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A UNIVERSAL NONPARAXIAL REFRACTION LAW FOR SPATIAL SOLITONS

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Abstract

The interaction of self-localizing and self-stabilizing wavepackets with interfaces is a fundamental nonlinear wave phenomenon. Indeed, interfaces play a crucial role as boundary conditions in a wide class of problem, ranging from guided-wave optics in photonic architectures to water waves interacting with coastal structures.

We consider scattering of spatial optical solitons at the planar boundary between dissimilar nonlinear materials. Our approach allows arbitrary angles of beam incidence, reflection and refraction (relative to the interface); conventional theories demand these angles (in the laboratory frame) be near-negligibly small. A range of analytical and semi-analytical methods is deployed in both domains, and respecting solution continuity at the boundary allows the derivation of a universal Snell's law governing beam refraction. Numerical analysis, in combination with fast computational techniques, tests the validity of theoretical predictions. Given the universality of soliton phenomena, we expect the methods developed here to be applicable in other nonlinear-wave based systems.

Conventional (paraxial) theory

A light beam impinging on the boundary between two dissimilar dielectric materials is a fundamental optical geometry. After all, the single-interface configuration is an elemental structure that facilitates more sophisticated device designs and architectures for a diverse range of applications. The seminal papers of Aceves, Moloney and Newell [1,2] in the late 1980s considered a simple scenario, where a spatial soliton was incident on the boundary between two different Kerr-type materials. Their intuitive approach reduced the full complexity of the electromagnetic interface problem to something far more tractable, namely the solution a scalar equation of the inhomogeneous nonlinear Schrödinger type. Over the past two decades, investigations of single [3] and multiple-layer [4] interface geometries have paved the way to deeper understandings of how light behaves inside patterned nonlinear structures such as coupled waveguide arrays and photonics crystals.

It is true to say that the analyses of Aceves *et al.* [1-6] and others [7-10] have provided an enormous level of insight, and they have heralded new research fields in nonlinear photonics. Oblique incidence effects (see Figure 1) are central in the understanding of nonlinear wave-interface interactions [11,12]. For instance, one can envisage the ingoing spatial soliton being arbitrarily inclined with respect to the boundary, either through changing the orientation of the light beam (relative to the interface) or by rotating the waveguiding structure itself (relative to the beam). It is thus desirable, essential even, for theoretical models to capture this type of intrinsic *angular* characteristic. Unfortunately, the ubiquitous assumption of slowly-varying envelopes renders traditional (paraxial) modelling applicable only when angles of incidence, reflection and refraction in the laboratory frame are negligibly (or near-negligibly) small.

Helmholtz (nonparaxial) theory

In 2007, we proposed the first scalar model of spatial soliton refraction that is valid across the entire angular range [11]. In a scalar environment, the beams-at-interfaces problem comprises two main themes: *propagation* and *material* aspects.

Propagation aspects

Our preliminary research concentrated on developing the propagation formalism by deploying Helmholtz spa-

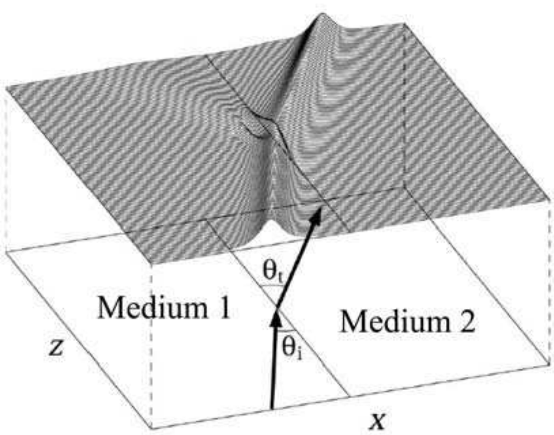

Figure 1: External spatial soliton refraction in the laboratory frame, where $\theta_t > \theta_i$

