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Al-Obaedi, JTS and Yousif, S

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EXAMINING THE LIMITATIONS FOR VISUAL ANGLE-CAR FOLLOWING MODELS

Jalal Al-Obaedi
PhD student
University of Salford

Saad Yousif
Senior Lecturer
University of Salford

Abstract

Visual angle car following models assume fixed values (thresholds) for the angular velocity ($\Delta\theta/\Delta t$) over which the driver of the following vehicle is affected by the leading vehicle. This paper explains the advantages and limitations of such models and provides possible solutions for these limitations. Sensitivity analysis has been carried out for these purposes as a part of this paper.

It was found that the main advantages of using visual angle models are their simplicity and ability to represent the effect of the widths of different types of vehicles (e.g. heavy goods vehicles, cars and motorbikes) on space headway (i.e. the clear distance between leading and following vehicles). Also, these models are simply capable of including the effect of driver's reaction time as a function of traffic density, relative speeds between leading and following vehicles and the action/behaviour of the leading vehicle (i.e. accelerating/decelerating following a change in local traffic conditions).

However, the main concern in using such models is related to the appropriate choice of the threshold values. The paper will illustrate that using a fixed threshold value for the angular velocity will be illogical since it eliminates the effect of variations in speed of the following vehicle for the same given relative speed between the leader and follower (ΔV). Moreover, other limitations are related to the effect of leader's width on the acceleration/deceleration rates of the follower.

Finally, and in order to deal with the above limitations and advantages that visual angle models have, an alternative car following model has been proposed. The proposed model will then be tested against data from sites to examine its applicability.

Keywords: Traffic micro-simulation models, car following, visual angle models

1. Background and literature review

1.1 Introduction

Car following models describe the reaction of the driver of the following vehicle travelling close to the vehicle ahead. This reaction is represented by his/her acceleration/deceleration rate based on the differences in speeds and spacing between these two vehicles.

From the middle of the twentieth century and until now, several car following models were developed. Gazis-Herman-Rothery (1960) model (GHR) represented the earlier car following model which was formulated in 1958 at the General Motors Research labs in Detroit. According to the model, the acceleration of the following vehicle is based on relative velocity, relative spacing and the following vehicle's velocity as shown in the following equation:

$$a_{f,t+\tau} = cv^m \frac{v_{f,t} - v_{l,t}}{x_{f,t} - x_{l,t}} \quad \dots \text{Equation 1}$$

Where:

$a^f(t + \tau)$ is acceleration of the follower

τ is driver's reaction time (for the following vehicle)

$v^l(t), x^l(t)$ is the velocity and position of the leading vehicle at time t , respectively

$v^f(t), x^f(t)$ is the velocity and position of the following vehicle at time t , respectively
and

c, m and l are model parameters.

1.2 Visual angle car following models

One of the earlier car-following models is the visual angle model. The visual angle as shown in Figure 1 is given by the following equation:

$$\theta = 2 \tan^{-1} \left(\frac{w}{2H} \right) \quad \dots \text{Equation 2}$$

Where:

w is a width of the leading vehicle.

H is the spacing between the leading and the following vehicles.

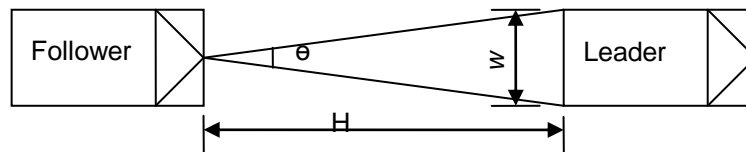


Figure 1 Illustration of the visual angle (θ)

Michaels (1963) observed that the detection of the relative velocity depends on the rate of change of angular motion (angular velocity) of an image across the retina of the eye of the follower driver (Fox and Lehman, 1967).

The angular velocity is found by differentiating Equation 2 with respect to the time (t)

$$\frac{\Delta \theta}{\Delta t} = -w \frac{V_L - V_F}{(X_L - \text{length}(l) - X_F)^2} \quad \dots \text{Equation 3}$$

Where:

V_L and V_F are speeds of leading and following vehicles respectively.

w is the width of leading vehicle.

