ are chosen according to theory. Since the accelerator posits a relationship between real investment and real output, the vector $X_t$ will contain a real investment and a real output variable. In addition to real output, the neoclassical theory stresses the importance of the user cost of capital. In Jorgenson’s pure version of the model, output and user cost enter the equation in a single composite term which measures the firm’s optimal stock of capital. In this case, $X_t$ will contain a real investment variable and a variable measuring optimal capital stock. In the putty-clay version of the model investment is responds to changes in output and user cost at differing rates. To account for this output and user cost variables are entered into traditional specifications separately. In the corresponding VAR formulation to be estimated here the vector $X_t$ contains three variables, namely, real investment, real output and the real user cost of capital. According to $Q$ theory the rate of investment can be fully explained by $Q$. Therefore, $X_t$ in the $Q$-type VAR consists of a rate of investment variable and a $Q$ variable. We consider the VAR specifications corresponding to these theories in the subsequent paragraphs.

The first task of any application of the Johansen procedure is to determine the lag length $k$ of the VAR. The lag length should be long enough to ensure normality and no serial correlation in the error term, $U_t$. At the same time it is essential that consideration be given to the number of degrees of freedom lost in estimating the number of parameters. In a system of five equations an increase in the lag length $k$ to $k+1$ will necessitate the estimation of five more parameters. Thus, it can be seen that the degrees of freedom are quickly lost. In practical terms, the lag length is determined by estimating the equations separately using OLS and testing the error on each equation. The lag length $k$, should be as long as is required for all errors to be uncorrelated and normally distributed. Thus, there is a trade-off between the properties of the residuals and the parsimony of the model.

Since we are using seasonally unadjusted data (as well as data thought to contain no seasonal component), it would seem sensible to set the lag length to $k=4$ initially. For the $Q$-type model this was sufficient to eliminate serial correlation. However, the output equations in the other systems required a lag length of $k=5$ to eliminate serial correlation. Thus, the accelerator, the neoclassical and the putty-clay systems were each estimated with five lags of each variable. The residual statistics for the
accelerator, neoclassical, putty-clay and Q-type VARs are respectively given in Tables 5.19 to 5.22. A 5% level of significance is adopted for these tests.

Table 5.19: Tests of normality and autocorrelation in an accelerator type VAR: lag length k=5.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi^2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta K$</td>
<td>-0.067(0.190)</td>
<td>0.359 (0.378)</td>
<td>0.77</td>
<td>14.03</td>
</tr>
<tr>
<td>$LY$</td>
<td>-0.925(0.190)</td>
<td>5.577 (0.378)</td>
<td>218.76</td>
<td>19.09</td>
</tr>
</tbody>
</table>

Note: CV denotes the test's critical value. Standard errors are given in parenthesis.

Table 5.20: Tests of normality and autocorrelation in a neoclassical type VAR: lag length k=5.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi^2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta K$</td>
<td>0.016(0.190)</td>
<td>0.164 (0.378)</td>
<td>0.11</td>
<td>12.33</td>
</tr>
<tr>
<td>$LOK$</td>
<td>-1.436(0.190)</td>
<td>30.805 (0.378)</td>
<td>6,099.04</td>
<td>8.83</td>
</tr>
</tbody>
</table>

Note: CV denotes the test's critical value. Standard errors are given in parenthesis. LOK denotes the logarithm of optimal capital stock defined as the ratio of output to the user cost of capital.

Table 5.21: Tests of normality and autocorrelation in a putty-clay type VAR: lag length k=5.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi^2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta K$</td>
<td>-0.092(0.190)</td>
<td>0.357 (0.378)</td>
<td>0.87</td>
<td>14.62</td>
</tr>
<tr>
<td>$LY$</td>
<td>-0.840(0.190)</td>
<td>5.489 (0.378)</td>
<td>208.51</td>
<td>19.54</td>
</tr>
<tr>
<td>$LC$</td>
<td>-0.169(0.190)</td>
<td>26.340 (0.378)</td>
<td>4,417.92</td>
<td>8.44</td>
</tr>
</tbody>
</table>

Note: CV denotes the test's critical value. Standard errors are given in parenthesis.

Table 5.22: Tests of normality and autocorrelation in a $Q$ type VAR: lag length k=4.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi^2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta K/K$</td>
<td>0.143(0.191)</td>
<td>0.562(0.380)</td>
<td>40.68</td>
<td>16.99</td>
</tr>
<tr>
<td>$Q$</td>
<td>0.720(0.191)</td>
<td>4.721(0.380)</td>
<td>151.85</td>
<td>24.78</td>
</tr>
</tbody>
</table>

Note: CV denotes the test's critical value. Standard errors are given in parenthesis.

The LM statistics corresponding to each equation indicate a lack of serial correlation. For all equations the test statistics are well below the $\chi^2_{23}$ critical values and thus, the null hypotheses of no serial correlation can not be rejected. Box-Pierce statistics and $t$-tests on lagged residuals (neither of which are reported here) are also suggestive of an absence of serial correlation.
Although the residuals are not serially correlated, they are non-normal in a number of equations. The null hypothesis in the Jarque-Bera test (see Jarque and Bera (1980)) is that of normality. The null hypothesis is clearly and consistently rejected in the residuals from equations for output, optimal capital stock and $Q$. The source of the non-normality appears to be due to both skewness and excess kurtosis. Moreover, the clear rejection can not be overcome by altering the lag length $k$. Although the Johansen maximum likelihood procedure assumes white noise normally distributed errors, the non-normality of errors in a number of equations is assumed not to seriously affect the performance of the estimation procedure. If the models were not to be used to generate forecasts, it may have been possible to drop one or more of these equations at a later stage by testing variables for exogeneity. This procedure is illustrated later in this subsection.

### Table 5.23: Determination of the number of cointegrating vectors in the accelerator type VAR

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic</th>
<th>Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda$-max</td>
<td>CV=0.95</td>
</tr>
<tr>
<td>$\rho = 0$</td>
<td>8.229</td>
<td>14.069</td>
</tr>
<tr>
<td>$\rho \leq 1$</td>
<td>0.146</td>
<td>3.762</td>
</tr>
</tbody>
</table>

### Table 5.24: Determination of the number of cointegrating vectors in the neoclassical type VAR

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic</th>
<th>Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda$-max</td>
<td>CV=0.95</td>
</tr>
<tr>
<td>$\rho = 0$</td>
<td>13.245</td>
<td>14.069</td>
</tr>
<tr>
<td>$\rho \leq 1$</td>
<td>3.881</td>
<td>3.762</td>
</tr>
</tbody>
</table>

### Table 5.25: Determination of the number of cointegrating vectors in the putty-clay type VAR

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic</th>
<th>Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda$-max</td>
<td>CV=0.95</td>
</tr>
<tr>
<td>$\rho = 0$</td>
<td>13.553</td>
<td>20.967</td>
</tr>
<tr>
<td>$\rho \leq 1$</td>
<td>8.302</td>
<td>14.069</td>
</tr>
<tr>
<td>$\rho \leq 2$</td>
<td>0.285</td>
<td>3.762</td>
</tr>
</tbody>
</table>

### Table 5.26: Determination of the number of cointegrating vectors in the $Q$ type VAR

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic</th>
<th>Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda$-max</td>
<td>CV=0.95</td>
</tr>
<tr>
<td>$\rho = 0$</td>
<td>4.461</td>
<td>14.069</td>
</tr>
<tr>
<td>$\rho \leq 1$</td>
<td>1.042</td>
<td>3.762</td>
</tr>
</tbody>
</table>
The test of the rank of \( \Pi \) (in equation 5.24) is performed using the likelihood ratio tests given in equations 3.60 and 3.61 in Section 3.4.4. Test results corresponding to the four models of investment are given in Tables 5.23 to 5.26, along with their respective critical values. The 10% critical values are given in addition to the 5% values since the tests have low power. For the accelerator type VAR, the null hypothesis of no cointegrating vectors (i.e. that the rank of \( \Pi=0 \)) can not be rejected: both the maximal eigenvalue and the trace statistics (which are respectively 8.229 and 8.375) are well below the critical values at either level of significance.

Similarly, the null hypothesis of no cointegrating vectors can not be rejected in the putty-clay and \( Q \) type VARs. These findings should be of no surprise given the results of cointegration tests conducted in Section 5.2. In Section 5.2, the data was examined for cointegrating relationships between net investment and the level of optimal capital stock (as measured by the ratio of real output to the real user cost of capital) and between net investment and \( Q \) (see Tables 5.4 and 5.7). The augmented Dickey-Fuller and Phillips-Perron tests found no evidence of long run relationships. As a result, the putty-clay and \( Q \) models were estimated in first differences. The same conclusion is reached with the Johansen tests performed here: for both the putty-clay (in which unlike in Section 5.2, the real output and real user cost are entered into the system as separate variables) and \( Q \) systems there is no evidence of cointegration between net investment and its determinants. Recall from the discussion of the accelerator in Section 5.2.1.1, the accelerator equation suggested by theory was unbalanced: net investment is I(1), whereas changes in output were I(0). Since it is not possible for an I(1) and an I(0) variable to be cointegrated, the Dickey-Fuller and Phillips-Perron tests were not carried out on investment and changes in output in Section 5.2. Direct comparisons between the estimation results on the accelerator in Section 5.2 and those presented in this section are not possible, since here we examine the relationship between net investment and the level of output, whereas in Section 5.2 we examine the relationship between net investment and first differences in output. In any case, since the rank of \( \Pi \) in equation 5.24 is found to be zero in the accelerator, putty-clay and \( Q \) type systems, each should be estimated in first differences.

In Section 5.2.2.2 for the pure neoclassical model, cointegration tests were not performed on net investment and the changes in optimal capital stock. This was
because these two variables have differing orders of integration. This rather odd result follows from the definition of optimal capital stock which, under the assumption of Cobb-Douglas technology, is given by the ratio of output to user cost, both of which were found to be I(1). The results of the Johansen tests (presented in Table 5.24) are also rather odd. The trace statistic rejects the null hypothesis of no cointegration (i.e. that the rank of $\Pi=0$ in equation 5.24) at both levels of significance. Moreover, the null of one cointegrating relation is also rejected. On the basis of this test alone one must conclude that $\Pi$ is of full rank, which implies that both net investment and optimal capital stock are stationary, contrary to the results of the unit root tests conducted in Section 4.4. The results of the maximal eigenvalue test are somewhat less clear cut. The test of the null of no cointegration yields a statistic of 13.245. Thus the null can not be rejected at the 95% level. However, the 90% critical value is exceeded and therefore suggests that the null should be rejected. If one takes the decision to reject the initial null, the maximal eigenvalue statistic indicates the presence of a second cointegrating relation. Given the test’s low power it may be more appropriate to use the 90% significance level. In this case, both tests imply that net investment and the user cost of capital are I(0) and that the system should be estimated in levels.

The plausibility of this result can be analysed by examining plots of the residuals of the cointegrating relations. These are generated as

$$CR = \hat{\beta}^t R_{kt}$$

(5.25)

where $\hat{\beta}$ is the matrix of eigenvectors and $R_{kt}$ is a matrix containing all the variables in the VAR, which in this case is just net investment and optimal capital stock. Given the premise of two cointegrating relations we would expect the residuals from both eigenvectors to appear stationary. The plots of the residuals of the cointegrating relations, corrected for short-run dynamics, are presented in Figures 5.9 and 5.10. Neither the residuals from the first or second relation appear to be stationary. Thus, the graphical evidence appears to contradict the results of the Johansen tests. To include a non-stationary eigenvector in a difference equation will lead to an unbalanced regression. The exclusion of a relevant variable on the other hand, implies the well known omitted variables problem. On the grounds that it is better to include
Figure 5.9: Residuals of cointegrating vector 1 (corrected for short-run dynamics) from the basic neoclassical model.

Figure 5.10: Residuals of cointegrating vector 2 (corrected for short-run dynamics) from the basic neoclassical model.
irrelevant variables than to exclude relevant ones, one might be tempted to conclude that the system should be estimated in levels, but this is inconsistent with earlier analysis and seems somewhat implausible. It is easy to see why the Johansen test suggests that the optimal capital stock is I(0). The outlying observations in this series in the mid 1970s give the appearance of strong mean reverting behaviour in the time series plots (see Figure 4.39). Recall, these outliers are caused by the fact that the measured user cost of capital is very close to zero at this time. However, both output and user cost were found to be I(1), and thus, it would seem likely that optimal capital stock is I(1) but for these outlying observations. Indeed, that actual capital stock was found to be I(2), a priori one would expect optimal capital stock to be I(2) also. It is not easy to see why net investment has been found to be I(0) here. Evidence in Section 4.4 comprehensively pointed towards this series being I(1). Given the inconsistency of these results with those obtained earlier, and the fact that the time series plots of the cointegrating vectors do not appear to be stationary, we dismiss the results of the Johansen tests as implausible and estimate the neoclassical system in first differences on the assumption that there are no cointegrating vectors in the system.

The accelerator, putty-clay and $Q$ type VAR specifications developed in this section are not inconsistent with the single equation analysis presented earlier. A zero rank for the $\Pi$ matrix is found in all three models and for the putty-clay and $Q$ models this finding is consistent with the results of the augmented Dickey-Fuller and Phillips-Perron cointegration tests used in Section 5.2. Both approaches suggest that the accelerator, the putty-clay, and $Q$ models should be estimated in first differences. For the neoclassical model, the analysis here contradicts the analysis of Section 5.2 suggesting estimation in levels rather than first differences. Given the implausibility of these results, we estimate a first difference model consistent with analysis in Section 5.2. The forecasting performance of these models is assessed in the Chapter 6.

5.4.3 Four Modifications to the Basic VAR Specifications

In this section four more VAR models of investment behaviour are estimated. None of the models are consistent with any one particular theory. As mentioned above, the first is an adaptation of the accelerator model in which allowance is made for firm’s ability
to meet increased demand by increasing utilisation of existing capacity. Manne (1945) Chenery (1952) and Meyer and Kuh (1957) find a role for capacity variables in accelerator type models (see Section 2.2.4). The second VAR is an adaptation of the pure neoclassical model in which allowance is made for the fact that perfect capital markets do not exist in reality. Indeed, one of the reasons investment did not take off in the mid-seventies (given a negative real user cost of capital) is that firms faced a very strong adverse effect from company liquidity. A similar modification is made to the putty-clay model which is also based on the same idea. Coen (1971), Sarantis (1979), Anderson (1981), and Sinai and Eckstein (1983) find a significant role for liquidity variables in neoclassical type models (see Sections 2.3.5 and 2.4.4). The fourth VAR specification to be estimated in this section is a three equation system consisting of variables for the rate of net investment, $Q$, and output. Although the theoretical literature states that $Q$ is sufficient to explain investment, the empirical performance of $Q$ has been poor. Typically, $Q$ explains a very low proportion of the variation in investment and estimated residuals are strongly serially correlated. Moreover, when other variables are included in $Q$ equations they are highly significant. In particular, many authors note that the fit of an equation and its residual properties drastically improve when output is included (see Section 2.6.7). Since most of the literature suggests that the relationship between net investment and $Q$ is not as predicted by theory, and since output obviously contains information in addition to $Q$, the $Q$ type system estimated in Section 5.4.2 is augmented with an accelerator effect provided by an output (over capital stock) variable. Blanchard and Wyplosz (1981) and Chirinko (1987) find a significant role for output in $Q$ type equations.

We begin by determining the lag length, $k$, for the four systems. Since the quarterly data is seasonal, $k$ is initially set equal to 4. However, residual serial correlation remained in the output equations of the augmented accelerator, $Q$ and putty-clay systems and in the optimal capital stock equation of the neoclassical system. This serial correlation is removed from the augmented putty-clay system when the lag length is increased to $k=6$. A lag length of $k=5$ was sufficient for the augmented accelerator, neoclassical and $Q$ systems. The corresponding residual statistics for the four models are given in Tables 5.27 to 5.30.
Table 5.27: Tests of normality and autocorrelation in the augmented accelerator VAR: lag length k=5.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi^2_2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L\Delta X$</td>
<td>0.003 (0.199)</td>
<td>0.305 (0.396)</td>
<td>0.39</td>
<td>15.51</td>
</tr>
<tr>
<td>$LY$</td>
<td>-0.955 (0.199)</td>
<td>6.105 (0.396)</td>
<td>233.82</td>
<td>19.72</td>
</tr>
</tbody>
</table>

Note: CV denotes the test's critical value. Standard errors are given in parenthesis.

Table 5.28: Tests of normality and autocorrelation in the augmented neoclassical VAR: lag length k=5.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi^2_2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L\Delta X$</td>
<td>-0.051 (0.202)</td>
<td>0.226 (0.401)</td>
<td>0.25</td>
<td>15.32</td>
</tr>
<tr>
<td>$LOX$</td>
<td>-1.037 (0.202)</td>
<td>23.68 (0.401)</td>
<td>3,150.49</td>
<td>10.95</td>
</tr>
<tr>
<td>$LLR$</td>
<td>-0.780 (0.202)</td>
<td>3.450 (0.401)</td>
<td>79.25</td>
<td>28.66</td>
</tr>
</tbody>
</table>

Note: CV denotes the test's critical value. Standard errors are given in parenthesis.

Table 5.29: Tests of normality and autocorrelation in the augmented putty-clay VAR: lag length k=6.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi^2_2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L\Delta X$</td>
<td>-0.101 (0.203)</td>
<td>0.178 (0.403)</td>
<td>0.34</td>
<td>15.32</td>
</tr>
<tr>
<td>$LY$</td>
<td>-0.459 (0.203)</td>
<td>4.439 (0.403)</td>
<td>112.92</td>
<td>22.15</td>
</tr>
<tr>
<td>$LC$</td>
<td>-0.047 (0.203)</td>
<td>8.453 (0.403)</td>
<td>401.19</td>
<td>23.33</td>
</tr>
<tr>
<td>$LLR$</td>
<td>-0.708 (0.203)</td>
<td>3.427 (0.382)</td>
<td>75.29</td>
<td>30.49</td>
</tr>
</tbody>
</table>

Note: CV denotes the test's critical value. Standard errors are given in parenthesis.

Table 5.30: Tests of normality and autocorrelation in the augmented Q type VAR: lag length k=5.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi^2_2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta K/K$</td>
<td>-0.306 (0.192)</td>
<td>2.182 (0.382)</td>
<td>31.21</td>
<td>17.24</td>
</tr>
<tr>
<td>$Q$</td>
<td>0.583 (0.192)</td>
<td>4.331 (0.382)</td>
<td>124.23</td>
<td>23.64</td>
</tr>
<tr>
<td>$Y/K$</td>
<td>-0.325 (0.192)</td>
<td>3.006 (0.382)</td>
<td>57.89</td>
<td>21.63</td>
</tr>
</tbody>
</table>

Note: CV denotes the test's critical value. Standard errors are given in parenthesis.

The LM statistics corresponding to each equation indicate a lack of serial correlation. For all equations the test statistics are well below the $\chi^2_{23}$ critical values and thus the null hypotheses of no serial correlation can not be rejected. Box-Pierce statistics and $t$-tests on lagged residuals (neither of which are reported here) are also suggestive of an absence of serial correlation.

As with the VARs estimated in the Section 5.4.2, a number of equations carry non-normal residuals. Only in the investment equations is the null hypothesis of normal residuals not rejected. The rejection of the null in the case of the accelerator,
user cost and \( Q \) models is largely due to excess kurtosis. Altering the lag length \( k \) does nothing to improve matters. As in the previous section, non-normal residuals are assumed not to seriously affect the performance of the Johansen maximum likelihood procedure, although it is fully appreciated that the procedure assumes white noise normally distributed errors.

It is noteworthy that the augmented accelerator specification consists of only two equations. Recall, the capacity utilisation variable is found to be \( 1(0) \) in the unit root tests carried out in Section 4.4. Since the variable is stationary it can not be included among the variables in \( X \). If capacity utilisation is to be included in the system it must be included as an additional right hand side variable.

The test of the rank of \( \Pi \) is again performed using the likelihood ratio tests given in equations 3.60 and 3.61 in Section 3.4.4. Test results are given in Table 5.31 for the augmented accelerator VAR and in Tables 5.34 to 5.36 for the augmented neoclassical, putty-clay and \( Q \) VARs. Again, 10% critical values are given in addition to the 5% values since the tests have low power. The null hypothesis of no cointegrating vectors (i.e. that the rank of \( \Pi=0 \)) is rejected in the augmented accelerator model. The maximum eigenvalue and trace test results reject the hypothesis of more than one cointegrating vector at both levels of significance. Thus, when capacity utilisation variable is included as an additional right hand side variable one cointegrating vector is found. This contrasts with the results of the basic accelerator model in which no evidence of cointegration was found.

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>The Max Statistic</th>
<th>The Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho = 0 )</td>
<td>( \lambda )-max</td>
<td>CV=0.95</td>
</tr>
<tr>
<td>( \rho \leq 1 )</td>
<td>0.057</td>
<td>3.762</td>
</tr>
<tr>
<td>( \rho \leq 1 )</td>
<td>16.135</td>
<td>14.069</td>
</tr>
</tbody>
</table>

The cointegrating vector and the remaining eigenvectors, along with their weights in each equation. This table is presented for illustrative purposes only: in this analysis the theoretical implications of the model are unimportant since the model is developed principally for forecasting.
Table 5.32: Eigenvectors and their corresponding weights in the augmented accelerator model

<table>
<thead>
<tr>
<th>Eigenvectors</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>$\mu_1$</td>
</tr>
<tr>
<td>0.634</td>
<td>-0.153</td>
</tr>
<tr>
<td>-0.395</td>
<td>0.396</td>
</tr>
</tbody>
</table>

It can be useful to test whether a particular cointegrating vector appears as a significant regressor in a particular equation. If it is insignificant in a particular equation, i.e. if the weight $\alpha$ in that equation is insignificantly different from zero, then the error correction term can be legitimately dropped from that equation. In this case, the variable determined by this equation can be taken as weakly exogenous and the equation can be dropped from the system. Such a procedure is not of interest here given our interest in the forecasting. The forecasts for one series requires past values of all other series in the model and thus the estimation of the system as a whole. However, the procedure is outlined in Section 3.4.4.

