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Fixed Echo Rejection in Sodar using non-coherent matched filter detection and Gaussian Mixture Model based post-processing

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1 Abstract

Doppler sodar (SOund Detection and Ranging) is a technology used for acoustic based remote sensing of the lower planetary boundary layer. Sodars are often used to measure wind profiles however, they suffer from problems due to noise (both acoustic and electrical) and echoes from fixed objects, which can bias radial velocity estimates.

An experimental bi-static sodar was developed with 64 independent channels. The device enables flexible beam forming; beams can be tilted at the same angle irrelevant of frequency, a limitation in most commercial devices.

This paper presents an alternative sodar signal processing algorithm for wind profiling using a multi-frequency stepped-chirp pulse. A non-coherent matched filter was used to analyse returned signals. The non-coherent matched filter combines radial velocity estimates from multiple frequencies into a single optimisation.

To identify and separate sources of backscatter, noise and fixed echoes, a stochastic pattern recognition technique, Gaussian Mixture Modelling, was used to post-process the non-coherent matched filter data. This allowed the identification and separation of different stochastic processes. After identification, noise and fixed echo components were removed a clean wind profile produced. This technique was compared with traditional spectrum-based radial velocity estimation methods and demonstrated an improvement in the rejection of fixed echo components; this is one of the major limitations of sodar performance when located in complex terrain and urban environments.
1 Introduction

Sodar (SOnic Detection and Ranging) operates by transmitting pulses of sound into the atmosphere and analysing the sound backscattered from moving turbulent fluctuations. Sodars have a variety of applications including wind velocity profiling (Peters 1997) and measurements of turbulence and stability parameters (Petenko et al. 2014). However, backscattered sound may originate from both atmospheric turbulence and stationary objects such as buildings or masts. Due to the zero-Doppler shift on these echoes, in wind velocity profiling this can biases estimates. Fixed echoes from side-lobes of the transmitted beam pattern are one of the main sources of error in sodar measurements (Bradley and von Hünerbein 2007). Fixed echoes are usually minimized by placing the sodar transponder away from tall objects such as trees, masts, buildings and slopes with large stone boulders (AQ500 windfinder User Guide, 2013). Most sodars use acoustic baffles to reduce side-lobes and thus control fixed echoes. Sodar manufacturers provide guidance as to how to reduce the risk from fixed echoes. The AQ500 windfinder User Guide (2013) suggests plotting complete (no data rejection) wind-speed profiles on a log-log scale. Should the relationship be non-linear, this indicates deviation from the expected power-law relationship and could indicate the presence of fixed echoes. Similarly, fixed echoes may be detected as discontinuities in radial velocity profiles of amplitude or frequency (Kalogiros and Helmis 1999). Kalogiros and Helmis (1999) also proposed a method utilising the wavelet transform to better locate fixed echoes in time. The variability of the radial wind velocity over a scattering volume causes broadening of the returned signal (Mayer 2005). Spectral broadening is
also linked to the beam-width of the device (Quintarelli and Bergstrom 2001). As such, commercial sodars may also use the width of Doppler-spectrum peaks to indicate the presence of fixed echoes (Antoniou et al. 2003). However, such methods are sensitive to the chosen threshold and may lead to poor data availability. Alternatively, a bistatic sodar can be used where the Doppler shift in the backscattered signal is greater than an equivalent monostatic implementation (S. Bradley et al. 2012); this makes the differentiation between fixed echo and turbulent backscatter easier.

This paper aims to address some of the issues associated with fixed echoes. A flexible bistatic sodar was developed with two, 32-channel, two-dimensional transducer arrays. Each transducer had its own independent signal path to allow more control over beamforming behaviour than other sodars. Backscattered return signals were analysed using a non-coherent matched filter, which combined the estimation of Doppler shifts from multiple sequential pulses of different frequencies into a single optimisation. Gaussian Mixture Models (GMM) were then used to analyse received signals; this processing method shows interesting advantages in fixed echo detection and rejection, one of the main limitations in sodar performance.

