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Beam Geometry Calibration of SODARs Without Use of a Mast

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16
17

Abstract

18 A new method for calibration of SODAR wind speed measurements is described. The
19 method makes no assumptions whatsoever about the SODAR operation and its hardware
20 and software, other than the assumption that only one beam is transmitted at a time.
21 Regardless of the complexity of the actual beam shape, the *effective* beam zenith angle is
22 accurately estimated: this is the angle which must be used in estimations of velocity
23 components. In a very simple experiment the effective beam zenith angle has been found
24 to within around 0.2° , which is as good as is required in the most stringent SODAR
25 calibration procedures. It has been found, even for such a short data run, that the
26 estimated beam angle is very close to that calculated from the SODAR array geometry.
27 The main limitation is the requirement for horizontally homogeneous flow, since the
28 regression methods use both a tilted beam and a vertical beam. Note that this is also a
29 fundamental limiting assumption in the normal *operation* of ground-based wind LIDARs
30 and SODARs.

31

32

1. Introduction

33 SODARs transmit a short pulse in at least three upward directions. Scattering from
34 atmospheric turbulent refractive index fluctuations results in a time series signal from
35 each direction. Spectral analysis of time-gated segments of these time series gives a
36 spectral peak whose frequency is a measure of the Doppler shift from the moving
37 scatterers. Using at least three independent acoustic beams assures a system of at least
38 three equations in the vector wind Cartesian components $\mathbf{V} = (u, v, w)$. Solving this set of
39 equations then gives a wind profile with estimates at the centre of each height represented
40 by the centre of each time gate (Bradley, 2007).

41 There is very little that can ‘go wrong’ with such a design. Nevertheless, large
42 efforts have been expended on comparisons between mast-mounted anemometers and
43 SODARs in such experiments as the Profiler Inter-comparison Experiment PIE (Bradley
44 *et al*, 2005), directed toward remote-sensing becoming a viable replacement for mast
45 instrumentation. The most important findings of PIE were that a SODAR gives similar
46 variability in wind speeds to a cup anemometer, but there remain small *systematic* errors
47 in wind speeds estimated by a SODAR. Such biases can be detected through SODAR-
48 mast comparisons, but these are in general rather inconvenient. Therefore we consider a
49 new method for doing *in-situ* field calibrations of wind measurements from a SODAR.
50 This method has the huge advantages of not requiring comparison against some other
51 ‘standard’, nor requiring any assumptions regarding SODAR geometry and operation.

52 The method is equally applicable to wind LIDARs. However, the emphasis on
 53 SODARs is warranted because it is difficult to test a full size SODAR system in an
 54 anechoic facility. Also, the acoustic beam from a SODAR has greater width than the
 55 optical beam from a LIDAR, and therefore the equivalent volume-averaged Doppler shift
 56 is likely to be less well known. This is rather difficult to estimate *a priori*, as opposed to
 57 the beam azimuth angle or the central pointing direction of a vertical beam, which are
 58 well determined by the SODAR antenna geometry.

59 2. SODAR wind measurement calibration

60 *Traditional calibration*

61 Monostatic SODARs use beams tilted from the vertical. The signal scattered back to the
 62 receiver in each tilted beam is Doppler-shifted according to the radial component V_r of
 63 wind velocity \mathbf{V} in the beam direction. For a thin beam in direction

64 $\boldsymbol{\Omega}_0 = (\cos \phi_0 \sin \theta_0, \sin \phi_0 \sin \theta_0, \cos \theta_0)$ and wind velocity $\mathbf{V} = (u, v, w)$

$$65 \quad V_r = \mathbf{V} \cdot \boldsymbol{\Omega}_0 = u \cos \phi_0 \sin \theta_0 + v \sin \phi_0 \sin \theta_0 + w \cos \theta_0. \quad (1)$$

66 At least 3 independent measurements are needed to solve for (u, v, w) . We will
 67 concentrate on the typical 3-beam design. The system of equations

68

$$69 \quad \mathbf{R} = \mathbf{B}\mathbf{V}$$

70 is solved, where \mathbf{R} is the 3x1 vector of measured radial velocity components, \mathbf{B} is the
 71 3x3 weighting matrix, and \mathbf{V} is the 3x1 vector of unknown wind velocity components.

72 The solution $\hat{\mathbf{V}} = \mathbf{B}^{-1}\mathbf{R}$ is used to form $(\hat{u}^2 + \hat{v}^2 + \hat{w}^2)^{1/2} = (\hat{\mathbf{V}} \cdot \hat{\mathbf{V}})^{1/2}$ for comparison

73 with $(u^2 + v^2 + w^2)^{1/2} = (\mathbf{V} \bullet \mathbf{V})^{1/2}$ measured by a mast-mounted anemometer. By this
 74 method a *single* calibration parameter

$$75 \quad m = (\hat{\mathbf{V}} \bullet \hat{\mathbf{V}})^{1/2} / (\mathbf{V} \bullet \mathbf{V})^{1/2} \quad (2)$$

76 is obtained.