In summary, the inclusion of a capacity utilisation variable as an additional right hand side variable results in a system with one cointegrating relation. Therefore, this system should be estimated with an error correction term on each equation. This contrasts with the basic accelerator VAR, in which no long run information can be retained. The tests for weak exogeneity are carried out purely for illustrative purposes: the fact that this model is developed primarily with forecasting in mind means that the system should be estimated in its entirety. The procedure is not illustrated for other specifications.

The data are now examined for evidence of cointegrating relationships in the augmented neoclassical, putty-clay and $Q$ systems. The test results for these models are given in Tables 5.33 to 5.35, respectively. For the neoclassical model (see Table 5.33) the null hypothesis of no cointegrating vectors (i.e. that the rank of $\Pi=0$) is rejected using the trace statistic at both levels of significance. The hypothesis is also rejected by the trace statistic at the 90% level of significance but not at the 95% level. Examination of the residuals from the cointegrating relationship (corrected for short run dynamics) appear to be stationary (see Figure 5.11). We therefore adopt the hypothesis that at least one cointegrating relation exists. This hypothesis can not be rejected in favour of the alternative that there is more than one cointegrating relation.
We conclude, that when a measure of corporate liquidity is included in a neoclassical type system, one cointegrating relation exists.

Table 5.33: Determination of the number of cointegrating vectors in the augmented neoclassical VAR

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic</th>
<th>The Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>λ-max</td>
<td>CV=0.95</td>
</tr>
<tr>
<td>ρ = 0</td>
<td>20.243</td>
<td>20.967</td>
</tr>
<tr>
<td>ρ ≤ 2</td>
<td>1.031</td>
<td>3.762</td>
</tr>
</tbody>
</table>

Table 5.34: Determination of the number of cointegrating vectors in the augmented putty-clay VAR

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic</th>
<th>The Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>λ-max</td>
<td>CV=0.95</td>
</tr>
<tr>
<td>ρ = 0</td>
<td>17.634</td>
<td>27.067</td>
</tr>
<tr>
<td>ρ ≤ 2</td>
<td>7.539</td>
<td>14.069</td>
</tr>
<tr>
<td>ρ ≤ 3</td>
<td>0.174</td>
<td>3.762</td>
</tr>
</tbody>
</table>

Table 5.35: Determination of the number of cointegrating vectors in the augmented Q type VAR

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic</th>
<th>The Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>λ-max</td>
<td>CV=0.95</td>
</tr>
<tr>
<td>ρ = 0</td>
<td>22.627</td>
<td>20.967</td>
</tr>
<tr>
<td>ρ ≤ 2</td>
<td>3.187</td>
<td>3.762</td>
</tr>
</tbody>
</table>

When corporate liquidity is included in the putty-clay VAR we find no evidence of cointegration in the augmented system (see Table 5.34). This implies that the augmented system, like the basic system, should be estimated in first differences.

There is evidence of a single cointegrating relationship in the augmented Q model (see Table 5.35). The trace and maximum eigenvalue test statistics of 24.654 and 22.627 respectively, clearly exceed the 95% critical values, hence the null of no cointegrating vectors must be rejected. Subsequent null hypotheses regarding the number of cointegrating vectors can not be rejected. Thus, although there is no cointegrating relationship between the rate of net investment and Q, there is such a relationship between the rate of investment, Q and output (over capital stock). Thus, the augmented model should be estimated with an error correction term on each equation, unlike the basic model which must be estimated in first differences.

In summary, these models differ from the basic models in a number of ways. The augmentation of the basic accelerator with an I(0) capacity utilisation variable results
Figure 5.11: Residuals of cointegrating vector 1 (corrected for short-run dynamics) from the augmented neoclassical model

![Figure 5.11: Residuals of cointegrating vector 1](image)

Figure 5.12: Residuals of cointegrating vector 2 (corrected for short-run dynamics) from the ACS(b)

![Figure 5.12: Residuals of cointegrating vector 2](image)
in a system with one cointegrating relationship. Thus, both the investment and output equations should be estimated with one error correction mechanism. The basic neoclassical and putty-clay systems are augmented with a variable measuring corporate liquidity. In the augmented neoclassical system a single cointegrating relationship is found, implying that each of the three equations in the system should be estimated with a single error correction mechanism. When the putty-clay model is augmented with the liquidity variable there is no evidence of cointegration. Thus, like the basic system, the augmented system should be estimated in first differences. The introduction of an output variable in the $Q$ model results in a system with one cointegrating relationship. Thus, the augmented system should be estimated with one error correction term on each of the three equations. Recall, that estimation in first differences was found to be appropriate for the basic $Q$ system.

5.4.4 An Atheoretical Approach to Modelling

None of the VARs estimated in this section are consistent with any one of the dominant theories of investment. However, they may provide a forecasting performance that is superior to that of the theoretically consistent models. In subsection 5.4.4.1 we develop a composite VAR model in which all the main determinants of investment suggested by the four dominant theories appear. In later subsections we develop a number of VAR models of private industrial and commercial construction output, some of which make use of leading indicator variables.

5.4.4.1 A Composite VAR Model

Here we consider a VAR which contains elements of each of the four dominant theories of investment. Whilst it is clear from the empirical investment literature discussed in Chapter 2 and the analysis of Section 5.2, that output, the user cost of capital and $Q$ each have some information content, as individual variables they do not provide a complete explanation of investment. On a purely intuitive level, firms may be guided by a number of variables in evaluating investment opportunities and consequently, they may not react to changes in output, user cost and $Q$ in the way that
the theory of the accelerator, the neoclassical and the \( Q \) models predict. Therefore, a four equation composite VAR is estimated in which the output, user cost and \( Q \) variables are all used to explain real net investment (or more accurately private industrial and commercial construction output). The resulting system is inconsistent with any single theory of investment, but given that each variable in the system has some information content, their mutual inclusion in a single model may not be inappropriate when that model is developed for the sole objective of forecasting.

The residual statistics corresponding to such a model are given in Table 5.36. The initial lag length of \( k=4 \) was increased to \( k=5 \) to remove serial correlation in the output equation. As with the systems estimated in Sections 5.4.2 and 5.4.3, the residuals of the output, user cost and \( Q \) equations display a substantial degree of non-normality. The user cost equation, in particular, generates a Jarque-Bera test statistic of 3480. This non-normality is due to excess kurtosis caused by a number of outliers in the mid 1970s corresponding to times when the real interest rate was negative. As with the VARs estimated earlier, we assume that this non-normality does not seriously affect the estimation procedure.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness</th>
<th>Coefficient of Excess Kurtosis</th>
<th>Jarque-Bera for Normality</th>
<th>LM for Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L\Delta K )</td>
<td>-0.081 (0.192)</td>
<td>0.634 (0.382)</td>
<td>2.39</td>
<td>15.61</td>
</tr>
<tr>
<td>( L\Delta Y )</td>
<td>-0.438 (0.192)</td>
<td>3.435 (0.382)</td>
<td>77.22</td>
<td>23.71</td>
</tr>
<tr>
<td>( L\Delta C )</td>
<td>0.027 (0.192)</td>
<td>23.616 (0.382)</td>
<td>3,479.98</td>
<td>9.24</td>
</tr>
<tr>
<td>( Q )</td>
<td>0.630 (0.192)</td>
<td>4.800 (0.382)</td>
<td>152.31</td>
<td>24.48</td>
</tr>
</tbody>
</table>

Note: CV denotes the test's critical value. Standard errors are given in parenthesis.

The results of the Johansen tests for cointegration are given in Table 5.37. According to the trace and maximum eigenvalue tests, the null hypothesis of no cointegrating vectors (i.e. that the rank of \( \Pi \) is zero) can not be rejected at either level of significance. This should not be surprising: the Johansen tests do not provide evidence of a long run bivariate relationship between net investment and any single determinant, and so there is no reason why such a relationship should exist between net investment and its determinants jointly. The implication of these results is that the composite VAR should be estimated in first differences.
Table 5.37: Determination of the number of cointegrating vectors in the composite VAR

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>The Trace Statistic</th>
<th>The Max Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda$-trace</td>
<td>CV=0.95</td>
</tr>
<tr>
<td>$\rho = 0$</td>
<td>20.882</td>
<td>27.067</td>
</tr>
<tr>
<td>$\rho \leq 1$</td>
<td>9.752</td>
<td>20.967</td>
</tr>
<tr>
<td>$\rho \leq 2$</td>
<td>4.579</td>
<td>14.069</td>
</tr>
<tr>
<td>$\rho \leq 3$</td>
<td>0.122</td>
<td>3.762</td>
</tr>
</tbody>
</table>

This composite model also provides a general framework for testing the efficacy of individual theories of investment. We can impose restrictions on this general VAR to derive a special case broadly consistent with each theory. For example, suppose we impose the restriction that the coefficients on the lagged investment, user cost and $Q$ variables are equal to zero. A failure to reject this joint hypothesis is suggestive of an accelerator type system, such as that discussed in Section 5.4.2. Similarly, suppose we were unable to reject the joint hypothesis that the coefficients on the lagged investment, output, and user cost terms were zero, then we may conclude that a $Q$ type specification is appropriate.

Dolado and Lutkepohl (1996) have suggested a procedure which allows restrictions on a VAR or VECM to be tested using a simple Wald test. They note that the Wald test can not be used to test restrictions on a VAR$(k)$ specification (of the form given by equation 3.30) since such a specification does not subsume the underlying model. Dolado and Lutkepohl suggest estimating a VAR$(k+1)$ specification for which the Wald test is valid. Adopting this procedure we re-estimate the composite VAR described above with $k$ set equal to six rather than five. The Wald test is used to examine the validity of special cases of the general model consistent the dominant theories of investment.

The results are not encouraging. We can not reject the joint hypothesis that all coefficients in the general model, except those on the lagged output terms, are zero. The Wald test statistic is 16.67 and the $\chi^2_{18}$ critical value is 28.9. This implies an accelerator type system. However, we can not reject the joint hypothesis that all coefficients, except those on the lagged $Q$ terms, are zero. The Wald test statistic is 25.08 and this is also distributed $\chi^2_{18}$. This implies a $Q$ type specification. Similarly, the special case implying a putty-clay type system can not be rejected. This involves testing the joint hypothesis that the coefficients on the lagged investment and $Q$ variables are zero. The Wald test statistic is 12.35 and the $\chi^2_{12}$ critical value is 21.0.
The main conclusion that we can draw from this analysis is that none of the variables suggested by the individual theories of investment are sufficient to explain private industrial and commercial construction output. The results of Sections 4.4 and 5.2 suggest that the empirical implementation of investment models in the literature is unsatisfactory. The results of this section however, suggest that the poor performance of these models is also due to the fact that the determinants of investment suggested by theory are inadequate.

5.4.4.2 Systems Containing Construction Specific Variables

Next, four systems are estimated using data specific to the construction sector. The first system is a six equation system in which private industrial and commercial construction output, $\Delta K$, is explained by new construction orders, $LO$, private sector output, $LY$, an index of tender prices, $LTPI$, a construction market conditions index, $MCI$ and a real interest rate variable, $RIR$. A description of the derivation of these variables is given in Section 4.3. The results of unit root tests performed on these variables are presented in Section 4.4. The basic six equation system is referred to hereafter as the basic construction system (BCS). Real construction new orders, the index of tender prices and the market conditions index are chosen since they may well provide information on likely movements in construction output in advance of these movements. In other words, they may be leading indicators of construction output. Trends in new orders, for example, clearly and consistently lead trends in construction output. Indeed, if all new orders were completed to budget, output data would simply reflect the payments for many new orders spread over respective duration of the building process. The size of the payment for a particular new order in any specified period accords with the value of the work done in that period. As such, the value of output corresponding to a particular new order can be thought of as a distributed lag relationship of that new order. Since most new orders placed are realised and completed reasonably closely to budget, the correlation between lagged new orders and output is very strong.

Similarly, an index of tender prices may provide an indication of future trends in output. Rising tender prices may be indicative of increased (or an expected increase
in demand for contractors’ services. If this leads to an increase in contractors’ current or expected work loads, output will increase. The market conditions index is constructed as the residuals from a regression of the log of tender prices on (a trend and) the log of building costs. In some sense, it measures the profitability of investment, although much less sophisticated than $Q$. The real interest rate variable can also be thought of as a cruder replacement for another theoretically consistent variable contained in the previous system, namely, the real user cost of capital. Real output of private sector firms remains in this system to capture the accelerator effect which, as discussed in the literature review, appears to be provide the biggest single determinant of investment. The residual statistics from this system (the BCS) are presented in Table 5.38 and the results of the Johansen tests for cointegrating relationships are given in Table 5.42. It is noteworthy that the estimation period is shorter than for the models estimated previously. This is because new orders data is not available prior to 1958q1.

The three remaining systems build on the BCS. In the first, denoted ACS(a), the BCS is augmented with a variable for the real wage in the construction sector, $LW$. In the second, denoted ACS(b), the BCS is augmented with a variable measuring employment in the construction sector relative to the economy as a whole, $RE$. These systems contain seven equations. For the third, denoted ACS(c), the BCS is augmented with a variable capturing manufacturing investment intentions, $II$. This variable was found to be stationary in Section 4.4 and is therefore included as an extra right hand side variable. Details of how these variables were constructed are given in Section 4.3. Again, these construction specific variables are included as (leading) indicators of construction output. Rising real wages, for example, will tend to reflect the difficulties encountered by contractors in obtaining workers, both skilled and unskilled, in periods when workloads are high. Similarly, increasing employment in the construction sector relative to the economy as a whole, may be indicative of the relative prosperity of this sector. By its very nature, the intentions variable is a leading indicator of investment and therefore construction output. The residual statistics of these three models are given in Tables 5.39, 5.40 and 5.41. The results of the Johansen tests for cointegration are given in Tables 5.43, 5.44 and 5.45. Due to a lack of data, these models are estimated over a shorter time period. Real wage and
construction employment data can not be obtained prior to 1965 and 1964, respectively. Intentions data is available from 1959.

Table 5.38: Tests of normality and autocorrelation in the BCS: lag length k=5.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi^2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LAK$</td>
<td>-0.334 (0.197)</td>
<td>0.908 (0.392)</td>
<td>7.18</td>
<td>10.48</td>
</tr>
<tr>
<td>$LO$</td>
<td>0.262 (0.197)</td>
<td>0.549 (0.392)</td>
<td>3.22</td>
<td>16.96</td>
</tr>
<tr>
<td>$LY$</td>
<td>-0.561 (0.197)</td>
<td>5.346 (0.392)</td>
<td>173.39</td>
<td>28.99</td>
</tr>
<tr>
<td>$LTPi$</td>
<td>-0.126 (0.197)</td>
<td>-0.147 (0.392)</td>
<td>0.59</td>
<td>11.34</td>
</tr>
<tr>
<td>$MCI$</td>
<td>-0.227 (0.197)</td>
<td>0.019 (0.392)</td>
<td>1.28</td>
<td>17.85</td>
</tr>
<tr>
<td>$RIR$</td>
<td>-0.519 (0.197)</td>
<td>1.266 (0.392)</td>
<td>15.47</td>
<td>20.46</td>
</tr>
</tbody>
</table>

Note: CV denotes the test’s critical value. Standard errors are given in parenthesis.

Table 5.39: Tests of normality and autocorrelation in the ACS(a): lag length k=5.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi^2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LAK$</td>
<td>0.016 (0.218)</td>
<td>0.247 (0.433)</td>
<td>0.19</td>
<td>12.33</td>
</tr>
<tr>
<td>$LO$</td>
<td>0.121 (0.218)</td>
<td>0.304 (0.433)</td>
<td>0.59</td>
<td>18.09</td>
</tr>
<tr>
<td>$LY$</td>
<td>-0.864 (0.218)</td>
<td>6.709 (0.433)</td>
<td>224.34</td>
<td>31.71</td>
</tr>
<tr>
<td>$LTPi$</td>
<td>-0.409 (0.218)</td>
<td>0.015 (0.433)</td>
<td>3.36</td>
<td>15.55</td>
</tr>
<tr>
<td>$MCI$</td>
<td>-0.339 (0.218)</td>
<td>-0.032 (0.433)</td>
<td>2.34</td>
<td>22.48</td>
</tr>
<tr>
<td>$RIR$</td>
<td>-0.426 (0.218)</td>
<td>0.619 (0.433)</td>
<td>5.15</td>
<td>18.93</td>
</tr>
<tr>
<td>$LW$</td>
<td>0.284 (0.218)</td>
<td>3.169 (0.433)</td>
<td>47.56</td>
<td>27.91</td>
</tr>
</tbody>
</table>

Note: CV denotes the test’s critical value. Standard errors are given in parenthesis.

Table 5.40: Tests of normality and autocorrelation in the ACS(b): lag length k=5.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi^2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LAK$</td>
<td>-0.083 (0.222)</td>
<td>0.131 (0.441)</td>
<td>0.16</td>
<td>10.31</td>
</tr>
<tr>
<td>$LO$</td>
<td>0.096 (0.222)</td>
<td>0.336 (0.441)</td>
<td>0.55</td>
<td>26.95</td>
</tr>
<tr>
<td>$LY$</td>
<td>-0.887 (0.222)</td>
<td>6.378 (0.441)</td>
<td>197.48</td>
<td>33.15</td>
</tr>
<tr>
<td>$LTPi$</td>
<td>-0.145 (0.222)</td>
<td>0.025 (0.441)</td>
<td>0.41</td>
<td>20.67</td>
</tr>
<tr>
<td>$MCI$</td>
<td>-0.317 (0.222)</td>
<td>-0.179 (0.441)</td>
<td>2.19</td>
<td>33.22</td>
</tr>
<tr>
<td>$RIR$</td>
<td>-0.442 (0.222)</td>
<td>1.193 (0.441)</td>
<td>9.69</td>
<td>20.69</td>
</tr>
<tr>
<td>$RE$</td>
<td>0.221 (0.222)</td>
<td>0.642 (0.441)</td>
<td>2.53</td>
<td>11.22</td>
</tr>
</tbody>
</table>

Note: CV denotes the test’s critical value. Standard errors are given in parenthesis.

The statistics presented in Tables 5.38 to 5.41 indicate that the residuals from these four models are reasonably well-behaved. A lag length of $k=5$ is sufficient to ensure an absence of serial correlation and only the equations for private sector output and the real wage display seriously non-normal residuals. The non-normality can not be
removed by increasing the lag length, whereas a reduction in $k$ induces substantial serial correlation.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality</th>
<th>LM for Autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L\Delta K$</td>
<td>-0.267(0.200)</td>
<td>0.691(0.398)</td>
<td>4.11</td>
<td>10.2</td>
</tr>
<tr>
<td>$LO$</td>
<td>0.356(0.200)</td>
<td>0.398 (0.398)</td>
<td>3.52</td>
<td>19.24</td>
</tr>
<tr>
<td>$LY$</td>
<td>-0.456(0.200)</td>
<td>4.761(0.398)</td>
<td>132.33</td>
<td>30.72</td>
</tr>
<tr>
<td>$LTPi$</td>
<td>-0.108(0.200)</td>
<td>0.171 (0.398)</td>
<td>0.54</td>
<td>12.34</td>
</tr>
<tr>
<td>$MCI$</td>
<td>-0.262(0.200)</td>
<td>-0.016 (0.398)</td>
<td>1.67</td>
<td>16.41</td>
</tr>
<tr>
<td>$RIR$</td>
<td>-0.506(0.200)</td>
<td>1.246 (0.398)</td>
<td>14.44</td>
<td>17.09</td>
</tr>
</tbody>
</table>

Note: CV denotes the test's critical value. Standard errors are given in parenthesis.

