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Spontaneous Spatial Fractals: Universal Contexts and Applications

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Reaction-diffusion systems can exhibit a Turing instability in which homogeneous states develop large-amplitude emergent patterns. These patterns are typically characterized by a *single* dominant length scale that corresponds to a single minimum in a modulational instability threshold curve. However, several nonlinear systems possess a hierarchy of local Turing minima. It was proposed [1] that this hierarchy could signature the spontaneous emergence of spatial fractal patterns (with detail spanning *many* decades of scale). The proposal was tested, and confirmed true [1], in an optical context involving a Kerr slice and a feedback mirror [2]. Firstly, we present the first evidence of spontaneous spatial fractals in ring cavities, and for a range of nonlinear materials [3]. This classic system is modelled in the thin-slice limit, with a pump detuning parameter determining the level of dispersion. Periodic pumping and losses are introduced via the conventional ring-cavity boundary condition, and a spatial filter allows the control of complex pattern formation. Characteristics of the resulting fractal patterns depend upon system parameters (such as diffusion length, pump intensity, and mirror reflectivity). One key result is that spatial fractals can emerge even in purely-absorptive regimes – see Fig. 1(a).

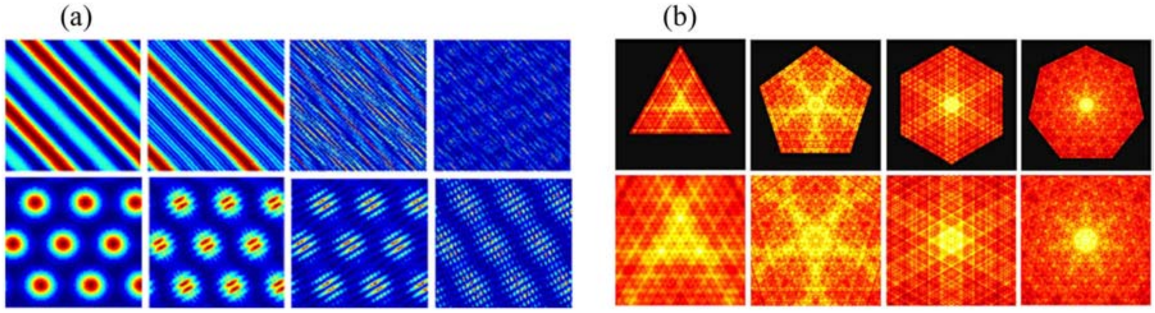

Fig. 1 (a) Spontaneous transformation of classic Turing patterns (top – stripes; bottom – hexagons) toward fractals in nonlinear ring cavities. (b) Lowest-loss eigenmode patterns for kaleidoscope lasers with N_{eq} = 30 and M = 1.5.

Secondly, we summarize examination of further optical fractal generating contexts. Generalizations involving *nonlinear* systems include: (i) Fabry-Perot cavities; (ii) the interaction of counter-propagating beams [4]; and (iii) narrow-beam nonparaxial considerations – permitting wavelength-scale optical structure over finite-medium propagation distances and thus investigation of bulk media geometries (i.e. beyond the thin-slice limit) [5]. Our earlier work on *linear* optical fractal generation, in which unstable cavity lasers were found to possess fractal eigenmodes [6], has also been extended. We outline the first detailed analysis involving arbitrary equivalent Fresnel number N_{eq} and magnification M. Previous analyses were restricted to regimes where either $N_{eq} = O(1)$ (where conventional ABCD matrix modelling can be deployed) or $N_{eq} >> O(1)$ (where asymptotic approximations apply [7]). Fractal eigenmode characteristics can now be surveyed in the important intermediate regime – corresponding to real-world systems with significant and exploitable fractality. We focus on cases of non-trivial transverse geometries ('kaleidoscope lasers') in which the defining aperture has the shape of a regular polygon; this has a profound impact on the structure of the modes patterns – see Fig. 1(b). One key advantage of our modelling it that a single calculation allows one to access entire families of modes; another is that any particular mode may be computed to any desired accuracy.

Both linear and nonlinear fractal generators hold enormous potential for novel laser designs for a wide range of applications (e.g., more efficient probing, scanning and ablation experiments). The huge spatial bandwidths associated with fractal sources also have potential exploitation within distinct novel information contexts. We conclude with a brief account of such potential new application technologies.

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