77 Consider the following simple example. A very narrow beam in the x - z plane,
 78 and with $w = 0$ has $V_r = u \sin \theta_0$ so the wind estimate is $\hat{u} = V_r / \sin \theta_0$. If there is an
 79 uncertainty or an error $\Delta\theta$ in the tilt angle θ_0 , then the uncertainty or error in estimated
 80 wind is $\Delta\hat{u} / \hat{u} = -\Delta\theta / \tan \theta_0$. For $\theta_0 = 15^\circ$, each 1° error in beam pointing angle gives a
 81 5% error in estimation of wind speed: Monostatic SODARs and LIDARs are **highly**
 82 **sensitive** to beam pointing.

83 *Complete wind measurement calibration*

84 The calibration parameter m in (2) contains combinations of elements from beam matrix
 85 \mathbf{B} , which are functions of the three zenith angles and three azimuth angles for a three-
 86 beam system. In obtaining estimates of u , v , and w , these elements are *assumed* known in
 87 the SODAR processing software. Incorrect values of any of these elements will give a
 88 variation in m . This variation in m will also be *wind-direction dependent* as can be seen
 89 from the very simple case of a beam tilted an angle θ_0 in the x - z plane, another beam
 90 tilted θ_0 in the y - z plane, and the third beam vertical. Then

91

$$92 \quad m^2 = \left(\frac{\hat{V}}{V} \right)^2 = \frac{\sin^2 \theta_0}{\sin^2 \hat{\theta}} + \frac{w(\cos \theta_0 - \cos \hat{\theta}) [2(u+v)\sin \theta_0 + w(2 + \cos \theta_0 + \cos \hat{\theta})]}{(u^2 + v^2 + w^2)\sin^2 \hat{\theta}}$$

93 where $\hat{\theta}$ is the tilt angle assumed by the software, and θ_0 is the actual tilt angle. This
 94 problem with traditional calibration methods has not been previously considered.

95 In practice however, the beam is not an angular delta-function and the weights in
 96 (1) are volume averages over the transmitted and received beams

$$97 \quad V_r = \overline{u \cos \phi \sin \theta} + \overline{v \sin \phi \sin \theta} + \overline{w \cos \theta}. \quad (3)$$

98 The elements of \mathbf{B} could be found in principle by measuring the beam angular
 99 intensity variations in an anechoic chamber, or perhaps in the field, but this effort would
 100 be large because of the need to capture beam details on a hemispherical surface in high
 101 angular resolution in 2D so that the proper volume averages can be calculated.

102 3. Tilt angle perturbation

103 *Basic perturbation concept*

104 Figure 1 shows the x - z plane for a SODAR having a beam at an initial effective tilt angle
 105 θ_1 . If there is also a beam in the y - z plane tilted at an angle of θ_2 to the vertical, the
 106 equations corresponding to (1) are

$$107 \quad V_{r1} = u \sin \theta_1 + w \cos \theta_1 \quad (4)$$

$$108 \quad V_{r2} = v \sin \theta_2 + w \cos \theta_2 \quad (5)$$

$$109 \quad V_{r3} = w. \quad (6)$$

110 Also shown is the entire SODAR rotated by an angle $\Delta\theta$ about the y axis. Now

$$111 \quad V_{r1}^* = u \sin(\theta_1 + \Delta\theta) + w \cos(\theta_1 + \Delta\theta) \quad (7)$$

$$112 \quad V_{r2}^* = u \sin(\Delta\theta) \cos \theta_2 + v \sin \theta_2 + w \cos(\Delta\theta) \cos \theta_2 \quad (8)$$

$$113 \quad V_{r3}^* = u \sin(\Delta\theta) + w \cos(\Delta\theta). \quad (9)$$

114 The $V_{r1}, V_{r3}, V_{r1}^*, V_{r3}^*$ quantities are measured, the tilt perturbation $\Delta\theta$ is known,
 115 and u, w, θ_1 and θ_2 are unknown. Equations (4) through (9) are non-linear in the
 116 unknowns, but can be solved by finding: w from (6); u from (9); $\sin\theta_1$ from (4) and (7);
 117 $\cos\theta_2$ from (5) and (8); and v from (5), giving

$$118 \quad u = \frac{V_{r3}^* - V_{r3} \cos \Delta\theta}{\sin \Delta\theta}. \quad (10)$$

$$119 \quad v = \frac{(V_{r2}V_{r3}^* - V_{r3}V_{r2}^*)(V_{r3}^* - V_{r3})}{\sqrt{(V_{r3}^* - V_{r3})^2 - (V_{r2}^* - V_{r2})^2}} \quad (11)$$

$$120 \quad w = V_{r3} \quad (12)$$

$$121 \quad \sin \theta_1 = \frac{V_{r1}V_{r3}^* - V_{r1}^*V_{r3}}{V_{r3}^2 - 2V_{r3}V_{r3}^* \cos \Delta\theta + V_{r3}^{*2}} \sin \Delta\theta \quad (13)$$

$$122 \quad \cos \theta_2 = \frac{V_{r2}^* - V_{r2}}{V_{r3}^* - V_{r3}}. \quad (14)$$