X_L and X_F are positions of leading and following vehicles respectively.

Visual angle models are described by previous researchers, such as Brackstone and McDonald (1999), and Panwai and Dia (2005), as one type of psychophysical or action point models since these models define the next vehicle's action on whether or not the follower exceeds certain thresholds. These assume fixed values (thresholds) for angular velocity. Once the absolute value of the angular velocity exceeds the threshold, the follower will accelerate or decelerate opposite to the sign of the relative angular velocity.

1.3 Angular velocity thresholds for visual angle models

Pipes (1967) reported that the most carefully controlled study by Michaels and Cozan (1963) of the absolute threshold of the angular velocity indicates that it is reasonable to use 0.0006 rad/sec as a mean value for the threshold. This means that if the relative speed is 10 km/h and the width of the

leading vehicle is 1.8m, the angular velocity can be detected if the relative spacing is less than 91 m (based on Equation 3 above).

Fox and Lehman (1967) described a car following model based on visual angle concept. According to the model, when the threshold value is exceeded, the driver is considered to be in a "velocity-detection mode". Below that threshold, the driver is assumed to be in a "distance-detection mode". If the driver of the following vehicle is in a velocity-detection mode, GHR model is used. In a distance-detection mode, the acceleration of the following vehicle is considered to be mainly depends on the relative spacing between the leading and the following vehicles. A base value of 0.0006 rad/sec (similar to that used by Michaels) was chosen.

Ferrari (1989) used a car following model with different threshold value which is 0.0003 rad/sec. The model based on the assumption that the follower will choose to decelerate or accelerate in case the threshold is exceeded. A minimum time gap of 1 second is assumed between the two successive vehicles.

Based on experiments carried out to scale the relative velocity between vehicles, Hoffmann and Mortimer (1994 and 1996) suggested a different threshold value of 0.003 rad/sec. They concluded that only when the subtended angular velocity of the lead vehicles exceeded that threshold were the subjects able to scale the relative velocity.

Table 1 presents a brief summary of the values used for angular velocity thresholds by various researchers. It is clear that the assumed range of values used was different in most cases. Therefore, this paper examines some of the limitations in using these visual angle thresholds for use in car following models.

Table 1 Summary of angular velocity thresholds used by various researchers

Researcher(s)	Threshold value ($\Delta\Theta/\Delta t$) (rad/sec)
Michaels and Cozan (1963)	0.0003 – 0.001
Fox and Lehman (1967)	0.0006
Ferrari (1989)	0.0003
Hoffman and Mortimer (1994 and 1996)	0.003

1.4 Possible applications of visual angle models

According to Traffic Technology Today (2008), researchers in Germany (i.e. Karlsruhe University with Fraunhofer Institute for Information and Data Processing) are carrying out instrumental work on co-ordinating manoeuvres of cars following each other without the need to drivers' intervention. This is done in an emergency situation by equipping cars travelling on the same lane with car-to-car communication and integrated sensors to enable them recognise their surroundings and avoid potential obstacles (such as a stopping vehicle in front).

The selection of appropriate angular velocity threshold values which suits such safe manoeuvres could help in providing the basis for the settings of such sensors used in the car-to-car communication system.

2. Visual angle model assumptions

This section presents the main assumptions for the visual angle model which is used in this theoretical study to show the advantages and limitations of such models.

The main assumption of the model is based on whether or not the angular velocity calculated from Equation 3 exceeds the assumed angular velocity threshold values. The effect of the visual angle model's parameters (i.e. relative velocity and relative spacing) on the angular velocity values is shown in Figure 2. A value of 1.8 m is assumed as the width of the leader.

