Irreversibility analysis for reactive third-grade fluid flow and heat transfer with convective wall cooling

Adesanya, SO, Falade, JA, Jangili, S and Beg, OA

http://dx.doi.org/10.1016/j.aej.2016.09.017

<table>
<thead>
<tr>
<th>Title</th>
<th>Irreversibility analysis for reactive third-grade fluid flow and heat transfer with convective wall cooling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Authors</td>
<td>Adesanya, SO, Falade, JA, Jangili, S and Beg, OA</td>
</tr>
<tr>
<td>Type</td>
<td>Article</td>
</tr>
<tr>
<td>URL</td>
<td>This version is available at: <a href="http://usir.salford.ac.uk/id/eprint/40213/">http://usir.salford.ac.uk/id/eprint/40213/</a></td>
</tr>
<tr>
<td>Published Date</td>
<td>2016</td>
</tr>
</tbody>
</table>

USIR is a digital collection of the research output of the University of Salford. Where copyright permits, full text material held in the repository is made freely available online and can be read, downloaded and copied for non-commercial private study or research purposes. Please check the manuscript for any further copyright restrictions.

For more information, including our policy and submission procedure, please contact the Repository Team at: usir@salford.ac.uk.
Inherent irreversibility in the flow of a reactive third grade fluid though a channel with convective heating is examined. It is well known that heat dissipated from the exothermic chemical reaction passes through fluid in an irreversible manner and as a result entropy is generated continuously within the channel. Analytical solutions of the resulting dimensionless nonlinear boundary-value-problems arising from the governing equations were obtained by using perturbation method. These solutions are utilized to obtain the entropy generation rate and Bejan number for the system. The influence of various important parameters on the entropy generation rate and Bejan number are shown graphically and discussed accordingly.

Corresponding author.
E-mail addresses: adesanyas@run.edu.ng (S.O. Adesanya), j.srinivasnit@gmail.com (S. Jangili).
Peer review under responsibility of Faculty of Engineering, Alexandria University.

http://dx.doi.org/10.1016/j.aej.2016.09.017
1110-0168 © 2016 Faculty of Engineering, Alexandria University. Production and hosting by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
variational iteration method (VIM). Further studies of strongly exothermic fluid flows in various geometries are documented in Refs. [10–16].

In recent years, entropy generation modeling has also become an active area of thermal engineering sciences since heat passes through a fluid in an irreversible manner and there is an obvious change in entropy of the fluid particles. This is attributable to the temperature difference in the fluid particle which will lead to a random or disordered motion of the fluid particles. Evidently, this will minimize the efficiency of the system. Motivated by the fact that many thermal processes occur at extremely high temperature, the principal objective of this analysis was to examine the thermal performance of the channel flow problem studied earlier by Makinde [5]. This is necessary to measure the efficiency of the setup since increased entropy depletes the energy level of the system. The approach here follows the second law of thermodynamics that is based on the original pioneering work of Bejan [17] which has revolutionized thermal optimization of engineering processes and subsequently used by Adesanya and Makinde [18], Adesanya and Falade [19], Adesanya and Makinde [20] and Adesanya et al. [21] for non-Newtonian liquids. Pakdemirli and Yilbas [22] considered the third grade fluid model with Vogel viscosity to study the entropy generation analysis in a pipe. They showed that different viscosity parameters may either increase or decrease the entropy generation number. Hooman et al. [23] obtained numerical solutions for entropy effects on forced convection flow in a channel. More recent works on entropy generation analysis for various flow configurations is documented in [24–31].

The plan of study is as follows: in Section 2, the mathematical formulation (reactive non-Newtonian fluid dynamics and heat transfer) of the problem is presented. In Section 3 perturbative series solutions are presented for the non-dimensional heat transfer) of the problem is presented. In Section 3 perturbative series solutions are presented for the non-dimensional boundary-value problem while in Section 4, the entropy generation analysis is described. In Section 5, graphical results and interpretation are provided. Section 6 concludes the paper.