123 *The effective tilt angle*

124 As indicated in (3), components of \mathbf{B} are volume averages. The volume averaging means
 125 that a normalized beam gain function $G(\Omega, \Omega_0)$ is averaged over solid angle Ω around a
 126 pointing direction Ω_0 in each of the terms on the right of (1):

127

$$128 \quad V_r = \int_{\Omega} \Omega \cdot \mathbf{V}G(\Omega, \Omega_0) d\Omega$$

$$= \int_{\Omega} (u \cos \phi \sin \theta + v \sin \phi \sin \theta + w \cos \theta) G(\Omega, \Omega_0) d\Omega$$

$$\begin{aligned}
129 \quad &= u \int_{\Omega} \cos \phi \sin \theta G(\mathbf{\Omega}, \mathbf{\Omega}_0) d\Omega + v \int_{\Omega} \sin \phi \sin \theta G(\mathbf{\Omega}, \mathbf{\Omega}_0) d\Omega + w \int_{\Omega} \cos \theta G(\mathbf{\Omega}, \mathbf{\Omega}_0) d\Omega \\
&= \overline{u \cos \phi \sin \theta} + \overline{v \sin \phi \sin \theta} + \overline{w \cos \theta} \\
130 \quad & \tag{15}
\end{aligned}$$

131 where

$$132 \quad \int_{\Omega} G(\mathbf{\Omega}, \mathbf{\Omega}_0) d\Omega = 1.$$

133 For a beam nominally in the x - z plane, there will be contributions from finite
134 azimuth angles ϕ . However, such beams are invariably symmetric in azimuth, so G is an
135 even function of ϕ and the integral

$$136 \quad \int_{\Omega} \sin \phi \sin \theta G(\mathbf{\Omega}, \mathbf{\Omega}_0) d\Omega = 0.$$

137 This means that

$$138 \quad V_r = \overline{u \cos \phi \sin \theta} + \overline{w \cos \theta} = u \sin \theta_1 + w \cos \theta_1.$$

139

140 The θ_1 appearing in (4) is therefore an *effective* beam tilt angle. If this is
141 perturbed by rotating the entire SODAR through $\Delta\theta$ about the y axis then, using an
142 angular coordinate system attached to the SODAR, $G(\mathbf{\Omega}, \mathbf{\Omega}_0)$ remains unchanged but the
143 beam direction with respect to the wind \mathbf{V} is now $(\cos \phi \sin[\theta+\Delta\theta], \sin \phi \sin[\theta+\Delta\theta],$
144 $\cos[\theta+\Delta\theta])$. The first term on the right of (15) becomes

145

$$\begin{aligned}
& u \int_{\Omega} \cos \phi \sin(\theta + \Delta\theta) G(\mathbf{\Omega}, \mathbf{\Omega}_0) d\Omega \\
&= u \int_{\Omega} \cos \phi (\sin \theta \cos \Delta\theta + \cos \theta \sin \Delta\theta) G(\mathbf{\Omega}, \mathbf{\Omega}_0) d\Omega \\
146 \quad &= u \cos \Delta\theta \int_{\Omega} \cos \phi \sin \theta G(\mathbf{\Omega}, \mathbf{\Omega}_0) d\Omega + u \sin \Delta\theta \int_{\Omega} \cos \phi \cos \theta G(\mathbf{\Omega}, \mathbf{\Omega}_0) d\Omega \\
&= u \cos \Delta\theta \overline{\cos \phi \sin \theta} + u \sin \Delta\theta \overline{\cos \phi \cos \theta} \\
&= u \cos \Delta\theta \sin \theta_1 + u \sin \Delta\theta \cos \theta_1 \\
&= u \sin(\theta_1 + \Delta\theta)
\end{aligned}$$

147 This means that, although θ_1 is an effective zenith angle and not necessarily the
148 same as the pointing zenith angle, we can validly do arithmetic such as
149 $\sin(\theta_1 + \Delta\theta) = \sin(\theta_1)\cos(\Delta\theta) + \sin(\Delta\theta)\cos(\theta_1)$ as in (4)-(14) above.

