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## **SOUND SPEED PROFILE STRUCTURE AND VARIABILITY MEASURED OVER FLAT TERRAIN**

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### **Abstract**

This paper describes a fundamental data base and data-variability guide for outdoor sound propagation models requiring sound speed profiles. The vertical sound speed profile has been measured directly over an extended period at a flat terrain site and to a height of 150m using a RASS (radio-acoustic sounding instrument). Additionally, vector wind profiles were available at 10m height intervals from a SODAR (an acoustic radar), and carefully calibrated wind and temperature data recorded at a number of fixed sites on a 120m mast. Combinations of these data sources are used to evaluate a number of influences, which include: applicability of the log-linear approximation; vertical variability of sound speed; change of wind direction with height; vector sound speed variability; effects of averaging intervals; short-term gust effects; longer-term diurnal effects; and fetch. These results demonstrate how variations in temperature, wind speed, and wind direction propagate through to sound speed profile changes, and into fitted parameter changes, thereby providing guidance on interpretation of comparisons between model output and propagation measurements.

### **INTRODUCTION**

Sound propagation models are used extensively for urban planning, traffic installations, and industry impact evaluations. These models typically take source data which is estimated or known from prior measurements and predict the sound intensity field under various meteorological conditions.

The main impact on propagation models by the meteorology is the profile of sound speed with height. Since this profile is unlikely to be measured throughout the model domain, or even above a single location, it must be estimated from meteorology either measured at the ground or predicted from a local-area meteorological model.

The function of the present on-going work is to provide guidance as to the quality of simple sound speed profiles which are based on conventional, and limited, meteorological observations or expectations. In this paper we describe some initial results.

The sound speed,  $c$ , depends on both (absolute) temperature,  $T$ , and wind speed  $U$ . For propagation in a direction making an angle  $\phi$  with the direction the wind is flowing toward,

$$c = c_a + U \cos \phi \quad (1)$$

since most propagation paths of interest are at shallow angles to the ground [1].

The adiabatic sound speed,  $c_a$ , is given by

$$c_a = \sqrt{\gamma \frac{R}{M} T} \quad (2)$$

where  $T$  is the absolute temperature of the air,  $R$  is the universal gas constant,  $\gamma$  is the ratio of specific heats for the air, and  $M$  is the molecular weight of the air. There is a very small influence on both  $\gamma$  and  $M$  due to water vapour.

The question is: How well can  $T$  and  $U$  be estimated as a function of height  $z$  based on simple observations or classifications of the weather?

## EXPECTED PROFILES OF SOUND SPEED

Very near the surface, turbulent processes are most likely friction-dominated and further from the surface turbulent processes are more likely to be buoyancy-dominated. A useful length scale which is an estimator of the transition between these regimes is the Obhukov length

$$L = -\frac{u_*^3 T_0 \rho c_p}{\kappa g H_0} \quad (3)$$

where  $\rho$  is the air density,  $c_p$  is the specific heat of air at constant pressure,  $H_0$  is the sensible heat flux at the surface,  $T_0$  is the air temperature near the surface,  $u_*$  is called the frictional velocity,  $\kappa = 0.4$  is von Karman's constant, and  $g$  is the acceleration due to gravity. For a stable atmosphere (in which air displaced vertically returns to its original position),  $L > 0$ , and  $L < 0$  for an unstable atmosphere. The Monin-Obukhov similarity theory postulates that the shapes of the profiles of  $U$  and potential temperature  $\Theta$  are functions only of the dimensionless buoyancy parameter  $z/L$ . The Businger-Dyer relations are empirical profiles based on a large body of meteorological data. A modification of these profiles gives

$$U = \frac{u_*}{\kappa} \left( \ln \frac{z}{z_0} + 5 \frac{z}{L} \right) \quad \text{for } \frac{z}{L} \geq 0 \quad (4a)$$

