



University of  
**Salford**  
MANCHESTER

## Helmholtz-Manakov solitons

Christian, JM, McDonald, GS and Chamorro-Posada, P

<b>Title</b>	Helmholtz-Manakov solitons
<b>Authors</b>	Christian, JM, McDonald, GS and Chamorro-Posada, P
<b>Publication title</b>	
<b>Publisher</b>	
<b>Type</b>	Conference or Workshop Item
<b>USIR URL</b>	This version is available at: <a href="http://usir.salford.ac.uk/id/eprint/18430/">http://usir.salford.ac.uk/id/eprint/18430/</a>
<b>Published Date</b>	2005

USIR is a digital collection of the research output of the University of Salford. Where copyright permits, full text material held in the repository is made freely available online and can be read, downloaded and copied for non-commercial private study or research purposes. Please check the manuscript for any further copyright restrictions.

For more information, including our policy and submission procedure, please contact the Repository Team at: [library-research@salford.ac.uk](mailto:library-research@salford.ac.uk).

# Helmholtz-Manakov Solitons

*J. M. Christian and G. S. McDonald,  
Joule Physics Laboratory, School of Computing, Science and Engineering, Institute of Materials Research,  
University of Salford, Salford M5 4WT, U.K.*

*P. Chamorro-Posada,  
Departamento de Teoría de la Señal y Comunicaciones e Ingeniería Telemática, Universidad de Valladolid,  
ETSI Telecomunicación, Campus Miguel Delibes s/n, 47011 Valladolid, Spain.*

## 1. Introduction

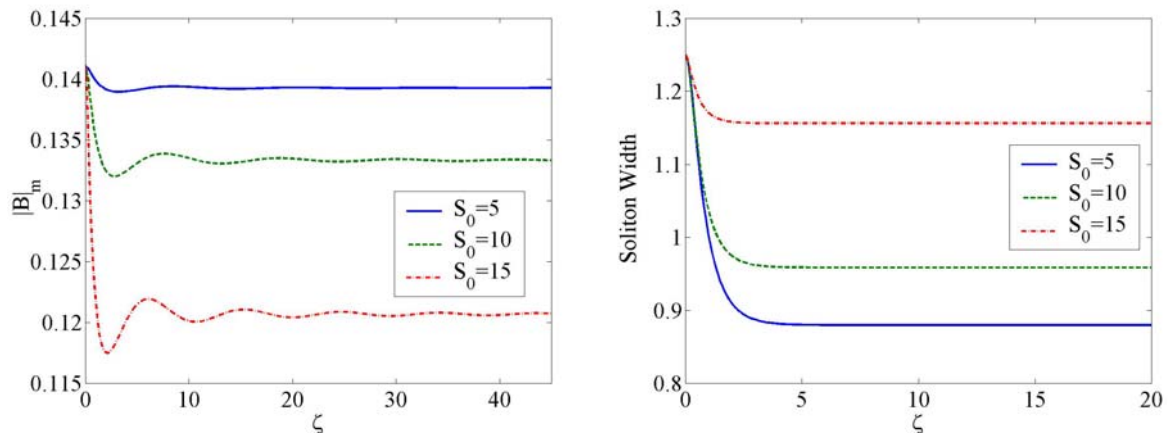
The propagation of a spatial vector soliton beam in a Kerr planar waveguide is typically described by the Manakov equation [1]. However, the assumption of beam paraxiality breaks the rotational symmetry of the wave propagation problem. Manakov-based descriptions are, for example, incapable of describing physical effects associated with off-axis propagation at non-trivial angles. We will report the *first* Helmholtz generalizations of the Manakov equation *and* its soliton solutions, along with a thorough investigation of the dynamical properties of the new solutions.

## 2. Helmholtz-Manakov Solitons

We introduce the Helmholtz-Manakov (H-M) equation as a vector generalization of the scalar Non-Linear Helmholtz (NLH) equation [2], whereby the guided electric field has two transverse orthogonal components. Exact analytical soliton solutions of the H-M equation will be derived for *both* focusing and defocusing media; the classic Manakov solitons are a subset of these new results. H-M solitons are found to exhibit non-trivial features that are absent from the paraxial-based descriptions (these new features will be shown to influence propagation characteristics).

## 3. Stability as Robust Attractors

Well-tested numerical perturbative techniques will be employed to demonstrate the role of H-M solitons as robust attractors (in a non-linear dynamical sense). Rich dynamical behaviour will be summarised, including evolution characteristics associated with both fixed-point (see Fig. 1) and limit-cycle attractors.



**Fig. 1.** Reshaping simulations using exact Manakov (paraxial) solitons as initial conditions for the H-M equation. Simulations correspond to the reshaping of (left) dark-bright, and (right) dark-dark solitons in a defocusing medium - both are classed as *stable fixed points*.

## 4. References

- [1] S.V. Manakov, "On the theory of two-dimensional stationary self-focusing of electromagnetic waves," *Sov. Phys. JETP* **38**, 248 (1974).
- [2] P. Chamorro-Posada, G.S. McDonald and G.H.C. New, "Exact soliton solutions of the nonlinear Helmholtz equation: communication," *J. Opt. Soc. Am. B* **19**, 1216 (2002).