150 4. The effect of beam geometry on Doppler shift

151 In the above, the Doppler shift is contained in the elements of vector \mathbf{R} . The weighting
152 on each of the wind velocity components is volume-averaged, but this does not give any
153 indication of the spread or shape of the Doppler spectrum from which, by detecting the
154 peak position, the components of \mathbf{R} are estimated.

155 The acoustic radar equation covers this in principle (Bradley, 2007). Including the
156 dependence on frequency and on volume averaging, the spectral density of received
157 power at the mono-static antenna equation becomes

$$158 \quad \frac{dP_R}{df} = c\tau\sigma_s \frac{e^{-2\alpha r}}{r^2} \int_{\Omega} \frac{dP_T}{df} G(\mathbf{\Omega}) d\Omega .$$

159 Here c is the speed of sound, τ is the pulse duration, σ_s is the scattering cross-section area
160 per unit volume and per unit solid angle, α is the acoustic absorption, r is the range to the

161 scattering volume, dP_T/df is the power per unit frequency interval transmitted into solid
 162 angle $d\Omega$, and G is an angle-dependent sensitivity kernel. The atmospheric absorption
 163 and scattering parts have been taken outside of the scattering volume integral since they
 164 are only weakly frequency-dependent and it is assumed that they do not vary much within
 165 a typical scattering volume. Assuming a Gaussian-shaped transmitted pulse of spectral
 166 width σ_f , and that the Doppler spectrum is centered on f_D rather than transmitted
 167 frequency f_T ,

$$168 \quad \frac{dP_R}{df} \propto \int_{\Omega} \exp\left[-\frac{1}{2\sigma_f^2}(f - f_D)^2\right] G(\Omega) d\Omega.$$

169 Note that all commercial SODARs use an approximately Gaussian pulse shape.

170 For example, if the acoustic beam has sensitivity G at a zenith angle θ and
 171 azimuth angle ϕ , then the integral is

$$172 \quad \int_0^{2\pi} \left\{ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \exp\left[-\frac{1}{2\sigma_f^2} \left[f - f_T \left(1 - 2\frac{u}{c} \sin\theta \cos\phi - 2\frac{v}{c} \sin\theta \sin\phi - 2\frac{w}{c} \cos\theta \right) \right]^2 \right] G \sin\theta d\theta \right\} d\phi. \quad (16)$$

173 The usual assumption is that the beam in the x - z plane is effectively an angular
 174 delta-function

$$175 \quad G(\theta, \phi) = \cos\theta \delta(\theta - \theta_0) \delta(\phi).$$

176 Then the above integral becomes

$$177 \quad \exp\left[-\frac{1}{2\sigma_f^2} \left[f - f_T \left(1 - 2\frac{u}{c} \sin\theta_0 - 2\frac{w}{c} \cos\theta_0 \right) \right]^2 \right] \sin\theta_0 \cos\theta_0$$

178 so that the spectrum peaks at

$$179 \quad f_x = f_T \left(1 - 2\frac{u}{c} \sin\theta_0 - 2\frac{w}{c} \cos\theta_0 \right)$$

180 giving the expected radial component as in (1) with $\phi_0 = 0$. Similarly, it is usually
 181 assumed that the beam in the $+z$ direction has the form $G(\theta, \phi) = \delta(\theta)\delta(\phi)$ so that that
 182 spectrum peaks at

$$183 \quad f_z = f_T \left(1 - 2 \frac{w}{c} \right).$$

184 More generally, it can be seen in (16) that there is a term in $\sin^2\phi$ so that there is a
 185 contribution from the traverse width of the beam in spite of G being even in ϕ . The
 186 influence of this term in v is to give a broader spectral peak but not to change the peak
 187 position substantially, so will be ignored in the following. Also, in general the effect of
 188 the $\sin\theta$ weighting on u is to bias the spectral peak to the equivalent of a larger effective
 189 θ_0 . There is therefore a small change in the effective tilt angle, as expected. However,
 190 this does not change the methodology of the new calibration concept when the effective
 191 tilt angle is unknown anyway.

192 5. Error analysis

193 Writing σ_V for the uncertainty in wind speed V , (13) gives

$$194 \quad \sigma_{\theta_1}^2 \approx \left(\frac{\tan \theta_1}{\tan \Delta\theta} \right)^2 \left[\sigma_{\Delta\theta}^2 + \left(\frac{\sigma_V \sin \theta_1}{V} \right)^2 \right].$$

195 To obtain a calibration accuracy of 1%, we need $\sigma_\theta \approx 0.2^\circ \approx 4 \times 10^{-3}$ radian. For $\theta_1 = \Delta\theta =$
 196 15° , and without any peak detection error, $\Delta\theta$ also needs to be measured to 0.2° . This is
 197 achievable with a linear actuator and a digital inclinometer. The accuracy of 10-minute
 198 averaged SODAR spectral peak estimation is typically $\sigma_V = 0.2 \text{ m s}^{-1}$, so the term in σ_V is
 199 typically a factor 10 larger than the $\sigma_{\Delta\theta}$ term. What this means is that around 10 trials of

200 10-minute duration must be conducted in order to reduce the typical errors from peak
 201 detection to an acceptable level.

