Multi-Turing instability spectra and spatial fractals in absorptive systems

J. M. Christian[1], C. Bostock[1], G. S. McDonald[1], and J. G. Huang[2]

[1] *Materials & Physics Research Centre, University of Salford, Greater Manchester M5 4WT, U.K.*

[2] *Faculty of Advanced Technology, University of Glamorgan, Pontypridd CF27 1DL, U.K.*

Spontaneous optical pattern formation in ring cavities containing saturable absorbers has been increasingly studied over the past two decades, with models often based on the two-level Maxwell-Bloch equations. Linear stability analysis of the weakly-absorptive system tends to be fraught with complexity, and leads ultimately to a threshold instability spectrum for Turing patterns that possesses comparable multiple minima [Patrascu *et al.*, Opt. Commun. **96**, 433 (1992)] – a *multi-Turing spectrum* that we have proposed as a universal signature for predicting a system's innate capacity to generate spontaneous spatial fractals [Huang and McDonald, Phys. Rev. Lett. **95**, 174101 (2005)]. Taking the mean-field limit decreases the overall complexity of the problem [Firth and Scroggie, Europhys. Lett. **26**, 521 (1994)] but the multi-Turing spectrum vanishes, leaving only a single instability lobe which provides no obvious mechanism by which fractal patterns can emerge.

Here, we consider two reduced models of a two-level saturable-absorber system without recourse to mean-field theory [Huang, Christian, and McDonald, J. Nonlin. Opt. Phys. Mat. **21**, 1250018 (2012)]. Our analysis begins with the thin-slice approximation, capturing the interplay between the circulating cavity field, a diffusive population difference (driven by local light intensity), and the classic ring cavity boundary condition (lumping together periodic pumping, coupling-mirror losses, and interferomic mistuning). Simulations have shown that emergent spatial patterns can be both simple (single-scale) and fractal (multi-scale) in character. More recently, we have gone beyond the thin-slice limit and applied our approach to bulk geometries (where light-medium interaction lengths become finite and can no longer be ignored). Theoretical analysis still predicts a multi-Turing spectrum under the conditions of an instantaneous local medium response (i.e., no diffusion) and paraxial diffraction (i.e., nonlinear Schrödinger-type governing equation). This key result suggests that the multi-Turing mechanism may be truly universal, arising in even the simplest models of a bulk absorptive cavity.