2 Background and Method

2.1 A flexible sodar design

Sodars may be implemented using horn antenna with parabolic dishes for capturing backscatter (Argentini et al. 2013) but are also often implemented as two-dimensional
transducer arrays where beam steering is achieved by introducing phase-shifts between groups of transducers, Bradley and von Hünerbein (2007) compare several sodar technologies. Introducing phase-shifts between groups rather than individual transducers minimises the required number of channels; however, it also means that tilt angle is dependent on frequency and as such, different frequencies will backscatter from entirely different volumes of atmosphere. An experimental bistatic sodar was designed with 32 separate transducers for both receiver and transmitter where each had an independent signal path (Figure 1). This allows for flexible signal processing approaches where tilt and angle and frequency can be de-coupled. A 32-channel DAC and a 32-channel ADC (rme RME M-32AD and RME M-32DA) were used with playback and capture carried out in Matlab. The sampling frequency was 44,100 Hz. The design was bistatic, though the receiver and transmitter were closely located so that the sodar could be operated as a monostatic device. This means there was no requirement for a switch between transmission and receive modes which minimised the blind range. Both transmitter and receiver used 32 identical transducers (Motorola KSN-1005A super-horn piezoelectric tweeters). More details of the implementation can be found in von Hünerbein et al. (2010).

Beamforming was implemented over the transmitter and receiver by applying independent time delays to each channel. A five-beam configuration was used including north, south, east, west and vertical with a tilt of 20 degrees. Beamforming was trivial to implement on the transmitter as individual delays could be applied to the pulse generating function for each speaker. For the receiver, delays were applied to each
signal by adding a linear phase term to the Fourier transform of the received signal. This was converted back into the time domain using an inverse Fourier transform, prior to summation over all channels.

2.2 Pulse design and matched filtering

2.2.1 Stepped-chirp

The flexible nature of the sodar design ensured the different frequencies could be steered in the same directions for both transmitter and receiver. Multi-frequency pulses can offer advantages in terms of flexibility and increased range in sodar (Rao et al. 2009). Therefore, a multi-frequency approach was adopted, where a train of sub-pulses of different frequencies was generated and transmitted. This is referred to as a stepped-chirp which is a form of frequency modulation known as Frequency Shift Keying (FSK). The stepped-chirp is a train of \( M \) single frequency pulses, each with frequency \( f_m \), and modulated by a Gaussian window,

\[
x(t) = \sum_{m=0}^{M-1} \sin[2\pi f_m(t - mT)] G(t)
\]  

(1)

where \( m \) is the pulse number, \( T \) is the pulse length and \( G(t) \) is a Gaussian window function;

\[
G(t) = e^{-\frac{(t-(mT+T/2))^2}{\sigma^2}}
\]  

(2)
The stepped chirp can be processed in multiple ways. It can be processed using a common approach adopted by most commercial sodars; the return signal is windowed into range gates and the Doppler power-spectrum computed for each frequency separately. Doppler shifts and thus radial velocities are estimated by locating peaks in the spectrum. A range offset is applied to account for the delay in sub-pulse transmission times for different frequencies. The stepped-chirp can also be analysed using a technique known as matched filtering; a technique that has been shown to improve performance in phased-array radar systems for weather radar (Alberts and Chilson 2011).

2.2.2 Matched filter receiver

Matched filters are associated with pulse compression, a technique used in radar to increase maximum range without compromising range resolution for moving point targets (Klauder et al. 1960). Longer waveforms are transmitted, this increases the transmitted power but unlike the simple pulse, the transmitted bandwidth is also increased. This enables echoes to be unwrapped; pulse compression allows an increase in range while preserving range resolution. Bandwidth extension in pulse compression is usually achieved by frequency or phase modulation (FSK in this case). Detection involves a matched filter. A matched filter is commonly used to detect scattered signals in both point target and weather radar. Formally, a matched filter is the optimal linear filter, for a signal, for maximising the signal to noise ratio within the presence of noise. It is defined as the correlation of the received signal with a local copy of the transmitted waveform,
\[ mf(t) = \int_{-\infty}^{\infty} y(\tau)x^*(\tau - t)d\tau \]  

where the matched filter magnitude \( mf(t) \) at time \( t \) represents the magnitude of the returned signal at a range of \( ct \) meters, \( x(t) \) is the local copy of the transmitted signal and \( y(t) \) is the received signal.