202 An alternative is to recast (13) in the form

$$203 \quad Y = aX$$

204 where

$$205 \quad Y = \frac{V_{r3}^2 - 2V_{r3}V_{r3}^* \cos \Delta\theta + V_{r3}^{*2}}{V_{r1}V_{r3}^* - V_{r1}^*V_{r3}}$$

206 and $X = \sin \Delta\theta$. The slope of the least-squares line through the origin is $a = 1/\sin\theta_1$.

207 A disadvantage of this method is that the radial velocity components may not be
 208 made available to the user by the SODAR manufacturer. They then need to be calculated
 209 based on the beam zenith angle assumed by the manufacturer, or the zenith angle
 210 calculated from the antenna parameters. An alternative, and much simpler procedure, is
 211 to assume that, in comparison with u and v , w is negligible, so

$$212 \quad \frac{u^*}{u} - \cos \Delta\theta = \left(\frac{1}{\tan \theta_1} \right) \sin \Delta\theta$$

213 which means that θ_1 can be estimated from the slope of the straight-line fit through the
 214 origin, via

$$215 \quad \tan \theta_1 = \frac{\sum_{n=1}^N (\sin \Delta\theta_n)^2}{\sum_{n=1}^N \left(\frac{u_n^*}{u_n} - \cos \Delta\theta_n \right) \sin \Delta\theta_n}.$$

216 In this case

$$217 \quad \sigma_{\theta_1}^2 \approx \frac{2}{\sum_{n=1}^N \sin^2 \Delta\theta_n} \left(\frac{\tan^2 \theta_1}{1 + \tan^2 \theta_1} \right)^2 \left(\frac{\sigma_V}{V} \right)^2$$

218 where N measurements are taken at $\Delta\theta_n$, $n=1,2,\dots,N$. For $\theta_1 = 15^\circ$, and $\sigma_V/V = 0.04$,
 219 three cycles of $\Delta\theta = 15^\circ$ and 38° should give $\sigma_\theta < 0.2^\circ$.

220 6. Field measurements

221 Field measurements on very flat land in western Denmark, have been completed on an
 222 ASC4000 SODAR mounted on a frame, which is then tilted using a 12V-powered linear
 223 actuator, as shown in Figure 2. The operator used a reversing switch to raise and lower
 224 the tilting platform in synchronism with the SODAR averaging time, so that one
 225 undisturbed averaging period was followed by an averaging period in which the actuator
 226 was moved. Tilt angle $\Delta\theta$ and 90-m wind speed vs time are shown in Fig. 3. The
 227 correlation between retrieved wind speed and tilt angle is strong. This is expected from
 228 (7), which shows that V_{r1}^* is essentially linear in $\Delta\theta$.

229 7. Data analysis

230 Wind vector components were recorded at 10 m height intervals from 30 m to 130 m.
 231 The beam zenith angle θ_1 was estimated from the least-squares slope of the line through
 232 the origin for both the $w = 0$ case and the full solution case. Variances of the Y values
 233 corresponding to each of the two tilt angles were used as least-squares weights, since it
 234 was expected that the radial wind variability would increase as the SODAR was tilted
 235 further. Figure 4 shows estimated θ_1 values at each height for the two cases. The lowest

236 height gives outlier values of angle, consistent with some clutter contamination from
237 beam side-lobes when the beam is tilted. The estimated angle at the upper height (130 m)
238 also appears to give an outlier, especially for the $w = 0$ case, consistent with the signal-to-
239 noise ratio for SODAR signals decreasing rapidly above 120 m (see Fig. 5).

240 The expected value of θ_1 can be calculated from the phased-array geometry for
241 this SODAR. An incremental phase shift of $\pi/2$ is used to change beam zenith angles.
242 The beam maximum will therefore be at a zenith angle of $\theta_1 = \sin^{-1}(\lambda/4d)$ where λ is the
243 wavelength and d is the array element spacing. In the case of this SODAR, the
244 transmitted frequency was 4500 Hz, and the speakers have a diameter of 0.085 m but are
245 used in diagonal rows of spacing $d = 0.085/2^{1/2} = 0.06$ m. Taking into account the mean
246 air temperature at SODAR height during the experiment, $\theta_1 = 18.32^\circ$. This compares
247 with the estimated zenith angle from the two cases given in Table 1.

248 8. Conclusions

249 Since Doppler measurement is inherently calculable, the main source of systematic
250 calibration errors for SODARs is uncertainty regarding the effective beam pointing angle.

