Validation of a mathematical model to estimate turning movements as roundabouts using field data

Yousif, S and Razouki, SS

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<tr>
<th>Title</th>
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<tr>
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</tr>
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</table>

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VALIDATION OF A MATHEMATICAL MODEL TO ESTIMATE TURNING MOVEMENTS AT ROUNDABOUTS USING FIELD DATA

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Abstract
In this paper, a mathematical model was presented and used to determine turning movements at roundabouts based on field data. Assumptions were made in order to simplify the model; such as no U-turns from and to the same arm of a roundabout, total traffic into the roundabout is equal to total traffic out of the roundabout and traffic is homogenous (i.e. mainly consisting of cars). Using Gaussian elimination, turning movements could be estimated for 3, 4 and 5-arm roundabouts for the indeterminate traffic stream movements when inflows and outflows for each arm of the roundabout is known together with a flow stream on one internal circulating (weaving) section between any two arms of the roundabout. The model has practical use in reducing the number of detectors or counters (whether automatic, videoing techniques or manual methods are in use) which are needed in collecting data to determine the estimated flows from and to the different parts of a roundabout. The reduction in the number of detectors (or traffic counts) could be due to site limitations caused by faulty or limited number of counters used, inaccessible sections for obtaining video images for later analysis (e.g. presence of sharp bends, buildings or large trees obscuring vision). The benefits in cut saving costs could be significant in terms of time and man-power needed on site, and this could depend on the amount of traffic flow through the roundabout. The model was validated against data obtained from different sites and the results were found to be satisfactory.

Key words: mathematical model, turning movement, roundabouts

1. INTRODUCTION
There is an increasing need to collect quality traffic data for a variety of uses. In some cases manual traffic counts are used and on others more sophisticated devices, such as automatic traffic counters, video cameras are in use to gather the necessary information on traffic such as traffic flow, composition of traffic and turning movements at junctions.

One of the widely used types of junctions in the UK is a roundabout. Roundabouts serve a major role mainly in regulating the turning movements of vehicles from one direction to another in a safe and efficient manner. These movements could be complex as the number of arms in a roundabout increase, and they need to be monitored and studied when considering further improvements to the operation of such roundabouts. Such improvements could involve changes to the way road markings are implemented on large roundabouts. McCann (1996) reported some benefits mainly in reducing drivers’ confusion which in turn results in improving safety through changes made to the angle of incidence by using spiral markings. In some cases, the presence of new
developments, such as new retail stores or other housing or industrial developments, trigger the need for collecting data for use in traffic impact assessment studies (before and after studies).

The task of collecting good quality data for such large roundabouts or even for heavily trafficked smaller sized roundabouts is daunting in terms of the resources needed in conducting such surveys. There is a need sometimes to reduce the number of manual workers/data collectors or automatic counters to cut the cost of such traffic surveys. Instead, one could make use of available data from and to the different arms while concentrating efforts on certain sections of the roundabout to have as accurate information as possible without the need to survey whole sections of a roundabout. This required the need for some methods of determining the missing data of turning movements from the different arms by using mathematical models.

2. REVIEW OF PREVIOUS WORK

Several researchers attempted to estimate turning movements at road junctions and a number of methods were developed over the years making use of traffic flows entering and leaving a junction (for example, see Jeffreys and Norman (1979), Mekky (1979), Van Zylen (1979), Razouki and Jadaan (1997) and Razouki (2000)). Eisenman and List (2005) used detailed information on individual vehicle's trajectories through the roundabout and proposed estimates of turning movements. The accuracy of turning flow estimates at road junctions from traffic counts was also examined by Bell (1984) who accepted a relative difference of about 22.5% for traffic flows in the region of about 1575 veh/hr and an absolute difference of 74 veh/hr for traffic flows of 454 veh/hr. Jadaan (1989) studied the accuracy of turning flows and accepted a relative difference between actual and estimated flow of 13.1% for traffic flows of 168 veh/hr.