Unfortunately Pulse compression for wind profiling is not possible in sodar as shown by Hargreaves et al. (2014). This is because Bragg scattering, the dominant scattering mechanism, does not provide coherent return signals. Scattering is dominated by randomly located turbulent eddies spaced by integer multiples of the transmission wavelength. Hence, differently-located eddies will dominate at different wavelengths. This causes the phase-response of a scattering volume to be both stochastic and non-linear; there is not a predictable relationship between the phases of each sub-pulse. This breakdown of the inter-sub-pulse phase relationship means that the advantages of pulse compression are not realised. However, a matched filter is still capable of non-coherent signal detection (Guimarães and de Souza 2015). Non-coherent matched filtering does not increase range or Doppler resolution compared with spectral estimation method, but, as it will be demonstrated, it helpfully combines the Doppler estimation from many discrete frequencies into one optimization problem.

2.2.3 Doppler estimation from matched filters

When an object is moving there is now a mismatch due to the Doppler shift between the scattered signal and the stored waveform. This mismatch results in a decrease in the matched filter output magnitude as the correlation between returned and transmitted
signal is reduced. If the stored waveform can be modified to better represent the scattered signal from the moving object; the mismatch is reduced. Therefore, it is common to use a bank of matched filters; each representing a different radial velocity. The filter with the largest output, at a time-lag (range), represents the likely radial velocity for an object at that range.

Radar matched filter banks such as those demonstrated by Othman et al. (2017) utilise a narrow-band model of Doppler shift. This assumes that the wave-speed is significantly greater than the target-speed and as a result doppler can be modelled as a simple shift in frequency. In sodar, wave speed and wind velocity are more similar orders of magnitude, thus, a wide-band model of Doppler-shift was employed. In this case both the shift in frequency and elongation (or compression) in time of the waveform were modelled. The degree to which the waveform is stretched is captured in a Doppler stretch parameter \( \alpha \) which has a monotonic relationship with the radial velocity, \( (v) \) as follows,

\[
\alpha(v) = \frac{c - v}{c + v}
\]  

A delayed, stretched version of the stepped chirp, representing an echo from target at velocity \( v \) can be expressed as,

\[
x(t, v) = \sum_{m=1}^{M} \sin[2\pi f_m \left( \alpha(v)(t - (m - 1)T) \right)] G(t, v)
\] 

where
\[ G(t, v) = e^{(\alpha(v)(t - ((n-1)T + T/2))/\sigma)^2} \]  

(6)

A matched filter bank was defined which contains waveforms representing a range of target radial velocities \( v_k \). These waveforms were then correlated with the scattered signal \( y(t) \), the matched-filter output for a particular range and velocity is,

\[ m_{f_k}(t, v_k) = \left| \sqrt{\alpha(v_k)} \int_{-\infty}^{\infty} y(\tau) x^*(\alpha(v_k)(\tau - t)) d\tau \right| \]

(7)

The matched filter bank was implemented as shown in Figure 2. A grid of trial-radial-velocities was defined over a realistic range. Assuming a maximum horizontal wind velocity of 20 ms\(^{-1}\) and a beam tilt of 20\(^o\) the maximum radial velocity is about 7 m s\(^{-1}\) so a grid of trial velocities was defined between -7 and 7 m s\(^{-1}\) in 0.05 m s\(^{-1}\) steps (resolution defined by the available RAM at a sampling frequency of 44100 Hz). A matched filter was computed for each trial radial-velocity, cross-correlation calculations were carried out in the frequency domain for speed,

\[ m_{f_k}(\tau, v_k) = F^{-1}\{F[y(t)]F[x_{d}(t, v_k)]^*\} \]

(8)

\( F \) indicates a Fourier transform and \( F^{-1} \) is the inverse Fourier transform. The resulting matched filter output is downsampled from 44100 Hz to 400 Hz to reduce memory requirements. This reduces the range resolution from 7.6 mm to 0.84 m; though the effective resolution is still determined by the sub pulse duration. The maximum value for each time lag was found,

\[ m_{f_{\text{max}}}(\tau) = \max_{v_k} \left( m_{f_{k \text{downsampled}}}(\tau, v_k) \right) \]

(9)
\[ v_r(\tau) = \arg\max_{v_k} \left( m_{f_{downsampled}}(\tau, v_k) \right) \] (10)

This results in two output vectors, \( m_{f_{max}}(\tau) \) and \( v_r(\tau) \), the time lag \( (\tau) \) represents the range \( (c \times \tau) \). \( m_{f_{max}}(\tau) \) indicates the returned signal strength for a particular range and \( v_r(\tau) \) is the estimated radial velocity for each lag. The signal to noise ratio (SNR) of the backscatter was estimated as follows,

\[ SNR_{matched}(\tau) = 20 \log_{10} \left[ \frac{m_{f_{max}}(\tau)}{N_{mf}} \right] \] (11)

where the noise level \( (N_{mf}) \) is the average of \( m_{f_{max}}(\tau) \) between the ranges 300 and 350 m. This was possible as the atmospheric conditions under which the experiment was carried out were relatively stable (evening), this is not a general procedure and other methods should be employed to estimate the background noise level in other conditions.