251 A new method for beam geometry calibration of SODARs is described. The
252 method makes no assumptions about the SODAR operation and its hardware and
253 software, other than the assumption that only one beam is transmitted at a time, and that
254 the flow is horizontally homogeneous. Regardless of the complexity of the actual beam
255 shape, the *effective* beam tilt angle is accurately estimated: this is the angle which must be
256 used in estimations of velocity components. In a very simple experiment the effective
257 beam zenith angle has been found to within around 0.2° , which is as good as is required

258 in the most stringent SODAR calibration procedures. It has been found, even for such a
259 short data run, that the estimated angle is very close to that calculated from the SODAR
260 array geometry.

261 Atmospheric refraction effects are not significant here. For example, with a beam
262 zenith angle of 45° , an adiabatic lapse rate, and a height range of 100 m, the change in
263 propagation angle is only around 0.1° . The main limitation evident at this stage is the
264 requirement for horizontally homogeneous flow, since the regression methods use both a
265 tilted beam and a vertical beam. Note that this is also a fundamental limiting assumption
266 in the normal *operation* of ground-based wind LIDARs and SODARs. However, since
267 horizontal homogeneity of the flow is assumed, this method should only be applied over
268 flat homogeneous terrain, and not when strong vertical gradients might be expected. The
269 vertical gradient restriction is because there is also the assumption that the wind at a
270 particular radial distance for the artificially tilted beam is the same as the wind at the
271 same range without artificial tilting. For example, with the 40° artificial tilt applied here,
272 this means that the wind at 100 m height should be similar to the wind at 80 m height.
273 Given the extended vertical sampling volume of the SODAR, this assumption will not
274 normally cause significant errors. Note that both SODARs and LIDARs are used with the
275 assumption (generally not stated) that the sampling in the vertical, via 'range gating', is
276 adequate to describe the vertical structure of the wind, and that spatial aliasing is not
277 occurring.

278 There are a number of reasons why the method described above is of practical
279 importance. These include the fact that there will be a bias in measured Doppler shift
280 compared to that calculated from simple beam geometry because, for a beam symmetric

281 around the central tilted direction, the angles between wind vector and portions of the
282 beam are not symmetrical about the central direction. Furthermore, there can be bias
283 arising from clipping of the beam by acoustic baffles surrounding the instrument, and
284 these effects are generally difficult to estimate or measure in other ways. Similarly, it is
285 challenging to calculate with confidence the beam shape of a SODAR based on small
286 parabolic dish reflectors, such as the AQ500. Even for a SODAR based on a phased array
287 of transducers, the beam shape details depend on the relative gains of the transducer
288 elements, which may not be known with confidence, especially after the SODAR has
289 been deployed in the field for some time.

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293 **References**

294 Bradley S. G., 2007: Atmospheric Acoustic Remote Sensing. CRC Press/Taylor and

295 Francis Group, 271pp.

296 Bradley S.G., I. Antoniou, S. von Hünerbein, D. Kindler, M. de Noord, and H. E.

297 Jørgensen, 2005: SODAR calibration for wind energy applications. Manchester, UK,

298 University of Salford. 69pp.

299

300 **List of Figures**

301 FIG. 1. The geometry of a SODAR beam tilted at an angle θ_1 (left diagram) and with the
302 SODAR rotated by an angle $\Delta\theta$ about the y axis (right diagram). The wind velocity
303 components in this plane are u and w , and the along-beam radial components for the two
304 beams in this plane are V_{r1} and V_{r3} .

305

306 FIG. 2. The mounting frame and linear actuator, with digital level (left photograph) and
307 measurements being taken (right photograph).

308

309 FIG. 3. Wind speed (crosses) and tilt angle (solid line) plotted versus time.

310

311 FIG. 4. Estimated beam zenith angles θ_1 from the $w=0$ case (filled circles) and the
312 unconstrained w case (plus signs).

313

314 FIG. 5. The mean ratio of signal power to noise power (SNR) for the w beam, as a
315 function of height.

316

317

318

319 TABLE 1. Comparison between estimated beam zenith angles and the calculated zenith
320 angle.

	Mean θ_1	$\sigma_{\text{mean } \theta}$	Estimated-calculated θ_1
Calculated θ_1	18.32°		
θ_1 estimated with $w=0$	18.27°	0.23°	-0.05°
θ_1 estimated with $w \neq 0$	18.55°	0.54°	0.23°

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330 FIG. 1. The geometry of a SODAR beam tilted at an angle θ_1 (left diagram) and with the331 SODAR rotated by an angle $\Delta\theta$ about the y axis (right diagram). The wind velocity332 components in this plane are u and w , and the along-beam radial components for the two333 beams in this plane are V_{r1} and V_{r3} .