Table 5.42: Determination of the number of cointegrating vectors in the BCS

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic</th>
<th>The Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = 0$</td>
<td>$\lambda_{\text{max}}$ CV=0.95 CV=0.90</td>
<td>$\lambda_{\text{trace}}$ CV=0.95 CV=0.90</td>
</tr>
<tr>
<td>$\rho \leq 1$</td>
<td>54.367 39.372 36.762</td>
<td>107.141 94.155 89.483</td>
</tr>
<tr>
<td>$\rho \leq 2$</td>
<td>25.575 33.461 30.901</td>
<td>52.774 68.524 64.843</td>
</tr>
<tr>
<td>$\rho \leq 3$</td>
<td>13.262 27.067 24.734</td>
<td>27.199 47.211 43.949</td>
</tr>
<tr>
<td>$\rho \leq 4$</td>
<td>9.649 20.967 18.598</td>
<td>13.937 29.681 26.785</td>
</tr>
<tr>
<td>$\rho \leq 5$</td>
<td>3.855 14.069 12.071</td>
<td>4.287 15.411 13.325</td>
</tr>
<tr>
<td>$\rho \leq 6$</td>
<td>0.432 3.762 2.687</td>
<td>0.432 3.762 2.687</td>
</tr>
</tbody>
</table>

Table 5.43: Determination of the number of cointegrating vectors in the ACS(a)

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic</th>
<th>The Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = 0$</td>
<td>$\lambda_{\text{max}}$ CV=0.95 CV=0.90</td>
<td>$\lambda_{\text{trace}}$ CV=0.95 CV=0.90</td>
</tr>
<tr>
<td>$\rho \leq 1$</td>
<td>59.768 45.277 42.317</td>
<td>160.363 124.243 118.501</td>
</tr>
<tr>
<td>$\rho \leq 2$</td>
<td>42.984 39.372 36.762</td>
<td>100.595 94.155 89.483</td>
</tr>
<tr>
<td>$\rho \leq 3$</td>
<td>25.145 33.461 30.901</td>
<td>57.612 68.524 64.843</td>
</tr>
<tr>
<td>$\rho \leq 4$</td>
<td>19.019 27.067 24.734</td>
<td>32.467 47.211 43.949</td>
</tr>
<tr>
<td>$\rho \leq 5$</td>
<td>8.038 20.967 18.598</td>
<td>13.448 29.681 26.785</td>
</tr>
<tr>
<td>$\rho \leq 6$</td>
<td>5.107 14.069 12.071</td>
<td>5.409 15.411 13.325</td>
</tr>
<tr>
<td>$\rho \leq 7$</td>
<td>0.303 3.762 2.687</td>
<td>0.303 3.762 2.687</td>
</tr>
</tbody>
</table>

Table 5.44: Determination of the number of cointegrating vectors in the ACS(b)

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic</th>
<th>The Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = 0$</td>
<td>$\lambda_{\text{max}}$ CV=0.95 CV=0.90</td>
<td>$\lambda_{\text{trace}}$ CV=0.95 CV=0.90</td>
</tr>
<tr>
<td>$\rho \leq 1$</td>
<td>58.122 45.277 42.317</td>
<td>149.351 124.243 118.501</td>
</tr>
<tr>
<td>$\rho \leq 2$</td>
<td>38.275 39.372 36.762</td>
<td>91.229 94.155 89.483</td>
</tr>
<tr>
<td>$\rho \leq 3$</td>
<td>26.951 33.461 30.901</td>
<td>52.955 68.524 64.843</td>
</tr>
<tr>
<td>$\rho \leq 4$</td>
<td>10.048 27.067 24.734</td>
<td>26.004 47.211 43.949</td>
</tr>
<tr>
<td>$\rho \leq 5$</td>
<td>8.852 20.967 18.598</td>
<td>15.955 29.681 26.785</td>
</tr>
<tr>
<td>$\rho \leq 6$</td>
<td>6.256 14.069 12.071</td>
<td>7.103 15.411 13.325</td>
</tr>
<tr>
<td>$\rho \leq 7$</td>
<td>0.847 3.762 2.687</td>
<td>0.847 3.762 2.687</td>
</tr>
</tbody>
</table>
From the evidence in Table 5.42 it is clear that the basic system contains one cointegrating relationship. Both the trace and maximal eigenvalue statistics support this hypothesis. When this basic model is augmented with a real wage variable, both statistics suggest that two cointegrating relationships exist. When the basic model is augmented with construction employment as a share of total employment the conclusions are less clear cut. Certainly, there is evidence of at least one cointegrating relationship at the 95% level of significance. The hypothesis of there being only one cointegrating relationship can not be rejected at the 95% level with either test. If we adopt the 90% level of significance, which might be justifiable given the tests' low power, the hypothesis of there being only one cointegrating relationship is rejected with both tests. In this case, as in previous cases in which there is disagreement between the tests, we plot the residuals of the eigenvectors; if the plot of the residuals from the second eigenvector appear to be stationary this is taken as evidence that this vector is in fact a cointegrating vector. The residuals (corrected for short run dynamics) are plotted in Figure 5.12 and certainly appear to be stationary. We therefore conclude that when the basic system is augmented with construction employment as a share of total employment two cointegrating relationships are present. From the evidence in Table 5.45 it is clear that when the basic system is augmented with the investment intentions variable, the resulting system, the ACS(c) contains one cointegrating relationship.

Thus, in summary, the BCS and the ACS(c) are estimated with one cointegrating relationship and the ACS(a) and ACS(b) are estimated with two cointegrating relationships. The forecasting performance of these four models is examined in Chapter 6.
5.4.4.3 Systems Containing Macroeconomic Variables

The final two models estimated in this section use mainly macroeconomic data rather than data specific to the construction sector. The orders series is retained in the system due to the fact that it is likely to be the best indicator of construction output. The real interest rate is also retained in the system. The output of the private sector is replaced by output of the economy as a whole as measured by real GDP, $Y^t$. Recall, the output equation has consistently failed the Jarque-Bera test for normal residuals. Its replacement with a more aggregate measure of output (or economic activity more generally) may result in a system where the residuals are more normal. The market condition index is replaced with a measure of ICCs liquidity, $LLR$. A description of this variable, which is defined as the ratio of ICCs liquid assets over liabilities, is given in Section 4.3. Essentially, the argument for including this variable in the system is simply that it provides some measure of a company’s ability to finance investment out of internal funds. The construction employment as a share of total employment variable is replaced with total employment, $LE$. This is included on the grounds it too may provide some indication of forthcoming trends in investment. For example, a large increase in employment may necessitate the building of larger premises to accommodate the new personnel.

### Table 5.46: Tests of normality and autocorrelation in the BMS: lag length $k=5$.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness $CV=1.96$</th>
<th>Coefficient of Excess Kurtosis $CV=1.96$</th>
<th>Jarque-Bera for Normality $\chi^2 = 5.99$</th>
<th>LM for Autocorrelation $\chi^2_{23} = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LAX$</td>
<td>-0.204 (0.202)</td>
<td>0.089 (0.401)</td>
<td>0.99</td>
<td>15.78</td>
</tr>
<tr>
<td>$LYA$</td>
<td>0.466 (0.202)</td>
<td>0.296 (0.401)</td>
<td>5.46</td>
<td>17.18</td>
</tr>
<tr>
<td>$LJ^t$</td>
<td>-0.222 (0.202)</td>
<td>1.158 (0.401)</td>
<td>8.11</td>
<td>27.02</td>
</tr>
<tr>
<td>$LE$</td>
<td>-0.149 (0.202)</td>
<td>0.144 (0.401)</td>
<td>0.58</td>
<td>13.69</td>
</tr>
<tr>
<td>$LLR$</td>
<td>-0.989 (0.202)</td>
<td>3.664 (0.401)</td>
<td>96.37</td>
<td>28.48</td>
</tr>
<tr>
<td>$RIR$</td>
<td>-0.429 (0.202)</td>
<td>-0.235 (0.401)</td>
<td>4.76</td>
<td>23.11</td>
</tr>
</tbody>
</table>

Note: CV denotes the test’s critical value. Standard errors are given in parenthesis.

### Table 5.47: Determination of the number of cointegrating vectors in the BMS

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic $\lambda$-max $CV=0.95$ $CV=0.90$</th>
<th>Trace Statistic $\lambda$-trace $CV=0.95$ $CV=0.90$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = 0$</td>
<td>45.101 39.372 36.762</td>
<td>108.884 94.155 89.483</td>
</tr>
<tr>
<td>$\rho \leq 1$</td>
<td>20.829 33.461 30.901</td>
<td>63.784 68.524 64.843</td>
</tr>
<tr>
<td>$\rho \leq 2$</td>
<td>18.756 27.067 24.734</td>
<td>42.954 47.211 43.949</td>
</tr>
<tr>
<td>$\rho \leq 3$</td>
<td>13.488 20.967 18.598</td>
<td>24.198 29.681 26.785</td>
</tr>
<tr>
<td>$\rho \leq 4$</td>
<td>8.744 14.069 12.071</td>
<td>10.709 15.411 13.325</td>
</tr>
<tr>
<td>$\rho \leq 5$</td>
<td>1.966 3.762 2.687</td>
<td>1.966 3.762 2.687</td>
</tr>
</tbody>
</table>

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Thus, a basic six equation system, referred to as the basic macroeconomic system (BMS), is estimated, the residual statistics for which are presented in Table 5.46. The estimation period begins in 1960q1 due to the lack of available data on the liquidity ratio prior to this. A lag length of $k=5$ is sufficient to ensure a lack of serial correlation in the residuals. Some non-normality exists in the residuals of the $L^\alpha$ equation but this is much less severe than in previous models estimated with private sector output. There is however, a more severe problem in the liquidity ratio equation. The results of the Johansen tests for cointegration are given in Table 5.47. These test results suggest that this system contains just one cointegrating relationship.

This basic six equation system is augmented with two further variables, both of which are I(0) and therefore included in levels on the right hand side of the system. These variables are capacity utilisation, $LCU$, and business optimism, $BO$. These variables are discussed in Section 4.3 and results of unit root tests are presented in Section 4.4. Current and one lag of capacity utilisation and business optimism are included in the system. This system is referred to as the augmented macroeconomic system (AMS). Business optimism in particular, is well known to be a leading indicator of economic activity. Indeed, business optimism is one component in the Office of National Statistics' longer leading index of cyclical activity. The residual statistics for this model are presented in Table 5.48. The estimation period begins in 1960q1 due to the lack of data on the liquidity ratio. A lag length of $k=5$ is sufficient to ensure an absence of serial correlation. However, as in the basic model, some non-normality exists in the liquidity ratio equation, and this remains whatever lag length is selected.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Coefficient of Skewness CV=1.96</th>
<th>Coefficient of Excess Kurtosis CV=1.96</th>
<th>Jarque-Bera for Normality $\chi_2^2 = 5.99$</th>
<th>LM for Autocorrelation $\chi_{23}^2 = 35.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L\Delta K$</td>
<td>0.017 (0.202)</td>
<td>0.302 (0.401)</td>
<td>0.38</td>
<td>12.47</td>
</tr>
<tr>
<td>$L O$</td>
<td>0.476 (0.202)</td>
<td>0.522 (0.401)</td>
<td>6.59</td>
<td>17.55</td>
</tr>
<tr>
<td>$L^\alpha$</td>
<td>-0.065 (0.202)</td>
<td>0.444 (0.401)</td>
<td>1.01</td>
<td>30.82</td>
</tr>
<tr>
<td>$L E$</td>
<td>-0.014 (0.202)</td>
<td>0.005 (0.401)</td>
<td>0.02</td>
<td>18.42</td>
</tr>
<tr>
<td>$L L R$</td>
<td>-0.991 (0.202)</td>
<td>3.227 (0.401)</td>
<td>80.13</td>
<td>24.53</td>
</tr>
<tr>
<td>$R I R$</td>
<td>-0.261 (0.202)</td>
<td>-0.339 (0.401)</td>
<td>2.42</td>
<td>23.37</td>
</tr>
</tbody>
</table>

Note: CV denotes the test's critical value. Standard errors are given in parenthesis.
Table 5.49: Determination of the number of cointegrating vectors in the AMS

<table>
<thead>
<tr>
<th>Hypothesis</th>
<th>Maximal Eigenvalue Statistic</th>
<th>The Trace Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max.</td>
<td>CV=0.95</td>
</tr>
<tr>
<td>ρ = 0</td>
<td>31.909</td>
<td>39.372</td>
</tr>
<tr>
<td>ρ ≤ 1</td>
<td>24.441</td>
<td>33.461</td>
</tr>
<tr>
<td>ρ ≤ 2</td>
<td>20.974</td>
<td>27.067</td>
</tr>
<tr>
<td>ρ ≤ 3</td>
<td>10.897</td>
<td>20.967</td>
</tr>
<tr>
<td>ρ ≤ 5</td>
<td>2.902</td>
<td>3.762</td>
</tr>
</tbody>
</table>

The results of the Johansen tests for cointegration are presented in Table 5.49. The two tests provide different conclusions. The maximum eigenvalue statistic for example, suggests that there is one cointegrating relation if the 95% level of significance is adopted and two if the 90% level is used. The trace statistic however, suggests that the rank of Π is zero, i.e. that there are no cointegrating relationships. An examination of the plots of the residuals from the first two eigenvectors corrected for short-run dynamics (see Figures 5.13 and 5.14) suggests that there is only one cointegration relationship: the residuals from the second eigenvector (see Figure 5.14) appear to be non-stationary. As such this system, like the basic system, is estimated with only one cointegrating relationship. The forecasting performance of these two models is examined in Chapter 6.

5.4.5 A Summary of VAR Results

In this section fifteen VAR models of private industrial and commercial construction output have been estimated. The first four of these are simply representations suggested by a rigorous interpretation of the four main theories of investment behaviour. In this sense, these four models are analogous to the single equation versions of theory estimated in Section 5.2. The dimensions of these models are determined in Section 5.4.2. These basic systems have each been augmented with an additional variable in Section 5.4.3. These augmentations are essentially a response to the deficiencies of the pure versions of the models and are consistent with suggestions made in the empirical literature.

The move away from rigorous theory continues in Section 5.4.4. In 5.4.4.1 investment is modelled using a system in which all the determinants suggested by theory appear. Of course, there is a certain amount of collinearity between these variables and it is
Figure 5.13: Residuals of cointegrating vector 1 (corrected for short-run dynamics) from the AMS

Figure 5.14: Residuals of cointegrating vector 2 (corrected for short-run dynamics) from the AMS
recognised that this may have an adverse effect on parameter estimates. In 5.4.4.2 and 5.4.4.3 the emphasis is on producing leading indicator models of private industrial and commercial construction output. In the former section, construction specific variables thought to have some indicating ability are used in the development of four VAR models. In the latter section the emphasis is on the use of macroeconomic variables. Here, two further systems are estimated.

In Section 5.4 we have sought to determine the dimensions of these systems. In particular, we are interested in determining the lag length necessary to ensure white noise normally distributed residuals and the number of cointegrating relationships existing in each system. No attempt is made to provide an economic interpretation of the cointegrating relationships since the theoretical implications of the systems are not of first order importance in this forecasting exercise. In any event, Wickens (1993) has shown that the interpretation of cointegrating relationships is impossible when a system contains multiple relationships but does not subsume a structural model. A summary of these findings is presented in Table 5.50.

Table 5.50: A synopsis of VAR results

<table>
<thead>
<tr>
<th>System</th>
<th>Variables</th>
<th>Lag length</th>
<th>Sample Period</th>
<th>No. of CVs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerator</td>
<td>$\Delta K, L_Y$</td>
<td>5</td>
<td>1955q1-1996q4</td>
<td>0</td>
</tr>
<tr>
<td>Neoclassical</td>
<td>$\Delta K, L_O K$</td>
<td>5</td>
<td>1955q1-1996q4</td>
<td>0</td>
</tr>
<tr>
<td>Putty-Clay</td>
<td>$\Delta K, L_Y, L_C$</td>
<td>5</td>
<td>1955q1-1996q4</td>
<td>0</td>
</tr>
<tr>
<td>$Q$</td>
<td>$\Delta K, K, Q$</td>
<td>4</td>
<td>1955q4-1996q4</td>
<td>0</td>
</tr>
<tr>
<td>Augmented Accelerator</td>
<td>$\Delta K, L_Y, (L_C U, 1(0))$</td>
<td>5</td>
<td>1959q1-1996q4</td>
<td>1</td>
</tr>
<tr>
<td>Augmented Neoclassical</td>
<td>$\Delta K, L_O K, L_L R$</td>
<td>5</td>
<td>1960q1-1996q4</td>
<td>1</td>
</tr>
<tr>
<td>Augmented Putty-Clay</td>
<td>$\Delta K, L_Y, L_C, L_L R$</td>
<td>6</td>
<td>1960q1-1996q4</td>
<td>0</td>
</tr>
<tr>
<td>Augmented $Q$</td>
<td>$\Delta K, Q, Y/K$</td>
<td>5</td>
<td>1955q4-1996q4</td>
<td>1</td>
</tr>
<tr>
<td>Composite Model</td>
<td>$\Delta K, L_Y, L_C, Q$</td>
<td>5</td>
<td>1955q4-1996q4</td>
<td>0</td>
</tr>
<tr>
<td>BCS</td>
<td>$\Delta K, L_O, L_Y, L_T P_I, M_C I, R_I R$</td>
<td>5</td>
<td>1958q1-1996q4</td>
<td>1</td>
</tr>
<tr>
<td>ACS(a)</td>
<td>$\Delta K, L_O, L_Y, L_T P_I, M_C I, R_I R, L_W$</td>
<td>5</td>
<td>1965q1-1996q4</td>
<td>2</td>
</tr>
<tr>
<td>ACS(b)</td>
<td>$\Delta K, L_O, L_Y, L_T P_I, M_C I, R_I R, R_E$</td>
<td>5</td>
<td>1964q1-1996q4</td>
<td>2</td>
</tr>
<tr>
<td>ACS(c)</td>
<td>$\Delta K, L_O, L_Y, L_T P_I, M_C I, R_I R, (I_I, 1(0))$</td>
<td>5</td>
<td>1959q1-1996q4</td>
<td>1</td>
</tr>
<tr>
<td>BMS</td>
<td>$\Delta K, L_O, L_Y, L, L_E, L_L R, R_I R$</td>
<td>5</td>
<td>1960q1-1996q4</td>
<td>1</td>
</tr>
<tr>
<td>AMS</td>
<td>$\Delta K, L_O, L_Y, L (BO, BO(-1), L_C U, L_C U(-1), 1(0))$</td>
<td>5</td>
<td>1960q1-1996q4</td>
<td>1</td>
</tr>
</tbody>
</table>
5.5 Concluding Remarks on Estimation

In this chapter a number of models of private industrial and commercial construction output have been estimated using three different techniques. In Section 5.2 the four basic theories of investment are modelled in a single equation framework. Although models of this kind are most frequently estimated in the literature, the analysis in this chapter and in Section 4.4 show that they are econometrically invalid, in the sense that they were unbalanced or likely to generate spurious results. As such, the theoretically consistent versions of the models were respecified to give econometrically sound equations. A summary of the within sample properties of the latter equations is given in Section 5.2.3. It is the econometrically sound versions of the models that we take in to the forecasting contests of Chapter 6.

In Section 5.3 a number of ARIMA models of private industrial and commercial construction output were developed. The initial intention was to develop a single ARIMA model, the forecasts from which would serve as a base with which to compare forecasts derived from models using other methods of estimation. However, a number of issues relating to the appropriate transformations of the data were not resolved and therefore, four model portfolios were developed each corresponding to a particular data transformation. Diagnostic testing enabled discrimination between specifications within model portfolios, thereby revealing a preferred model. The preferred models from each of the four portfolios are taken into the forecasting exercise of Chapter 6, where it is hoped that further discrimination based on forecasting performance will result in a ‘winning model’ which will provide a benchmark for comparison.

There is no reason why the models estimated in Section 5.2, which were developed from sound theoretical premises, should do particularly well in the forecasting exercise of Chapter 6. In the models of Section 5.2, net investment is typically determined by current and lagged values of a single determinant (i.e. changes in output, optimal capital stock or \(Q\)). In the case of the \(Q\) model in particular, a large part of investment is unexplained by the \(Q\) variable. This suggests that it should be possible to develop models of private industrial and commercial construction output that provide superior forecasting performance by departing from a rigorous implementation of investment theory. This provides the motivation for the VAR
models developed in Section 5.4. Here, fifteen models of private industrial and commercial construction output are estimated, some of which have elements in common with the single equation models of Section 5.2. These VARs provide the bridge between Sections 5.2 and 5.4 in that they use the variables suggested by theory but are estimated using the relatively non-structural VAR procedure. The other VARs estimated in Section 5.4 represent a complete departure from theory and have been developed using leading indicators of investment, construction output or economic activity in general, with forecasting performance in mind. To this end, a priori, one should expect these models to do relatively well in the forecasting contests of Chapter 6.
6.1 Introduction

In this chapter we use the models estimated in Chapter 5 to forecast private industrial and commercial construction output. In Section 5.3 a number of ARIMA models of private industrial and commercial construction output were estimated. It was intended that the model providing the best explanation of private industrial and commercial construction output would provide a benchmark model against which the forecasts from other models estimated in Chapter 5 could be compared. ARIMA models were developed with actual and logged data. Since the appropriate degree of differencing could not be unambiguously determined for the actual or logged data, four model portfolios were developed each corresponding to a particular data transformation. Within each portfolio were a number of models that provided an adequate explanation of the data. Diagnostic tests were used to discriminate between specifications within a portfolio yielding a preferred specification for each of the four data transformations. It was noted in Section 5.3 that discrimination between the four preferred models would be conducted on the basis of forecast performance. The specification providing the most accurate forecasts of private industrial and commercial construction output will be adopted as the benchmark model. A comparison of the forecasting performance of these four ARIMA models is given in Section 6.2.

In Section 6.3, forecasts of private industrial and commercial construction output are generated using the models of investment behaviour estimated in Section 5.2. These are compared with each other and with the forecasts from the benchmark ARIMA model. In Section 6.4 we assess the forecasting performance of the VAR models estimated in Section 5.4. An overall evaluation of the forecasting ability of these models is given in Section 6.5.

In Chapter 3 we considered a number of issues surrounding the evaluation of forecasts (see Sections 3.5 and 3.6). Before evaluating the forecast performance of the models estimated in Chapter 5, we summarise the approach to forecast evaluation to be adopted here. The models estimated in Chapter 5 are re-estimated using data up to 1993q4. Given that our sample period extends to 1996q4, this leaves twelve quarters
of actual data with which to evaluate forecast performance. In this sense, the forecasts generated in this chapter are *ex post* forecasts. We use the models to generate one-step-ahead, four-step-ahead and twelve-step-ahead forecasts for private industrial and commercial construction output. We place more importance on the four-step-ahead and twelve-step-ahead forecast performance since users of such models are likely to be more interested in forecasts over the longer horizon. For all forecasts we calculate the values of the MSFE, the MAPFE and Theil's $U_2$. For the one-step-ahead and four-step-ahead forecasts we are able to use the modified Diebold-Mariano procedure to examine whether one forecast generating procedure is significantly better than another. We also test the one-step-ahead forecasts for conditional efficiency. Since we have only one twelve-step-ahead forecast for each model, evaluation is undertaken on the basis of analysis of time series plots of forecasts and forecast errors and simple comparisons of actual evaluation criteria characterising the sample evidence. It is noteworthy that different models are estimated with different transformations of the private industrial and commercial construction output data. For example, the econometric models (estimated in Section 5.2) use first differenced data whereas the VAR models (estimated in Section 5.4) use first differenced logged data. In order that the forecasts generated by these models are comparable, we transform all forecasts such that they are comparable with the actual level of private industrial and commercial construction output before evaluating accuracy. The final point to note about this evaluation exercise is that the forecast performance of a particular model is first compared with the performance of other models of the same type. That is, the accuracy of a particular econometric model for example, is first compared with the performance of the other econometric models estimated in this work. The performance of these models is then compared with that of the benchmark ARIMA model (to be determined in the following section). Only then are comparisons between the forecast performance of econometric models and VAR models made.