Other efforts went into developing complex mathematical models for estimating turning flows at intersections and comparing results obtained from each model (for example, see Maher (1984)). Other studies, such as Cremer and Keller (1987), used dynamic methods where traffic flow through a facility is considered as a dynamic process and these have shown high accuracy results.

Marshall (1979) proposed labour-saving methods for counting traffic movements at 3- and 4-arm junctions and showed that two observers together with two automatic counters are sufficient for 3-arm junctions. However, this task is rather difficult when traffic flow is high and where observers need to trace the path of vehicles individually. Mountain et al. (1986) studied 4-arm intersections and reported on the potential of cost saving from replacing direct observation of turning flows with estimates based on counts taken from entry and exits flows. Adebisi (1987) proposed the use of algorithms in manual traffic counts of turning flows at road junctions to save on labour costs. Goethe et al. (1989) presented multiobjective programming formulations for estimating O-D matrices, while Lam and Lo (1990) used direct traffic counts to estimate an O-D matrix. Nihan and Davis (1987) used a single set of traffic data (based on input/output counts) to estimate a static O-D matrix rather than using time-series of counts to track a dynamic O-D matrix. Dixon and Rilett (2002) used automatic vehicle identification in traffic data counts for real-time O-D estimation.

3. MODELLING TURNING MOVEMENTS USING O-D MATRICES

Several researchers attempted the use of O-D matrices to describe the turning movements from and to the various arms at road junctions (for example, see Fisk and Boyce (1983, Mountain and Steele (1983) and Mountain and Westwell (1983)). The process of obtaining accurate measurements of turning movements could be particularly complex when dealing with more than 3-arm junctions whether they are roundabouts or any other type of intersections. Any attempts to reduce the need to obtain counts from each and every entry/exit sections as well as other circulating sections from a roundabout will become very attractive indeed especially when traffic...
flows are high. This may lead to the fact that the number of equations used in forming the matrices (i.e. independent algebraic equations) is less than the number of unknowns representing the different turning movements in a roundabout.

To solve this problem, an analogy has been made for estimating turning flows at road junctions with that used in solving “statically indeterminate” structures in Civil and Structural Engineering where the number of unknowns exceeds the number of independent algebraic equations (see for example, Bhatt and Nelson (1990)). The Gaussian elimination procedure (Kreyszig, 2006) is applied for this purpose and the following terms, as defined by Razouki (1997) and Razouki and Jadaan (1997) are used:

- “traffically determinate” road junction: which is one that provides a number of linearly independent algebraic equations equal to the number of traffic streams in that junction, and
- “traffically indeterminate” road junction: where the number of linearly independent algebraic equations is less than the number of traffic streams in that junction.

For any roundabout, each equation corresponds to flow counts at a certain cross section. These flow counts could be obtained easily by unskilled labourers, automatic counters or video cameras. However, certain assumptions should be made in arriving at a logical and simple model for use in accurately estimating turning movements at roundabouts. These assumptions are as follows:

a. Traffic flow is continuous through the junction (i.e. no stopping or parking within the roundabout). Accordingly,
\[ \Sigma E_i = \Sigma L_i \quad \text{... Equation 1} \]
where
\[ E_i \text{ and } L_i \text{ are traffic entering and leaving arm } i, \text{ respectively.} \]

b. No U-turns were made (i.e. from and to the same arm). Accordingly,
\[ T_{ii} = 0 \text{ (for all } i \text{ values) \quad ... Equation 2} \]
where,
\[ T_{ii} \text{ is traffic from and to arm } i. \]

c. Traffic is homogenous (i.e. traffic composition is the same for all links).

In order to be able to solve an O-D matrix, one needs a “traffically determinate” junction. However, when dealing with a “traffically indeterminate” junction (as defined above) and because of the missing data from and to certain arms or unavailability of data (which could be due to shortage in the number of data collectors or counters on site), extra data is required. Such data could be obtained by considering extra information on flows for a selected weaving section within the circulating traffic between roundabout arms (and possibly the use of information from other “redundant” traffic streams which may easily be obtained from observations).