2. Mathematical reactive non-Newtonian thermofluid model

Steady-state laminar flow of a chemically-reactive fluid between two rigid horizontal parallel plates containing an isotropic, homogeneous porous medium saturated with the fluid is considered. The exothermic chemical reaction is simulated following Arrhenius kinetics and a third grade non-Newtonian Rivlin-Ericksen (“differential” fluid) model is implemented to simulate the viscoelastic effects. The channel depth is 2a. The fluid flow is induced by a constant applied axial pressure gradient and is hydrodynamically and thermally fully developed. The physical model is illustrated below in Fig. 1. The equations governing the transport, following Makinde [5] are as follows:

\[-\mu \frac{dP}{dx} + \mu \frac{du}{dy} + 6\beta_3 \frac{d^2 u}{dy^2} \left( \frac{du}{dy} \right)^2 = 0\]  

\[k \frac{dT}{dy^2} + \mu \left( \frac{du}{dy} \right)^2 \left( 1 + 2\beta_3 \frac{du}{dy} \right)^2 + Qc_0 A e^{-\frac{T}{a}} = 0\]  

The appropriate boundary conditions imposed are as follows:

\[\text{At } y' = +a : u = 0, \quad \frac{dT}{dy} = -h(T - T_o)\]  

\[\text{At } y' = 0 : \frac{du}{dy} = \frac{dT}{dy} = 0\]  

Here \( u \) is the fluid velocity, \( T \) is the fluid temperature, \( h \) is the heat transfer coefficient, \( k \) is the thermal conductivity of the material, \( Q \) is the heat of reaction, \( A \) is the rate constant, \( c_0 \) is the initial concentration of the reactant species, \( R \) is the universal gas constant, \( \beta_3 \) is the material coefficient, \( E \) is the activation energy, \( P \) is the modified fluid pressure and \( \mu \) is the dynamic viscosity of the non-Newtonian fluid. The following dimensionless quantities and parameters are deployed to obtain the non-dimensional form of the governing equations:

\[\theta = \frac{E(T - T_o)}{RT_o}, \quad y' = \frac{y}{a}, \quad \lambda = \frac{QEA^2 c_0 e^{-\frac{T}{a}}}{RT_o^2 k}, \quad w = \frac{u}{UG}, \quad Bi = \frac{ha}{k}, \quad G = -\frac{\mu U}{\mu U_0}, \quad m = \frac{\mu G^2 U_0 e^{-\frac{T}{a}}}{QA_a C_0}, \quad \epsilon = \frac{RT_o}{E}, \quad \gamma = \frac{\beta_3 U_0 G^2}{\mu a^2}, \quad N_t = \frac{a^2 E^2 E_c}{kR T_o^2}\]  

Here \( U \) is the fluid characteristic velocity, \( T_o \) is the ambient temperature, \( \theta \) denotes non-dimensional temperature, \( y' \) is dimensionless channel width (depth), \( w \) is dimensionless axial velocity, \( Bi \) is the Biot number, \( G \) is the non-dimensional pressure gradient parameter, \( \gamma \) is the dimensionless non-Newtonian (viscoelastic material) parameter within which \( \beta_3 \) is the third grade viscoelastic fluid material parameter [32], \( \lambda \) represents the Frank-Kamenetskii chemical kinetic parameter, \( m \) is the viscous heating parameter and \( \epsilon \) is activation energy parameter.

In view of Eq. (5), the dimensionless boundary-value problem assumes the form:

\[1 + \frac{d^2 w}{dy^2} + \frac{dw}{dy} \left( \frac{dw}{dy} \right)^2 = 0\]  

\[\frac{d^2 \theta}{dy^2} + \lambda \left[ e^{\frac{\gamma}{\epsilon}} \left( 1 + 2\gamma \left( \frac{dw}{dy} \right)^2 \right) \right] = 0\]  

The corresponding normalized boundary conditions become the following:
\[ w = 0, \quad \frac{dw}{dy} = -Bi\theta \quad \text{on} \quad y = 1 \]
\[ \frac{d\theta}{dy} = 0, \quad \frac{d^2\theta}{dy^2} = 0 \quad \text{on} \quad y = 0 \]  \hspace{1cm} (8)
\[ u(y) = a_0 - \int_0^y \int_0^y \left\{ 1 + 6\gamma \frac{du}{dY} \right\} dYdY \]  \hspace{1cm} (13)
\[ \theta(y) = b_0 - \lambda \int_0^y \int_0^y \left\{ e^{\frac{\gamma}{2}} + m \left( \frac{du}{dY} \right)^2 + 2\gamma \left( \frac{du}{dY} \right)^2 \right\} dYdY \]  \hspace{1cm} (14)