2.2.4 Waveform design

The ambiguity function is a design tool introduced by Woodward (1951) to help understand the Doppler-range ambiguity problem. Doppler-range ambiguity arises when multiple frequencies are transmitted. Consider the case of a train of simple pulses where each subsequent pulse is a higher frequency. For a received reflection, there is now an ambiguity between the estimated range and Doppler-frequency-shift. In simple terms a distant fast object and a closer slower object could be indistinguishable. The wide-band ambiguity function, \( \psi_n(t, v) \), is a two-dimensional function of time lag and
radial-velocity for a waveform \( x(t) \), and Doppler-shifted waveform \( x_d(t) \) (Cahlander et al. 1964),

\[
\Psi_n(t, v) = \left| \sqrt{\alpha(v)} \int_{-\infty}^{\infty} x^*(t) x_d(\alpha(v)(\tau - t)) \, d\tau \right|
\]

(12)

A waveform with an ambiguity function that is a Dirac Delta function at \( t = 0 \) and \( v = 0 \) has no Range-velocity ambiguity. Thus minimising \( \Psi_n(t, v) \) where \( t \neq 0 \) and \( v \neq 0 \), reduces the range-velocity ambiguity. Range-velocity ambiguity manifests in a stepped chirp when the Doppler shift is equal to the frequency spacing: For example, if the first transmitted sub-pulse frequency, matches the second frequency of the returned signal then they are indistinguishable.

Doppler scales with frequency; therefore the ambiguity is more problematic with higher frequencies. Hence non-linear frequency spacing makes best use of the available bandwidth. The experimental chirp sodar has a usable frequency range of 3-6 kHz. To optimise the available bandwidth the transmitted frequencies was chosen using equation (13). Where, \( f_1 \) is the maximum operating frequency (6 kHz) and each subsequent frequency \( (f_{n+1}) \) is determined from the previous using the maximum expected radial velocity \( (V_{rmax}) \) as follows

\[
f_{n+1} = f_n - f_n \left( 1 - \frac{c - 2V_{rmax}}{c + 2V_{rmax}} \right)
\]

(13)

The experimental sodar has a usable bandwidth of 3 - 6 kHz, assuming \( V_{rmax} = 7 \) \( m/s \) this means that a step-chirp may have up to 9 frequencies without ambiguity and
assuming $c=340 \text{ m/s}$ this yields the following frequencies were used, 6000, 5525, 5088, 4686, 4315, 3974, 3660, 3370, and 3104 Hz.

2.2.5 Non-coherent detection

Due to the nature of the scattering volume, the phase of the returned signal is unpredictable. Therefore, coherent signal detection is not possible. However, a matched filter receiver can still be used to detect backscatter, but the lack of coherence will influence the performance. To investigate this, an approximate model of Bragg scattering was employed. The ambiguity function in equation (12) was evaluated for the step chirp, but the phase of each of the sub-pulse as randomised for the Doppler-shifted signal,

$$x_d(t, v) = \sum_{m=1}^{M} \sin[2\pi f_m (\alpha(v)(t - (m - 1)T)) + \phi_m] G_d(t, v)$$

where $\phi_n$ is a uniformly distributed random phase between $-\pi$ and $\pi$, generated independently for each sub-pulse. The ambiguity function was averaged from 40 repeated simulations for a 9-frequency stepped-chirp with sub-pulse length of 20 m. This average represents the expected range-velocity ambiguity when a set of randomly located moving objects scatter the sound simultaneously but, a different set of objects scatter each frequency. The ambiguity function for a single discrete moving object was also calculated; this is where the phase relationship is predictable and linear (coherent).