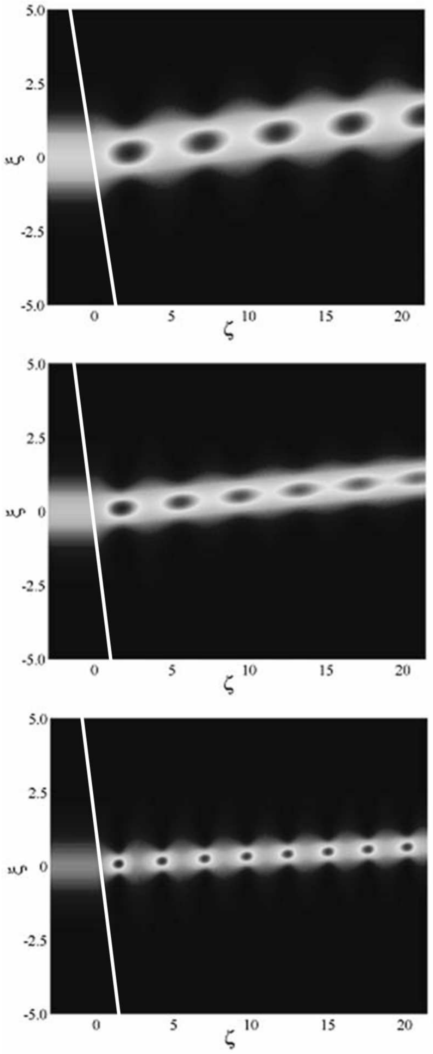

Figure 2: Refraction of spatial solitons at a nonlinear interface between power-law type media with similar nonlinearity exponents (the refractive index varies as $\delta n^2_{\mathrm{NL}} \sim I^p$). The incidence angle in each part is 30° (well beyond what is attainable from paraxial theory [1-6]). Top: $p = 1/2$; middle: $p = 1$ (Kerr); bottom: $p = 3/2$.

tial soliton theory. The angular restriction of paraxial models was lifted, and a manageable envelope equation emerged.

For simplicity, Kerr-type materials were considered first; these have the most straightforward nonlinearities, where induced refractive-index changes are directly proportional to the local light intensity I. By deriving exact analytical solitons, and enforcing solution continuity at the interface, a Snell's law was obtained for bright [11,12] and dark [13,14] Kerr beam refraction. At first glance, this new law strongly resembles the classic refraction rule for *plane waves* at the interface between *linear* media. However there appears an additional multiplicative factor that captures the interplay between finite-beam effects and medium discontinuities. Extensive numerical simulations tested, and confirmed the validity of, theoretical predictions.

Material aspects

Our more recent research has been geared toward systematic generalizations that describe novel material configurations. The first steps in this direction have considered scenarios where the intensity dependence of the refractive index is preserved across the boundary, but this dependence is non-Kerr in character. The simplest generalization to consider is that of arbitrary power-law materials (i.e., nonlinear refractive indexes like $\delta n^2_{\mathrm{NL}} \sim I^p$, where $0 < p < 2$, and $p = 1$ describes the Kerr effect). Media falling into this category include some semiconductors (e.g., InSb and GaAs / GaAlAs), doped filter glasses (e.g., $\mathrm{CsS}_x\mathrm{Se}_{x-2}$) and liquid crystals [15,16]. In this regime, we have uncovered new qualitative and quantitative phenomena (see Figure 2).

We are currently analysing, for the first time, arbitrary angle effects in combination with cubic-quintic interfaces [17,18].

Generalized interfaces model

It is also possible to construct a scalar model to describe the scattering of spatial solitons incident on the boundary between dissimilar nonlinear media. We consider a TE-polarized time-harmonic scalar electric field $E(x, z, t) = E(x, z) \exp(-i\omega t) + \text{c.c.}$, where x and z are the spatial coordinates, t is the time coordinate, and ω is the angular frequency. If the spatial part of the field varies slowly (in the transverse direction) on the scale of the free-space optical wavelength λ, then $E(x, z)$ satisfies a Helmholtz equation on each side of the boundary:

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}\right) E(x, z) + \frac{\omega^2}{c^2} n^2(E) E(x, z) = 0, \quad (1)$$

where c is the vacuum speed of light. The total refractive index n is routinely taken to be the sum of two terms so that $n_j^2 = n_{0j}^2 + \delta n^2_{\mathrm{NL}j}(E)$, where n_{0j} is the linear index of medium j = 1, 2 (which labels the two domains see Figure 1) at frequency ω, and $\delta n^2_{\mathrm{NL}j}(E)$ the small field-dependent correction on each side of the interface. In contrast to all our previous analyses, the $\delta n^2_{\mathrm{NL}j}(E)$ functions may be have *different* intensity dependences. The carrier wave component of E can be factored out according to

$E(x,z) = E_0 u(x,z)\exp(ik_1 z)$, so that z and x are the longitudinal and transverse coordinates, respectively, E_0 scales the field amplitude, $k_1 = (\omega/c)n_{01}$, and $u(x,z)$ is the dimensionless envelope, [equally, one could have factored out the complex-exponential factor $\exp(ik_2 z)$]. After substitution into Eq. (1), it can be shown that u satisfies the inhomogeneous equation,

$$\kappa\frac{\partial^2 u}{\partial\zeta^2} + i\frac{\partial u}{\partial\zeta} + \frac{1}{2}\frac{\partial^2 u}{\partial\xi^2} + f_1\left(|u|^2\right)u = \left[\frac{\Delta}{2\kappa} + f_1\left(|u|^2\right) - f_2\left(|u|^2\right)\right]H(\xi,\zeta)u. \quad (2)$$

