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Summary

We report on research concerning spontaneous spatial fractal pattern formation in passive nonlinear optical cavities (both ring-resonator *and* Fabry-Pérot geometries). A new model for light propagation inside discrete coupled cavities is proposed and we predict, for the first time, a multi-Turing instability spectrum for such a system.

Turing instabilities: *single vs. multiple minima*

Alan Turing's seminal analysis of morphogenesis [1] was ahead of its time and laid the foundations for a modern understanding of the origin of pattern and form in Nature. He discovered that when a class of reaction-diffusion system is sufficiently stressed, arbitrarily-small disturbances to its uniform states may give rise to spontaneous self-organization into a simple pattern whose dominant scale-length is (inversely) related to the minimum in the threshold instability spectrum.

Turing's single-minimum mechanism plays a key role in describing simple pattern emergence in, for example, nonlinear optical models [2]. However, there also exists a class of system whose threshold instability spectrum comprises a hierarchy of many comparable minima. We have proposed that such a multiple-minimum characteristic may provide a universal signature for predicting a system's innate capacity to generate spontaneous spatial fractals (that is, patterns possessing proportional levels of detail spanning decimal orders of scale-length) [3].

Continuous cavities: *from ring resonator to Fabry-Pérot*

Recently, we demonstrated that the same multi-Turing instability signature predicting spontaneous fractal patterns in the classic single feedback-mirror system [3] was also present in a ring cavity containing a thin slice of diffusive material that may be either dispersive or purely-absorptive [4]. This key result has provided strong supporting evidence that the multiple-minimum signature has independence with respect to the details of the system nonlinearity. More recently, we have considered a Fabry-Pérot (FP) cavity [see Fig. 1(a)] to capture a different type of feedback loop. Even a simple model epitomizes complexity through the interplay between diffraction,

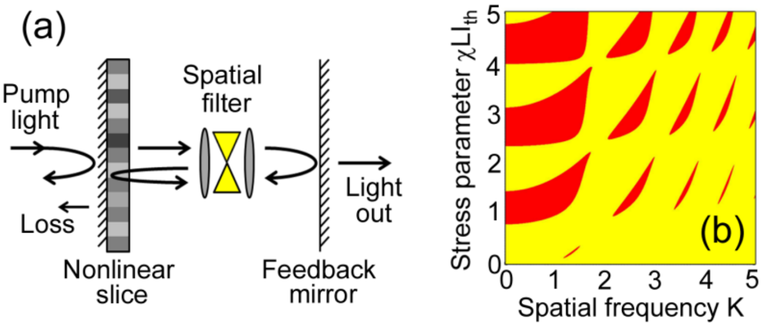

Fig. 1. (a) A schematic diagram illustrating the basic geometry of an FP cavity with a thin slice of self-focusing Kerr type material (χL = +1). (b) Linear analysis predicts a multi-Turing threshold instability spectrum for the FP slice, where the "lobes" of the single feedback-mirror system break-up into a set of discrete instability "islands".

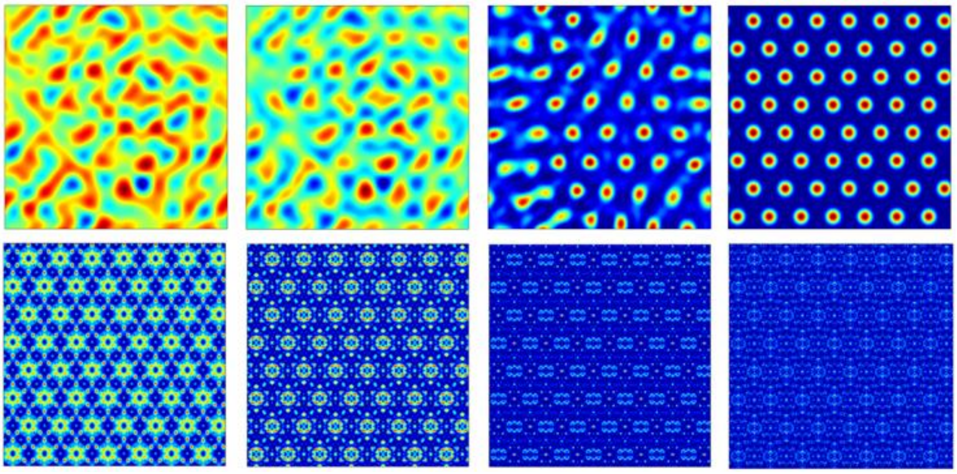

Fig. 2. Simulation of pattern formation in a simple model of a dispersive nonlinear FP cavity. Top row: simple pattern emergence, where the perturbed plane wave (stationary) state evolves into a static hexagon pattern. Bottom row: evolution of a hexagon pattern toward a fractal.

diffusion, nonlinearity, counter-propagation, and a host of cavity effects. Here, the FP geometry will be reported to possess a multi-Turing instability spectrum [see Fig. 1(b)], and simulations have revealed that the mechanism for simple and fractal pattern emergence exists with both one and two transverse dimensions (see Fig. 2).

Discrete cavities: *Multi-Turing spectra & spatial patterns*

We have also revisited the classic discrete nonlinear Schrödinger (dNLS) equation [5] with a view to modelling spatial patterns in coupled microcavities. While other authors have considered a mean-field approach to the related problem of cavity solitons [6,7], we retain the traditional boundary condition for capturing lumped ring-resonator feedback [8] (the mean-field limit tends to suppress the possibility of multi-Turing spectra). Linear analysis has investigated the susceptibility of the stationary states of the dNLS cavity to spontaneous pattern-forming instabilities, and predicted a multi-Turing threshold spectrum (see Fig. 3). An overview of simulation results illustrating pattern emergence will be given.

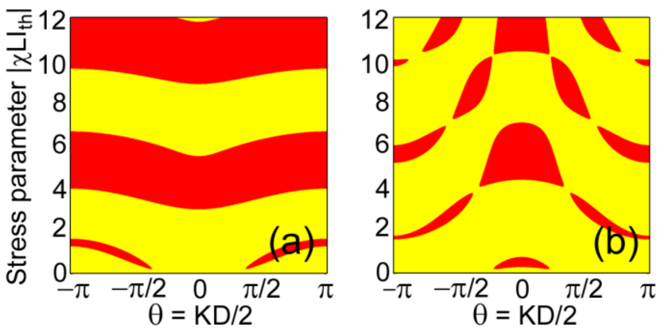

Fig. 3. Two typical examples of multi-Turing threshold instability curves for coupled cavities based on the discrete nonlinear Schrödinger equation for (a) self-focusing (χL = +1) and (b) self-defocusing (χL = –1) Kerr-type materials. The spectrum is 2π-periodic in θ, and in the long-wave limit (given by $KD \to 0$) one recovers the classic continuum result corresponding to the NLS in a ring cavity [8] (as must be the case).

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