6.2 Forecasts from the ARIMA Models

In Section 5.3 a number of ARIMA models of private industrial and commercial construction output were estimated. It was intended that the model providing the best
explanation of private industrial and commercial construction output would provide a benchmark model against which the forecasts from other models estimated in Chapter 5 could be compared. ARIMA models were developed with actual and logged data. Since the appropriate degree of differencing could not be unambiguously determined for the actual or logged data, four model portfolios were developed, each corresponding to a particular data transformation. Within each portfolio were a number of models that provided an adequate explanation of the data. Diagnostic tests were then used to discriminate between specifications within a portfolio, yielding a preferred specification for each of the four data transformations. We have an ARIMA(5,1,5) and a SARIMA(0,1,4)(0,1,0) for the actual data. For the logged data we obtained an ARIMA(4,1,4) and a SARIMA(0,1,4)(0,1,0). With the exception of the ARIMA(5,1,5), all models satisfied the diagnostic checks. It was noted in Section 5.3.5 that the models with logged data were preferred over those using actual data, on the grounds that the residuals from latter displayed some evidence of increasing variance. Of the two logged models, the ARIMA(4,1,4) model was preferred on the grounds that the private industrial and commercial construction output series was unlikely to contain a seasonal unit root.

6.2.1 Discriminating Between Four ARIMA Specifications

It was noted in Section 5.3 that formal discrimination between the four preferred models would be conducted on the basis of forecast performance. The specification providing the most accurate forecasts of private industrial and commercial construction output will be adopted as the benchmark model. As discussed in Section 3.5.4, forecast performance is judged using a selection of objective forecast evaluation criteria. We focus attention on the MSFE, the MAPFE and Theil’s $U_2$ statistic and employ the modified Diebold-Mariano test. For dynamic forecasts, we also use graphical analysis of the forecast and forecast error series.

As discussed in Section 3.6, the forecast period is twelve quarters long, beginning in 1994q1 and ending in 1996q4. The models are reestimated using data up to 1993q4. Since the one-step-ahead forecasts from a model are simply its predicted values, the MSFE is simply a measure of the model’s fit over this period. The respective
evaluation criteria for the four ARIMA models is given in Table 6.1. It is noteworthy that in models estimated with logged data, the exponent of forecasts are taken prior to calculating the value of the criteria, in order to allow direct comparisons between specifications.

Table 6.1: MSFE, MAPFE and $U_2$ for four ARIMA models

<table>
<thead>
<tr>
<th>Specification</th>
<th>one-step-ahead</th>
<th>four-step-ahead</th>
<th>dynamic forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSFE</td>
<td>MAPFE</td>
<td>$U_2$</td>
</tr>
<tr>
<td>$\Delta\Delta\Delta K$</td>
<td>(0,4)</td>
<td>10,339</td>
<td>3.3014</td>
</tr>
<tr>
<td>$\Delta\Delta\Delta L$</td>
<td>(0,4)</td>
<td>11,185</td>
<td>3.4741</td>
</tr>
<tr>
<td>$\Delta K$</td>
<td>(5,5)</td>
<td>9,465</td>
<td>3.0137</td>
</tr>
<tr>
<td>$\Delta L K$</td>
<td>(4,4)</td>
<td>9,870</td>
<td>3.2647</td>
</tr>
</tbody>
</table>

On the basis of the implied one-step-ahead forecasts, the ARIMA(5,1,5) model of private industrial and commercial construction output appears to perform best, although the performance is only marginally better than the ARIMA(4,1,4) model with logged data. The modified Diebold-Mariano test reveals that the differences in the MSFE corresponding to these two models are insignificant (the statistic is calculated as 0.157 and the 5% $t$-distribution critical value for a two sided alternative is 2.20). The seasonally differenced models appear to be inferior to the models estimated with first differenced data. However, the modified Diebold-Mariano statistic suggests that the difference between the MSFE on the ARIMA(5,1,5) model and that on the SARMA(0,1,4)(0,1,0) model with logged data is again insignificant. It is noteworthy that the MSFEs are consistent with the $R^2$ statistics given in Table 5.17. The fact that the ARMA(5,1,5) data appears to fit the data best may be due the relatively large number of parameters estimated.

The ranking of models changes, however, when we consider the dynamic forecasting performance of these models. This is, of course, the more important test given our aim is to select one of these four models on the basis of longer term forecasting performance. According to the three evaluation criteria, the ARIMA(4,4) specification with logged data generates much more accurate forecasts over the twelve period forecasting horizon. For example, the MSFE for the ARIMA(4,1,4) with logged data is 6563 which is much lower than that of its closest rival, the ARIMA(5,1,5), which has an MSFE of 30607. The three measures unanimously agree as to the ranking of the four models based on the accuracy of the dynamic forecasts. We can not use the
Diebold-Mariano test here since we only have the one twelve-period-ahead forecast for each model and so, strictly speaking, it does not make sense to calculate the mean of the squared forecast errors. The MSFE presented in Table 6.1 is calculated as the sum of squared error averaged over the quarter twelve forecast horizon.

Time series plots of the dynamic forecasts generated from the four models are given in Figure 6.1 along with the out-turn data. The SARIMA models generate very similar forecasts, but both grossly under predict private industrial and commercial construction output. The forecast error variances can be clearly seen to increase throughout the forecast period. The ARIMA(5,1,5) model over predicts consistently after 1994q2. In this case, however, the forecast error variance first increases and then begins to fall after 1996q1. Of the four forecasts, this model generates the smallest forecast error in the final period of the forecast horizon, 1996q4. The ARIMA(4,1,4) model of the log of private industrial and commercial construction output certainly appears to perform best of the four models. Although there is a tendency for over prediction, the forecast errors are both positive and negative. The forecast errors from this model are plotted in Figure 6.2. The variance of the forecast errors appears to be reasonably constant with the largest errors occurring in 1994q4 and 1996q3. The error mean, however, does not appear to be zero but this can not easily be tested since the forecast errors associated with any multiple-step-ahead forecast are unlikely to be white noise.

The ARIMA(4,1,4) model of logged private industrial and commercial construction output also appears to provide superior four-step-ahead forecasts. Given that we have nine four-step-ahead forecasts for each model, we can calculate the MSFEs and make comparisons between them using the Diebold-Mariano test. The MSFE corresponding to forecasts from the ARIMA(4,1,4) model with logged data is significantly lower than that corresponding to the SARIMA(0,1,4)(0,1,0) model with actual data (the value of the Diebold-Mariano test statistic is 2.648).

Thus, we can conclude that although there is not much to choose between the models in terms of one-step-ahead forecast performance, the ARIMA(4,1,4) model generates four-step-ahead forecasts that are significantly more accurate than those generated by the other models considered here. Moreover, examination of Figure 6.1 reveals that the model is likely to track private industrial and commercial construction output over
Figure 6.1: Dynamic forecasts generated by the four ARIMA models

Forecast period

- $\Delta \Delta \Delta K(0,4)$
- $\Delta \Delta 4K(0,4)$
- $\Delta \Delta K(5,5)$
- $\Delta L \Delta K(4,4)$

Out-turn data

1991q1 1992q1 1993q1 1994q1 1995q1 1996q1

Figure 6.2: Dynamic forecast errors from the ARIMA(4,1,4) model

1993q4 1994q4 1995q4 1996q4
a longer forecast horizon. Recall, the objective of this section was to discriminate between models on the basis of their forecast performance in order to obtain a benchmark model. We therefore select the ARIMA(4,1,4) model as our benchmark model. The fact that the dynamic forecast errors do not appear to have a zero mean implies that there is scope for improved forecasts of private industrial and commercial construction output from other models. In the subsequent sections of this chapter, forecasts generated from other models will be compared with the forecasts from this benchmark model.

6.2.2 A Comment on Forecasts from the Benchmark Model

The ARIMA(4,1,4) model of logged private industrial and commercial construction output is to provide benchmark forecasts with which forecasts generated from other models are to be compared. The model has been selected from many estimated in Section 5.3. From the four model portfolios developed in Section 5.3, four preferred models were determined on the basis of the AIC and diagnostic testing. Each of the preferred models is estimated for a different transformation of the private industrial and commercial construction output series. The ARIMA(4,1,4) model, estimated with logged data, provides the best forecasts of the four preferred models (for the period under consideration).

The ex post (1994q1 to 1996q4) and ex ante (1997q1 to 2001q1) forecasts from the ARIMA(4,1,4) model of logged private industrial and commercial construction output are plotted in Figures 6.3 and 6.4 (with confidence intervals). Note that these are respectively 12 and 17 step ahead forecasts. The ex ante forecasts (in Figure 6.3) display a regular annual cycle and predict constant annual growth up to 2001q1. Over the four year period from 1996q4 to 2000q4, private industrial and commercial construction output is forecast to grow by a total 15.9%. Even after such growth, private industrial and commercial construction output will still be substantially lower than its peak value in 1990.

One notable aspect of these forecasts (both ex post and ex ante) is the very wide confidence intervals. These are in fact so wide that they dominate the time series plots in Figure 6.4. The forecast standard errors indicate that the central estimates have not
Figure 6.3: Ex post and ex ante forecasts from the ARIMA(4,1,4) model

Out-turn data

Figure 6.4: Ex post and ex ante forecasts from the ARIMA(4,1,4) model (with confidence intervals)
been estimated with a high degree of precision. This problem is common to many long-term dynamic forecasts. We adopt the ARIMA(4,1,4) as our benchmark model on the grounds that the forecasts it generates are more accurate than those generated by the other specifications considered in this section. However, given the naive technique used to generate these forecasts and the fact that the dynamic forecasts errors appear not to have a zero mean, we expect the investment models estimated in Section 5.2 and the VAR models estimated in Section 5.4 to generate superior forecasts.

6.3 Forecasting with Traditional Models

In this section we use the traditional models of investment estimated in Section 5.2 to generate forecasts of private industrial and commercial construction output for the period 1994q1 to 1996q4. The models are re-estimated using data only up to 1993q4 so that the ex post static and dynamic forecasts accuracy can be assessed. In Section 6.3.1 the relative forecasting performance of each of the traditional models is assessed. The accuracy of these forecasts is compared in Section 6.3.2 with the accuracy of forecasts generated from similar models published in the empirical literature. Then, in Section 6.3.3, the forecasting performance of each of the traditional models is compared with that of the benchmark ARIMA model. Comparisons with the forecasts from the VAR models, discussed in Section 6.4, are reserved until Section 6.5.

6.3.1 The Relative Forecast Performance of the Traditional Models

The one-step-ahead (or static) and multiple-step-ahead (or dynamic) forecasts for private industrial and commercial construction output generated by the accelerator, neoclassical, putty-clay and $Q$ models are given in Table 6.2, along with the out-turn data, for the period 1994q1 to 1996q4. The models are given in equations 5.4, 5.8, 5.12 and 5.17, respectively. The forecasts and out-turn data in Table 6.2 are used to calculate the MSFE, the MAPFE, and Theil's $U_2$ statistic. The values of these criteria are given in Table 6.3 for static, four-step-ahead and twelve-step-ahead dynamic forecasts. It is noteworthy that forecasts from the $Q$ model (which has $\Delta K/K$ as its
dependent variable) have been multiplied by $K$ prior to calculating the value of the accuracy criteria. In so doing, forecasts from the $Q$ model are directly comparable with the forecasts generated from the accelerator, neoclassical and putty-clay models. The MSFE, the MAPFE and the $U_2$ statistic for the benchmark ARIMA model, discussed in Section 3.5.2, are also given in Table 6.3.

Table 6.2: Out-turn and forecast data generated by the traditional models

<table>
<thead>
<tr>
<th>Specification</th>
<th>Out-turn</th>
<th>Accelerator (5.4)</th>
<th>Neoclassical (5.8)</th>
<th>Putty-Clay (5.12)</th>
<th>$Q$ (5.17)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>static 4-step</td>
<td>static 4-step</td>
<td>static 4-step</td>
<td>static 4-step</td>
</tr>
<tr>
<td>1993q4</td>
<td>2421.1</td>
<td>2421.1</td>
<td>2421.1</td>
<td>2421.1</td>
<td>2421.1</td>
</tr>
<tr>
<td>1994q1</td>
<td>2249.8</td>
<td>2303.0</td>
<td>2266.5</td>
<td>2354.1</td>
<td>2354.1</td>
</tr>
<tr>
<td>1994q2</td>
<td>2486.4</td>
<td>2333.5</td>
<td>2384.5</td>
<td>2400.9</td>
<td>2469.0</td>
</tr>
<tr>
<td>1994q3</td>
<td>2530.2</td>
<td>2535.7</td>
<td>2438.9</td>
<td>2545.2</td>
<td>2546.7</td>
</tr>
<tr>
<td>1994q4</td>
<td>2637.3</td>
<td>2520.3</td>
<td>2434.6</td>
<td>2512.0</td>
<td>2546.7</td>
</tr>
<tr>
<td>1994q5</td>
<td>2313.4</td>
<td>2578.3</td>
<td>2345.9</td>
<td>2387.6</td>
<td>2535.5</td>
</tr>
<tr>
<td>1994q6</td>
<td>2469.1</td>
<td>2444.4</td>
<td>2604.9</td>
<td>2539.6</td>
<td>2555.5</td>
</tr>
<tr>
<td>1995q1</td>
<td>2633.2</td>
<td>2568.2</td>
<td>2691.5</td>
<td>2573.7</td>
<td>2555.5</td>
</tr>
<tr>
<td>1995q2</td>
<td>2626.4</td>
<td>2633.4</td>
<td>2800.0</td>
<td>2524.3</td>
<td>2555.5</td>
</tr>
<tr>
<td>1995q3</td>
<td>2300.2</td>
<td>2587.4</td>
<td>2543.1</td>
<td>2438.9</td>
<td>2555.5</td>
</tr>
<tr>
<td>1995q4</td>
<td>2520.3</td>
<td>2320.6</td>
<td>2387.6</td>
<td>2545.2</td>
<td>2555.5</td>
</tr>
</tbody>
</table>

Table 6.3: MSFE, MAPFE and $U_2$ for the traditional models

<table>
<thead>
<tr>
<th>Specification</th>
<th>one-step-ahead</th>
<th>four-step-ahead</th>
<th>dynamic forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSFE MAPFE $U_2$</td>
<td>MSFE MAPFE $U_2$</td>
<td>MSFE MAPFE $U_2$</td>
</tr>
<tr>
<td>Accelerator (5.4)</td>
<td>15,085 3.925 0.0482</td>
<td>14,167 3.9158 0.0459</td>
<td>11,103 3.3451 0.0413</td>
</tr>
<tr>
<td>Neoclassical (5.8)</td>
<td>8,941 2.9507 0.0371</td>
<td>19,169 4.3047 0.0534</td>
<td>26,730 4.9134 0.0641</td>
</tr>
<tr>
<td>Putty-clay (5.12)</td>
<td>17,833 4.5917 0.0523</td>
<td>8,824 3.2233 0.0362</td>
<td>11,751 3.3769 0.0425</td>
</tr>
<tr>
<td>$Q$ (5.17)</td>
<td>6,035 2.5478 0.0304</td>
<td>7,444 2.7611 0.0332</td>
<td>8,208 2.6532 0.0355</td>
</tr>
<tr>
<td>ARIMA(4,1,4)</td>
<td>9,870 3.2647 0.0389</td>
<td>10,415 3.2267 0.0394</td>
<td>6,563 2.4245 0.0318</td>
</tr>
</tbody>
</table>

The first point to note about Table 6.3 is the fact that the square root of the MSFEs are generally larger than the standard errors of the regressions during the sample (see Table 5.11). In view of the serial correlation in these models, this is to be expected. Recall, the within sample fit of these models is artificially enhanced by the correction for serial correlation.

Considering the one-step-ahead forecasts, the $Q$ model of investment (given by equation 5.17) yields a MSFE, a MAPFE, and a $U_2$ statistic that are lower than the accelerator, neoclassical and putty-clay models. On the basis of these criteria, the neoclassical model appears to generate the second most accurate within sample forecasts. All three criteria suggest that the putty-clay model performs least well. We
use the modified Diebold-Mariano statistic to see if the differences in performance, indicated by the MSFEs, are significant. The MSFE generated by the $Q$ model is not significantly lower than that generated by the neoclassical and accelerator models (the respective statistics are 1.571 and 1.816). However, the MSFE generated by the $Q$ model is significantly lower than that generated by the putty-clay model (the test statistic is 2.27). These findings are broadly consistent with the within sample fit of the models as measured by the $R^2$'s (see Table 5.11).

The ranking of models changes when we consider the dynamic forecast performance. The criteria in Table 6.3 suggest that the $Q$ model provides the most accurate dynamic forecasts and the accelerator and putty-clay models are ranked second and third respectively. Despite its reasonably good within sample performance, the neoclassical model performs least well in terms of the accuracy of the dynamic forecasts it generates. The dynamic forecasts generated by the four models are presented graphically in Figure 6.5. It can be clearly seen from this plot that the neoclassical and $Q$ models predict private industrial and commercial construction output in the first quarter of each year very well, but under predict in all other quarters. The accelerator and putty-clay models on the other hand, predict private industrial and commercial construction output in the first quarter of each year quite poorly and both tend to over predict throughout the forecast period. All models pick up the shape of the seasonal pattern, but not the extent of the variation. This may be a result of the fact that the models are not estimated with logged data. In this case, the coefficients on the seasonal dummies pick up average seasonal variation, which is likely to be less pronounced than the seasonal variation in the latter part of the sample. However, if this were true, one would expect tests for heteroskedasticity to reveal this. Another possible explanation is provided by Granger and Newbold (1986, p. 283) who show that the variance of even an optimal predictive series must be less than the variance of the actual series. The forecast errors from each of the four models are plotted in Figure 6.6. The forecast error variances tend to increase throughout the forecast horizon. This, of course, is to be expected when forecasting multiple periods ahead. The error mean appears to be positive for the neoclassical and $Q$ models and negative for the putty-clay. The mean of the errors from the accelerator model, on the other hand, appears to be close to zero in Figure 6.6. We can not check to see whether the
Figure 6.5: Dynamic forecasts generated by the four traditional models

<table>
<thead>
<tr>
<th>Forecast period</th>
<th>Accelerator (5.4)</th>
<th>Neoclassical (5.8)</th>
<th>Putty-clay (5.12)</th>
<th>Q (5.17)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-turn data</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.6: Dynamic forecast errors from the four traditional models
differences in dynamic forecast performance are significant since we have only one twelve-step-ahead forecast for each model. However, we can check to see if differences in the four-step-ahead forecast performance are significant and it is to these that we now turn.

The evidence in Table 6.3 suggests that the $Q$ model also provides the best four-step-ahead forecast performance. Again, the neoclassical model appears to do least well. Using the modified Diebold-Mariano test, we find that the four-step-ahead forecasts from the $Q$ model are significantly better than those generated by the accelerator and neoclassical models (the respective test statistics are 4.99 and 2.58), but they are not significantly better than those generated by the putty-clay model (the test statistic is 0.468). However, since the four-step-ahead forecasts generated by the putty-clay model are not significantly better than those generated by the accelerator model (the test statistic is 1.71), we conclude that on the $Q$ model provides the best four-step-ahead forecasts.

Overall, the evidence presented in this subsection suggests that the $Q$ model outperforms the other investment models in this forecasting competition. This is rather unexpected, given that contemporaneous $Q$ explains such a small proportion of variations in net investment (see the results of Section 5.2.2.4). Given the theoretical and empirical problems with the neoclassical model, it is not surprising to find that this model performs least well in forecasting exercises of more than one-step-ahead.

In Section 6.3.3 we compare the forecasting performance of these models with that of the benchmark model. In particular, we examine whether the static forecasts from these models are conditionally efficient with respect to those from the ARIMA(4,1,4) model. First, we reconsider the results from a number of studies in which the relative forecast performance of investment models is investigated.

6.3.2 A Comparison with Evidence in the Empirical Literature

In this section, we compare the results of the previous section with results of similar studies of forecasting performance published in the empirical literature. We focus our attention on five studies, all of which attempt to compare the models of investment behaviour systematically. In each of these studies authors go to considerable lengths
to ensure that fair comparisons of model performance are made. However, the
collection of the results of one study with those of another can only be done on a
superficial basis. Different studies focus on a different preferred set of models and
whilst the choice of models is broadly consistent across the five studies under
consideration here, subtle differences in the theoretical specification and
implementation of the models do exist. Even if the principal aim of all of the studies
is to compare the forecasting performance of a fixed set of identically specified
models, there are a number of other ways in which the studies will necessarily differ.
For example, the definition of the investment variable under consideration may differ
between studies. One study may use investment as its dependent variable, whereas
another may use investment as a share of capital stock or output. Moreover, one study
may be interested in gross investment, whereas another may use a net construct. The
country under consideration, estimation period, estimation technique, periodicity and
seasonality of the data used, the length of the forecasting horizon, and the criteria used
to assess forecast accuracy are all subject to variation between studies. Given these
sources of difficulty, only broad comparisons can be made across studies.

The studies under consideration here are those of Bischoff (1971b), Clark (1979),
Jenkinson (1981), Kopcke (1985) and Bernanke et al (1988). We discuss these studies
only within the context of comparisons of models’ forecasting performance. The
wider implications of these papers are discussed in Chapter 2. A synopsis of the
results of these studies is given in Table 6.4.

Bischoff (1971b) actually estimates five models of investment. These models are the
generalised accelerator model, the cash flow model, the $Q$ model (which he refers to
as the securities value model), the standard neoclassical model and a putty-clay
version of the neoclassical model. These five models are estimated for equipment and
construction. Only the results relating to the latter are reported in Table 6.4. For
construction, the standard neoclassical and generalised accelerator models provide the
best forecasts. The respective RMSFE are 0.5 and 0.7. The standard neoclassical
model, in particular, follows the movement in the actual series very well, although it
does predict the 1969q3 peak in investment one quarter too early. The cash flow
model is definitely inferior. Bischoff notes that this is unexpected since the cash flow
model provided the second best explanation of both equipment and structures
investment within sample. The \( Q \) (securities value) equations tend to over predict the actual series and estimated peaks are predicted one period late. The root mean square error on the securities value equation is 1.9, and this exceeds the root mean squared error on all other equations. Thus, the general conclusion from these tests is that, for construction, the output based models, namely the accelerator and standard neoclassical models, perform best over the forecast period under consideration, though no single model is clearly superior to the others.