$$U = \frac{u_*}{\kappa} \left\{ \ln\left(\frac{z}{z_0}\right) - \ln\left[\left(\frac{1+x^2}{2}\right)\left(\frac{1+x}{2}\right)^2\right] + 2 \tan^{-1} x - \frac{\pi}{2} \right\} \quad \text{for } \frac{z}{L} < 0 \quad (4b)$$

where

$$x = \left(1 - 15 \frac{z}{L}\right)^{\frac{1}{4}}$$

and  $z_0$  is the roughness length which is related to the size of the individual features protruding from the surface. Except above forest and urban canopies  $z_0 < 0.1$  m.

Although the unstable case has a more complex dependence on  $z$ , logarithmic profiles of the form

$$U = a_1 + a_2 z + a_3 \ln(z) \quad (5)$$

apply approximately in all cases (see Fig. 1).

For potential temperature

$$\Theta = \Theta_0 + \frac{\theta_*}{\kappa} \left( \ln \frac{z}{z_0} + 5 \frac{z}{L} \right) \quad \text{for } \frac{z}{L} \geq 0 \quad (6a)$$

$$\Theta = \Theta_0 + \frac{\theta_*}{\kappa} \left( \ln \frac{z}{z_0} - 2 \ln \frac{1+x^2}{2} \right) \quad \text{for } \frac{z}{L} < 0 \quad (6b)$$

where  $\theta_* = -\frac{H_0}{\rho c_p u_*}$  and  $\Theta_0$  is effectively the potential temperature at height  $z_0$ . The

temperature profile is related to the profile of the potential temperature through

$$\frac{\partial T}{\partial z} = -\frac{g}{c_p} + \frac{T}{\Theta} \frac{\partial \Theta}{\partial z} \quad (7)$$

but for sound profiles near the surface

$$\frac{\partial T}{\partial z} \approx -\frac{g}{c_p} + \frac{\partial \Theta}{\partial z}. \quad (8)$$

Therefore

$$T = T_0 + \frac{\theta_*}{\kappa} \left( \ln \frac{z}{z_0} + 5 \frac{z}{L} \right) - \frac{g}{c_p} z \quad \text{for } \frac{z}{L} \geq 0 \quad (9a)$$

$$T = T_0 + \frac{\theta_*}{\kappa} \left( \ln \frac{z}{z_0} - 2 \ln \frac{1+x^2}{2} \right) - \frac{g}{c_p} z \quad \text{for } \frac{z}{L} < 0. \quad (9b)$$

Again, both profiles are closely approximated by a profile of the form  $c_a = b_1 + b_2 z + b_3 \ln(z)$ .

The combination of temperature variation and wind speed variation with height means that the sound speed profile can be expected to vary in a log-linear form with height. This conclusion is based on empirical evidence from meteorological data collected in relatively unchanging conditions over level terrain at numerous sites. In practice, the situation might be more complicated because of temporal variations or because the surface conditions over which the wind is blowing are changing. This is the focus of the current work.