Figure 3a shows the marginalised velocity ambiguity function and Figure 3b shows the marginalised range ambiguity both for coherent and non-coherent scattering. The levels
of both are normalised to the maximum level for the coherent detector. The coherent
detector offers an increased resolution, and a 9 dB increase in signal level, this is a
result of pulse compression. However, the non-coherent detection still shows a broad
peak in the ambiguity function. By analysing the return signal and locating this broad
peak, a non-coherent matched filter can still be used to assess the range-velocity, but,
the increased resolution and range associated with pulse compression will not be
realised.

2.3 Analysis of sodar returns using GMMs

While matched filter-based processing of sodar signals does not improve range or
Doppler resolution, there are several other interesting advantages. It provides radial
velocity profiles from multi-frequency data without the need for separate processing for
each frequency; conventional methods often use ad-hoc methods for combining
frequency information. Additionally, range gating is not required. This does not increase
the resolution of the data but removes the need for smoothing or excessive overlapping
if smoother data is required. It is also possible that reflections from fixed objects will
maintain a coherent sub-pulse phase relationship; this means that pulse compression
may enable fixed echoes to be identified with increased resolution compared with
backscatter.

Analysis of sodar signals has traditionally involved a statistical moment analysis of the
peaks in spectral data. Within each range gate the mean and standard deviation of the
radial velocity are computed and post-processing methods are used to reduce the
influence of noise and fixed echoes on the radial velocity estimates. This usually involves rejecting data from range gates when either the Signal to Noise Ratio (SNR) is too low, or the spectrum is narrow. This reduces the data availability; it would be advantageous if the presence of fixed echoes did not mean rejection of data but could be robustly ignored. It is with this goal that Gaussian Mixture models are applied to the matched filter result.

2.3.1 Background to Mixture Models

A mixture model is a probabilistic tool where it is assumed that the probability density function (PDF) of a random process can be approximated as a sum of other, simpler PDFs (e.g. Gaussian). Melnykov and Maitra (2010) traces back the history of statistical mixture modelling to Newcomb (1886) and Pearson (1894). The most popular form of mixture model is where the component mixtures are Gaussian distributions (GMM) (Day 1969; Wolfe 1970; McLachlan and Peel 1999; McLachlan and Peel 1999; Banfield and Raftery 1993). GMMs are used for cluster analysis, segmentation and density estimation in many areas including, finance (Lindemann et al. 2004), audio signal processing (Reynolds and Rose 1995), image processing (Permuter et al. 2003) and medical applications (Schlattmann 2009) amongst many others.

Taylor’s classical "frozen turbulence" hypothesis assumes that the atmosphere consists of many randomly-located discrete scatterers. The velocity, range and scattering strength of these objects is assumed to vary according to some underlying random process. By fitting a GMM to the matched-filter output the joint probability density function of this underlying processing can be estimated. From this, components which
exhibit particular properties can be rejected. For instance, components with low variance in range and velocity are likely to be from fixed echoes.

The resulting GMM fit is a joint probability density function that represents the probability there is a scatterer at a particular range, radial-velocity and scattering strength. The joint probability density function, $p_x(z, v_r, SNR_{matched})$, was modelled as a sum of Gaussian probability density functions,

$$p_x(z, v_r, SNR_{matched}) = \sum_{k=1}^{K} w_k N(x | \mu_k, \Sigma_k)$$

(15)

$x$ is the output from the matched filter, $N$ is a Gaussian probability density function, $k$ is the component number. $K$ is the total number of components, $w_k$ is the component weight, $\mu_k$ is a 3-dimensional vector of mean values for the $k^{th}$ component, $\Sigma_k$ is a $3 \times 3$ covariance matrix for the $k^{th}$ component. If the number of components ($K$) is known, the parameters of the GMM can be estimated using Expectation maximization (EM) (Dempster 1977). Using conditional probability, the likelihood function is derived,

$$\ln L(\mu_k, \Sigma_k, w_k | x_1 \ldots x_M) = \sum_{m=1}^{M} \sum_{k=1}^{K} w_k N(x_m | \mu_k, \Sigma_k)$$

(16)

The maximum of equation (16) was found by optimising the model parameters, $\mu_k, \Sigma_k$ and $w_k$ over $M$ measurements. The EM algorithm is an iterative algorithm which searches for local maxima in the log-likelihood function to estimate the most likely set of model parameters. The number of components $K$ is chosen by computing the Akaike information Criterion (AIC) (Akaike 1974); this is a measure of goodness of fit for a
model. The EM algorithm is carried out for a range of total number of components, $K$, and the model with the lowest AIC is selected.