Table 6.4: The relative forecast performance of alternative investment models

<table>
<thead>
<tr>
<th>Study</th>
<th>Dependent Variable</th>
<th>Estimation Period</th>
<th>Forecast Horizon</th>
<th>Evaluation Criterion</th>
<th>Accelerator</th>
<th>Neoclassical</th>
<th>Putty-Clay</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bernanke et al (1988)</td>
<td>net US investment in nonresidential structures over potential output</td>
<td>51q1-73q2 73q3-83q4</td>
<td>RMSFE</td>
<td>0.06</td>
<td>0.06</td>
<td>0.07</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Bischoff (1971b)</td>
<td>gross US investment in nonresidential structures</td>
<td>53q1-68q4 69q1-70q4</td>
<td>RMSFE</td>
<td>0.7</td>
<td>0.5</td>
<td>1.5</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>Clark (1979)</td>
<td>gross US investment in nonresidential structures over potential output</td>
<td>54q1-73q2 73q3-78q4</td>
<td>forecast standard error</td>
<td>0.54</td>
<td>0.46</td>
<td>0.45</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>Jenkinson (1981)</td>
<td>gross UK aggregate investment by ICCs</td>
<td>67q2-76q3 76q4-78q3</td>
<td>not given</td>
<td>not given</td>
<td>not estimated</td>
<td>not given</td>
<td>not given</td>
<td></td>
</tr>
<tr>
<td>Kopcke (1985)</td>
<td>gross US investment in nonresidential structures</td>
<td>56q1-79q4 80q1-84q4</td>
<td>RMSFE</td>
<td>3.9</td>
<td>not estimated</td>
<td>5.3</td>
<td>7.5</td>
<td></td>
</tr>
<tr>
<td>Authors</td>
<td>net UK investment in industrial and commercial buildings</td>
<td>55q1-93q4 94q1-96q4</td>
<td>MSFE</td>
<td>16.664</td>
<td>6.053</td>
<td>11,751</td>
<td>8,208</td>
<td></td>
</tr>
</tbody>
</table>

Notes: 1. Actual forecast standard error as a percentage of potential GDP. 2. Except for the \( Q \) model in which investment is deflated by capital stock. 3. Except for the \( Q \) model in which investment is deflated by a measure of capacity utilisation.

But for the inclusion of an accelerator term in a cash flow model, Clark (1979) has estimated the same models as Bischoff (1971b). These five models are applied to investment expenditure in equipment and structures separately, for the period 1954-73. The equations are then used to forecast from 1973 to 1978 and their relative performance is assessed against the actual data. Clark (1979) uses the estimated
within sample standard error as his measure of expected forecasting ability. On this criteria he expects the putty-clay model to forecast best, with the hybrid accelerator-cash flow model doing slightly better than the accelerator. The estimated standard error on the neoclassical model leads Clark to expect that this model will perform least well. The first point to note about the actual forecasts for structures is that all models give rise to negative errors through most of the forecast horizon. With the possible exception of the accelerator-cash flow model, all models substantially over predict investment. Structures investment fell much more rapidly between 1973 and 1975 than any of the models predicted. Only towards the end of the forecast horizon do the actual values begin to move closer to the predicted values. With the exception of the hybrid accelerator-cash flow model, the actual forecast standard errors are much larger than the expected values, estimated within sample. Based on the actual forecast standard errors, there is not much to choose between the accelerator, neoclassical and putty-clay models with slight preference being given to the latter. The $Q$ model appears to do least well, but this is a result of the dependent variable being investment, deflated by capital stock, rather than potential output. In fact, the $Q$ equation performs better than the accelerator, neoclassical and putty-clay models over the five year forecasting horizon.

Bernanke, Bonn and Reiss (1988) note that given the arbitrariness of within or out of sample goodness of fit criteria used to rank model performance, it is hardly surprising that investigators have been in disagreement on the merits of alternative models. Bernanke et al estimate the same models as Bischoff (1971b) and Clark (1979) (without the cash flow model), again with US data, with a view to discriminating between them using the more rigorous techniques of non-nested testing. These were used in addition to criteria such as root mean squared error, Durbin-Watson and $R^2$ statistics, and tests for structural stability that have frequently been used to discriminate between models in earlier studies. For structures, these conventional criteria indicate that no model uniformly outperforms the other models, both within sample or out of sample. The out of sample RMSFEs given in Table 6.4, for example, are very similar (around 0.06 or 0.07). These are almost identical to the within sample RMSFEs. When these models are compared on the basis of non-nested tests that take into account serial correlation in the residuals, all the models are rejected by at least
one of the other models. However, Monte Carlo simulations carried out by Bernanke \textit{et al} indicate that the non-nested tests they employ are biased in favour of a rejection of the null, and consequently, they state that test results should be viewed with caution.

Kopcke (1985) compares the accelerator, the cash flow, the neoclassical, the $Q$ and a univariate autoregressive model of investment in terms of within sample fit and out-of-sample forecasting performance. Kopcke’s neoclassical model allows for the fact that investment can respond to changes in output differently to changes in user cost, and thus may be more accurately described as a putty-clay model in the context of this work. The models are estimated with 24 years of quarterly data from 1956 to 1979 for US investment expenditure in equipment and non-residential structures separately. These models were then used to forecast both series from 1980 to 1984 and these forecasts were compared with actual data. Both within and out-of-sample performance was evaluated using mean absolute error and root mean squared error statistics. All of Kopcke’s equations fit the data very well within sample but, on the basis of these criteria, no equation dominates. When the models are used to produce static forecasts, i.e. one-step-ahead forecasts, no single model significantly outperforms the others significantly. The main differences in the performance of the models are revealed most clearly in the dynamic forecasts. For structures, the $Q$ equation performs worst on the basis of the adopted criteria. These results contrast with those of Clark (1979). The forecast statistics indicate that the autoregressive, accelerator and neoclassical models (in that order) provide the best forecast of investment in structures, but visual inspection of the actual and forecast data indicates that none of the models fit the cyclical course of actual investment well.

Jenkinson (1981) has also examined the forecasting performance of alternative models of investment. Using quarterly data, he estimates a $Q$ model, a neoclassical model and an accelerator model for investment undertaken by UK industrial and commercial companies over the period 1967-76. In fact, although Jenkinson uses the term neoclassical model, the model that he actually estimates is a version of the model first proposed by Feldstein and Flemming (1971) in which investment is allowed to respond differently to changes in output and user cost. In the context of the terminology used in this work, this model is better described as a putty-clay model,
although it is not derived as such from first principles. Although Jenkinson is not able to say that the $Q$ model outperforms the other two models, he does state that the results of the $Q$ and neoclassical models, assessed in terms of goodness of fit criteria, are at least as good as those derived from the accelerator model. Jenkinson evaluates forecast performance only by visual inspection of the actual and forecast data.

In conclusion, we can say that although a number of authors have attempted to form a consensus model of investment demand by evaluating the performance of alternative models on the same set of data, these attempts have not been particularly successful. In some cases, authors have not been able to discriminate between models (eg Jenkinson (1981) and Bernanke *et al* (1988)), either within sample or out of sample, and in other cases, in which the ranking of alternative models is determined, the margin of determination is not convincing. Kopcke (1985) suggests that the accelerator model narrowly beats the putty-clay model, but both are beaten by a simple autoregressive model. In contrast, Clark (1979) ranks the $Q$ model above the putty-clay, accelerator, and neoclasical models, but all are beaten by a hybrid accelerator cash-flow model. Only Bischoff (1971b) suggests that the best forecasting performance is given by the neoclassical model, narrowly beating the accelerator. The results presented in the previous section suggest a ranking of investment models that is not consistent with the rankings given in any of these studies. As in Clark (1979), the $Q$ model does relatively well. However, Clark's $Q$ model, like the $Q$ models estimated in other studies considered in this section, is a distributed lag model. In this work, the change in contemporaneous $Q$ (and lagged net investment) explains the change in net investment. We have already noted that lagged net investment is the dominant explanator of net investment in all of the models estimated here (see Section 5.2.3). Unlike in the Bischoff study, the neoclassical model performs least well over the 1994q1 to 1996q4 forecasting horizon. More generally, we conclude that models containing quantity variables do least well in the task of forecasting private industrial and commercial construction output. This result contrasts with the conclusion reached following analysis of the empirical investment literature given in Chapter 2. In the next section, we compare the forecasting performance of our investment models with the performance of the ARIMA(4,1,4) model estimated in Section 5.3 and discussed in Section 6.2.
6.3.3 Comparisons with the Benchmark Forecast

In this section we compare the forecasting performance of the traditional models of investment with the performance of the ARIMA(4,1,4) model. In Section 6.3.1, it was determined that the \( Q \) model generates the most accurate static and dynamic forecasts of investment over the horizon 1994q1 to 1996q4.

The forecasts from the ARIMA(4,1,4) model of the log of investment provide the benchmark against which the forecasts generated from the traditional or VAR models are compared. The MSFE, the MAPFE and the \( U_2 \) statistics for the traditional models and the ARIMA(4,1,4) model are given in Table 6.3. On the basis of these criteria, it appears that the one-step-ahead forecasts generated by the \( Q \) model are slightly superior than those generated by the ARIMA model. For example, the \( Q \) model and ARIMA model give rise to MSFEs of 6035 and 9870, respectively. This is simply a reflection of the fact that the \( Q \) model fits the data better than the ARIMA model over this period. The criteria in Table 6.3 also suggest that the neoclassical model generates more accurate one-step-ahead forecasts than those generated by the ARIMA model. We can examine whether these apparent differences are statistically significant using the Diebold-Mariano test. Applying this test, we find that neither the \( Q \) model, nor the neoclassical model, provide one-step-ahead forecasts that are significantly more accurate than those generated by the ARIMA model (the test statistics are 1.47 and 0.56 for the \( Q \) and neoclassical models respectively).

We also examine whether the one-step-ahead forecasts from these models are conditionally efficient with respect to the ARIMA model using the procedure suggested by Granger and Newbold (1986) and described in Section 3.5.1. We find that both the \( Q \) model and the neoclassical model generate one-step-ahead forecasts that are conditionally efficient with respect to the forecasts from the ARIMA(4,1,4) model (the test statistics are 0.67 and -0.15 for the \( Q \) and neoclassical models respectively). The one-step-ahead forecasts generated by the accelerator and putty-clay models are conditionally inefficient with respect to the ARIMA model at the 5% level of significance (the respective test statistics are -2.64 and -3.48).

For the dynamic forecasts, the evidence in Table 6.3 suggests that none of the models outperform the ARIMA model. The MSFE on the \( Q \) model is 8208, whereas on the
ARIMA model the MSFE is 6563. The question is whether the MSFE on the ARIMA model is significantly lower than that on the $Q$ model. Unfortunately, given that we have only one twelve-step-ahead forecast for each model, we can not make comparisons based on the usual criteria of statistical significance. However, we can make such comparisons between the four-step-ahead forecasts generated from the two models. The MSFE suggests that the $Q$ model generates more accurate four-step-ahead forecasts than the ARIMA model. The modified Diebold-Mariano test suggests that this difference is not statistically significant at the 5% level (the test statistic being 2.18), although it is at the 10% level. We also use this test to examine whether the four-step-ahead forecasts from the ARIMA model are significantly more accurate than those from the neoclassical model. The results suggest that they are not (the test statistic is 0.717).

On balance, we conclude that only the neoclassical and $Q$ models generate static forecasts that are conditionally efficient with respect to the ARIMA(4,1,4) model. However, we can not say that the one-step-ahead forecasts from the $Q$ model are significantly more accurate than those from the ARIMA model. The $Q$ model also generates significantly more accurate four-step-ahead forecasts than the ARIMA model (at the 10% level). In terms of dynamic forecast performance, there is not much to choose between the $Q$ model and the benchmark ARIMA model, although the descriptive measures of accuracy presented in Table 6.3 suggest that the latter is superior.

On the whole the results are very disappointing, but not surprising given the theoretical and empirical difficulties associated with these models. Only the $Q$ model could be thought of as providing an adequate forecast performance, but even this can not be said to outperform the benchmark ARIMA model in this exercise. The one-step-ahead forecasts from the accelerator and putty-clay models are not even conditionally efficient with respect to the forecast from the ARIMA model. We now turn to an examination of the forecast performance of the VAR models of private industrial and commercial construction output that were estimated in Section 5.4.
6.4 Forecasting with VAR Models

In this section we use the VAR models estimated in Section 5.4 to generate forecasts of private industrial and commercial construction output for the period 1994q1 to 1996q4. A synopsis of these VARs and the associated estimation results are given in Table 5.50. The models are reestimated using data only up to 1993q4 so that the ex post static and dynamic forecasts accuracy can be assessed. There are three points to note about the derivation of these forecasts. First, forecasts for private industrial and commercial construction output are generated using actual data for other variables in the systems rather than model generated data. Thus, the forecasts we obtain for private industrial and commercial construction output are those that would be obtained had the values of other variables in the system been forecast perfectly. Actual data is used so that fair comparisons can be made with the benchmark and the traditional model forecasts. Secondly, where possible, a logarithmic transformation has been applied to all series used in the estimation of these VAR models. Finally, since private industrial and commercial construction output (and ΔK/K) is I(1), it enters the VARs in first differences. This results in forecasts for changes in private industrial and commercial construction output. Therefore, forecasts are transformed prior to assessing accuracy, in such a way as to make them comparable with levels of the actual private industrial and commercial construction output. This allows these forecasts to be compared with those derived from the traditional and ARIMA models.

In Section 6.4.1 we evaluate the forecast performance of four basic VAR models, each of which is broadly consistent with one of the theories of investment. Recall from Section 5.4.3, these VARs were modified in order to take account of observations made in the empirical literature. The forecasting performance of these modified VARs is assessed in Section 6.4.2. In Section 6.4.3 we consider the performance of the atheoretical VARs developed in Section 5.4.4. An overview of the results from this forecasting exercise is presented in Section 6.4.4. Then, in Section 6.4.5, the forecasting performance of some of these VARs is compared with that of the benchmark ARIMA model.
6.4.1 Forecasts Generated by VARs Based on Investment Theory

The static and dynamic forecasts for net investment in industrial and commercial buildings generated from the accelerator, neoclassical, putty-clay and $Q$ type VARs are given in Table 6.5, along with the out-turn data for the period 1994q1 to 1996q4. The corresponding MSFE, MAPFE, and $U_2$ statistics for each model are given in Table 6.6. It is noteworthy that forecasts from the $Q$ type VAR, in which net investment is deflated by capital stock, have been multiplied by $K$ prior to calculating the value of the accuracy criteria. Thus, the forecasts from the $Q$ type VAR are directly comparable with those generated by the accelerator, neoclassical and putty-clay VARs.

Table 6.5: Out-turn and forecast data generated by the basic VAR models

<table>
<thead>
<tr>
<th>Actual</th>
<th>Accelerator</th>
<th>Neoclassical</th>
<th>Putty-Clay</th>
<th>$Q$</th>
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</thead>
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Table 6.6: MSFE, MAPFE and $U_2$ for the basic VAR models

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<th>four-step-ahead</th>
<th>dynamic forecasts</th>
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<td>MSFE $MAPFE$</td>
<td>$U_2$</td>
<td>MSFE $MAPFE$</td>
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<td>11,703 5.3567</td>
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<td>12,562 3.6466</td>
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<td>Neoclassical</td>
<td>13,456 3.7756</td>
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<td>39,148 6.5843</td>
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<td>Putty-clay</td>
<td>12,358 3.6575</td>
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<td>$Q$</td>
<td>7,899 2.8966</td>
<td>0.0348</td>
<td>15,048 4.0554</td>
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<td>ARIMA(4,1,4)</td>
<td>9,870 3.2647</td>
<td>0.0389</td>
<td>10,415 3.2267</td>
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On the basis of the evidence in Table 6.6, the $Q$ type VAR appears to generate the most accurate static forecasts of private industrial and commercial construction output. Recall, the traditional $Q$ model (given by equation 5.17) also generated a lower MSFE than the other traditional models. On the basis of the MSFE, the MAPFE and the $U_2$, the accelerator type VAR appears to provide the second most accurate static forecasts of private industrial and commercial construction output, although
these may not be significantly more accurate than those generated by the putty-clay type VAR. All three criteria suggest that the static forecasts from the neoclassical type VAR are least accurate. We use the modified Diebold-Mariano procedure to test whether the differences in the MSFEs, associated with the one-step-ahead forecasts from these models, are statistically significant. The test suggests that the differences are not significant. In testing the equality of the MSFE from the $Q$ type VAR and accelerator type VAR, we obtain a test statistic of 0.96. Applying the same test to the $Q$ and neoclassical type VARs we obtain a test statistic of 1.56. It is worth noting that the values of the accuracy criteria are similar to those on the traditional models given in Table 6.3 (for example, the MAPFE on the $Q$ type VAR is 2.897 as compared with 2.548 on the traditional version).

The MSFEs, MAPFEs and $U_2$ statistics are much larger for the dynamic forecasts. Moreover, on the basis of these criteria, the ranking of models changes: the putty-clay type VAR generates a slightly lower MSFE than the accelerator type VAR, the $Q$ type VAR is ranked third and the neoclassical VAR is a very poor fourth. This poor showing of the neoclassical model is not surprising given the poor performance of the traditional neoclassical model (see Table 6.3). The dynamic forecasts from these four VARs are shown in Figure 6.7. The neoclassical type VAR can clearly be seen to under predict investment: a steady fall in investment is forecast rather than a steady increase. Similarly, the $Q$ type VAR under predicts investment but forecasts an upturn in the last year of the forecast horizon to approach out-turn investment. The accelerator and putty-clay type VARs tend to over predict investment and, curiously, forecast an almost identical path for investment. This suggests that the addition of user cost to a VAR containing investment and output does very little to improve forecasting ability. Again, these results are consistent with the forecast performance of the traditional models. On the whole, the dynamic forecast performance of these models is poor. The most accurate forecasts are given by the accelerator and putty-clay type VARs. These VARs forecast almost identical investment profiles, both substantially over predicting. Given that we have only one twelve-step-ahead forecast from each model, we can not test whether the differences in performance are significant. In all cases, the expected value of mean error is substantially different from zero, implying that the forecasts are biased. Therefore, they can not be optimal.
A broadly similar picture emerges from the four-step-ahead forecasts. The accelerator and putty-clay type VARs do best and the neoclassical type VAR does least well. We use the modified Diebold-Mariano test to examine whether the differences in the MSFE are statistically significant. We find that the MSFE on the four-step-ahead forecasts from the accelerator type VAR is not significantly lower than that on the putty-clay type VAR or that on the $Q$ type VAR (the respective test statistics are 1.41 and 0.30) but it is significantly lower than that on the neoclassical type VAR (the test statistic is 2.67).

6.4.2 Results from the Modified VARs

The four VARs considered above were each augmented with a single variable suggested by authors in the empirical literature, with a view to improving the forecast performance of these models. Recall, the accelerator type VAR is augmented with a capacity utilisation variable to account for the fact that firms can meet increased demand for output by utilising existing capacity more fully. The neoclassical and putty-clay type VARs are augmented with a liquidity variable to account for the
possibility that firms might be unable to increase capacity as a result of liquidity constraints. The implicit assumption here is that capital markets are not perfect. The $Q$ type VAR is augmented in line with the observation that output tends to be significant in $Q$ equations. The static and dynamic forecast and out-turn data from the resulting VARs is given in Table 6.7 and the associated forecast evaluation criteria are given in Table 6.8.

On the basis of the static forecasts there is not much to choose between the four models: the augmented $Q$ type VAR appears to perform slightly better than the augmented accelerator or neoclassical type VARs and the augmented putty-clay type VAR does least well. The values of the MSFE, the MAPFE and the $U_2$ statistic are broadly consistent with those associated with the basic VARs. The modified Diebold-Mariano test reveals that the MSFE associated with forecasts generated by the augmented $Q$ type VAR is not significantly lower than that of the augmented accelerator, putty-clay and neoclassical type VARs (the respective statistics are 0.60, 0.46 and 1.75).

Table 6.7: Out-turn and forecast data generated by the augmented VAR models

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<th>$Q$</th>
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Table 6.8: MSFE, MAPFE and $U_2$ for the augmented VAR models

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<th>dynamic forecasts</th>
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<td>3.2558</td>
<td>0.0419</td>
</tr>
<tr>
<td>Neoclassical</td>
<td>11,665</td>
<td>3.5077</td>
<td>0.0423</td>
</tr>
<tr>
<td>Putty-clay</td>
<td>14,553</td>
<td>3.9929</td>
<td>0.0473</td>
</tr>
<tr>
<td>$Q$</td>
<td>10,462</td>
<td>3.1833</td>
<td>0.0401</td>
</tr>
<tr>
<td>ARIMA(4,1,4)</td>
<td>9,870</td>
<td>3.2647</td>
<td>0.0389</td>
</tr>
</tbody>
</table>
However, the augmentations have led to a dramatic change in dynamic forecast performance of these VARs. The MSFEs on the augmented accelerator and $Q$ type VARs are slightly higher than those on their basic versions, whereas the MSFEs on the putty-clay and neoclassical type VARs are much lower. Given that we have only one twelve-step-ahead forecast for each model, we can not determine whether the improvements in the dynamic forecast performance of neoclassical and putty-clay type VARs are statistically significant. The increase in the MSFEs following the augmentation of the basic accelerator VAR with a capacity utilisation variable and the augmentation of the basic $Q$ type VAR with an output variable, suggests that these augmentations are inappropriate. If these variables are inappropriate the variance of parameter estimates will increase as will the variance of the forecasts.