3. Methods of solution

3.1. Perturbation technique

Following [5], to obtain the analytical solution of Eqs. (6)-(8), we assume a perturbative series solution of the form:
\[ w = \sum_{n=0}^{\infty} u_n y^n, \quad \theta = \sum_{n=0}^{\infty} \theta_n y^n \]  \hspace{1cm} (9)
where \( 0 < \gamma < 1 \) and \( 0 < \lambda < 1 \) are assumed small parameters.
Substituting (9) in (7) and (8) and equating coefficients, we get the following:
\[ \sum_{n=0}^{\infty} u_n y^n = a_0 - \int_0^y \int_0^y \left\{ 1 + 6\gamma A_n \right\} dYdY \]
\[ \sum_{n=0}^{\infty} \theta_n y^n = b_0 - \lambda \int_0^y \int_0^y \left\{ B_n + m \left( \frac{du}{dY} \right)^2 + 1 + 2\gamma \left( \frac{du}{dY} \right)^2 \right\} dYdY \]
\[ \frac{d^2 u}{dy^2} + 1 = 0; \quad \frac{du}{dy} (0) = 0 = w_0 (1) \]
\[ \frac{d^2 u}{dy^2} + 6\gamma \frac{du}{dy} \left( \frac{du}{dy} \right)^2 = 0; \quad \frac{du}{dy} (0) = 0 = w_1 (1) \]
\[ \frac{d^2 u}{dy^2} + 12\gamma \frac{du}{dy} \left( \frac{du}{dy} \right)^2 = 0; \quad \frac{du}{dy} (0) = 0 = w_2 (1) \]
\[ \text{...} \]
\[ \frac{d^2 \theta}{dy^2} = 0; \quad \frac{d^2 \theta}{dy^2} + \left[ \frac{\gamma}{1 + \alpha y} + m \left( \frac{du}{dY} \right)^2 \right] = 0; \quad \frac{d^2 \theta}{dy^2} (0) = 0, \quad \frac{d^2 \theta}{dy^2} (1) + Bi\theta_0 (1) = 0 \]
\[ \frac{d^2 \theta}{dy^2} = 0; \quad \frac{d^2 \theta}{dy^2} + \left[ \frac{\gamma}{1 + \alpha y} \right] = 0; \quad \frac{d^2 \theta}{dy^2} (0) = 0, \quad \frac{d^2 \theta}{dy^2} (1) + Bi\theta_1 (1) = 0 \]
\[ \text{...} \]

Eqs. (10) and (11) are coded in the DifferentialSolve algorithm in a symbolic package - Mathematica. This gives the solutions as partial sums:
\[ w = \sum_{n=0}^{M} u_n y^n, \quad \theta = \sum_{n=0}^{M} \theta_n y^n \]  \hspace{1cm} (12)
where \( M \) is the point of truncation for the series solution. Due to large output of the symbolic solutions only graphical results are presented in Section 5 and the series expansions are omitted for brevity.

3.2. Adomian Decomposition Method (ADM)

To validate the accuracy of the solutions obtained by regular perturbation above, we need to validate the solution using another rapidly converging semi-numerical method [33]. To implement this method, the dimensionless boundary-valued-problems (6)-(8) are integrated as follows:

\[ B_0 = e^{\frac{\gamma}{2}} \]
\[ B_1 = \frac{\gamma}{1 + \alpha y} e^{\frac{\gamma}{2}} \]
\[ B_2 = \frac{\gamma}{1 + \alpha y} e^{\frac{\gamma}{2}} \]
\[ \text{...} \]
The following partial sum
\[ n = \sum_{n=0}^{M} u_n, \quad \theta = \sum_{n=0}^{M} \theta_n \] (22)
are used to evaluate the unknown constants and consequently the approximate solutions. Eqs. (18)–(22) are carefully coded in Mathematica software giving out huge symbolic solutions. Convergence of the two solution methods is shown in Table 1.