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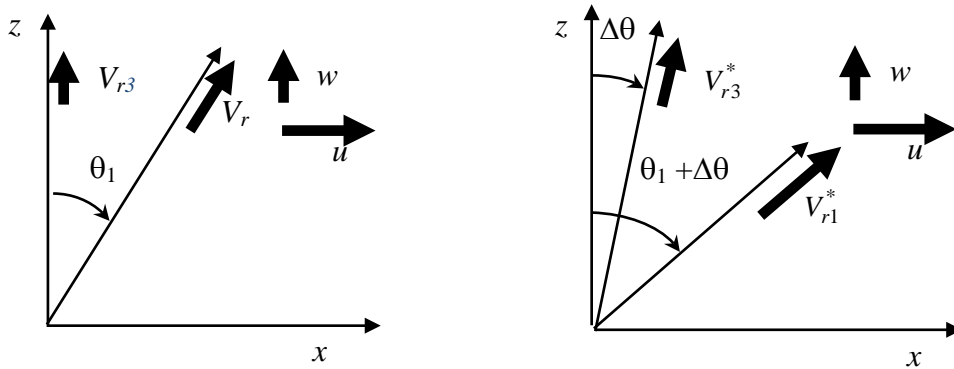


FIG. 1. The geometry of a SODAR beam tilted at an angle θ_1 (left diagram) and with the SODAR rotated by an angle $\Delta\theta$ about the y axis (right diagram). The wind velocity components in this plane are u and w , and the along-beam radial components for the two beams in this plane are V_{r1} and V_{r3} .



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337 FIG. 2. The mounting frame and linear actuator, with digital level (left photograph) and

338 measurements being taken (right photograph).

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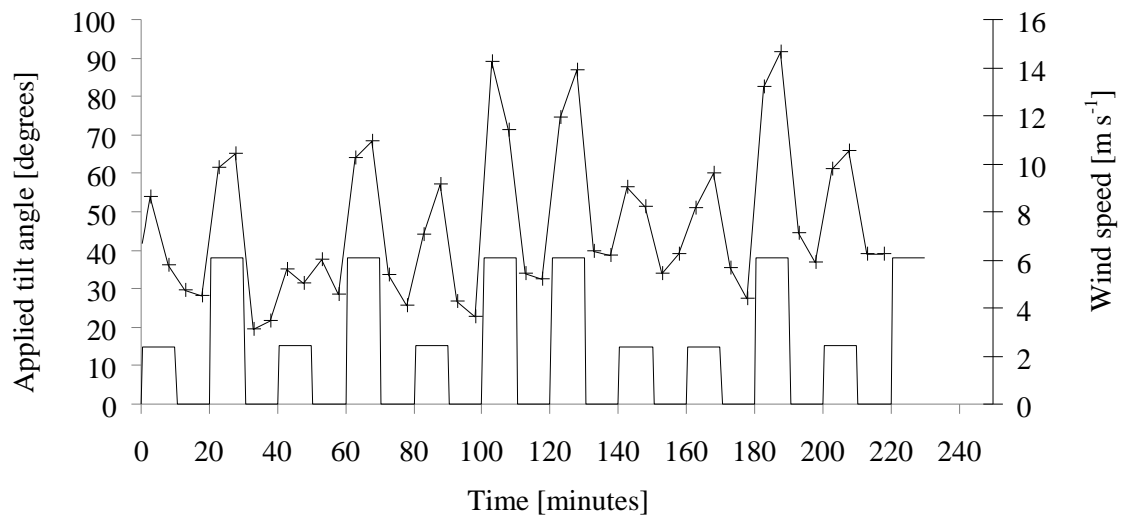
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346 FIG. 3. Wind speed (crosses) and tilt angle (solid line) plotted versus time.

