Spontaneous spatial fractal patterns in simple linear and nonlinear optical cavities

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Summary

We present an overview of our research on the fractal-generating properties of two distinct optical cavities: fractal eigenmodes of linear systems with inherent magnification, and spontaneous spatial fractals in nonlinear systems. Our latest research focuses on "kaleidoscope" lasers and nonlinear ring-resonator geometries.

Spatial fractals in linear optical systems

Kaleidoscope lasers are an intuitive generalization of classic unstable strip resonators [1] to fully-2D transverse geometries, where the defining aperture has the shape of a regular polygon [2]. The non-orthogonal edges of this element have a profound impact on the structure of the cavity eigenmodes, which exhibit striking complexity and beauty. Most obviously, N-sided regular-polygon boundary conditions impose N-fold rotational symmetry on the intensity pattern. Transverse aperture symmetry also has a strong influence on the excess noise properties of the cavity.

We report on the first detailed analysis of kaleidoscope lasers through accommodation of *arbitrary equivalent Fresnel number* N_{eq} (which quantifies the cavity aspect ratio) *and round-trip magnification M*. All previous analyses have been restricted to regimes where either: $N_{eq} = O(1)$ (when conventional ABCD paraxial matrix modelling, in combination with Fast Fourier Transforms can be deployed [3]), or $N_{eq} >> O(1)$ (in which case asymptotic approximations may be used [4]). Our approach is based on a fully-2D generalization of Southwell's Virtual Source (2D-VS) method [5], and exploits exact (Fresnel) mathematical descriptions of the constituent edge-wave patterns [6]. Mode patterns from 2D-VS calculations are shown in Fig. 1.

One key advantage of our technique is that a single calculation allows one to access entire families of higher-order modes; another is that any particular mode may be computed to any desired accuracy. We also quantify the convergence properties of kaleidoscope laser modes (eigenvalue spectra and patterns themselves) in the limit that $N \to \infty$, where the feedback mirror becomes circular. Important steps forward in the understanding of eigenmode fractal dimension are also detailed.

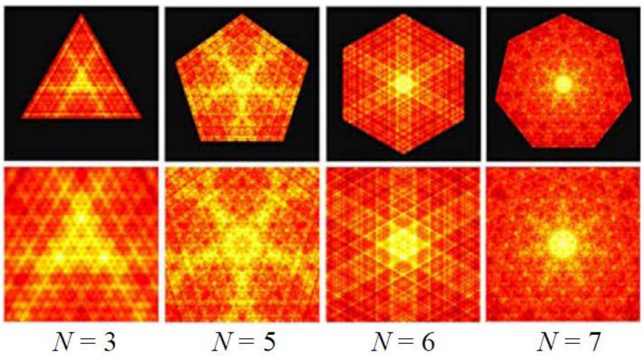

Fig 1. Computations (using the 2D-VS method) of the lowest-loss kaleidoscope laser mode patterns for a cavity with $N_{eq} = 30$ and $M = 1.5$.

Spatial fractals in nonlinear optical systems

Over the last two decades, spontaneous spatial pattern formation [7] has blossomed into a huge field of research in nonlinear photonics. However, the overwhelming majority of theoretical investigations have been rooted in the mean field approximation [8], where light propagation effects are averaged out and the spatiotemporal complexity is consequently reduced. Such models tend to possess, at most, only a single Turing minimum [7] and hence are unlikely to predict multiscale spatial structures. Here, we present the first evidence of spontaneous spatial fractals in ring cavities, *beyond mean field dynamics*, and for a range of nonlinear materials [9]. Linear stability analysis has uncovered multi-Turing threshold minima that are precisely those proposed as indicative of spontaneous fractal generation [10].

We begin by demonstrating simple pattern formation through numerical computations. A small level of background noise is added to a stationary-homogeneous state, and a spatial filter is set so that only those spectral components within the first instability band may propagate freely around the cavity (waves outside this band are attenuated). When the intensity of the stationary state exceeds threshold, spontaneous self-organization (the feedback between diffraction, diffusion, and nonlinearity) drives the system toward a simple static pattern. Once the new single-*K* stationary state is established, we remove the spatial filter and allow all spectral components to propagate freely. One finds that the simple patterns evolve into scale-dependent fractals (see Fig. 2) whose characteristics depend upon system parameters (e.g., diffusion length, pump intensity, and mirror reflectivity).

Both linear and nonlinear fractal generators hold enormous potential for inspiring novel laser designs and a wide range of applications (e.g., more efficient probing, scanning and ablation experiments). Moreover, the huge spatial bandwidths associated with fractal sources may have potential exploitation within distinct novel information contexts. We will conclude with a brief account of prospective new device application technologies.

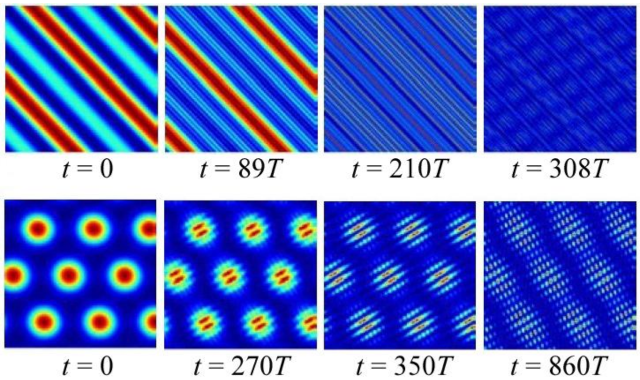

Fig 2. Transition from a simple pattern to a scale-dependent fractal pattern in a (top) dispersive and (bottom) purely-absorptive cavity. *t* denotes the number of transit-times *T* after the filter is removed.

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