4. Entropy generation analysis

The heat transfer process within the channel is irreversible [18]. Then the random motion of the fluid particles within the flow channel and heat transfer encourages entropy production. Mathematically, the equation for the total entropy generation arising from heat transfer and fluid friction irreversibilities can then be written as

\[ E_o = \frac{k}{T_a} \left( \frac{dT}{dy} \right)^2 + \mu \left( \frac{du}{dy} \right)^2 \left( \mu + 2\beta_1 \left( \frac{du}{dy} \right)^2 \right) \] (23)

Using (14), we get \( N_1 \), \( N_2 \) represent heat transfer and fluid friction irreversibilities respectively:

\[ N_1 = \left( \frac{dl}{dy} \right)^2, \quad N_2 = \frac{\lambda m}{\varepsilon} \left( \frac{du}{dy} \right)^2 \left( 1 + 2\gamma \left( \frac{du}{dy} \right)^2 \right) \] (25)

Next, we get the Bejan number (\( Be \)) as follows:

\[ Be = \frac{N_1}{N_2} = \frac{N_1}{N_1 + N_2} = \frac{1}{1 + \Phi}, \quad \Phi = \frac{N_2}{N_1} \] (26)

The graphical results for \( N_s \) are shown in Figs. 2–6 and those corresponding to \( Be \) and \( \Phi \) are illustrated in Figs. 7–11.

5. Results and interpretation

In this section, the graphical results associated with Eqs. (22) and (23) are presented for various values of the flow parameters.

Fig. 2 describes the effect of Frank-Kamenetskii parameter on velocity field. As this parameter increases, the velocity decreases. Fig. 3 displays the variation of Frank-Kamenetskii parameter on temperature distribution. Increased Frank-Kamenetskii parameter values lead to increase in the reaction and viscous heating source terms and therefore increases the temperature.

Fig. 4 illustrates the entropy generation rate at various values of the Frank-Kamenetskii parameter (\( \gamma \)). As seen from the plot, elevating the Frank-Kamenetskii parameter encourages entropy generation in the flow channel. This is physically true since internal heat generation increases with the increasing concentration of the reagents. The exothermic chemical interactions encourage the rate of heat transfer from the combustion zone to the cool wall. In addition, the heat is transferred through the fluid has melting effect on the fluid viscosity, and this means that inter-particle collision will be encouraged; thus, more heat is being generated by viscous interaction of the fluid particles. Therefore, concentration of the reagent is one factor to be monitored to achieve the best result.

Fig. 5 depicts the effect of third grade material parameter (\( \gamma \)) on the entropy generation in the channel. From the graph, entropy production in the fluid layer closer to the walls decreases as the non-Newtonian material parameter increases. This is because as the third grade material effect increases, the fluid becomes more viscoelastic as a result of stronger bonding forces between the fluid particles in the laminar flow. Also, the heat released due to exothermic chemical reaction is absorbed to excite the fluid particles. Hence entropy generation must be on the decrease as presented in the graph. The downward trend

Table 1  Convergence of solutions when \( M = 3, \varepsilon = 0.1 = m = \lambda, Bi = 10, \gamma = 0.01 \).

<table>
<thead>
<tr>
<th>( y )</th>
<th>( u_{\text{Perturbation}} )</th>
<th>( u_{\text{ADM}} )</th>
<th>( Abs \ Error )</th>
<th>( \theta_{\text{Perturbation}} )</th>
<th>( \theta_{\text{ADM}} )</th>
<th>( Abs \ Error )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0</td>
<td>1.7130E–17</td>
<td>1.71E–17</td>
<td>0.0108030</td>
<td>0.0108057</td>
<td>2.6886E–06</td>
</tr>
<tr>
<td>-0.75</td>
<td>0.215486</td>
<td>0.215486</td>
<td>0</td>
<td>0.0343656</td>
<td>0.0343748</td>
<td>9.18258E–06</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.370497</td>
<td>0.370497</td>
<td>0</td>
<td>0.0511111</td>
<td>0.0511258</td>
<td>1.46827E–05</td>
</tr>
<tr>
<td>-0.25</td>
<td>0.463957</td>
<td>0.463957</td>
<td>0</td>
<td>0.0611240</td>
<td>0.0611424</td>
<td>1.83873E–05</td>
</tr>
<tr>
<td>0</td>
<td>0.495188</td>
<td>0.495188</td>
<td>0</td>
<td>0.0644559</td>
<td>0.0644756</td>
<td>1.96960E–06</td>
</tr>
<tr>
<td>0.25</td>
<td>0.502141</td>
<td>0.502141</td>
<td>0</td>
<td>0.0662769</td>
<td>0.0662926</td>
<td>2.05247E–06</td>
</tr>
<tr>
<td>0.5</td>
<td>0.502141</td>
<td>0.502141</td>
<td>0</td>
<td>0.0662769</td>
<td>0.0662926</td>
<td>2.05247E–06</td>
</tr>
<tr>
<td>0.75</td>
<td>0.502141</td>
<td>0.502141</td>
<td>0</td>
<td>0.0662769</td>
<td>0.0662926</td>
<td>2.05247E–06</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1.7130E–17</td>
<td>1.71E–17</td>
<td>0.0108030</td>
<td>0.0108057</td>
<td>2.6886E–06</td>
</tr>
</tbody>
</table>
observed here is due to the imbalance between the nonlinear heat and the linear convective cooling at the walls as the third grade material parameter increases.