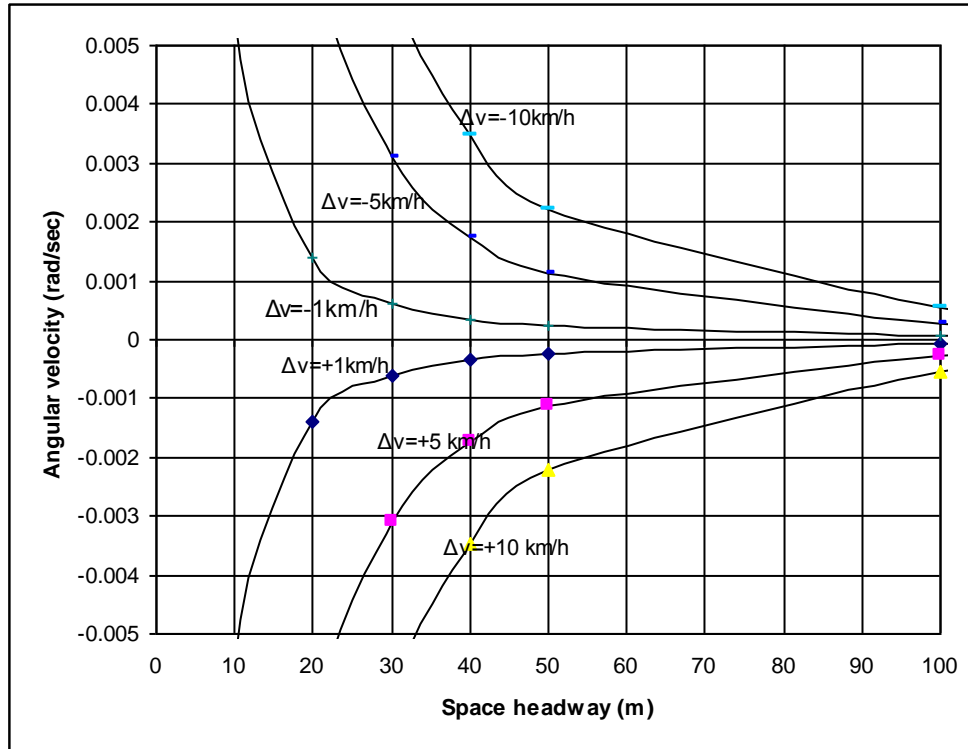


Figure 2 Relationship between the angular velocity and the space headway for different relative speeds between the leader and its follower (ΔV)

In general, the Figure shows that the absolute angular velocity increases with an increase in the absolute relative velocity between the leader and its follower, and decreases with an increase in the relative spacing between them. If the absolute angular velocity becomes higher than a certain selected threshold, the follower starts to accelerate or decelerate opposite in sign to that of the angular velocity value. For example, if the angular velocity threshold is + 0.003 rad/sec (as suggested by Hoffman and Mortimer (1994 and 1996), the follower will decelerate if his/her angular velocity is higher than 0.003 rad/sec and accelerate if this angular velocity is below -0.003 rad/sec.

The selected values for acceleration or deceleration are the minimum of the following rates (see for example Fox and Lehman, 1967 and Ferrari, 1989):

- the acceleration rate which is required to reach the desired speed,
- the acceleration/deceleration rate required to reach the leader's speed, and
- the acceleration/deceleration rate to maintain the minimum desired spacing.

When the angular velocity value is within the selected range for the angular velocity threshold (i.e. + 0.003 rad/sec), the acceleration/deceleration of the follower will not be based on factors described in Equation 3 above. Instead, the acceleration/deceleration of the follower will be based on the desired spacing (or desired time headway). The acceleration or deceleration rates are as shown in Equation 4 and they are a function of both relative distance and relative speed between the leader and follower. Also, they are a function of the desired distance the follower wishes to maintain based on his/her speed. This equation is derived based on the same assumptions reported by Hidas (1996).

$$ac_f = \frac{[(x_l(t + \Delta t) - x_f(t)) - v_f(t) * \Delta t - DTHead * v(f,t) - length(l)]}{\frac{\Delta t^2}{2} + DTHead * \Delta t} \dots \text{Equation 4}$$

Where:

ac_f is the acceleration (or deceleration) rate of the follower

Δt is the scanning time.

DT_{Head} is the desired time (spacing) for the follower.

$x(t + \Delta t)$ is the position of the leader at time $t + \Delta t$

(other terms are as defined before).

