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Complex Ginzburg-Landau equations with space-time symmetry: attenuation and amplification, solitons and shockwaves

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Summary

We present an overview of our research into space-time-symmetrized complex Ginzburg-Landau equations, going beyond the traditional assumption of slowly-varying envelopes. Exact analytical solitary solutions are detailed, and their stability properties explored computationally through sets of initial-value problems.

Introduction: *Ginzburg-Landau theory & instabilities*

Ginzburg-Landau (GL) models play a fundamental role as complex-amplitude equations in the arena of universal wave phenomena, describing the interplay between dispersion, diffraction, gain, and loss [1]. In nonlinear optics, they predict the emergence of stationary wavepackets (dissipative solitons) when group-velocity dispersion (GVD) is balanced by self-phase modulation, and attenuation (from two-photon absorption and gain dispersion) is compensated by amplification (typically doping the host medium with fluorescent ions) [2]. For purely-cubic nonlinearity, uniformly-distributed linear growth tends to introduce instability in the zero-amplitude state such that finite-amplitude localized waves are rendered unstable in the long term [1–4]. Inclusion of quintic effects is a route toward suppressing any unphysical collapse [5].

Analysis: *space-time symmetric model*

We will report on recent results for a generalization of the classic cubic-quintic GL model [5]. A mathematical formalism based on the spirit of special relativity is proposed [6], whereby the space and time coordinates, denoted by ζ and τ , respectively (as measured with respect to the laboratory frame), appear with equal status in the governing equation for the dimensionless wave envelope u :

$$\begin{aligned} \kappa \frac{\partial^2 u}{\partial \zeta^2} + i \left(\frac{\partial u}{\partial \zeta} + \alpha \frac{\partial u}{\partial \tau} \right) + \frac{s}{2} \frac{\partial^2 u}{\partial \tau^2} + \gamma_2 |u|^2 u + \gamma_4 |u|^4 u \\ = iD \frac{\partial^2 u}{\partial \tau^2} + i\varepsilon_{\text{lin}} u - i\varepsilon_2 |u|^2 u - i\varepsilon_4 |u|^4 u. \end{aligned} \quad (1)$$

Here, $\kappa \ll O(1)$ determines the level of spatial dispersion, $s = \pm 1$ defines the GVD regime (+1 for anomalous, -1 for normal), and α is a ratio of group velocities. Parameters $\gamma_{2,4}$ and $\varepsilon_{2,4}$ control the intensity-dependent dispersion and losses, respectively, while gain dispersion is set by D and linear amplification by ε_{lin} .

Our approach is based on coordinate transformations that are directly analogous to those encountered in special relativity. Frame-of-reference considerations and Lorentz-type velocity combination rules also play key roles. Moreover, the predictions of conventional GL theory appear asymptotically by way of a limit process similar to that used for recovering Newtonian mechanics as the low-speed limit of relativity [7].

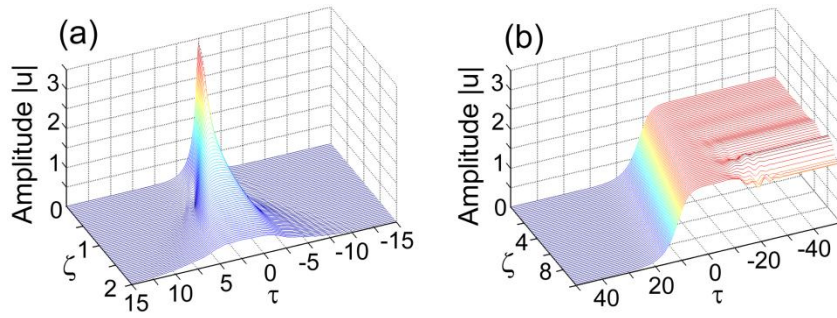


Fig. 1. Simulations illustrating the inherent instability of dissipative solitary solutions [(a) bright hyperbolic soliton, and (b) shockwave] to Eq. (1). The stabilization of such symmetric nonlinear waveforms may become possible in the presence of finite gain dispersion.

Dissipative solutions: *solitons & shockwaves*

Gain dispersion [the term in Eq. (1) at $iD\partial^2 u/\partial \tau^2$] is omitted from our preliminary analysis – while desirable from a physical standpoint (e.g., to help stabilize the pulse in the Fourier domain [5]), its inclusion tends to frustrate the derivation of exact solitary solutions in the context of fully-second-order space-time symmetry. For $D = 0$, three classes of interconnected stationary states can be shown to exist: hyperbolic solitons, algebraic solitons, and shockwaves. Each class possesses forward- and backward-propagating solution branches by virtue of the spatial dispersion term $\kappa\partial^2 u/\partial \zeta^2$, which ascribes Eq. (1) either elliptic or hyperbolic characteristics. It is clearly desirable to find exact dissipative solitons to the full version of model (1), where finite- D effects are included, and hence to provide spatiotemporal generalizations of those corresponding conventional solutions derived by Soto-Crespo *et al.* [5]. Developing mathematical and numerical techniques to look for such solitary waves remains a central objective of our research.

When considering slowly-varying envelopes, and after Galilean-boosting to the local-time frame, one can show that (zero gain dispersion) soliton families derived by Soto-Crespo *et al.* [5] are subsets of our more general solutions. The space-time-symmetric dissipative waves [which satisfy Eq. (1)] are subsequently deployed in computational initial-value problems with a view to addressing stability issues in the system's fully-developed nonlinear dynamics.

Simulations: *from instability toward stability*

A summary of results from an extensive set of simulations will be given, with attention focusing predominantly on bright-hyperbolic solitons. Even in the absence of linear gain (e.g., scenarios where $\varepsilon_{\text{lin}} < 0$), the solitary solutions are typically unstable and tend to undergo dispersive spreading in finite ζ . Numerical investigations are currently considering our new solitons as input waves for Eq. (1) when finite- D effects are incorporated. Such simulations may predict the emergence of stationary states, similar to those reported numerically in Ref. [5], for the space-time-symmetric model.

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