In Fig. 6, the influence of viscous heating parameter \((m)\) on the entropy generation rate within the flow channel is presented. It is clear that an increase in the viscous heating parameter \((m)\) leads to an increase in entropy generation.
eter enhances the entropy production. This is correct in view of
the fact that more and more heat is liberated from inter-
particle collisions; in other words, kinetic energy generates
heat energy since it is an additional heat source.

Fig. 7 depicts the entropy generation rate as the symmetric
Biot number increases at the walls. As shown in the graph the entropy generation rate decreases with an increase in the con-
vective cooling parameter. This is due to the irreversible heat
flow from the surface of hot walls to the ambient, in a manner
that satisfies the Newtonian cooling law. This results in a min-
imization of the entropy in the fluid region close to the cool
walls. Biot numbers considered are much greater than 0.1 i.e.
this corresponds to the “thermally thick” scenario as elabo-
rated by Prasad et al. [34] and Bég et al. [35]. The case where
\( \text{Bi} \leq 0.1 \) is known as the “thermally-thin” scenario and is
not relevant to polymeric flows since heat conduction inside
the body is much faster than the heat convection away from
its surface implying that temperature gradients are negligible
inside the body. The Biot number \( \text{Bi} = \frac{h\alpha}{k} \) is directly pro-
portional to the convection heat transfer coefficient at the
channel wall and inversely proportional to thermal conductiv-
ity, for a given channel width. Lower thermal conductivity is
associated therefore with higher Biot numbers and significant
cooling.

Fig. 8 displays the effect of the activation energy parameter
\( (\epsilon) \) on the entropy generation rate. As seen from the plot an
increase in the activation energy parameter is observed to
lower the entropy generation rate within the flow channel. This
arises since a rise in the fluid activation energy parameter cor-
responds to a decrease in the fluid activation energy. Hence,
the higher the activation energy parameter, the lower the tem-
perature. Therefore this reduction in temperature is expected
to discourage entropy generation within the channel. Similarly
an increase in the activation energy parameter physically
means an associated decrease in the fluid friction due to a rise
in fluid viscosity as shown in Eq. (15) (the viscoelastic param-
ter, \( \gamma \), featuring in Eq. (15) is inversely proportional to the
dynamic viscosity).

Fig. 9 demonstrates that with increasing activation energy
parameter \( (\epsilon) \), Bejan number is consistently enhanced across
the channel span. Evidently, as the irreversibility due to fluid
friction continues to decrease, the Bejan number tends to be
elevated and eventually will attain unity value. This is physi-
cally correct since increase in activation energy parameter as
shown in earlier implies decreased activation energy and
reduced fluid viscosity. When this happens, heat transfer is
expected to dominate over fluid friction irreversibility.
Irreversibility analysis for reactive third-grade fluid flow and heat transfer

Fig. 12 Bejan number at different values of Biot number.

(i) an increase in both viscous heating and Frank-Kamenetskii parameter discourages entropy generation across the channel. (ii) an increase in the fluid activation energy and Frank-Kamenetskii was seen to encourage the dominance of heat transfer irreversibility over fluid friction. On the other hand, increasing values of the Biot number and viscous heating parameter encourages fluid friction irreversibility over heat transfer irreversibility in the flow channel.

The present analysis has neglected time-dependent effects, which will be addressed soon.

References