Here $\zeta = z/L_{D1}$ and $\xi = 2^{1/2}x/w_0$, where $L_{D1} = k_1 w_0^2/2$ and w_0 are the diffraction length and waist of a reference Gaussian beam. Mismatches in the linear refractive index are described by $\Delta \equiv 1 - n_{02}^2/n_{01}^2$, and the nonparaxial parameter $\kappa = 1/(k_1 w_0)^2 = \varepsilon^2/4\pi^2 n_{01}^2$ quantifies the (inverse) beam width. The validity of Helmholtz modelling requires $\varepsilon \equiv \lambda/w_0 \ll O(1)$, so that beam waists are much larger than the free-space light wavelength. Hence, $\kappa \ll O(1)$ is always taken to be a small parameter throughout. The Heaviside unit function $H(\xi,\zeta)$ is defined so that H = 0 (+1) in domain 1 (2). The functions $f_1(|u|^2)$ and $f_2(|u|^2)$ describe the normalized nonlinear response on either side of the interface.

Snell's law: theory and computation

Generalized Snell's law

By exploiting quadrature techniques that we established in previous publications [19], Eq. (2) can be integrated exactly and solitons obtained (in principle) on both sides of the material boundary. These solutions provide a nonlinear basis for deriving a generalized Snell's law:

$$\gamma n_{01}\cos\theta_i = n_{02}\cos\theta_t. \quad (3)$$

Here, γ is a function that depends upon κ (finite transverse effects), the linear mismatch Δ, and the two beam phase-shift parameters (which explicitly incorporates nonlinearity mismatches). Crucially, all our earlier analyses are recovered from this universal γ.

Simulations of soliton scattering

The universal Snell's law embodied in Eq. (3) has been verified by direct numerical integration of Eq. (2) [20] (see Figure 3). A variety of technologically important material combinations has been considered, including dissimilar power-law materials [$\delta n_{\mathrm{NL1}}^2(I) = \alpha_1 I^{p_1}$ and $\delta n_{\mathrm{NL2}}^2(I) = \alpha_2 I^{p_2}$, where $p_1 \neq p_2$] and Kerr/cubic-quintic materials [$\delta n_{\mathrm{NL1}}^2(I) = \alpha_1 I$ and $\delta n_{\mathrm{NL2}}^2(I) = \alpha_2 I + v_2 I^2$]. In both cases, we find very good theory-numerics agree-ment across a wide range of parameter regimes.

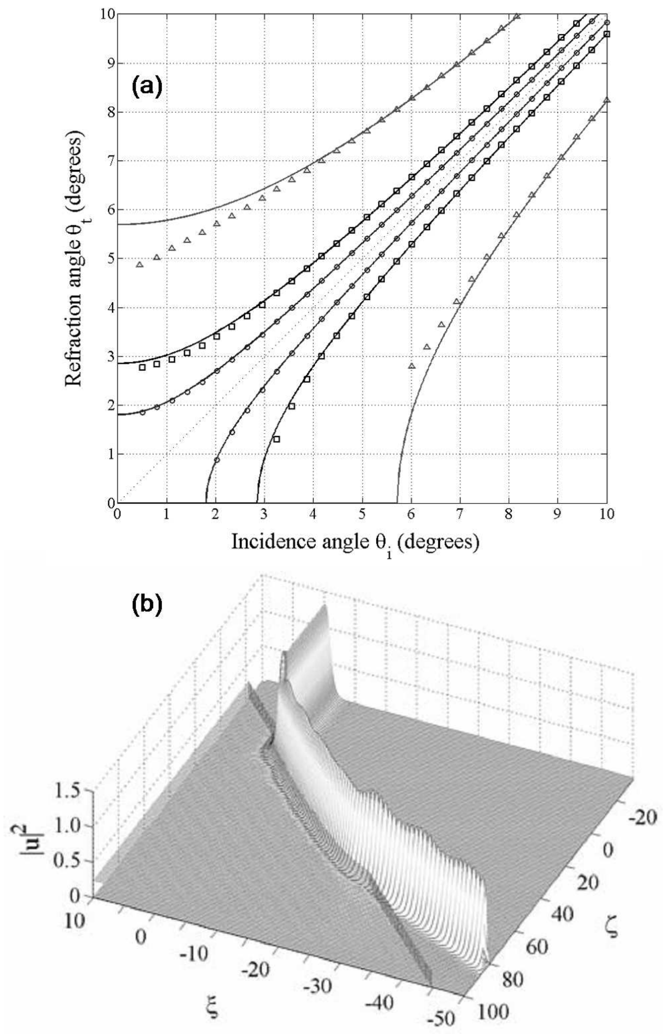

Figure 3: (a) Extensive computations have tested the validity of Snell's law (3), here for Kerr/cubic-quintic interface scenarios. (b) Goos-Hänchen shifts can occur when incidence angles are close to the critical angle.

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