Table 1 gives the number of movements within a chosen weaving section for different numbers of roundabout arms, say W_{12} as shown in Figure 1. Considering a particular weaving section within a circulating traffic and assuming no U-turns, it is clear from Table 1 that as the number of roundabout arms increases, the number of movements within the weaving section increases sharply. This shows that the complexity of monitoring turning movements at roundabout increases.
sharply as the number of arms increases adding to the complexity of the data collection process as well as the analysis.

<table>
<thead>
<tr>
<th>Number of roundabout arms</th>
<th>Number of movements within a weaving section (including those from adjacent arms)</th>
<th>Details of traffic movements</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2</td>
<td>$W_{12}=E_1+T_{32}$</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>$W_{12}=E_1+T_{32}+T_{42}+T_{43}$</td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>$W_{12}=E_1+T_{32}+T_{42}+T_{43}+T_{52}+T_{53}+T_{54}$</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>$W_{12}=E_1+T_{32}+T_{42}+T_{43}+T_{52}+T_{53}+T_{54}+T_{62}+T_{63}+T_{64}+T_{65}$</td>
</tr>
</tbody>
</table>

4. MATRICES FORMATION FOR “TRAFFICALLY INDETERMINATE” ROUNDABOUTS

In this section, a simple example is chosen for a 3-arm roundabout to describe the proposed mathematical model to form the matrices used in solving “traffically indeterminate” roundabouts. Here the assumptions described in previous sections (i.e. homogenous traffic, $\Sigma E_i = \Sigma L_i$ and $T_i = 0$) were used together with the assumption that some data on traffic flow movements were missing due to absence of counters.

The following matrix describes the algebraic equations representing turning flows if the weaving section $W_{12}$ is chosen for the 3-arm roundabout shown in Figure 1:
\[
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
T_{13} \\
T_{12} \\
T_{31} \\
T_{32} \\
T_{21} \\
T_{23} \\
\end{bmatrix}
= 
\begin{bmatrix}
E_1 \\
E_3 \\
E_2 \\
E_1 \\
L_2 \\
L_3 \\
W_{12} \\
\end{bmatrix}
\]

\[\text{...... Equation 3}\]

Note that \(W_{12}\) is equal to \(T_{12} + T_{13} + T_{32}\) while \(E_1\) is equal to \(T_{12} + T_{13}\). Here the required counts are \(E_1, E_2, E_3, L_1, L_3\) and \(W_{12}\) (assuming that the missing flow counts for this case is \(L_2\)).

In a similar way, other matrices for 4-, 5- or 6-arm roundabouts could be formed to describe the algebraic equations representing traffic movements using a selected weaving section. As mentioned earlier, the matrices are much more complex as the number of arms increases. For example, the 5-arm roundabout shown in Figure 2 possesses 20 traffic streams.

![Figure 2 Sketch of a 5-arm roundabout](image_url)
The required matrices for this 5-arm roundabout are:

\[
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
\begin{bmatrix}
T_{14} \\
T_{15} \\
T_{21} \\
T_{25} \\
T_{31} \\
T_{32} \\
T_{43} \\
T_{54} \\
\end{bmatrix}
= 
\begin{bmatrix}
E_1 - T_{12} - T_{13} \\
E_2 - T_{23} - T_{24} \\
E_3 - T_{34} - T_{35} \\
E_4 - T_{45} - T_{41} \\
E_5 - T_{51} - T_{52} \\
L_2 - T_{12} - T_{13} \\
L_3 - T_{13} - T_{23} \\
L_4 - T_{24} - T_{34} \\
L_5 - T_{35} - T_{45} \\
W_{12} - T_{12} - T_{13} - T_{52} \\
\end{bmatrix}
\]  