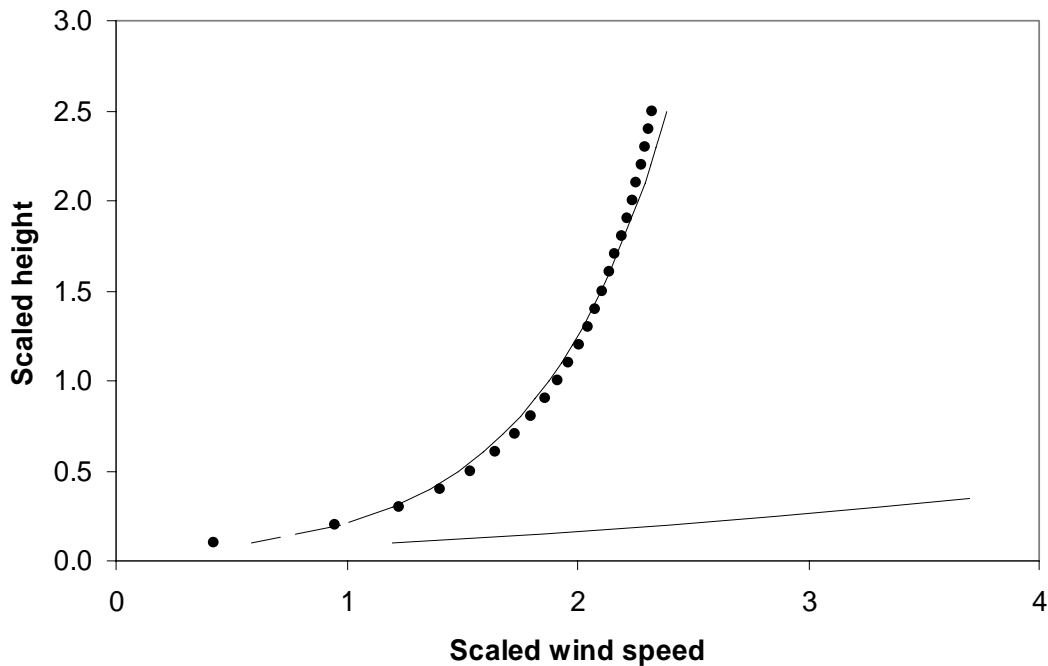


Figure 1. Log-linear variation of scaled wind speed  $\kappa U/u_*$  with scaled height  $z/L$  for  $z_0 = 0.05m$ . Unstable atmosphere (dots); log-lin fit (dashed line); stable atmosphere (solid line).

In fact, even the similarity-based behaviour of the fitted coefficients  $b_1$ ,  $b_2$ , and  $b_3$  is quite complex, as shown in Fig. 2.

## REMOTE ATMOSPHERIC PROFILING METHODOLOGY

It is generally quite difficult to obtain profiles of the *actual* sound speed for use in model validations. However, a combination of SODAR and RASS instruments provides such a profile. A RASS sends a pulse of sound vertically and uses a microwave transmitter and receiver to obtain reflections off the upward-propagating sound wave. Suitable design gives a real-time vertical profile of  $c_a$ : this is a direct measurement of how quickly the sound propagates upward. A SODAR sends a pulse of sound upward slightly off-vertical. Echo signals are received from scattering by turbulence and the Doppler shift gives the radial wind speed component. By suitable choice of three acoustic beam directions, the three vector components of the wind can

be obtained typically every 10 m up to a height of more than 100 m. The combination of RASS and SODAR therefore gives  $c$  estimates every 10 m or so, but of course with measurement errors.