In this paper, the threshold values for angular velocity were tested and compared with those suggested by other researchers. In particular, a comparison has been made between the values of 0.0006 rad/sec used by Michaels and Cozan (1963) and that suggested by Hoffman and Mortimer (1994 and 1996) of 0.003 rad/sec as will be discussed in Section 3.

3. Advantages of visual angle models

3.1 Representing the effect of the width of leading vehicle

Based on real data from UK motorway sites, Yousif (1993) reported that some passenger car drivers try to leave sufficient space to avoid visual problems associated with obstructed traffic signs or other traffic control devices on the road. This could contribute to forcing drivers following heavy goods vehicles (HGVs) to leave a much larger space. Parker (1996), when studying the effect of HGVs at three motorway roadwork sites, reported that the presence of HGVs in the traffic stream increases average headways, thus reducing the capacity of the road section.

It was shown that most of other car following models, such as collision avoidance and desired spacing models, could not directly include the effect of the size of the leading vehicle. Visual angle models can take into consideration the effect of the size of vehicles without making any further assumptions since the width of the leader is included in the main equation used for these models.

Figure 3 shows the effect of having different widths of the leader on the starting distance for its follower to be affected by the leader for different angular velocity threshold values. The Figure is based on the assumption that there is a 10 km/h relative speed difference between the two vehicles. From the Figure, it can be shown that the follower starts applying his/her deceleration earlier if the leader is an "HGV" (i.e. width equals to 2.55 m) compared with the case when the leader is a "Car" (i.e. width equals to 1.8 m). However, this direct representation of the width of the leader may seem unnecessary, as will be discussed in Section 4.1.

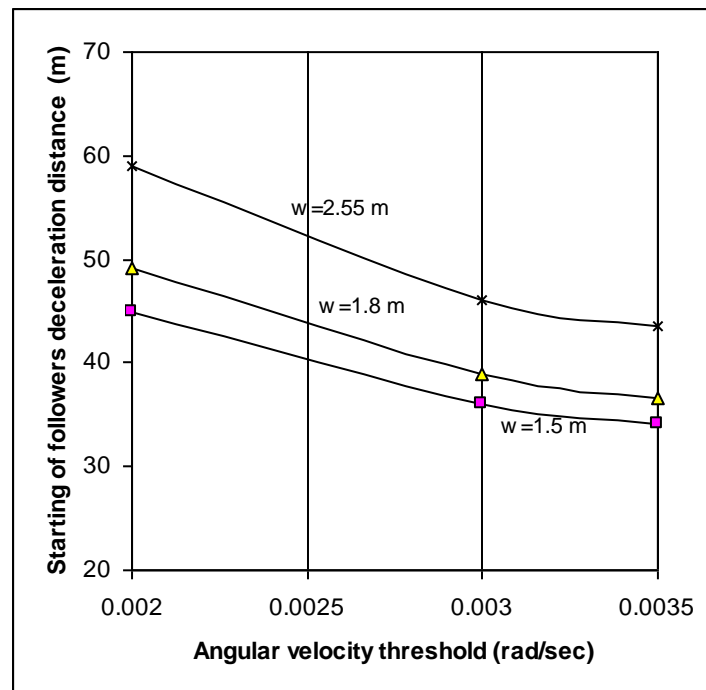


Figure 3 Starting distance for the follower to be affected by its leader

3.2 Modelling of driver's reaction time

3.2.1 Background information on reaction time

Reaction time indicates a time lag between the detection of a stimuli and the application of a response. O'Flaherty (1986) stated that the length of perception time varies considerably since it depends upon factors such as, the distance to object, the natural rapidity with which the driver reacts and the optical ability of the driver.

The main limitation in most of the existing car following models is related to the representation of drivers' reaction time. Most models assign a pre-defined single value for each driver as his/her reaction time. Some researchers used two values for each driver to represent the alerted and surprised (unaltered) cases for congested and non-congested traffic conditions, respectively. The majority of such models could not represent the follower's reaction time to show how it varies with traffic conditions.

Table 2 shows a summary of some of the main work in determining driver's reaction time. It is clear from the different trials to estimate driver's reaction time that there are some difficulties in doing so accurately.