### 2.4 Data capture and analysis

The chirp-sodar was operated at a site near Manchester, UK (Basell Polyolefins, Carrington, Manchester) on 28th March 2012 at 20:50 for ten minutes under stable atmospheric conditions. The sodar transponders were placed around 2 m apart on a flat concrete surface, the nearest object was a single storey building 60 m from the transponders. Additionally, a cooling tower, buildings, and a number of pipes were located between 100 and 200 m away. The presented data represents a limited time period and type of atmospheric condition; in future studies it will be important to understand how the method performs in a wider range of conditions such as when convection occurs.

The device was operated using a 9-frequency stepped-chirp. The length of each subpulse was 20 m. Five beams were transmitted labelled 1 to 5; South, East, Vertical, North and West using a tilt angle of 20°. The receiver captured 3.5 s of audio synchronised from the start of playback and beam-forming was carried out on the received signals. Radial velocities were estimated using both the non-coherent matched filter method and frequency estimation from power spectra.

For the power-spectrum method the return signal was windowed into range gates using 20 m, 50 % overlapping Hanning windows. A power spectrum was computed for each range gate, and a Gaussian function was fitted to the spectrum amplitude to estimate
the frequency; this is a method commonly utilised in commercial sodar. The Gaussian fit was restricted to within 100 Hz of the $m^{th}$ subpulse frequency and $(m - 1) \times T/c$ seconds was removed from the start of the signal to account for the delayed transmission of each subpulse. Once the Doppler shift ($d$) was estimated for each frequency and range, the radial velocity was computed using the following relationship,

$$v = -\frac{\lambda d}{2}$$  \hspace{1cm} (17)

The signal to noise ratio of the backscatter was estimated as follows,

$$SNR_{\text{spec}}(z) = 20 \log_{10} \left[ \frac{Spec_{\text{peak}}(z)}{N_{\text{spec}}} \right]$$  \hspace{1cm} (18)

where $Spec_{\text{peak}}(z)$ is the peak level of the amplitude spectrum in range gate $z$, $N_{\text{spec}}$ is the noise level computed as the average peak spectrum amplitude from range gates between 300 and 350 m. For each subpulse and range-gate the radial velocities were averaged over frequencies where $SNR_{\text{spec}}(z)$ was greater than 3 dB, all range gates with insufficient SNR were rejected.

3 Results and Discussion

3.1 Non-coherent signal detection

Figure 4 shows the matched filter radial velocity estimates from the westerly beam, overlaying ten minutes of data (30 pulses). A matched filter bank resolution of $0.05 \, ms^{-1}$ was used. The x-axis shows the range and the y-axis shows the radial velocity; the
colour indicates the matched filter output magnitude ($SNR_{matched}$). Figure 4 shows data points when $SNR_{matched}$ was greater than 3 dB.

Figure 4 has captured a radial wind velocity profile, showing an increase in radial velocity up to 70 m (the range of the device was generally limited by electronic noise). However, there were also several scatterers with radial velocities around 0 m s$^{-1}$. These are fixed echoes due to several tall structures nearby. Fixed echoes were visible in the data up to 200 m away.

Figure 5 shows the radial velocity computed using the power spectrum method. As with the matched filter method, only data points which have an SNR 3 dB above the background noise level are plotted.

Comparing the data from the matched filter and spectral methods there is no difference in the maximum range where backscattered sound from turbulence can be detected (around 70 m). If the pulse compression was possible there would be a clear advantage in maximum range. This is consistent with Hargreaves et al. (2014); the matched filter is functioning only as a non-coherent detector.

Some fixed echoes visible in the Figure 4 are not apparent in Figure 5. For example, echoes at a range of 175 and 200 m in the matched filter data cannot be seen in the spectral data. This is indicative of pulse compression. The returned sound is a reflection from a solid object, unlike turbulence which exhibits backscattering; the reflected sound maintains the inter-subpulse phase relationship; for fixed echoes pulse compression appears successful.
In addition, there is a fixed echo at 60 m, visible at 0 ms\(^{-1}\) in Figure 4. In Figure 5, the same fixed echo at 60 m is not clearly visible, however it appears that the radial velocity estimates using the spectral method are lower. This is likely to be because of the spectral peak estimation; if there are two closely spaced peaks the Gaussian fit could be sub-optimal, and the radial velocity estimate biased.

