Eigenmodes of Next-Generation Unstable Cavity Lasers: Kaleidoscopes, Snowflakes, Pentaflakes, and Fractal Dimension

***J. M. Christian*[1]*, S. J. Wickham*[1]*, I. Begleris*[2]*, G. S. McDonald*[1]*, J. G. Huang*[3]**

1. Materials & Physics Research Centre, University of Salford, Greater Manchester M5 4WT, U.K.

2. Computational Modelling Group, University of Southampton, Highfield SO17 1BJ, U.K.

3. Faculty of Advanced Technology, University of South Wales, Pontypridd CF37 1DL, U.K.

We will report on our latest research into fractal lasers (linear systems which involve geometrically-unstable resonators with inherent magnification) [1], introducing two new classes of transverse cavity configuration. These devices are of fundamental interest as table-top generators of tunable fractal light that can be used in a wide range of applications – from optical probes (e.g., increased sensitivity in surface roughness measurements) to secure information encoding/transmission to medical imaging. Moreover, we expect them to play a pivotal role in new Nature-inspired optical architectures and devices (e.g., writing fractal structures into new materials).

Attention will be paid not only to the classic kaleidoscope geometries [2], but also to recently-proposed designs which incorporate a feedback mirror whose outer boundary corresponds to increasing iterations of the von Koch *snowflake* [3] (a six-fold-symmetric iterated function system involving self-similar sequences of equilateral triangles – see Fig. 1) and its five-fold-symmetric counterpart, the von Koch *pentaflake* (constructed in a similar way to the more traditional snowflake, but using isoceles triangles). All three systems can be modelled using a two-dimensional virtual source (2D-VS) method [4], which unfolds the.unstable empty cavity into a equivalent sequence of virtual apertures of increasing size [5]. A selection of mode patterns and eigenvalue spectra will be presented, and convergence issues considered in some detail.

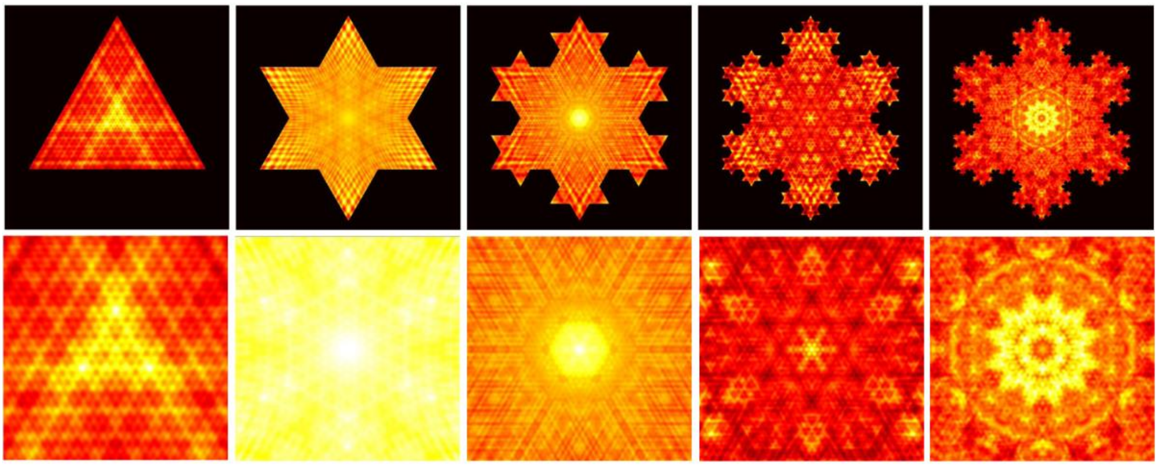

Fig. 1. 2D-VS computations of the lowest-loss mode patterns for a snowflake laser (parameters: $N_{\mathrm{eq}} = 30$ and $M = 1.5$) whose feedback mirror progresses through the first four application of the generator algorithm. The number of edges N_{edge} increases (left to right) geometrically with iteration number n according to $N_{\mathrm{edge}} = 3 \times 4^n$ (so $N_{\mathrm{edge}} = 3, 12, 48, 768, 3072$).

A key issue to be addressed in detail is the fractal dimension of unstable-resonator modes for cavities with arbitrary parameters (i.e., equivalent Fresnel number N_{eq} and round-trip magnification M). Previously, Berry has made similar considerations but only for the lowest-loss modes of kaleidoscope cavities, and in the limit $N_{\mathrm{eq}} \rightarrow \infty$ (where asymptotic approximations may be deployed) [6]. We will conclude with a summary of results from the first detailed exploration of fractal dimension in kaleidoscope resonators. Specialist software [7] has been deployed in parallel with our suite of 2D-VS codes to investigate potential anisotropy in the dimension using various different measures: power spectrum, roughness-length, rescaled range, and variogram. One-dimensional cross-sections through the lowest-loss (and several higher-order) mode patterns are computed, and direct comparisons with a strip resonator for the same cavity parameters [8] uncover some intriguing results.

References

[1] G. P. Karman *et al.*, "Fractal modes in unstable resonators," Nature **402**, 138 (1999).
[2] G. S. McDonald *et al.*, "The kaleidoscope laser," J. Opt. Soc. Am. B **17**, 524 (2000).
[3] K. Falconer, *Fractal geometry: mathematical foundations and applications*, 2nd ed., John Wiley and Sons, New York (2003).
[4] J. G. Huang *et al.*, "Fresnel diffraction and fractal patterns from polygonal apertures," J. Opt. Soc. Am. A **23**, 2768 (2006).
[5] W. H. Southwell, "Virtual-source theory of unstable resonator modes," Opt. Lett. **6**, 487 (1981).
[6] M. V. Berry, "Fractal modes of unstable lasers with polygonal and circular mirrors," Opt. Commun. **200**, 321 (2001).
[7] BENOIT 1.3, TruSoft International Inc. www.trusoft-international.com.
[8] G. H. C. New *et al.*, "Diffractive origin of unstable resonator modes," Opt. Commun. **193**, 261 (2001).