Table 2 Summary of brake reaction time based on previous research

Researcher	Median reaction time (sec.)	Situations
Johansson and Rumer (1971)	0.73, 0.54	Surprised, Alerted
Lerner <i>et al.</i> (1995)	1.44	Surprised
Maycock <i>et al.</i> (1999)	1.2	Unaltered
Zhang and Bham (2007)	0.6	From Micro-simulation

3.2.2 Brake reaction time based on visual angle models

An attempt has been made to represent driver's reaction time based on visual angle models. Figure 4 shows the relationship between the angular velocity and the time for different initial spacings between pairs of vehicles. Here, the time is that of the follower needed from the start of deceleration of the leader, assuming a constant deceleration of the leader of -2m/s^2 with an initial speed of 90 km/h for both leader and follower.

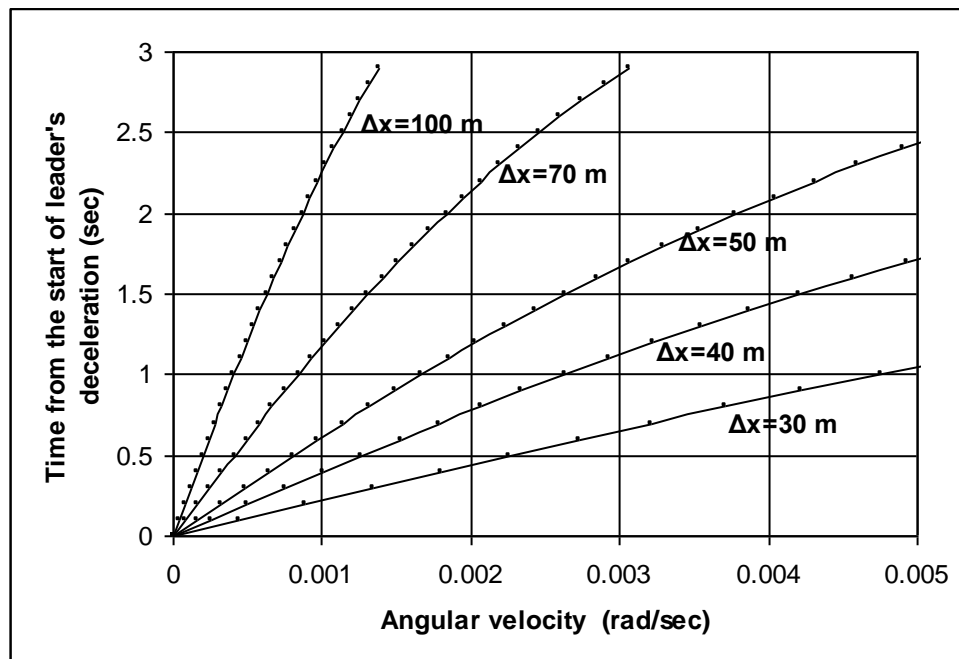


Figure 4 Relationship between angular velocity and time (assuming that leader's constant deceleration of -2m/sec^2 for different initial space headways)

The Figure shows that when traffic density is high (i.e. spacing is relatively small), the follower will react to the deceleration of the leader within a shorter time than for the case of lower traffic density. For example, when the initial spacing is 40 m, the follower will react to the leader's deceleration after 1.1 sec (assuming a threshold value of 0.003 rad/sec). For the case of 50 m initial spacing, the follower will react after 1.6 sec.

Figure 5 (which is derived from Figure 4), illustrates the relationship between initial spacing between the leader and its follower and the follower's reaction time. The Figure is based on two selected angular velocity threshold values of 0.003 and 0.0006 rad/sec as suggested by Hoffmann and Mortimer (1994 and 1996) and Michaels (1963), respectively.

For relatively high density conditions (i.e. small spacings of say 40 m) Hoffmann and Mortimer's thresholds suggest that the reaction time is about 1.1 sec, whereas the threshold suggested by Michaels yield a value of reaction time of 0.2 sec. If these two reaction times of 1.1 and 0.2 sec are compared with those in Table 1, the results show that the threshold value of 0.003 rad/sec (as suggested by Hoffmann and Mortimer) gives more reasonable representation for the reaction time. Therefore, in order to use the threshold values suggested by Michaels (1963), there should be another parameter (namely an extra brake reaction time) which needs to be considered in visual angle car following models.