### 3.2 Analysis of sodar returns using Gaussian Mixture models

Figure 6 shows the result of the GMM fit to the data from Figure 4. The AIC was evaluated for up to 60 components, the lowest AIC was when the GMM contained 50 components. To visualise the GMM, Figure 6a shows each component plotted as an ellipse where the location of the centre is the mean and the surface represents one standard deviation. The colour represents strength of the backscattered signal \(\text{SNR}_{\text{matched}}\). The probability density function (pdf) for the GMM was plotted in Figure 6b, where the shading represents the probability when the pdf was marginalised with respect to the matched filter output magnitude; this shows the probability of a moving scatterer at a particular range and radial-velocity.

Figure 6a shows a number of components with very low velocity magnitude and velocity variance. The direct sound is visible up to around 20 m. At 90 m there is a single small component with low variation in velocity and range and from around 120 to 200 meters there was component with a large range variance but low velocity variance; these two components represent fixed echoes from nearby objects. The GMM pdf in Figure 6b can be helpful in making sense of the response. As the GMM fit was computed over 30
pulses, it captures any consistencies in the response over the measurement period. Figure 6b indicates that there is a high probability that a stationary object is located at around 90 m. The use of the GMM pdf to analyse the response was further demonstrated by considering, in Figure 4, the presence of scatterers between 50 and 100 m away with a radial velocity of around 5 m/s. In Figure 6b, this same region shows that the components have a relatively low probability; this is because the returns were not consistent across the 30 pulses and are likely just due to noise.

Some of the more distant components show a very high variance. These components were due to background noise in the system; the matched filter was not detecting any strong returns, this is the matched filter output to a broadband random noise input. Additionally, many of the components from noise also do not decay with distance as one would expect the backscattered return to; this could be used to identify noise.

### 3.2.1 Cluster analysis of sodar returns

Based on the statistical parameters of each component \((\mu_k, \Sigma_k, w_k)\) extracted from the GMM, rules were defined to classify each component as either, backscatter, fixed echo or noise. Experience and guidance from Peters et al. (1997) were used to determine these rules. Classification rules were:

- Components are classed as fixed echoes if, the magnitude of the mean velocity \(\mu_v\) is less than 0.1 m/s and the standard deviation of the radial velocity, \(\sigma_v\) is less than 0.1 m/s. The rationale for this is that while backscatter may have a low Doppler shift, it will have a much higher variance due to turbulence fluctuations.
Components are classified as noise if the covariance matrix indicates that the matched filter output increases with range (scattering magnitude should decrease with range) and if the standard deviation in the radial velocity dimension ($\sigma_v$) is greater than 1 m/s.

After noise/fixed echo component classification all remaining components were classified as backscatter. Using these rules fixed echoes and noise were removed (Figure 7). This has removed the direct sound, the strong reflection at 90 m, the fixed echo between 120 and 200 m, and most of the noise components.

The same process was carried out for each of the 5 beam directions. Figure 8a shows the result of the GMM fit to the vertical beam (beam 3). Similar to beam 5, Figure 8b shows a high probability of scatter from objects with a low velocity magnitude and variance at 80-100 m and around 120m. It is important in the cluster analysis that the fixed echo rejection algorithm does not also reject backscatter. For backscatter with low radial velocity, this relies on the velocity variance being less than 0.1 m/s. From Figure 9, it can be seen that the fixed echoes at 80-100 m and around 120m have been successfully rejected. A couple of components between 20 and 80 m have been rejected. Further refining of procedure is likely required, for instance a Machine learning approach to identification and rejection of fixed echoes may be more robust, though this was beyond the scope of this initial investigation.
3.2.2 Extraction of wind velocity parameters

The GMM pdf appears to be a useful tool in identifying fixed echoes but it can also provide an estimate of the radial wind velocity profile. The matched filter results were analysed, all data points with a SNR less than 6 dB were rejected. After fixed echo components were removed, the GMM pdf was marginalised by summation over the matched-filter output magnitude dimension. The 50\textsuperscript{th} percentile (the median) of the radial velocity was computed for each range.

The standard deviation of the radial velocity is a commonly quoted meteorological parameter. As the standard deviation requires a normal distribution and this is not, it can be approximated by averaging the distance from the median to the 15.9\textsuperscript{th} and 84.1\textsuperscript{th} percentiles, which for a normal distribution would represent one standard deviation around the mean.