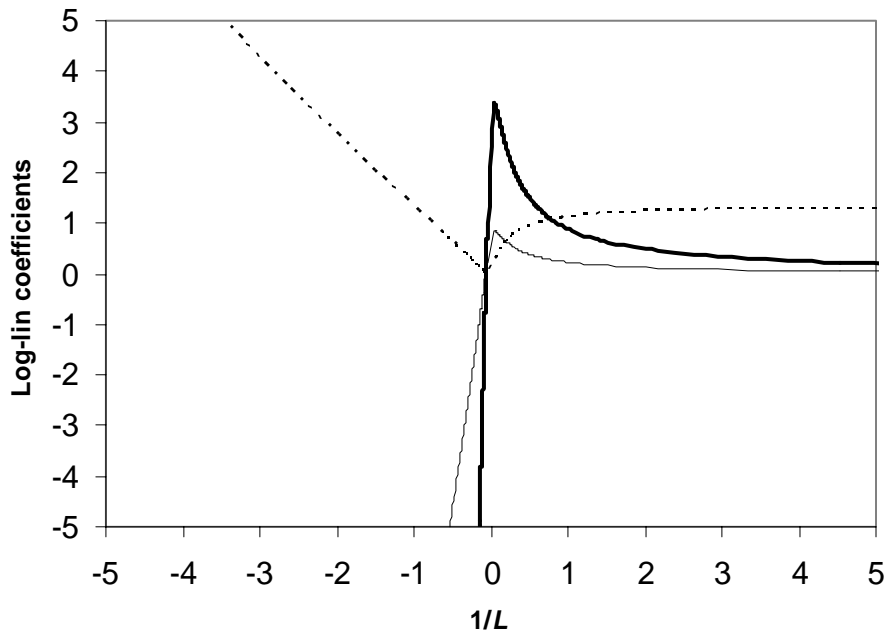


Figure 2. Predicted behaviour of log-lin coefficients as the atmosphere varies from unstable through neutral to stable conditions. Coefficients  $b_1-c_0$  (dark line),  $b_2$  (dashed line), and  $b_3$  (light line).

During the WISE EU Project [2], the Profiler Inter-comparison Experiment (PIE) involved very careful measurements of wind speed made with Metek, AeroVironment, and Scintec SODARs in comparison with a 120 m instrumented tower at Hovsoere in Denmark.

A Metek RASS was also used to record vertical profiles of temperature, obtained through the relationship between  $c_a$  and  $T$ . Because of the careful validation of the remotely-sensed database, this experiment provides a particularly good source of sound speed data under a range of meteorological conditions over a flat terrain site. Here we concentrate on data recorded with the Metek PCS2000-64 SODAR with the Metek 1290MHz RASS.

The test site is the National Danish Test Station for Large Wind Turbines situated in the northwest of Denmark close to the North Sea. The test site is flat, surrounded by grassland, with no major obstacles in the immediate neighbourhood and at a distance of 1.7 km from the west coast of Denmark. The prevailing wind direction is from the west. Reliable measurements were recorded from 1/4/2004 until 20/8/04.

## FITTING LOG-LINEAR PROFILES TO MEASURED DATA

Central to the current work is the fitting of the parameters  $b_1$ ,  $b_2$ , and  $b_3$  to measured data and also considering the associated uncertainties in the fitted parameters. Since the log-lin profile is linear in its parameters, conventional linear least-squares fitting can be used. SODAR measurement errors generally increase with height because of the spherical spreading of the signal and greater influence of the background acoustic noise in the Doppler shift determination. However, for the low heights considered for most outdoor sound propagation, the quality of the Doppler spectrum peak estimation is generally very good, so we will not consider weighted least-squares at this stage.

The parameters are estimated in the usual least-squares manner, using basis functions of 1,  $z/z_0$ , and  $\ln(z/z_0)$  for the model  $U = b_1 + b_2 \frac{z}{z_0} + b_3 \ln\left(\frac{z}{z_0}\right)$ . The errors in the parameters can also be estimated *a priori* by going through the least-squares methodology and approximating the various sums by integrals over  $z$  from  $z_0$  to the upper measurement height  $z_m$ . This gives

$$\begin{aligned}
 \sigma_{b_1}^2 &\approx 4 \frac{\sigma_U^2}{N} \left( \ln^2 \frac{z_m}{z_0} - 5 \ln \frac{z_m}{z_0} + 7 \right) \\
 \sigma_{b_2}^2 &\approx 48 \frac{\sigma_U^2}{N} \frac{z_0^2}{z_m^2} \\
 \sigma_{b_3}^2 &\approx 4 \frac{\sigma_U^2}{N}
 \end{aligned} \tag{10}$$

where  $N$  is the number of heights at which  $U$  is measured, and  $\sigma_U^2$  is the variance in  $U$  measurement. From this the variance in  $U$  can be estimated when a log-lin profile is used.

As an initial case study, 33 measured profiles of  $U$  and  $T$  were fitted with the log-lin profile model. Fig. 3 shows all profiles. There is generally a maximum around 70m. This may be due to a static echo from the instrumented mast, which is at about this distance. On the other hand the profiles are consistently monotonically increasing to this height and also decrease beyond this height.