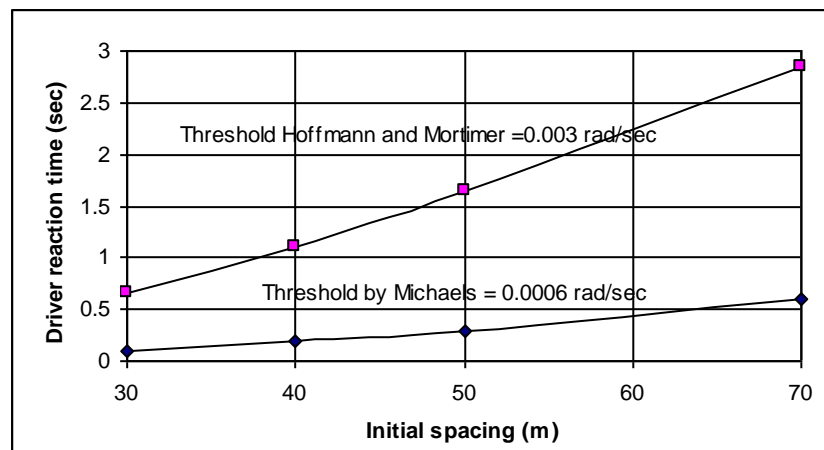


Figure 5 Relationship between driver reaction time and initial spacing for two values of angular velocity thresholds

4. Limitations of visual angle models

There are several limitations involving the use of visual angle models. This section discusses the main limitations and how they are dealt with.

4.1 Representing the width of vehicles

As shown in Equation 3 and Section 3.1 above, the width of the leading vehicle is directly included within visual angle models. Therefore the follower will react to the vehicle ahead at a distance influenced by that width as shown in Figure 3.

Figure 3 shows that the follower will start his/her deceleration earlier if he/she is preceded by a "Car" having a width of 1.8 m instead of 1.5 m. The difference in the starting distance for deceleration between the two "Cars" in this case is about 4 m (this is based on the assumption that there is a 10 km/h relative speed between the follower and its leader and without reference to the actual operating speed). This does not seem to be logical considering the fact that there is only a 0.3 m difference in width between the two "Cars".

One possible solution to this is by assigning one specific width for each type of vehicles. For example, if the leader is a "Car", the width is assigned to be 1.8 m even if the actual width of the leader is lower or higher than that value. This is based on the assumption that on average the width of "Cars" is equal to 1.8 m. Similarly, when the leader is an "HGV", the assigned width is 2.5 m for all "HGVs". These selected values could be examined further during the calibration and validation process of the model.

There is another limitation in representing the size of vehicles. The follower will not be able to react to the effect of the width of his/her leader if the angular velocity ($\Delta\theta/\Delta t$) is within the range of threshold values of say ± 0.003 rad/sec (as described in Section 3.2.2). This is because the follower will make his/her acceleration/deceleration based on the desired spacing.

When the differences in speeds (i.e. relative speeds) between successive vehicles are low (see Figure 2), it is expected that the angular velocity to be below the selected thresholds. Therefore, at these circumstances and according to this limitation, the space headway between the follower and the leader will be the same if the leader is a "Car" or an "HGV". This does not seem to be logical since studies by Yousif (1993) and Parker (1996) suggest that this distance is influenced by the type of vehicles in the traffic stream.

One possible solution for this limitation is to modify the desired time headway of the follower by including the width of the leader as a factor. This will be discussed further in Section 5.

4.2 Ignoring the effect of the leader/follower speeds

A sensitivity analysis conducted on the visual angle model has shown that the model is not sensitive to the leader/follower actual speeds and it is just sensitive to the difference between these two speeds. Figure 6 shows the relationship between angular velocity and reaction time for any assumed speeds of the traffic stream (i.e. leader's and follower's speeds). The Figure is based on the assumption that the leader is assumed to apply a constant deceleration of -3 m/sec² with initial spacing between the two vehicles of 50 m.