To provide an indication of when the signal disappears below the noise floor, the cumulative distribution function (cdf) is evaluated with respect to range. The range where this falls below 0.85 was chosen after empirical investigations. One of the idiosyncrasies with the EM algorithm is that by randomly initialising the starting point, the GMM will be different for each fit. To eliminate the variability due to the stochastic nature of the fitting algorithm the whole algorithm is repeated, 30 times and the resulting velocity magnitude and velocity standard deviation profiles averaged. Figure 10 shows the resulting radial velocity profiles for beams 1 to 5 with dashed lines representing the standard deviation.
Sodar devices often assess data quality in the form of the percentage availability over a measurement period for a range gate (from 0 % to 100%). It may be possible to use the magnitude of the GMM pdf to indicate data quality; however, the collection and analysis would need to be carried out over a wider range of atmospheric conditions than addressed in the current study to assess this.

3.2.3 Comparison with spectral methods

Radial velocity profiles for the 5 beams were computed using the power spectrum method. Data was rejected when the SNR is less than 6 dB. Spectral peaks which were the same or close to the spectral width of the transmitted sound were likely to be from fixed echoes. The width of the spectral envelope of the transmitted peak was 7.7 Hz (standard deviation $\sigma$ of the Gaussian fitted to the spectrum). Data was rejected when the spectral peak width ($\sigma$) was less than 9 Hz, this threshold was determined empirically. Figure 11, shows the spectrally estimated radial velocity profiles.

Comparing the spectrum method with GMM methods (Figures 10 and 11), the maximum ranges are similar, around 70 m, with the spectral method apparently offering slightly further range for some beams. However, the spectral method appears to consistently estimate a lower radial velocity magnitude compared with the GMM method. The reason for this is indicated comparing beam 5 (West) and beam 2 (East) in Figures 10 and 11. In the Matched-filter GMM results (Figure 8) the east and west beams show similar wind velocity profiles but in opposite directions and at an equal distance from the vertical radial velocity. This was as expected; the device is capturing the same wind radial-velocity field (East and West) but in opposite directions. However,
in Figure 11 beam 5 shows a sharp drop in the radial velocity at 65 m that is not replicated in beam 2; this is evidence that this profile was biased by fixed echoes from nearby objects. In general, the presence of clutter and nearby tall structures causes the spectrally estimated profiles to be under-estimated. The similarity of radial velocity profiles in opposite directions indicates that the matched-filter / GMM method is more robust to these fixed echoes. It is possible that sodar manufacturers have optimised their signal processing algorithms to be more resilient to this problem, so it would be important to validate this result on commercial system.

The advantage provided by the GMM method comes from the greater number of dimensions in which the statistical analysis is performed. The GMM procedure captures how the range, radial velocity and scattering magnitude all vary together, while the spectral analysis examines the radial velocity independently for each range gate. The covariance captured by the GMM allowed the definition of a set of empirical clustering rules. In further developments performance may be improved by learning the best set of clustering rules experientially from simulations or in measurements with known fixed echo locations.

4 Conclusion

This paper presents a novel sodar analysis technique with some advantages in fixed echo detection and rejection compared with more traditional techniques. A flexible 64 channel bi-static sodar device was developed where each transducer has its own signal
path, this enabled flexible beamforming, where different frequencies can be steered electronically in the same direction; this is not possible on most commercial devices.

Backscattered return signals were analysed using a non-coherent matched filter. This analysis method combined the estimation of Doppler shift from multiple pulses of different frequencies into a single optimisation. Post-processing of the matched filter data was carried out using, Gaussian Mixture Modelling. This is a statistical analysis technique which utilises the full dimensionality of the feature space to enable better identification and separation of unwanted sources from the backscattered signal. The proposed post processing method via GMM combines several aspects of sodar post-processing previously carried out separately, where profile estimation, fixed echo and noise rejection are all combined into a single framework. In a limited study (over ten minutes during stable atmospheric conditions) the processing method showed better performance in fixed echo detection and rejection over spectral methods, addressing one of the main limitations in sodar performance. However, careful siting of the instrument away from tall structures should still be the first consideration in the elimination of fixed echoes.

5 Acknowledgements

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\[ m_{f_{\text{max}}}(\tau) = \max_v (m_{f_k}(\tau, v_k)) \]

\[ v_{r}(\tau) = \arg\max_v (m_{f_k}(\tau, v_k)) \]
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