When least-squares fitting is performed, the residuals shown in Fig. 4 are obtained. Here we have fitted both a log profiles and a log-lin profile for comparison. It can be seen that the log-lin residuals are smaller (closer to zero). Also shown are the standard deviations in residuals, which give a measure of measure and fitting variability. The errors in the sound speed gradient are found to be greatest near the ground: if weighted least-squares fitting is used, with weighting decreasing with height, this conclusion may change.

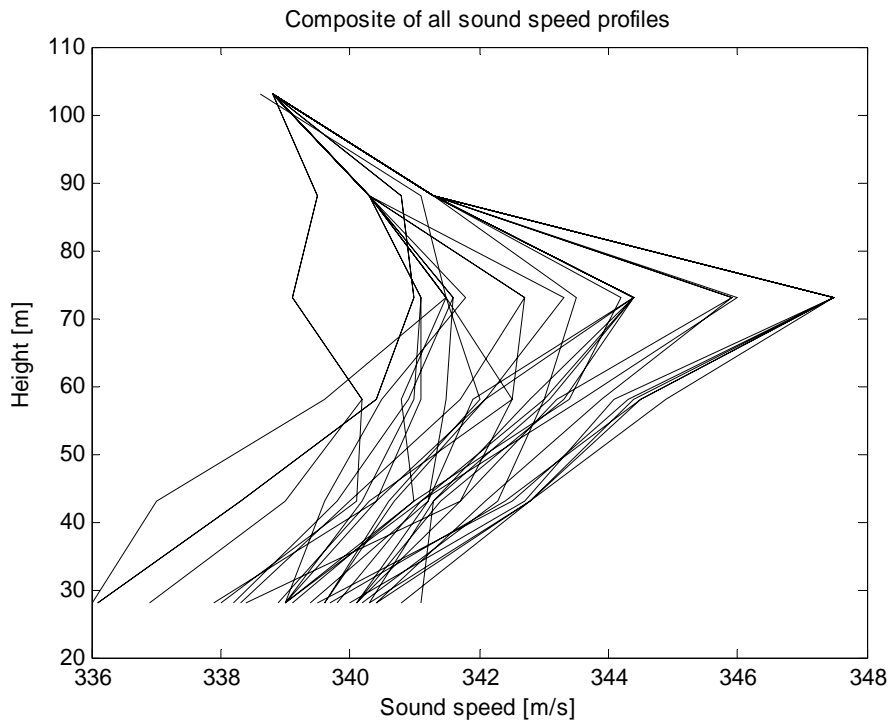


Figure 3. A composite of all profiles in the case study.