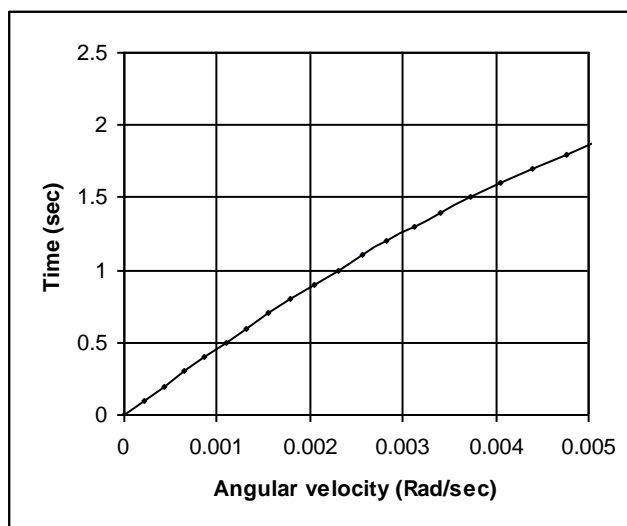


Figure 6 Relationship between angular velocity and reaction time for any speed (assuming leader's and follower's speeds are initially the same before the leader starts to decelerate)

The Figure shows that for an angular velocity of 0.003 rad/sec, the follower requires 1.3 seconds to react to the deceleration of the leader. This means that the follower will react to its leader regardless of the values of their initial speeds. This is in disagreement with the finding by O'Flaherty (1986) in which it was stated that the perception time at high speeds is usually less than that at low speeds (i.e. speed is a factor).

One possible solution for this limitation is to make either the angular velocity or the angular velocity threshold as a function of the leader/follower's speed. Since the angular velocity value comes from a mathematical calculation (see Equations 2 and 3), it is not possible to include these speed values into the equations without altering the whole equation. Therefore, further investigation is required to look into how the effect of the speeds is taken into consideration when determining the angular velocity threshold values. Also, the effect of other parameters influencing the calculation of the angular velocity threshold values (such as difference in spacing between vehicles) should be included in the investigation.

4.3 Representing driver's reaction time

In addition to the limitations described above with regards to the inability of the visual angle model to represent driver's reaction time as a function of leader/follower speeds, there is another limitation in representing this factor. The model is incapable of representing how driver's reaction time differs if the leader makes his/her deceleration with or without using the brakes (i.e. whether or not brake lights are obvious to the follower). However, this issue is not specific to visual angle models only. It is one of the existing limitations in other car following models and requires further investigation.

5. Proposed model

5.1 Introduction

As discussed above, the main limitations in visual angle models are related to the proper selection of suitable values for the angular velocity thresholds ($\Delta\theta/\Delta t$) for making a decision on acceleration or deceleration. This difficulty is due to the fact that angular velocity thresholds have no obvious connection with traffic characteristics and cannot be directly measured from site. Therefore, it is decided to propose a car-following model which should be able to deal with the above limitations and take into consideration other advantages for such visual angle models.

The main assumption of the proposed model is based on the use of the "Just Noticeable Difference – JND" threshold in determining the time headway. The "JND" is related to Weber's law which states that any change in behaviour is not noticeable until this change exceeds a certain percentage from its original state. This percentage is reported in literature to be between 10 and 15%. For example, Hoffmann and Mortimer (1994 and 1996) mentioned 12% for the JND value, while Brackstone and McDonald (1999) reported typical values of 10%.

The proposed model depends mainly on the preferred time headway (PTHead). This headway is a function of the follower's speed. If the speed of the follower is high, the distance between the leader and follower should increase.

In spite of having the width of the leader as a factor in the equation used for visual angle models, the effect of the height of the leader is not represented in these models. This needs to be investigated further since there has been an increase in the last decades in the overall size of vehicles using the road, particularly in relation to height (DETR, 1999 and Department of Transport, 2007).