Of the augmented models, the augmented putty-clay type VAR generates the lowest dynamic MSFE followed by the augmented neoclassical type VAR. The augmented $Q$ and accelerator type VARs generate forecasts with the highest MSFE. This underlying picture is borne out in the plots of the dynamic forecasts given in Figure 6.8. The augmented putty-clay and neoclassical type VARs track investment very well throughout the forecast horizon. The forecast errors from these two models are plotted in Figure 6.9. The forecast error of the augmented neoclassical type VAR is positive for most of the horizon, but the error at the end of the forecast horizon is less than 2.5% of the actual series. The mean error for the augmented putty-clay type VAR is insignificantly different from zero and variance is constant. Again, the error at the end of the forecast horizon is less than 2.5% of the actual series. The augmented accelerator and $Q$ type VARs, on the other hand, tend to over predict throughout the forecasting horizon. Notice that none of the models generate forecasts that capture the full extent of the seasonal variation. Granger and Newbold (1986, p. 283) note that the variance of even an optimal predictor series will always be lower than the variance of the actual series. However, it is not clear that this comment should include seasonal variations, which, given deterministic seasonality are forecastable. It may be the case that deterministic seasonality is not the best descriptor of the series, but we found no evidence of stochastic seasonality in the analysis of Section 4.4.1.1.

In the four-step-ahead forecasting exercise the augmented neoclassical type VAR generates forecasts with the lowest MSFE. The modified Diebold-Mariano test,
Figure 6.8: Dynamic forecasts generated by the four modified VAR models

Figure 6.9: Dynamic forecast errors from two of the modified VAR models
however, reveals that the MSFE associated with this VAR is not significantly different from that associated with the other augmented VARs. The test generates statistics of 0.90, 0.66 and 1.15 for the accelerator, putty-clay and $Q$ VARs respectively. Despite this lack of statistical significance, the evidence in Figure 6.8 suggests that one ought to be more at ease forecasting private industrial and commercial construction output with the augmented neoclassical or putty-clay VARs.

6.4.3 Results of Forecasting with Atheoretical Models

The VAR models used to forecast private industrial and commercial construction output in Section 6.4.2 are loosely based on the four dominant theories of investment behaviour. In this section, we examine the forecasting performance of the VAR models estimated in Section 5.4.4. The composite model, the forecasting performance of which is discussed in Section 6.4.3.1, is a four equation system containing all the variables suggested by the mainstream theories of investment. The remaining VARs are based on intuition rather than any well defined theory. The forecasting performance of these VARs is examined in two sections. The first, Section 6.4.3.2, contains an assessment of the forecasting performance of those VARs that are estimated largely with data on construction sector variables. The second, Section 6.4.3.3, contains an assessment of the accuracy of forecasts from two VARs that are estimated largely with macroeconomic data. The VARs in these two sections also make use of information provided by leading indicator variables.

6.4.3.1 Forecasts from the Composite Model

The set of forecasts for private industrial and commercial construction output generated by the composite VAR is given in Table 6.9 along with the out-turn data, forecast errors and the associated forecast evaluation criteria. In this system, contemporaneous first changes in logged private industrial and commercial construction output is determined by its own lagged values and lags of logged changes in private sector output, user cost, and $Q$. The Johansen tests find no evidence of any cointegrating relationships in this system.
Table 6.9: Out-turn and forecast data from the composite VAR

<table>
<thead>
<tr>
<th>Year</th>
<th>Out-turn forecast</th>
<th>static</th>
<th>error</th>
<th>Forecasts four-step-ahead forecast</th>
<th>dynamic</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993q4</td>
<td>2421.1</td>
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<td>0.0</td>
<td></td>
<td>2421.1</td>
<td>0.0</td>
</tr>
<tr>
<td>1994q1</td>
<td>2249.8</td>
<td>2337.2</td>
<td>-87.4</td>
<td></td>
<td>2337.2</td>
<td>-87.4</td>
</tr>
<tr>
<td>1994q2</td>
<td>2486.4</td>
<td>2381.5</td>
<td>104.9</td>
<td></td>
<td>2491.7</td>
<td>-5.2</td>
</tr>
<tr>
<td>1994q3</td>
<td>2530.2</td>
<td>2631.2</td>
<td>-101.2</td>
<td></td>
<td>2606.9</td>
<td>-76.9</td>
</tr>
<tr>
<td>1994q4</td>
<td>2637.3</td>
<td>2487.7</td>
<td>149.6</td>
<td>2593.4</td>
<td>2593.4</td>
<td>43.9</td>
</tr>
<tr>
<td>1995q1</td>
<td>2313.4</td>
<td>2568.2</td>
<td>-254.9</td>
<td>2408.0</td>
<td>2501.8</td>
<td>-188.4</td>
</tr>
<tr>
<td>1995q2</td>
<td>2469.1</td>
<td>2407.6</td>
<td>61.5</td>
<td>2649.7</td>
<td>2657.9</td>
<td>-188.8</td>
</tr>
<tr>
<td>1995q3</td>
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<td>50.3</td>
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<td>2748.1</td>
<td>-114.9</td>
</tr>
<tr>
<td>1995q4</td>
<td>2626.4</td>
<td>2613.0</td>
<td>13.4</td>
<td>2754.8</td>
<td>2711.3</td>
<td>-84.9</td>
</tr>
<tr>
<td>1996q1</td>
<td>2390.3</td>
<td>2482.0</td>
<td>-91.7</td>
<td>2400.3</td>
<td>2596.0</td>
<td>-205.7</td>
</tr>
<tr>
<td>1996q2</td>
<td>2,657.6</td>
<td>2515.3</td>
<td>142.3</td>
<td>2,555.3</td>
<td>2755.2</td>
<td>-97.6</td>
</tr>
<tr>
<td>1996q3</td>
<td>2,814.7</td>
<td>2816.3</td>
<td>-7.5</td>
<td>2,739.5</td>
<td>2864.0</td>
<td>-49.3</td>
</tr>
<tr>
<td>1996q4</td>
<td>2,736.9</td>
<td>2791.8</td>
<td>-54.9</td>
<td>2,763.4</td>
<td>2851.0</td>
<td>-114.1</td>
</tr>
</tbody>
</table>

| MSFE  | 12866 | 8,684 | 14527 |
| MAPFE | 3.7464 | 3.0217 | 4.1986 |
| $U_2$ | 0.0445 | 0.0361 | 0.0473 |

The MSFE on the static forecasts is 12866. This is broadly comparable with the MSFE on the basic VARs and augmented VARs, discussed in the previous two sections. Only the basic and augmented $Q$ type VARs generate MSFEs for static forecasts that appear to be substantially lower (7899 and 10462 respectively). However, the modified Diebold-Mariano test suggests that these are not significantly lower (the respective test statistics are 1.26 and 1.79). The MSFE increases to 14527 for dynamic forecasts of private industrial and commercial construction output. This is lower than the MSFEs associated with the basic VAR models and the augmented accelerator and $Q$ type VARs. Only the augmented neoclassical and putty-clay type VARs generate lower MSFEs (10133 and 5884 respectively). However, given that we have only one twelve-step-ahead forecast for each model, we can not test whether these differences are statistically significant. A glance down the column containing the dynamic forecast errors indicates that the composite model tends to over predict private industrial and commercial construction output throughout the forecast horizon. The relationship between the actual and forecast data generated by this model can be more clearly seen in Figure 6.10. It is noteworthy that the dynamic forecasts would have been generally more accurate but for the error in the first forecast period. A downward shift in the forecast data by this amount (87.4) would result in a forecast error...
that tracked the actual data much more closely, although it must be conceded that even then the error mean would probably be significantly less than zero.

In the four-step-ahead exercise, this composite VAR appears to perform well generating forecasts with a MSFE of 8684. Only the augmented neoclassical model generates forecasts with a lower MSFE, but a modified Diebold-Mariano test statistic of 0.12 reveals that this is not significantly lower. We apply the modified Diebold-Mariano test to examine whether the MSFE associated with four-step-ahead forecasts from the composite model is significantly lower than that associated with the other VARs. Only the augmented accelerator VAR is significantly outperformed by the composite model (the test statistic is 2.45).

Given this rather ordinary forecast performance, there appears to be little value to including the three main determinants of investment in a single VAR. This is not surprising, however, since these variables are likely to be collinear. According to the accelerator theory, changes in output are sufficient to explain net investment. The $Q$ theory, on the other hand, states that $Q$ alone should provide a sufficient explanation of (the rate of) investment. Thus, we should expect some collinearity between output
and $Q$. Therefore, adding one of these variables to a system already containing the other is likely to result in unreliable parameter estimates.

6.4.3.2 Forecasts from the Construction Specific VARs

In this section, we evaluate the forecast performance of the four VARs which were estimated with construction related data. The estimation results are discussed in Section 5.4.4.2. The basic system, denoted BCS, contains six equations. The variables included in this system are private industrial and commercial construction output, construction new orders, real private sector output, the tender price index, the market conditions index, and the real interest rate. In the analysis of Sections 4.4.1 and 4.4.2, each of these variables has been found to be $I(1)$. There are three augmented construction systems. In the first system, denoted ACS(a), the BCS is augmented with a measure of the real wage in the construction sector. In the second system, ACS(b), the BCS is augmented with a variable measuring construction employment relative to total employment. ACS(a) and ACS(b) are seven equation systems. In the third system, ACS(c), the basic system is augmented with a variable measuring the manufacturing industry's investment intentions. This variable was found to be stationary in Section 4.4.2 and is, therefore, added to the right hand side of the system. The real wage, relative construction employment, and the intentions variables are included as possible leading indicators of private industrial and commercial construction output.

The forecasts and out-turn data from these four systems are given in Table 6.10. The associated evaluation criteria are given in Table 6.11. According to the three criterion presented in Table 6.11, there is not much to choose between the static forecasts from the BCS, the ACS(b) and the ACS(c). The ACS(a) generates static forecasts with the lowest MSFE, 9633, and we use the modified Diebold-Mariano test to determine whether this is significantly lower than that associated with the BCS, ACS(b), and ACS(c). The test statistics of 0.68, 0.84 and 0.67 for comparisons with the BCS, the ACS(b) and ACS(c) respectively, reveal that the differences are insignificant.

There is greater disparity between the four models' dynamic forecast performance. The BCS performs relatively poorly: the MSFE is 56958 and the average absolute
The error is 8.38% of the average actual value. The forecast performance of this model is slightly enhanced by augmenting it with the intentions variable (the MSFE, for example, falls from 56958 to 34520). However, the resulting model, ACS(c), still performs relatively poorly compared to the other two augmented construction systems. According to the three criteria in Tables 6.11, the ACS(a), which, it will be recalled, contains a variable measuring the real construction wage, generates somewhat more accurate forecasts than ACS(b): the MSFE for the former is half that of the latter.

Table 6.10: Out-turn and forecast data from the construction related VARs

<table>
<thead>
<tr>
<th>Actually</th>
<th>BCS</th>
<th>ACS(a)</th>
<th>ACS(b)</th>
<th>ACS(c)</th>
</tr>
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<tr>
<td></td>
<td>static</td>
<td>4-step</td>
<td>static</td>
<td>4-step</td>
</tr>
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<td>2421.1</td>
<td>2421.1</td>
<td>2421.1</td>
</tr>
<tr>
<td>1994q1</td>
<td>2249.8</td>
<td>2381.5</td>
<td>2343.9</td>
<td>2343.9</td>
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<tr>
<td>1994q2</td>
<td>2486.4</td>
<td>2415.8</td>
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<td>2657.8</td>
<td>2574.8</td>
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</tr>
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<td>1996q1</td>
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</tr>
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</tr>
<tr>
<td>1996q3</td>
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<tr>
<td>1996q4</td>
<td>2736.9</td>
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<td>3073.1</td>
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</tbody>
</table>

Table 6.11: MSFE, MAPFE and U2 for the construction related VARs

<table>
<thead>
<tr>
<th>Specification</th>
<th>one-step-ahead</th>
<th>four-step-ahead</th>
<th>dynamic forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSFE</td>
<td>MAPFE</td>
<td>U2</td>
</tr>
<tr>
<td>BCS</td>
<td>11.813</td>
<td>3.5903</td>
<td>0.0426</td>
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<tr>
<td>ACS(a)</td>
<td>9.633</td>
<td>3.3748</td>
<td>0.0385</td>
</tr>
<tr>
<td>ACS(b)</td>
<td>12.884</td>
<td>3.6251</td>
<td>0.0445</td>
</tr>
<tr>
<td>ACS(c)</td>
<td>11.693</td>
<td>3.4973</td>
<td>0.0424</td>
</tr>
<tr>
<td>ARIMA(4,1,4)</td>
<td>9.870</td>
<td>3.2647</td>
<td>0.0389</td>
</tr>
</tbody>
</table>

This ranking is confirmed by the time series plots of the forecast and out-turn data given in Figure 6.11. The BCS generates dynamic forecasts that consistently over predict private industrial and commercial construction output. The ACS(c), and to a lesser extent the ACS(b), also generate forecasts that tend to over predict the outcome data. These forecasts are particularly poor for the first quarter of each year of the three year forecast horizon: private industrial and commercial construction output falls much more quickly in this quarter than any of the models predict. The ACS(a), on the other hand, shows little tendency to over predict throughout the forecast horizon, the
Figure 6.11: Dynamic forecasts from the construction specific VARs

Figure 6.12: Dynamic forecast errors from the construction specific systems
mean of the errors is -16.87 and the variance of errors is constant. This is reflected in the plots of the dynamic forecast errors from these models given in Figure 6.12. Since we have only one twelve-step-ahead forecast for each model, we cannot determine whether the apparent difference in dynamic forecast performance is statistically significant.

The four-step-ahead forecast exercise determines a similar ranking of models. The ACS(a) generates four-step-ahead forecasts with a much lower MSFE than any of the construction systems. The modified Diebold-Mariano test reveals that the difference between the MSFE on this model is significantly lower than that generated by the BCS and the ACS(c) (the respective test statistics are 4.47 and 2.96). The MSFE on the ACS(a) is not significantly lower than that on the ACS(b) (the test statistic is 0.79). On the basis of this evidence, we conclude that of the four VARs considered in this section, the ACS(a) provides superior forecasting performance over this forecast horizon.

6.4.3.3 Forecasts from VARs Estimated with Macro Data

Finally, we evaluate the forecast performance of the two VARs estimated with macroeconomic data in Section 5.4.4.3. The basic system, denoted BMS, contains six equations. The variables included in this system are private industrial and commercial construction output, construction new orders, GDP, total employment, a liquidity ratio and the real interest rate. Each of these variables has been found to be I(1) in the analysis of Section 4.4.1 and 4.4.2. The augmented system, denoted AMS, contains two I(0) variables, business optimism and capacity utilisation, in addition to the six I(1) variables in the basic system. The two I(0) variables are included as leading indicators of investment. These variables augment the right hand side of the system.

The forecasts and out-turn data from these two VAR models are given in Table 6.12 and the associated forecast evaluation criteria are given in Table 6.13. According to the three criterion given in Table 6.13, the basic macro system appears to provide slightly more accurate one-step-ahead forecasts than the augmented system over the 1994q1 to 1996q4 forecast horizon. The difference in the MSFEs on forecasts from
the two models turns out not to be statistically significant (the modified Diebold-Mariano test statistic is 0.93).

Table 6.12: Out-turn and forecast data from the macroeconomic VARs

<table>
<thead>
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<th>Out-turn</th>
<th>Forecasts</th>
</tr>
</thead>
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<td>BMS four-step</td>
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<td>2421.1</td>
</tr>
<tr>
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</tr>
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</tr>
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<td>1994q4</td>
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<tr>
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<td>2848.0</td>
</tr>
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</table>

Table 6.13: MSFE, MAPFE and U₂ for the macroeconomic VARs

<table>
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<tr>
<th>Specification</th>
<th>one-step-ahead</th>
<th>four-step-ahead</th>
<th>dynamic forecasts</th>
</tr>
</thead>
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<tr>
<td></td>
<td>MSFE</td>
<td>MAPFE</td>
<td>U₂</td>
</tr>
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<td>4.1954</td>
<td>0.0451</td>
</tr>
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<td>3.2647</td>
<td>0.0389</td>
</tr>
</tbody>
</table>

There is little to choose between the models in terms of dynamic forecast performance either. The MAPFEs, for example, are 2.76 and 2.64 for the basic and augmented systems respectively. The MSFE suggests the reverse ranking but, again, the difference appears to be marginal. Given that we have just one twelve-step-ahead forecast for each model, we can not determine whether the difference is statistically significant. On the basis of the MSFE, the MAPFE and the $U_2$ statistics the dynamic forecasts appear to be more accurate than the static forecasts (the respective $U_2$ statistics on the basic system, for example, are 0.0311 and 0.0419). The dynamic forecasts are presented graphically in Figure 6.13.

The first point to note about Figure 6.13 is the fact that both models tend to over predict private industrial and commercial construction output in the first quarter of each year in the forecast horizon and under predict in the third and fourth quarters of each year. In other words, the models do not capture the full extent of seasonal
Figure 6.13: Dynamic forecasts from the macro VAR models

Figure 6.14: Dynamic forecast errors from the macro systems
variation, although both pick up the shape of the seasonality. This should not be surprising, since even an optimal predictor series has a variance that is lower than that of the actual series. Both models also generate a smaller forecast error in the last quarter of the forecast horizon. This is due largely to the fact that the errors tend to cancel out rather than accumulate over the forecast horizon. It is also noteworthy that both models track the profile of private industrial and commercial construction output very well. For example, the moderate increase in 1995 and the more rapid growth in 1996 are picked up accurately. The dynamic forecast errors from the two models are shown in Figure 6.14. For both models the error mean is positive, but relatively small. There is also little to choose between the four step-ahead forecast performance of two macro VARs. Indeed, the modified Diebold-Mariano test statistic of 0.20 reveals that the difference between the respective MSFEs is insignificant. On the basis of these results, we can conclude that the augmentation of the right hand side of the basic system with current and lagged capacity utilisation and business optimism, both thought to be leading indicators of investment, has not lead to an improvement in forecasting performance.

6.4.4 An Overview of Forecasts from the VAR Models

So far in Section 6.4, we have examined the static and dynamic forecasting performance of fifteen VAR models. Four of these models can loosely be thought of as VAR representations of the four traditional models of investment under consideration in this thesis. Each of these four models was augmented with a single variable to produce four further VARs. A ninth VAR combines elements of each of the traditional models. Six further VARs make use of leading indicators, macro data, and data specific to the construction sector. The results of this exercise are summarised below.

We begin by comparing the forecasting performance of the VARs representations of the four theories of investment considered in Section 6.4.1 with that of their single equation variants. Recall from Section 6.3.3, we determined that the single equation investment models perform quite poorly against the benchmark ARIMA model. In terms of static forecast performance, there is not much to choose between these VARs
and their single equation counterparts (compare the forecast evaluation criteria in Table 6.3 for the traditional models with that in Table 6.6 for the VAR representations). Notice that the accelerator and putty-clay type VARs generate slightly lower static MSFEs than their single equation variants, whereas the neoclassical and \( Q \) type VARs generate somewhat higher MSFEs. The modified Diebold-Mariano statistics reveal that the MSFEs associated with static forecasts from the accelerator, putty-clay and \( Q \) type VARs are not significantly different from those associated with their single equation counterparts (the test statistics are 1.05, 2.17 and 0.65 for the accelerator, putty-clay and \( Q \) models respectively). The MSFE associated with static forecasts generated by the traditional neoclassical model is significantly lower than the MSFE generated by the basic neoclassical type VAR (the test statistic is 2.37). However, one-step-ahead forecast performance is not necessarily a good predictor of forecast performance over a longer horizon.

Using the modified Diebold-Mariano test, we can determine whether there is any significant difference in the four-step-ahead forecast performance of these VARs and their single equation counterparts. The single equation neoclassical, putty-clay and \( Q \) models give rise to lower four-step-ahead MSFE than their VAR counterparts. The single equation accelerator model generates a higher MSFE than its VAR variant. In no case is the difference significant (the test statistics are 0.15, 1.43, 0.88 and 0.98 for the accelerator, the neoclassical, putty-clay and \( Q \) model respectively). It is in terms of dynamic forecasts that the basic VARs perform weakest. In every case, the MSFE of the VARs is inferior to the single equation variants and in two cases, the neoclassical and \( Q \) type VARs, the difference in performance appears to be particularly severe. Given only one twelve-step-ahead forecast for each model, we can not determine whether differences in MSFEs are significant.

The augmentation of these four basic VARs with an additional variable has very little effect on their static forecast performance. We use the modified Diebold-Mariano procedure to test whether the MSFE corresponding to static forecasts generated by each of the augmented VARs are significantly different from that corresponding to each of the basic VARs. In each case, the difference is not significant (the test statistics are 0.17, 0.82, 1.60, and 0.70 for the accelerator, neoclassical, putty-clay and \( Q \) models respectively).
The augmentation of the basic VARs has a more marked effect on dynamic forecast performance. The modified accelerator type VAR, which includes an I(0) capacity utilisation variable and the modified Q type VAR which includes an output (over capital stock) variable, generate forecasts that are appear to be somewhat less accurate than the basic VARs (compare the data in Table 6.6 with that in Table 6.8). The augmentation of the neoclassical and putty-clay type VARs with a liquidity variable spectacularly improves the accuracy of the dynamic forecasts (the respective MSFEs fall from 97585 to 10133 and from 20939 to 5884). The dynamic forecast performance of these modified VARs appears to be better than their single equation counterparts, although we can not test the differences for significance. In terms of four-step-ahead forecast performance, only the augmented neoclassical type VAR generates forecasts with a significantly lower MSFE than its basic version (the test statistic is 2.24 for the neoclassical type VARs and 0.98, 0.44 and 0.19 for the accelerator, putty-clay and Q type VARs respectively).