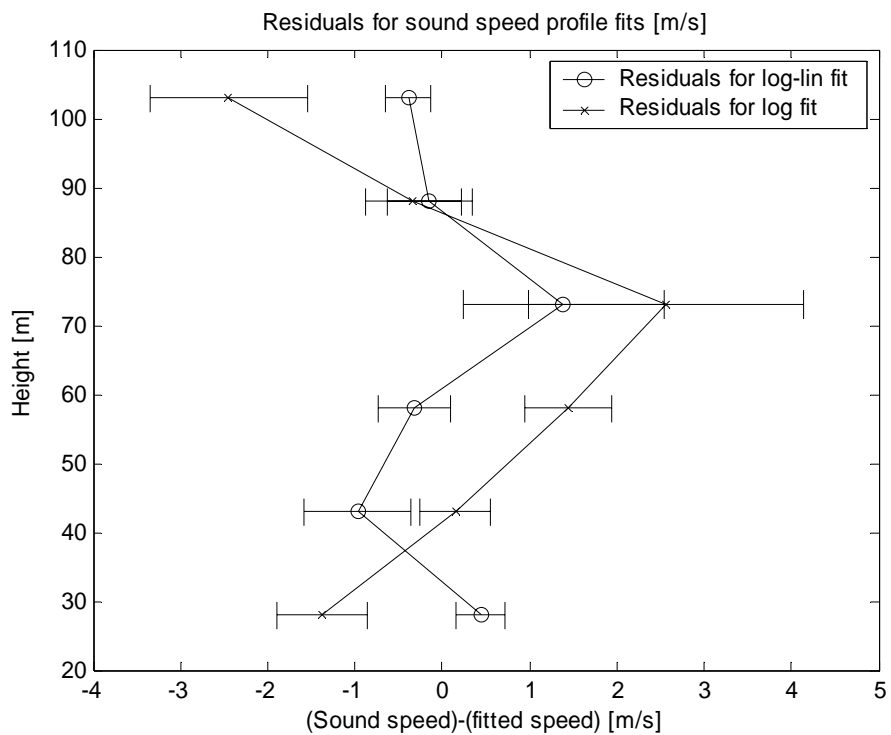


Figure 4. Mean residuals in fitting sound speed profiles, and variability in the residuals.



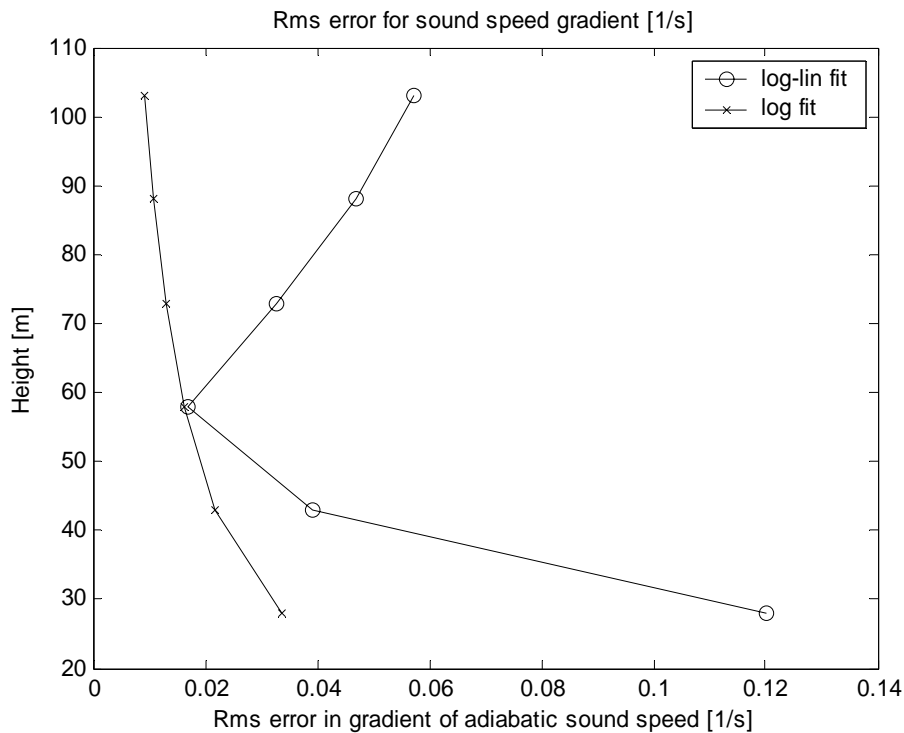


Figure 5. The rms error in the sound speed gradient, based on the errors in fitted sound speed profile.

## SUMMARY

In this work we are seeking to answer a number of questions: applicability of the log-linear approximation; vertical variability of sound speed; change of wind direction with height; vector sound speed variability; effects of averaging intervals; short-term gust effects; longer-term diurnal effects; and influence of fetch.

The material above gives a first introduction to the approach taken, the data set available, and some early results on a very limited case study of 33 profiles. Given that profiles are obtained every few seconds, and data were recorded for over four months, there is a very large dataset available for addressing the questions listed above.

## REFERENCES

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