Spontaneous pattern formation with Salerno equations: ring-cavity feedback, static instabilities, and mean-field theory

Christian, JM and Moorcroft, TM

Spontaneous pattern formation with Salerno equations: ring-cavity feedback, static instabilities, and mean-field theory

J. M. Christian and T. T. Moorcroft
Joule Physics Laboratory, University of Salford, Greater Manchester M5 4WT, United Kingdom

TALK ABSTRACT

In physics, the discrete nonlinear Schrödinger (dNLS) equation plays a key role in modelling wave propagation in periodic optical systems [Christodoulides and Joseph, Opt. Lett., vol. 13(9), 794–796 (1988)]. Architectures typically involve light confined to a set of waveguide channels with nearest-neighbour coupling and whose dielectric response has a local cubic nonlinearity. While the widely-used dNLS model is non-integrable, it possesses an integrable counterpart—the Ablowitz-Ladik (AL) equation [J. Math. Phys. vol. 17(6), 1011–1018 (1976)]—which is often of greater interest in applied mathematics research. The price paid for integrability is a nonlinear response that remains cubic but becomes nonlocal in a way that defies straightforward physical interpretations. In this presentation, our interest lies with the Salerno equation [Phys. Rev. A, vol. 46(11), 6856–6859 (1992)], which facilitates a simple linear interpolation between the dNLS and AL regimes.

Here, we consider the Salerno equation in the context of spontaneous pattern formation involving a discrete waveguide array and a ring-cavity arrangement. Feedback from the cavity—which comprises external periodic pumping, coupling-mirror losses, and mistuning relative to the pump wave—is accommodated via a single 'lumped' boundary condition applied on the input plane. The stationary plane-wave solutions of the cavity are detailed, and a linearized perturbation theory deployed to predict their robustness against small-amplitude periodic modulations. In this way, the most-unstable spatial frequency (hence the dominant length-scale of any emergent static patterns) can be identified from the threshold instability spectrum. The dNLS and AL spectra appear as special cases, and the long-wavelength asymptotics of all three models are consistent with the continuum nonlinear Schrödinger equation.

Extensive simulations of discrete cavities with a single transverse dimension have been carried out, with initialization corresponding to a plane-wave stationary state perturbed by low-level coloured noise. Those numerical calculations demonstrate the emergence of static cosine-type patterns, in good agreement with theory. We have also extended our considerations to capture a second transverse dimension in the Salerno equation. Simulations have yielded static hexagon patterns that appear to be stable across time.

We conclude with a foray into mean-field theory, which is typically used to model the space-time dynamics of a longitudinally-averaged cavity field. The resulting Salerno equation is of the discrete Ginzburg-Landau class, where cavity effects appear as additional forcing terms rather than through repeated application of a formal boundary condition. Results from pattern formation in both one and two transverse dimensions will be detailed.

Keywords: Discrete equations, instabilities, nonlinear waves, pattern formation.