The four main theories suggest real output, the real user cost of capital and Q as determinants of investment. These main determinants are combined in a single VAR model in Section 6.4.3.1. Although the dynamic forecasts from this VAR give rise to a MSFE that is considerably lower than that on any of the four basic VAR models, the model consistently over predicts throughout the forecast horizon (see Figure 6.10). Moreover, the dynamic forecasts appear to be substantially less accurate than those generated by the modified neoclassical and putty-clay type VARs. In the four-step-ahead forecasting exercise, the composite VAR performs well generating a MSFE of 8684. Only the augmented neoclassical model generates forecasts with a lower MSFE, but this proves not to be significantly lower. Of the augmented VARs, only the augmented accelerator VAR is significantly outperformed by the composite VAR (the test statistic is 2.45).

In Section 6.4.3.2 the relative forecast performance of four more VARs is assessed. These VARs make use of construction sector data. The basic six equation system, in which private industrial and commercial construction output is determined by new orders, output, the tender prices index, a market conditions index and the real interest rate, tends to over predict private industrial and commercial construction output throughout the forecast horizon (see Figure 6.11). The associated MSFE on the
dynamic forecasts is 56968 (see Table 6.11). Of the three variants of this model considered in Section 6.4.3.2, that containing the real wage variable, denoted ACS(a), generates dynamic forecasts with the lowest MSFE (7040) and the dynamic forecast error mean is close to zero. The ACS(a) also provides superior four-step-ahead forecast performance. The MSFE associated with the four-step-ahead forecasts from the ACS(a) is significantly lower than that on the BCS and ACS(c).

In Section 6.4.3.3 the forecasts from two macroeconomic VARs are assessed. The first, denoted BMS, contains variables for private industrial and commercial construction output, new orders, real GDP, total employment, liquidity and the real interest rate. This performs well over the forecast horizon (see Figure 6.11) and the associated dynamic MSFE is 6241. The second system contains current and lagged business optimism and capacity utilisation, both of which are I(0), in addition to the six I(1) variables in the basic system. This model generates similar dynamic forecasts and MSFEs (see Table 6.13). The difference between the MSFEs associated with the four-step-ahead forecasts generated by the two models also proves to be insignificant.

Table 6.14: The forecast evaluation criteria from four VAR models

<table>
<thead>
<tr>
<th>Specification</th>
<th>one-step-ahead</th>
<th>four-step-ahead</th>
<th>dynamic forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSFE MAPFE U₂</td>
<td>MSFE MAPFE U₂</td>
<td>MSFE MAPFE U₂</td>
</tr>
<tr>
<td>Modified neoclassical</td>
<td>11,665 3.5077 0.0423</td>
<td>8,332 2.7027 0.0352</td>
<td>10,133 3.2421 0.0395</td>
</tr>
<tr>
<td>Modified putty-clay</td>
<td>14,553 3.9929 0.0473</td>
<td>10,851 3.3794 0.0404</td>
<td>5,884 2.6108 0.0301</td>
</tr>
<tr>
<td>ACS(a)</td>
<td>9,633 3.3748 0.0385</td>
<td>3,981 2.0556 0.0243</td>
<td>7,040 3.1018 0.0329</td>
</tr>
<tr>
<td>BMS</td>
<td>13,153 4.1954 0.0451</td>
<td>5,933 2.3051 0.0297</td>
<td>6,335 2.6427 0.0312</td>
</tr>
<tr>
<td>ARIMA(4,1,4)</td>
<td>9,870 3.2647 0.0389</td>
<td>10,145 3.2267 0.0394</td>
<td>6,563 2.4245 0.0318</td>
</tr>
</tbody>
</table>

Thus, in summary, we can say that of the fifteen VAR models considered in this section the forecasting performance of four merit further consideration. Of the VAR models loosely based on theory, the modified neoclassical and putty-clay type VARs are preferred. Of the four construction VARs, the ACS(a) is preferred. Finally, of the macro systems, both appear to do equally well, although the augmentation of the BMS with capacity utilisation and business optimism variables appears to do nothing to enhance forecasting performance. The forecast evaluation criteria from these four models are represented in Table 6.14, along with those pertaining to the benchmark ARIMA model. The forecasting performance of these models is compared with that of the ARIMA model in the next section.
6.4.5 Comparisons with the Benchmark Forecast

The \textit{ex post} dynamic forecasts from the four preferred VARs are shown in Figure 6.15, along with the forecasts from the ARIMA(4,1,4) benchmark model. The associated dynamic forecast errors are plotted in Figure 6.16. It is clear from these plots that there is not much to choose between these VAR forecasts, or indeed between any of these VAR forecasts and the ARIMA model forecasts. The MSFEs in Table 6.14 suggest that the modified putty-clay type VAR provides the most accurate dynamic forecasts, followed by the BMS, the ARIMA model, the ACS(a), and the modified neoclassical type VAR. The MAPFE statistic, however, suggests an alternative ranking with the ARIMA model coming top. We cannot use the procedure suggested by Diebold and Mariano (1995) and Harvey \textit{et al} (1997) to determine the ranking of the models more rigorously, since we have only one twelve-step-ahead forecast from each of these models. However, we can examine the relative four-step-ahead forecast performance using this procedure.

First, we use this procedure to compare the four-step-ahead MSFE generated by each of the VAR models with that associated with the ARIMA model. The results can be summarised as follows. The MSFEs associated with the ACS(a) (3981) and the BMS (5933) are significantly lower than that on the ARIMA model (10415) (the respective test statistics are 3.04 and 2.60). The MSFEs on the modified neoclassical model (8332) and the modified putty-clay model (10851) are not significantly different from that on the ARIMA model (10415). We also use the procedure to test whether the MSFE on the ACS(a) is lower than that on the BMS. We find that the difference is insignificant (the test statistic is 0.35). Thus, on the basis of this test, we can conclude that in terms of these models’ four-step-ahead forecast performance, only the ACS(a) and the BMS outperform the ARIMA model.

Notice that only the ACS(a) generates static forecasts with a lower MSFE than that associated with static forecasts generated by the benchmark ARIMA(4,1,4) model. The modified Diebold-Mariano test suggests that the difference is not significant (the test statistic is 0.09). Although the MSFE associated with static forecasts generated by the ARIMA(4,1,4) model is lower than that corresponding to the other VARs, the difference is again insignificant (the test statistics are 1.21, 1.96, and 1.28 for the
Figure 6.15: Dynamic forecasts from the preferred VAR models

Forecast period
- Modified neoclassical
- Modified putty-clay
- ACS(a)
- BMS
- ARIMA(4,1,4)
- Out-turn data

Figure 6.16: Dynamic forecast errors from the preferred systems

MNC
MPC
ACS(a)
BMS
ARIMA
modified neoclassical type model, the modified putty-clay type model and the BMS respectively).

We also determine whether the static forecasts generated by these VARs are conditionally efficient with respect to the static forecasts from the ARIMA model. Recall, conditional efficiency implies that $\beta$ in equation 3.65 is insignificantly different from unity. The static forecasts from the modified neoclassical type VAR, the ACS(a) and the BMS are all conditionally efficient with respect to the ARIMA model (the respective $t$-statistics are 1.53, 0.75, and 1.76). However, the static forecasts from the modified putty-clay type VAR appear to be conditionally inefficient with respect to the ARIMA model (the $t$-statistic is 3.42). Granger and Newbold (1986) suggest that one should be dissatisfied with any model that generates forecasts that are not conditionally efficient with respect to those generated by an ARIMA model.

The dynamic forecasts for private industrial and commercial construction output generated by these models are quite similar and it is, therefore, difficult to discriminate between them. Recall, these *ex post* forecasts were generated using actual data for other variables. In generating *ex ante* forecasts this information is not available and so the inputs into the equation for private industrial and commercial construction output must be forecast themselves. A more stringent test of a model’s forecasting abilities may be provided if forecasts of other variables are used as inputs into the private industrial and commercial construction output equation rather than using actual data. In this way, it may be possible to distinguish between these models more effectively. Even if two models generate similar forecasts of private industrial and commercial construction output, we may still be able to discriminate between them on the basis of the forecasts they generate for other variables. We merely note this as an alternative means of discrimination. Here, the *ex post* forecasts for private industrial and commercial construction output are generated using actual data for other variables and this was sufficient to discriminate between most models.

Unlike in the traditional models, private industrial and commercial construction output in these VAR models depends only on lagged values of other variables. We can, therefore, use these models to generate *ex ante* forecasts of the series without the need for extraneous information. Recall in Section 6.2.2, we generated *ex ante*
forecasts for the benchmark ARIMA model and these were plotted in Figure 6.3. For the VAR models, *ex ante* forecasts of private industrial and commercial construction output depend on *ex ante* forecasts of other variables. This results in extremely wide confidence intervals. The mathematical derivation of exact confidence intervals (i.e. that capture this interdependency) is very complex and not attempted here.

The *ex ante* forecasts of private industrial and commercial construction output from these four VARs are plotted in Figure 6.17 for the period 1996q4 to 2000q4. The *ex ante* forecasts generated by the benchmark ARIMA model are also plotted in Figure 6.17 to enable comparisons to be made. Notice that of these models, the benchmark model forecasts a higher level of private industrial and commercial construction output over this period than any of the other models. The forecasts from the modified putty-clay model, modified neoclassical type VARs and the BMS are very similar and follow the ARIMA forecasts quite closely. The ACS(a) model, on the other hand, forecasts a very slight decline in private industrial and commercial construction output over the period.

![Figure 6.17: Ex ante forecasts from the VAR and ARIMA models](image-url)
6.5 VARs versus Traditional Models: A Comparison of Forecast Performance

In Section 6.4.4 we compared the performance of the VARs based loosely on the theory of investment with their single equation variants. We found that the VARs performed relatively poorly in the four-step-ahead and dynamic forecast exercises, although many of the differences in performance proved not to be statistically significant. In terms of static performance, the traditional single equation neoclassical model outperforms its VAR counterpart but this equation, it will be recalled, performed particularly poorly in the four-step-ahead and dynamic forecasts. Differences between the static performance of the other traditional models and their VAR counterparts turned out not to be statistically significant.

In Section 6.4.4 we also determined that the static forecast performance of the augmented VARs was not significantly different from their unaugmented counterparts. The MSFEs on dynamic forecasts generated by the augmented neoclassical and putty-clay type VARs are much lower than those corresponding to the unaugmented VARs, whereas augmentation of the accelerator and $Q$ models appears to lead to a slight deterioration in dynamic forecast performance. In the four-step-ahead forecasting exercise only the augmented neoclassical type VAR significantly outperforms its unaugmented version. We can also compare the forecast performance of the augmented VARs with the performance of their traditional single equation counterparts. We use the modified Diebold-Mariano test to determine whether the MSFEs associated with the static and four-step-ahead forecasts from the single equation traditional models are significantly different to those generated by the augmented VARs. For static forecasts, the differences prove to be insignificant (the test statistics are 1.68, 1.36, 1.17 and 1.18 for the accelerator, neoclassical, putty-clay and $Q$ models respectively). For four-step-ahead forecasts, only the augmented neoclassical type VAR significantly outperforms its single equation alternative (the test statistic is 2.98 for the neoclassical models, and 0.74, 0.88 and 0.62 for the accelerator, putty-clay and $Q$ models respectively). In terms of dynamic forecast performance, the augmented neoclassical and putty-clay type VARs generate forecasts with lower MSFE than their single equation alternatives. However, given that we have
only one twelve-step-ahead forecast for each model, we can not test whether the differences are significant.

In this section we compare the relative performance of the best of the traditional models, namely the $Q$ model, with the best of the VARs, namely the ACS(a) and the BMS. Table 6.15 contains the forecast evaluation criteria for the traditional $Q$ model (given by equation 5.17), the ACS(a), the BMS and the benchmark ARIMA model. In Section 6.3.2 it was determined that the $Q$ model, given by equation 5.17, yields significantly more accurate four-step-ahead forecasts and apparently more accurate dynamic forecasts than any of the other traditional models of investment. In Section 6.3.3 it was determined that, whilst the $Q$ model provides static forecasts that are conditionally efficient with respect to static forecasts from the benchmark ARIMA model, the four-step-ahead forecasts are not significantly more accurate than those generated by the benchmark ARIMA model. In Section 6.4.5 it was determined that, of the fifteen VAR models considered, the ACS(a) and BMS do best in the four-step-ahead forecasting exercise. Both do better significantly better than the benchmark ARIMA model. These two models also did well in the dynamic forecasting exercise, as did the augmented putty-clay type VAR, although subsequent analysis reveals that this latter model does not generate static forecasts that are conditionally efficient with respect to the ARIMA model. The question is whether or not these VAR models outperform the single equation $Q$ model.

<table>
<thead>
<tr>
<th>Specification</th>
<th>one-step-ahead</th>
<th>four-step-ahead</th>
<th>dynamic forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSFE</td>
<td>MAPFE</td>
<td>$U_2$</td>
</tr>
<tr>
<td>$Q(5.21)$</td>
<td>6,035</td>
<td>2.5478</td>
<td>0.0304</td>
</tr>
<tr>
<td>ACS(a)</td>
<td>9,633</td>
<td>3.3748</td>
<td>0.0385</td>
</tr>
<tr>
<td>BMS</td>
<td>13,153</td>
<td>4.1954</td>
<td>0.0451</td>
</tr>
<tr>
<td>ARIMA(4,1,4)</td>
<td>9,870</td>
<td>3.2647</td>
<td>0.0389</td>
</tr>
</tbody>
</table>

The MSFE associated with static forecasts generated by the $Q$ model is lower than that generated by the other models. Only when compared with the BMS is the difference significant (the test statistic is 2.49 for the BMS, 1.69 for the ACS(a) and 1.47 for the ARIMA model). Notice that the MSFE generated by four-step-ahead forecasts from the ACS(a) and BMS are lower than that generated by the $Q$ model. However, the modified Diebold-Mariano test statistic reveals that the differences are significant.
insignificant (the test statistics are 1.59 and 0.38 for the ACS(a) and the BMS respectively). We have already determined that the MSFE associated with four-step-ahead forecasts generated by the ARIMA model is significantly higher than that associated with the $Q$ model, the ACS(a) and the BMS (albeit only at the 10% level of significance in the case of the $Q$ model). The MSFEs associated with the dynamic forecasts generated by these models are quite similar and since we have just one twelve-step-ahead forecast for each of the models, we can not test whether the differences are statistically significant.

Thus, we can conclude that these three models provide more accurate four-step-ahead forecasts for the period 1994q1 to 1996q4 than any of the other models estimated in this work. However, discrimination between the four-step-ahead forecast performance of these particular models is not possible using the modified Diebold-Mariano test. The dynamic forecasts from these three models are plotted together in Figure 6.18. There is not much to choose between these models in terms of dynamic forecast performance, but we can say that these models generate forecasts that track private industrial and commercial construction quite well over the three year forecast horizon.
both absolutely and relative to the other models developed in this thesis. We have not been able to discriminate between the static forecast performance of these models using the modified Diebold-Mariano test statistic.

6.6 Summary and Concluding Remarks

In this chapter we evaluate the forecast performance of a number of models of private industrial and commercial construction output. We examined the performance of static, four-step-ahead and dynamic forecasts. The models can be conveniently divided into two types, namely econometric models and VAR models. Four econometric models are considered. Each is based on a rigorous interpretation of one of the four main theories of investment, expositions of which are provided in Chapter 2. The estimation of these four models is discussed in Section 5.2 and an evaluation of their forecasting performance is given in Section 6.3. Whilst four of the fifteen VAR models correspond to the four econometric models, the VAR models are, in general, tied to theory much more loosely. These VAR models are estimated in Section 5.4 and their forecasting performance is evaluated in Section 6.4. A summary of the VAR specifications is given in Table 5.50.

Forecasts from these models are assessed for a common forecasting horizon (1994q1 to 1996q4) using a selection of objective criteria, in addition to more informal techniques such as the examination of time series plots of forecasts and forecast errors. It is the four-step-ahead and dynamic forecast performance of the models that is of most interest, since typically, we are interested in \textit{ex ante} forecasts of a series more than one-step-ahead. Attempts are made to ensure that comparisons between forecasts from different models were fair. To this end, forecasts of changes in, or logarithms of, private industrial and commercial construction output are transformed in such a way that they are comparable with actual level of private industrial and commercial construction output, prior to the calculation of the forecast evaluation criteria. For the static and four-step-ahead forecasts we examine whether the MSFE associated with forecasts generated by one model is significantly different from that associated with forecasts generated by another.
The forecasts generated from these models are also compared with those generated from a benchmark model. The benchmark model, determined in Section 6.2, is a univariate time series model in which private industrial and commercial construction output depends only on its value in previous periods. That is to say, it does not embody any economic theory. The particular time series model found to provide the most accurate forecasts of private industrial and commercial construction output over the forecast horizon of interest here, is an ARIMA(4,1,4) model with logged data. We compare the MSFE generated by forecasts from this model with those generated by other models and examine whether the forecasts from other models are conditionally efficient with respect to this ARIMA model. Although this model provides reasonably accurate static and dynamic forecasts, we should expect models derived with the benefit of economic theory to generate 'better' forecasts.

Of the econometric models (which have been referred to as traditional models in this work), the Q model generates dynamic forecasts that appear to be more accurate that those generated by the accelerator, neoclassical and putty-clay models. It is noteworthy that these models tend not to capture the full extent of the seasonal variation (although the general pattern is picked up). This may be a result of the fact that the data is not logged prior to estimation. In this case, the seasonal variation at the end of the estimation period will be greater than that at the beginning of the period, but the coefficients on the seasonal dummies will measure the average variation. However, we would expect this to be highlighted in the results of tests for heteroskedasticity. Another possible explanation is supplied by the fact that the variance of even an optimal predictor series is lower than that of the actual series. It is also noteworthy that the root of the MSFEs on the static forecasts from these models tend to be higher than the within sample standard errors. This, of course, is no surprise since the within sample fit is artificially enhanced by the corrections for residual serial correlation. The four-step-ahead forecasts generated by the Q model are found to be significantly better than those generated by the ARIMA model (at the 10% level). We are not able to draw any firm parallels between these findings and those of other studies which aim to systematically compare the forecast performance of various single equation models of investment. Although the static forecasts generated by the Q model are not significantly better than those generated by the ARIMA model, they
are conditionally efficient with respect to it, unlike the accelerator and putty-clay models.

The neoclassical model, given by equation 5.8, performs least well in this forecasting exercise. This should not be surprising given the associated theoretical difficulties. Recall, the capital gains term in the user cost was scaled to ensure that the real user cost of capital remained positive throughout the sample period. A negative real user cost of capital implies that investment will be infinite and thereby invalidates the neoclassical model. Scaling results in very large values for optimal capital stock in the early to mid 1970s and these extreme values are likely to have distorted parameter estimates.

The forecasts of private industrial and commercial construction output from the VAR models were generated using actual rather than model-generated data on other variables. The resulting forecasts are then comparable with those generated by the econometric models. The forecasts from the basic VAR representations of the four main theories of investment (especially the neoclassical model) are rather poor and certainly not significantly better than those generated by their single equation variants. However, when the models are modified to take into account observations made in the literature, the accuracy of dynamic forecasts from the neoclassical and putty-clay models improve dramatically. Recall, the modification to these two models consisted of adding a liquidity variable to the systems to allow for the fact that capital markets are imperfect. The dynamic forecast performance of the accelerator and $Q$ type VARs, on the other hand, appears to deteriorate somewhat on augmentation. Whilst the modified putty-clay VAR generates dynamic forecasts with a lower MSFE than that of the benchmark ARIMA, none of these models are found to generate a significantly lower MSFE for four-step-ahead forecasts. Given that the forecasts from the basic accelerator and basic putty-clay models are found to be very similar, we conclude that the real user cost variable added very little to the forecast performance (over the given period) of a system that consisted simply of investment and private sector output. If the accelerator model had been augmented with a liquidity variable (to produce a kind of profits VAR model of investment) rather than a capacity utilisation variable, we might find a similar improvement in performance. However, such a model is not estimated.
The accuracy of forecasts from the remaining VARs is evaluated in Section 6.4.3. Of these, two are worthy of note. Of the systems based on construction sector data, the ACS(a), which contains a real wage variable, generates the most accurate forecasts of private industrial and commercial construction output. This implies that the real wage variable contributes more to the forecasting performance of the BCS than the construction over total employment variable used in ACS(b), or the investment intentions variable used in ACS(c). Moreover, the four-step-ahead forecasts generated by the ACS(a) are found to be significantly more accurate than those from the ARIMA model. There is little to choose between the accuracy of forecasts from the two systems estimated with macro data. In terms of four-step-ahead forecasts, both models significantly outperform the benchmark ARIMA model. The forecasts from the BMS are preferred on the grounds that the inclusion of the leading indicator variables (current and lagged capacity utilisation and business optimism) does nothing to enhance forecasting performance over the 1994q1 to 1996q4 horizon.

Thus, we can summarise as follows: of the fifteen VAR models considered, the ACS(a) and BMS provide the most accurate forecasts of private industrial and commercial construction output over the given horizon. From the econometric models the $Q$ model provides the most accurate forecasts. We could not discriminate between the forecast performance of these three models. We were able to ascertain that the four-step-ahead forecasts from the ACS(a) and the BMS were significantly more accurate than the forecasts from the ARIMA model.

Recall, considerable effort is spent ensuring that the traditional models of investment were estimated in a form that is as consistent with theory as possible, given the need to maintain econometric integrity. It is somewhat surprising, therefore, that of the four traditional models, only the $Q$ model outperforms the ARIMA model in terms of four-step-ahead forecast performance (and this only at the 10% level of significance). The ARIMA model, of course, was estimated without the benefit of theory. Although surprising, such a paradoxical result is not without precedence in the empirical literature. Recall, from the discussion in Section 6.3.2, Kopcke (1985) found that his autoregressive model of investment outperforms all of the theoretically consistent models he considered. A similar argument may be applied to the VAR models. Although the VAR models are developed from intuitive, rather than theoretically
rigorous, foundations we would expect the larger information set to result in superior forecasting performance. Of course, it is possible that the forecasts from the ARIMA model are particularly good over this horizon relative to other horizons. After all, private industrial and commercial construction output over this period is fairly stable and, *a priori*, we would expect forecasts from such a model to be more accurate in times of stability. Similarly, forecasts from the econometric and VAR models may be particularly poor over this horizon, but it is more difficult to see why this might be so. In either case, the econometric and VAR models may do better in comparisons over alternative horizons. The robustness of models’ forecasting performance has not been evaluated for alternative horizons.