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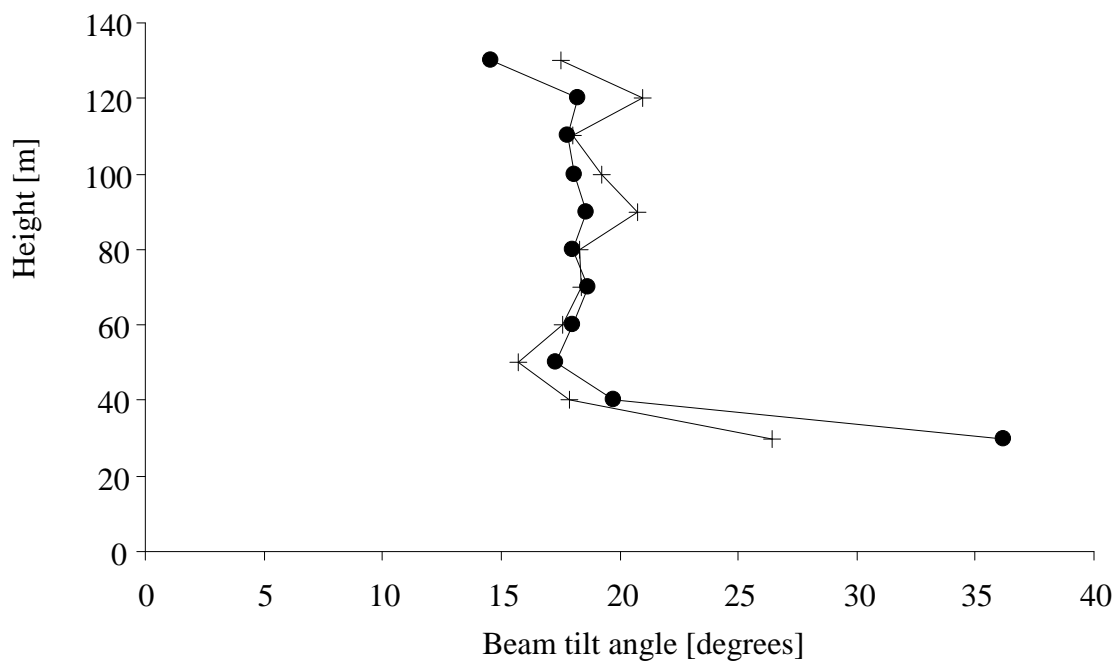
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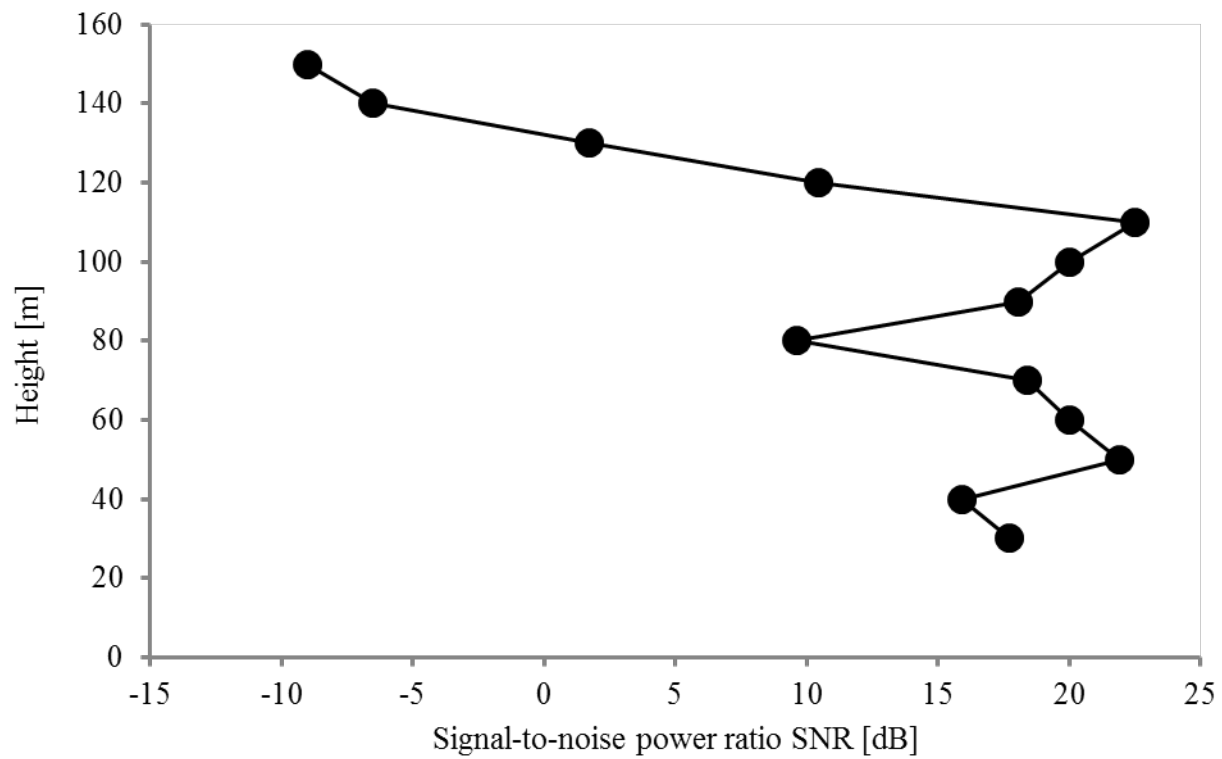
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371 FIG. 4. Estimated beam zenith angles θ_1 from the $w=0$ case (filled circles) and the372 unconstrained w case (plus signs).

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376 FIG. 5. The mean ratio of signal power to noise power (SNR) for the w beam, as a

377 function of height.

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