... Equation 4

Note that \( W_{12} \) is equal to \( T_{12} + T_{13} + T_{14} + T_{15} + T_{52} + T_{53} + T_{54} + T_{42} + T_{43} + T_{32} \). Here the required counts are \( W_{12} \) together with all entry counts (i.e. \( E_1, E_2, E_3, E_4 \) and \( E_5 \)), all exit counts except \( L_1 \) (i.e. \( L_2, L_3, L_4 \) and \( L_5 \)) and ten other “redundant” individual counts (i.e. \( T_{12}, T_{13}, T_{23}, T_{24}, T_{34}, T_{35}, T_{45}, T_{41}, T_{51} \) and \( T_{52} \)). This results in having a total of ten other counts which could be assumed missing from the actual survey but could be calculated using the Gaussian elimination procedure (i.e. with 10 unknowns namely, \( T_{14}, T_{15}, T_{21}, T_{25}, T_{31}, T_{32}, T_{42}, T_{43}, T_{53} \) and \( T_{54} \)).

It should be noted here that when choosing the “redundant” traffic streams for data collection (i.e. ten in the case of 5-arm roundabout) they should be selected so that they could be obtained as easily as possible on site. Considering a site similar to that in Figure 2, the easiest five left turning movements that a traffic observer/surveyor on site could obtain are those from all arms to the ones adjacent to them (i.e. \( T_{12}, T_{23}, T_{34}, T_{45} \) and \( T_{51} \)). The next set of traffic movements which require a slightly more effort to obtain on site (all depends on other factors such as visibility, location of observers, distance between adjacent arms … etc.) are \( T_{13}, T_{24}, T_{35}, T_{41} \) and \( T_{52} \).

The Gaussian elimination procedure is used to solve these matrices. However, it is more convenient to transform the calculations using the Gaussian elimination procedure into a spreadsheet using, for example, Microsoft Excel when processing such calculations for such matrices.

5. MODEL VALIDATION FOR 3-ARM ROUNDBOUDT

In order to check the validity of the mathematical model, a 3-arm roundabout in Oldham Town Centre was selected as one of the sites. The data was originally used as part of an MSc research project studying roundabout capacity. Therefore, peak hour periods were considered for that purpose. Table 2 shows the evening peak hour data based on 15 minute counts. The total traffic flow is in the region of 2000 veh/hr.

For the validation of the “traffically determinate” 3-arm roundabout, the relevant matrix information presented in Section 4, Equation 3 above was used. It has been assumed that certain traffic counts are missing from the original data. Here the required counts are \( E_1, E_2, E_3, L_1, L_3 \) and \( W_{12} \).
Table 2 Average flow per hour (based on 15 minutes counts) for a 3-arm roundabout (evening peak)

<table>
<thead>
<tr>
<th>Time of day</th>
<th>Entry Arm 1 (St Marys Way from the North West) to arm</th>
<th>Entry Arm 2 (Egerton St from the North East) to arm</th>
<th>Entry Arm 3 (St Marys Way from the South) to arm</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>16:30</td>
<td>145</td>
<td>34</td>
<td>107</td>
<td>486</td>
</tr>
<tr>
<td>16:45</td>
<td>133</td>
<td>21</td>
<td>109</td>
<td>466</td>
</tr>
<tr>
<td>17:00</td>
<td>159</td>
<td>28</td>
<td>118</td>
<td>562</td>
</tr>
<tr>
<td>17:15</td>
<td>137</td>
<td>20</td>
<td>105</td>
<td>509</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>574</strong></td>
<td><strong>103</strong></td>
<td><strong>439</strong></td>
<td><strong>2023</strong></td>
</tr>
</tbody>
</table>

Table 3 shows the required traffic counts and those assumed missing. A Microsoft Excel spreadsheet was used to calculate the required missing data (in this case $T_{13}$, $T_{12}$, $T_{31}$, $T_{32}$, $T_{21}$ and $T_{23}$) using the Gaussian elimination procedure.