The relative stability of private industrial and commercial construction output growth over the 1994q1 to 1996q1 forecasting horizon may also have implications for the relative rankings of models. In this exercise, the margins of superiority or inferiority are really quite small and we are unable to determine a definitive ranking between the preferred models. Given that year to year growth in private industrial and commercial construction output is relatively stable, and that the seasonal cycle is regular over this three year period, it might be argued that this forecast horizon does not provide a test of sufficient stringency to discriminate between models’ forecasting performances. Moreover, the same exercise conducted over a different forecasting horizon may result in alternative preferred models or a more definite ranking of these models. On the basis of the analysis in this chapter we can do no more than comment on the relative performance of these models over this forecast period.

In this chapter considerable effort has been spent attempting to determine a ranking of models. In the forecasting literature however, much attention has been devoted to the idea of combining alternative forecasts of the same variable. Bates and Granger (1969), for example, demonstrate how two unbiased forecasts can be combined to produce a new forecast that is at least as accurate as the better of the individual forecasts from which it is derived. The weights given to each of the component forecasts are shown to depend on the variances and the covariances of the forecast errors. Granger and Ramanathan (1984) have generalised this procedure and extended it to include biased forecasts. There are two reasons why forecasts from the preferred models are not combined here. Firstly, the resulting gains in accuracy are unlikely to
be large. This is because the forecasts generated from these models are ‘similar’. Large gains in accuracy are possible when the component forecasts utilise independent information and so do not lead to multicollinearity in the combining equation. The more variation they include, the lower the expected variance of the combined forecast error. This implies that forecasts generated by different techniques, such as econometric models, time series models and surveys, might be included. However, despite the employment of a variety of techniques here, Figure 6.18 reveals that the forecasts generated by these models display a high degree of collinearity. Secondly, the performance of the resulting forecast can only be judged outside the sample used to determine the optimal weights on each forecast. This requires data for private industrial and commercial construction output outside the available sample period.
7 Conclusions

7.1 An Overview

In this thesis we model an important and growing component of construction output, namely private industrial and commercial construction output. To the extent that new construction output represents capital formation, output can be modelled as an investment problem. Some of the models we develop here are consistent with the traditional theories of investment. Other models of private industrial and commercial construction output are based on VAR analysis and are consistent with theory only in a loose sense. We compare the static, four-step-ahead and dynamic forecasting performance of these models and evaluate the forecasts from these models against forecasts generated by a univariate ARIMA model which serves as a benchmark.

A large part of Chapter 2 is spent outlining the four dominant theories of investment behaviour with a view to deriving, for each theory, an investment equation of the kind frequently estimated in the empirical literature. The theoretical investment literature is disparate and confusing. However, in this work we present the theories in a unified framework which allows the differences and similarities between them to be easily identified. We have also explore the relationship between these dominant models and a number of peripheral models. From the exposition of theories provided in Chapter 2 it is clear that each of the four dominant theories attempts to overcome potential weaknesses in its rivals. The neoclassical model, for example, attempts to remedy some of the defects of the accelerator model by relating investment to relative prices (and thereby interest rates and taxation) in addition to output. The putty-clay model allows for the possibility that the response of investment to changes in output is faster than the response to changes in relative prices. Adjustment cost models, of which the \( Q \) model is one type, address one of the major weaknesses with the early neoclassical models, that being the instantaneous adjustment of capital stock to its desired level. \( Q \) models have a number of additional advantages over the earlier models. For example, the \( Q \) model explicitly recognises the dynamics due to expectations and technology and, moreover, it isolates their influences. In addition, the parameters of the \( Q \) model are the structural parameters of the adjustment cost function and thus do not depend
on the process by which firms form expectations of future variables. The relevant expectations variables are captured through the measured regressor, $Q$, and are therefore not subject to the Lucas critique. These advantages have made the $Q$ model extremely popular in the empirical literature.

In addition to outlining the theory underlying the four dominant models, we also examine the empirical performance of each of these models. Like the theoretical literature, the empirical literature on nonresidential fixed investment is vast. Of greatest interest to us are UK studies in which models of construction investment are estimated with a view to forecasting. Unfortunately, studies of this kind are too few in number to draw inference on the relative performance of investment models. We therefore consider the literature more widely. We review studies of equipment expenditure and of aggregate expenditure. Most of the studies we survey are based on US data and in these studies models are typically estimated with a view to analysing the effects of changes in policy. In an attempt to make the survey more manageable we focus on time series studies diverting attention to studies using cross-sectional and panel data only when their results merit mentioning. Moreover, a discussion of estimation issues is largely absent from this chapter, but we consider these in more detail in Chapter 3. There are a handful of mainly US studies whose primary objective is to systematically compare the forecasting performance of alternative econometric models of investment. Given that the objective of these studies overlaps our objective, we consider these in some detail. In general, the results of studies attempting to model investment expenditure have been disappointing. Whilst proponents of each theory can point to a number of studies which purport to uncover evidence in favour of their model, none of the models we consider perform consistently and systematically well in tests against actual data or against alternative models.

Although a number of the theoretical deficiencies inherent in the early theories of investment are addressed in more recent models, the empirical performance of investment models has not improved. Despite the theoretical appeal of the $Q$ model, for example, its usefulness has been called into question by its generally disappointing empirical performance. There are two caveats pertaining to the $Q$ model, the first relates to the possible measurement of the components of $Q$ and the second concerns the conditions under which average $q$ can be used as a proxy for marginal $q$. However,
empirical evidence suggests that neither of these are responsible for its poor performance. Whilst the $Q$ variable is usually significant, equations generally fit the data poorly with low $R^2$s. The presence of serially correlated errors and the significance of lagged dependant and lagged $Q$ variables indicate that the dynamics appear to be inadequate. Further, according to theory, no other variables should have a systematic relationship with investment but quantity variables, such as output and liquidity, are frequently statistically significant. Finally, $Q$ equations generate estimates of costs of adjustment parameters that are unfeasibly large.

Although there is clearly no uniformity in the results of these studies, it is possible to draw one major conclusion from the review of the empirical literature contained within Chapter 2. The response of investment to changes in price variables tends to be small and unimportant relative to the effects of quantity variables. This suggests that the direct response of investment to changes in fiscal parameters is modest, as is the direct response to changes in the tools of monetary policy, such as interest rates. This conclusion has implications for our study: quantity variables such as output are likely to prove the dominant determinants of private industrial and commercial construction. We also conclude that the recurring significance of financial factors in the literature suggests that liquidity effects are important, although the underlying theoretical justification for including these variables in models of investment behaviour is not well developed. Moreover, the precise channels through which these variables operate is not clear. This suggests that future research on investment behaviour should expand the view of the firm and the margins along which it operates.

We also consider a number of studies from the field of surveying and building which use statistical techniques to develop forecasting models of construction variables. Although these studies tend not to be based on secure theoretical foundations, they are included in this survey on the grounds that the objective of these studies is very much related to ours. However, since little attention is paid to model specification or estimation issues in these studies, one must interpret the associated results with a degree of caution.

In Chapter 3 we consider estimation and forecasting issues. The survey of the literature on investment models contained within Chapter 2 is largely devoid of discussion of econometric issues and, to a large extent, this reflects an absence of
discussion in the literature. The fact that many economic time series are known to be non-stationary implies that testing for non-stationarity is essential to the development of valid econometric equations. Also, it is important that the procedures for dealing with non-stationarity are followed. It is noteworthy that much of the empirical literature was developed prior to the realisation of the importance of non-stationarity in econometric analysis. In Chapter 3 we describe a number of popular tests for non-stationarity and describe the procedures for dealing with it. To the extent that we examine data for non-stationarities and estimate equations consistent with the findings from such tests this work represents an advance on the existing investment literature.

Also in Chapter 3, we outline the VAR approach to modelling. In particular, we focus on a description of the Johansen procedure. Although this procedure (and for that matter, the various tests for non-stationarity and the means of dealing with it) are well known in the econometrics literature, they are outlined in this work since they have not previously been applied or considered in the context of investment behaviour or in analysis of the construction sector.

In the final part of Chapter 3 we consider issues surrounding the evaluation of forecast performance. There are a number of studies in the empirical investment literature which aim to systematically compare the forecast performance of alternative investment models. On the whole the methods of comparison in these studies are crude, being based on statistics summarising the sample evidence (e.g. the mean square forecast error) and visual inspection of plots of forecasts and forecasts errors. In this thesis we adopt more rigorous methods of comparing forecasts. In Chapter 3 we describe the methods of forecast evaluation that are used here. We discuss the underlying objectives of an evaluation exercise and introduce the MSFE, the MAPFE, Theil’s $U_2$ statistic, the concepts of conditional efficiency and a test for evaluating whether one set of forecasts is significantly more accurate than another. All of these, together with informal graphical analysis of forecast and out-turn data and forecast errors, are used to assess forecast accuracy in this thesis. We also outline attempts to create conditions under which forecasts derived from different generating mechanisms can be fairly compared and discuss the parameters of the forecasting contest in Chapter 6.
Chapter 4 relates to the data. In the first part of the chapter we provide definitions and sources of the data used in this thesis, details of how variables are constructed and limitations of the resulting measures. We begin by considering the data suggested by the four dominant theories of investment behaviour. We also examine the relationship between construction output data and investment expenditure data with a view to determining the suitability of using theories of the latter to model the former. The data requirements are considerable. Among the determinants suggested by theory are the user cost of capital and Tobin’s $q$. Considerable effort is spent constructing these measures and adjusting them for the effects of corporate taxation and the various investment incentives that have been available on private industrial and commercial construction over the forty year sample period. We also develop a number of measures of the stock of privately owned industrial and commercial buildings using data on new private industrial and commercial construction output and repair and maintenance data.

We then consider other variables thought to have an impact on private industrial and commercial construction output. These variables are used as inputs into the VAR models. Some of these variables are suggested by other theories of investment, some are found to be significant in the empirical literature, and some are thought to be leading indicators of investment. The derivation of these measures is somewhat more straightforward than the derivation of those variables suggested by investment theory. Although we attempt to maximise the length of the sample period, it is not possible to obtain data back to the start of the sample period for some of these variables. Therefore, the estimation period is truncated for models in which these variables appear.

In the second part of the chapter, we examine the statistical properties of all series used in subsequent analysis. In particular, we examine the data for evidence of nonstationarity using a number of unit root tests. In cases where these tests fail unambiguously to determine the order of integration, we make use of time series and spectral techniques to provide more evidence. For some variables the orders of integration are somewhat surprising. For example, the stock of privately owned industrial and commercial buildings is found to be I(2). This implies that net investment (new private industrial and commercial construction output) is I(1). This
result is at odds with much of the empirical literature which implicitly assumes that net investment is I(0). Of course, the orders of integration of investment and capital stock found here may be specific to this type of investment, but this is counter intuitive. We consider the possibility that this is an artefact of the construction output data and test the order of integration of alternative measures of capital stock and investment which, although official, include factors such as the stock of, or expenditure on, infrastructure. These official series for capital stock and investment are also found to be I(2) and I(1) respectively. In addition to being at odds with the empirical literature, these results suggest that some of the investment models are econometrically invalid. For example, the accelerator model posits a relationship between net investment and changes in output. Since output is I(1), changes in output are I(0). Thus, the accelerator implies that an I(1) variable is determined by an I(0) variable. Such a strategy is bound to fail, since the variables will diverge by ever increasing amounts. A similar problem arose for the neoclassical model. Optimal capital stock is found to be I(1) and therefore its changes are I(0). That optimal and actual capital stock are of different orders of integration is rather odd, but the result suggests that the investment function implied by Jorgenson’s version of the neoclassical model is also unbalanced. We also find the rate of net investment and the $Q$ variables to be I(1). That $Q$ is I(1) is also inconsistent with the implicit assumption in the empirical literature. If investment is assumed to be I(0), as in much of the empirical literature, $Q$ must also be I(0) for the $Q$ equation to be econometrically balanced. Given that we find both the rate of net investment and $Q$ to be I(1), we expect (a priori) these variables to be cointegrated. We test for cointegration in Chapter 5.

Models of private industrial and commercial construction output are estimated in Chapter 5. In the first part of this chapter we consider the issues surrounding the estimation of the four dominant models and analyse the results of estimation and diagnostic testing. We estimate the models in a manner consistent with the findings of unit root tests presented in Chapter 4 in order to avoid spurious or unbalanced regressions. As noted above, this implies that the resulting equations are not those suggested by theory and developed in Chapter 2. We also provide a comparison of the within sample properties of each of these models. There is not much to choose
between the models, although we expect the accelerator to perform best in the forecasting exercises. Of the distributed lag models, this provides the best within sample fit, unconstrained parameter estimates are plausible and residual statistics are satisfactory. However, we note that the lagged dependent variable in all these models is the dominant regressor and this indicates that the determinants of net investment suggested by theory are not sufficient to explain net investment in industrial and commercial buildings. These results are consistent with the results of much of the empirical literature. It is noteworthy that the rate of net investment and $Q$ appear not to be cointegrated. This result provides additional support for the hypothesis that $Q$ is not sufficient to explain the rate of net investment in industrial and commercial buildings.

In the second part, we develop a number of ARIMA models of construction output. Several models are found to provide an adequate description of the data and four models, each corresponding to a different transformation of the data, satisfy the diagnostics. That is to say the residuals from these models appear to have a zero mean, constant variance, an absence of serial correlation and the models have significant parameter estimates. Further discrimination between models is conducted, in Chapter 6, in terms of forecast performance. An ARIMA(4,1,4) model, estimated with logged data for the period between 1955q1 and 1993q4, generates the most accurate forecasts of private industrial and commercial construction output over the 1994q1 to 1996q4 horizon. This model is therefore adopted as the benchmark model against which the forecasting performance of the econometric and VAR models is compared.

In the third part of the chapter, fifteen VAR models are estimated. Some of these VAR models are loosely consistent with theory, others make use of leading indicators, data from the construction sector and macro variables. In estimating these models we seek to determine the dimensions of each system. That is to say, we determine the number of lags for each variable in the VAR and the number of cointegrating vectors. In determining the lag length we aim to ensure that the residuals are white noise whilst being conscious of the need to preserve degrees for freedom. The number of cointegrating vectors is determined in accordance with the trace and maximum eigenvalue tests devised by Johansen. We make no attempt to interpret the
cointegrating vectors, since interpretation is invalid if the system does not subsume a structural model.

Among the VARs estimated in Chapter 5 is a composite VAR which contains elements of each of the four dominant theories of investment. In addition to the private industrial and commercial construction output (or net investment) variable, the system contains an output variable, a user cost variable and a $Q$ variable. Using this general model we test the efficacy of individual theories of investment. In particular, we impose restrictions on the general VAR to derive a special case broadly consistent with each theory. The main conclusion that we draw from this part of the analysis is that none of the variables suggested by the individual theories of investment is sufficient to explain private industrial and commercial construction output. The results of the unit root tests and the estimation results relating to the traditional models of investment suggest that the empirical implementation of investment models in the literature is unsatisfactory. Analysis of this general VAR however, suggests that the poor performance of these models is also due to the fact that the determinants of investment suggested by theory are inadequate.

In Chapter 6 we use the models estimated in Chapter 5 to generate one-step-ahead, four-step-ahead and twelve-step-ahead forecasts. The models are re-estimated up to 1993q4 to allow twelve quarters of data for ex post forecast evaluation. More importance is attributed to four-step-ahead and twelve-step-ahead forecast performance. Static forecasts simply provide a measure of fit over the forecast horizon. Four-step-ahead and twelve-step-ahead forecast performance provides a better insight into how well these models perform over the longer term. These are useful since we are usually interested in forecasts of private industrial and commercial construction output more than one-step-ahead. The twelve-step-ahead forecasts are fully dynamic forecasts and, as such, provide an insight into how well a forecaster would have fared with these models from the vantage point of the 1993q4. It is noteworthy that all forecasts for private industrial and commercial construction output are generated using actual data on all other variables over the forecast horizon. In other words, these forecasts are those that would be derived if the values of other variables over the forecast horizon are forecast perfectly.
As discussed in Chapter 3, forecasting performance is assessed by objective and subjective means. Forecast evaluation criteria such as the MSFE, the MAPFE and Theil’s $U_2$ are employed. We also examine whether the differences in MSFEs for different models are statistically significant. We determine whether the forecasts generated by the econometric and VAR models are conditionally efficient with respect to the forecasts generated by the benchmark ARIMA model. Comparisons of the twelve-step-ahead forecasts rely heavily on visual inspection of time series plots of forecast and out-turn data and forecast errors.

First, we examine the forecasting performance of the econometric models. Of these models, only the $Q$ model generates forecasts that can be said to be ‘adequate’. This is rather surprising given that $Q$ and the rate of net investment in industrial and commercial buildings are not cointegrated. The static forecasts from the accelerator and putty-clay models are not even conditionally efficient with respect to forecasts from the benchmark ARIMA model. Of the four econometric models, the four-step-ahead and twelve-step-ahead forecasts generated by the neoclassical model track the out-turn data least well. Given the theoretical difficulties and empirical inconsistencies associated with this model of investment, its poor performance should come as no surprise. None of the models are found to generate significantly more accurate static forecasts than the ARIMA model and only the $Q$ model outperforms the ARIMA model in terms of four-step-ahead forecasts. The forecasts generated by the VAR representations of these theories of investment are also rather poor and certainly not significantly better than those generated by their single equation variants. This suggests that the poor forecasting performance of the traditional models is due, at least in part, to the inadequacy of the determinants of investment posited by theory. Previous analysis (contained within Chapters 4 and 5) reveals that the way these models are typically implemented in the literature is also flawed. We augment these basic VARs in line with observations from the empirical literature. When the neoclassical and putty-clay type VARs are augmented with a liquidity variable forecasting performance dramatically improves, but the static and four-step-ahead forecasts from these models are not significantly more accurate than those generated by the benchmark ARIMA model. Moreover, the static forecasts from the modified
putty-clay VAR are not conditionally efficient with respect to those generated by the benchmark ARIMA model.

In general, more encouraging results are obtained from the VARs not based on the theory of investment. We consider two basic systems. The first, the construction system, consists of variables for private industrial and commercial construction output, private industrial and commercial new orders, private sector output, tender prices, market conditions and real interest rates. The second, the macro system, contains variables for private industrial and commercial construction output, private industrial and commercial new orders, aggregate output, total employment, ICCs' liquidity, and the real rate of interest. From the basic construction system, we develop three further systems by augmenting it in turn with the real construction wage, employment in construction relative to total employment, and a variable measuring investment intentions. From the basic macro system, we develop another system by augmenting it with capacity utilisation and business optimism variables. Of the construction systems, that containing the real wage variable performs best. Although there is little to choose between the forecasting performance of the two macro systems, we prefer the basic system on the grounds that augmentation with leading indicator variables adds nothing to performance. Both the preferred construction system and the preferred macro system generate significantly more accurate four-step-ahead forecasts than the benchmark ARIMA model. Moreover, the static forecasts generated by each model are conditionally efficient with respect to the static forecasts generated by the ARIMA model. We are not able to discriminate between the forecasting performance of these two VARs or indeed between the forecasting performance of either model and the $Q$ model.

Given the difficulties associated with identifying parameters of the investment equations and the dynamics of the investment problem, it would seem that a less structural approach of modelling, such as VAR analysis, is worth pursuing. VARs have an additional advantage in any forecasting exercise, since the forecaster need not make any assumptions about the values of exogenous variables in the forecast period. From this work it seems that the determinants of investment behaviour suggested by theory are not sufficient to explain net private sector investment in industrial and commercial buildings. This provides more support for the use of atheoretical
modelling techniques for forecasting purposes. Given the theoretical inconsistencies in these standard investment models, the difficulties associated with their empirical implementation, and the general lack of success of studies aiming to forecast investment expenditure, we should expect VAR models to outperform traditional models of investment. These expectations are fulfilled in this analysis of private industrial and commercial construction.

7.2 Limitations of this Work and Avenues of Future Research

The first point to note here is that the investment models estimated in Section 5.2 are not necessarily the best investment models that could have been estimated within the framework of the four dominant theories of investment. For example, more successful neoclassical models for forecasting investment may well be arrived at by considering alternative specifications of technology, or by considering alternative treatments of capital gains in the user cost of capital variable. In addition, given the determinants of investment suggested by theory, more successful forecasts might be obtained by adopting a more general single equation modelling strategy such as general to specific modelling. The investment models we estimate are intended to be typical of those estimated in the empirical literature.

Of course, there exists the possibility that the poor forecasting performance of the investment models estimated here is due to the fact that modelling new private industrial and commercial construction output as an investment problem is inappropriate. Although new construction output represents capital formation, the differences between the investment expenditure and construction output data may be more important than we have credited. However, as we discussed in Section 4.2, we believe the construction output data to provide a better measure of investment in industrial and commercial buildings than the official investment expenditure data.

There exists the possibility that differences in timing between the recording of investment expenditure and construction output data are important. This, and the fact that both lag changes in the determinants of investment, suggest that there may be some value to modelling new orders for industrial and commercial buildings placed by the private sector as an investment problem. Certainly, the dynamics of this
problem are likely to be more straightforward than the dynamics of the problem with investment expenditure or construction output data. However, our goal was to model new private industrial and commercial construction output and not new orders.

There also exists the possibility that the determinants of private industrial construction are different to the determinants of private commercial construction. Also, the dynamics of investment in the two types of asset may differ. This would suggest that parameter estimates from equations that aggregate across these types of assets will be distorted. Thus, one obvious avenue of future research would be to analyse these two types of construction output separately.

We do not claim to have estimated the best models for forecasting private industrial and commercial construction output. However, this work suggests that it is likely to be more profitable to model new private industrial and commercial construction output using VAR analysis than using the traditional investment models.


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