5.2 Effect of vehicle's size

5.2.1 Effect of vehicle's width

In order to introduce the effect of vehicle's width into the proposed model, the following equations may be used:

$$PTHead_1 = DHead * \left(\frac{w}{1.8} \right)^c \quad \dots \text{Equation 5}$$

Where:

c is the constant of the equation

DHead is the desired headway (seconds) for the case of Car following Car

PTHead1 is the preferred time headway in seconds

w is the width of leader (for $w > 1.8$ m)

5.2.2 Effect of vehicle's height

Similarly, the effect of vehicle's height may be entered into the model as follows:

$$PTHead_2 = DHead * \left(\frac{h}{1.5} \right)^d \quad \dots \text{Equation 6}$$

Where

d is the constant of the equation

h is the height of the leader (for $h > 1.5$ m)

PTHead2 is the preferred time headway in seconds

5.2.3 Summary

Finally, the maximum value of either $PTHead_1$ or $PTHead_2$ obtained from Equations 5 and 6 will be selected as the preferred headway $PTHead$ for that driver.

The values for the width (w) and the height (h) should be obtained from site. However, and in order to overcome such limitations that are discussed in Section 4.1, these two values are recommended to be based on average values representing vehicle's type (i.e. "Cars" or "HGVs").

The two constants in the Equations (i.e. c and d) should be also obtained based on the calibration and validation process. If the majority of drivers are not affected by the size of the leaders, zero values for these two constant should be assigned.

Therefore, the following Equation is suggested to replace Equation 4.

$$a_c(f, t) = \frac{v(f, t + \Delta t) - v(f, t) - v(f, t) * \Delta t - PTHead * v(f, t) - length(l)}{\frac{\Delta t^2}{2} + PTHead * \Delta t} \dots \text{Equation 7}$$

5.3 Headway thresholds and assumptions

There are two threshold values which can be used in the proposed model (based on Weber's law). These are the minimum and maximum preferred headway thresholds by a driver.

a. Minimum headway threshold: Once the actual headway is less than the minimum, the follower will decelerate to maintain his/her preferred headway based on Equation 7. This is given by Equation 8.

$$\text{min headway} = PTHead * 0.88 \dots \text{Equation 8}$$

b. Maximum headway threshold: Once the actual headway is higher than the maximum, the driver will accelerate to maintain his desired speed or to maintain his/her preferred headway based on which one of those gives a minimum acceleration.

$$\text{max headway} = PTHead * 1.12 \dots \text{Equation 9}$$

If none of the above thresholds are exceeded, the follower will keep a constant speed (i.e. acceleration is zero).

However, if it is shown during the calibration and validation process that the minimum and maximum headways are not following the above formulas, the constants values (0.88, 1.12) should be changed.

Finally, further tests are needed to show the main advantages of this proposed model.

6. Conclusions

This paper presented the main advantages and limitations in using visual angle models to simulate real traffic conditions. The main advantage is the model's ability to represent brake reaction time as a function of traffic density and relative speeds between leading and following vehicles. Another advantage is in representing the effect of the width of the leader as part of the calculations used in this model.

It was found that if the angular velocity threshold values ($\Delta\theta/\Delta t$) of about ± 0.003 rad/sec were used, these gave equivalent driver's reaction times which were generally within acceptable ranges.

The selection of appropriate threshold values could help in providing the settings needed for the car-to-car communication system in future trials on co-ordinating car following manoeuvres without the need to drivers' intervention.

In this paper two main limitations were discussed, namely the effect of a slight change in the width of the leader and the other is the fact that the equation for visual angle models should not be used when the angular velocity is within the selected threshold values (say between ± 0.003 rad/sec).

Another limitation for using such models is related to the appropriate choice of threshold values for different operating speeds. This is due to the fact that such models are only sensitive to relative speeds between the leader and its follower rather than the actual speeds that they are travelling with.

The proposed model described in this paper deals with these limitations. A suggestion has been made on how the size of vehicles could be represented in the model. Also, the preferred time headway thresholds (minimum and maximum) are obtained for the proposed model.

Future work is needed in order to test the assumptions made for the proposed model against real traffic data.

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