Table 3 Traffic count matrix for the 3-arm roundabout

<table>
<thead>
<tr>
<th>From</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td>1090</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0</td>
<td></td>
<td>235</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>0</td>
<td>698</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>571</strong></td>
<td><strong>619</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$W_{12} = 1349$

Substituting the above values into Equation 3 results in the following:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_{13} \\ T_{12} \\ T_{31} \\ T_{32} \\ T_{21} \\ T_{23} \end{bmatrix} = \begin{bmatrix} 1090 \\ 698 \\ 235 \\ 571 \\ 619 \\ 1349 \end{bmatrix}$$

...... Equation 5

The solution to Equation 5 using Gaussian elimination with the help of Microsoft Excel gives:

$$\begin{bmatrix} T_{13} \\ T_{12} \\ T_{31} \\ T_{32} \\ T_{21} \\ T_{23} \end{bmatrix} = \begin{bmatrix} 516 \\ 574 \\ 439 \\ 259 \\ 132 \\ 103 \end{bmatrix}$$

...... Equation 6
When comparing values of flow counts from Equation 6 with those of Table 2 the results show complete agreement. This indicates that the mathematical method used for solving a “traffically determinate” 3-arm roundabout results in a useful and significant saving in both time and labour cost.

6. MODEL VALIDATION FOR 5-ARM ROUNDBOUT

Further validation of the mathematical model was carried out using another site. This time the site consists of a 5-arm roundabout in Warrington in the North West of England for both a.m. and p.m. peak periods. Figure 3 shows a schematic layout of the large 5-arm roundabout together with the various turning movements from and to the different arms. Although one of the arrows (i.e. arrow representing T_{44}) is showing a U-turn movement from the western side of the A57 Manchester Road, the selected data for this analysis from the morning peak indicates that no U-turn was performed for that period. This fits well with the assumptions made earlier for the “traffically indeterminate” roundabout under consideration.

Table 4 shows the morning peak hour data based on 15 minute counts. The data in this Table relates to Figure 3 and should be read together. The total traffic flow is in the region of about 2600 veh/hr.
It is clear from Table 4 above that the flows from the different arms are totally unbalanced. This is expected in such situations were large roundabouts exists in close proximity to major sources of traffic (in this case the M6 Junction 21 and the A57 Manchester Road) feeding from one or two arms, together with minor sources of traffic from other arms (in this case a pub and a site access). The mathematical model proposed for the “traffically indeterminate” roundabout has no limitations to whether traffic flows from the different arms are balanced or not.

For the validation of the “traffically indeterminate” 5-arm roundabout, the relevant matrix information presented in Section 4, Equation 4 above was used. It has been assumed that certain traffic counts are missing from the original data.

Table 5 shows the required traffic counts and those assumed missing. Similar procedure to that used for 3-arm roundabout was adopted making use of Microsoft Excel spreadsheet in order to calculate the required missing data using the Gaussian elimination procedure. Missing data in this case represents T_{14}, T_{15}, T_{21}, T_{25}, T_{31}, T_{32}, T_{42}, T_{43}, T_{53} and T_{54} (i.e. 10 in total).

<table>
<thead>
<tr>
<th>Arm 1</th>
<th>Arm 2</th>
<th>Arm 3</th>
<th>Arm 4</th>
<th>Arm 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_{15}</td>
<td>T_{14}</td>
<td>T_{13}</td>
<td>T_{12}</td>
<td>T_{21}</td>
</tr>
<tr>
<td>7:45</td>
<td>0</td>
<td>62</td>
<td>0</td>
<td>49</td>
</tr>
<tr>
<td>8:00</td>
<td>0</td>
<td>77</td>
<td>0</td>
<td>59</td>
</tr>
<tr>
<td>8:15</td>
<td>1</td>
<td>101</td>
<td>0</td>
<td>46</td>
</tr>
<tr>
<td>8:30</td>
<td>1</td>
<td>79</td>
<td>2</td>
<td>55</td>
</tr>
<tr>
<td>Total</td>
<td>2319</td>
<td>2209</td>
<td>294</td>
<td>4579</td>
</tr>
</tbody>
</table>

