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# Turing instability: a universal route to spontaneous spatial fractals

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## Summary

We present the *first predictions* of spontaneous spatial fractal patterns in *nonlinear ring cavities*. New analyses reveal multi-Turing spectra characteristic of susceptibility for spontaneous fractals. Extensive computer simulations confirm theoretical predictions.

Turing instability is the tendency of the uniform states of a system to become spontaneously patterned in the presence of any small fluctuation [1]. Archetypal Turing-instability patterns include hexagons, squares, stripes, and rings. These simple structures are universal in Nature, and characterized by a *single* dominant scale-length. Recently [2], we proposed that a *multi-Turing instability* may result in another type of universal pattern: fractals. Fractals possess proportional levels of detail spanning decades of scale-length, and are thus inherently *scaleless*. This prediction was confirmed in analysis of a classic photonic system (the diffusive Kerr slice with a single feedback mirror). The growth of such multi-scale patterns is entirely due to intrinsic nonlinear dynamics. They are thus physically distinct from fractal mode patterns of unstable-cavity lasers [3], and optical fractals that rely on system changes for introducing each scale-length [4].

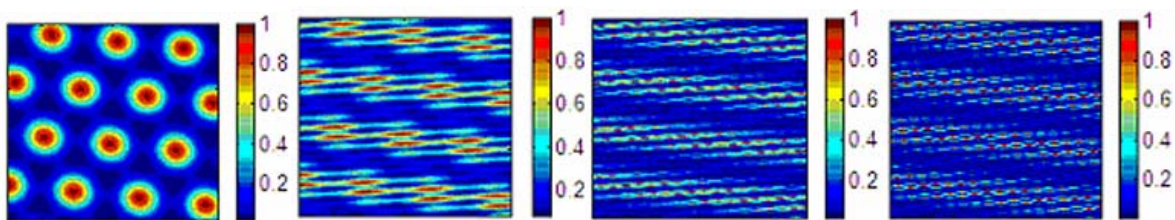


Fig1. Transverse intensity distributions. Transition from a simple Turing-instability pattern (hexagon) to a fractal mode in a pure-absorptive nonlinear cavity. Self-similar structure persists down to scale-lengths of the order of the optical wavelength.

Here, we present the *first predictions* of spontaneous spatial fractal patterns in *nonlinear ring cavities*. This includes the first reported spatial fractals arising from *purely-absorptive nonlinearity*. New analyses reveal multi-Turing spectra characteristic of susceptibility for spontaneous fractals. Computer simulations

consider both one and two transverse dimensions (see Fig. 1), and quantify the fractal properties of the generated patterns. A range of results will be reported, including the proposal of a new kind of “fractal soliton”.

## References

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