It is clear from Table 4 above that the flows from the different arms are totally unbalanced. This is expected in such situations were large roundabouts exists in close proximity to major sources of traffic (in this case the M6 Junction 21 and the A57 Manchester Road) feeding from one or two arms, together with minor sources of traffic from other arms (in this case a pub and a site access). The mathematical model proposed for the “traffically indeterminate” roundabout has no limitations to whether traffic flows from the different arms are balanced or not.

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<table>
<thead>
<tr>
<th>From</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>209</td>
<td>2</td>
<td>------</td>
<td>------</td>
<td>532</td>
</tr>
<tr>
<td>2</td>
<td>------</td>
<td>0</td>
<td>1</td>
<td>579</td>
<td>------</td>
<td>878</td>
</tr>
<tr>
<td>3</td>
<td>------</td>
<td>------</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>785</td>
<td>------</td>
<td>------</td>
<td>0</td>
<td>5</td>
<td>1217</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0</td>
<td>------</td>
<td>------</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td>636</td>
<td>3</td>
<td>900</td>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ W_{12} = 961 \]
Substituting the above values into Equation 4 results in the following:

\[
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
T_{14} \\
T_{15} \\
T_{21} \\
T_{25} \\
T_{31} \\
T_{32} \\
T_{42} \\
T_{43} \\
T_{53} \\
T_{54}
\end{bmatrix} =
\begin{bmatrix}
321 \\
298 \\
1 \\
427 \\
2 \\
427 \\
0 \\
321 \\
6 \\
750
\end{bmatrix}
\]

\[\ldots\text{ Equation 7}\]

The solution to Equation 7 using Gaussian elimination with the help of Microsoft Excel gives:

\[
\begin{bmatrix}
T_{14} \\
T_{15} \\
T_{21} \\
T_{25} \\
T_{31} \\
T_{32} \\
T_{42} \\
T_{43} \\
T_{53} \\
T_{54}
\end{bmatrix} =
\begin{bmatrix}
319 \\
2 \\
294 \\
4 \\
1 \\
0 \\
427 \\
0 \\
0 \\
2
\end{bmatrix}
\]

\[\ldots\text{ Equation 8}\]

Again, when comparing the values of flow counts from Equation 8 with that of Table 4, the results show complete agreement. In turn, this indicates that the mathematical model adopted here for solving a “trafficly indeterminate” 5-arm roundabout is useful and provides significant saving in both time and labour cost especially if the analysis of the data is made in a systematic spreadsheet (i.e. using Microsoft Excel).

7. CONCLUSIONS AND RECOMMENDATIONS

The main conclusions of this work can be summarised as follows:

- The Gaussian elimination procedure is applied to find the solution to a “trafficly indeterminate” roundabout (i.e. with the number of available linearly independent equations being less than the number of unknowns).
- The assumptions used for this “trafficly indeterminate” roundabout are that traffic flow is continuous through the junction, no U-turns are performed from/to the same arm of a roundabout and traffic is considered homogenous.
The validation of the proposed mathematical model yields complete agreement with the field data. It can be applied whether or not there are unbalanced traffic flows.

The use of the proposed mathematical model could save time and cut on labour costs as well as savings due to time required for the analysis especially when systematic spreadsheets are used (e.g. Microsoft Excel) in calculating the required information.

Recommendations for future work:
- The accuracy of the model when U-turns are present requires further testing.
- Also, the effect of having non-homogenous traffic (i.e. presence of heavy goods vehicles, buses and other types of vehicles) needs to be tested.

8. REFERENCES


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