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Spatiotemporal pulse propagation: connections to special relativity theory

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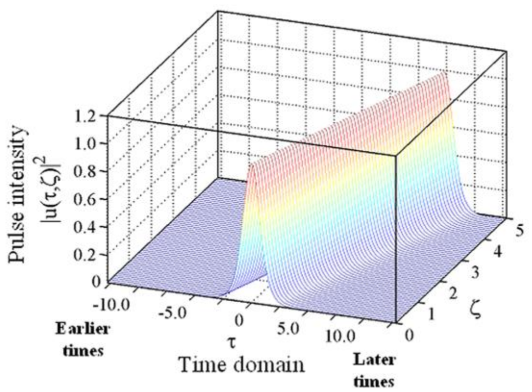
We propose a more complete model for describing the evolution of scalar optical pulses in nonlinear waveguides. The electromagnetic wave envelope u satisfies a dimensionless spatiotemporal governing equation that is of the form

$$\kappa \frac{\partial^2 u}{\partial \zeta^2} + i\left(\frac{\partial u}{\partial \zeta} + \alpha \frac{\partial u}{\partial \tau}\right) + \frac{s}{2}\frac{\partial^2 u}{\partial \tau^2} + f\left(|u|^2\right)u = 0\,. \tag{1}$$

The space/time coordinates are ζ/τ, while (α, s, κ) are system parameters, and the function $f(|u|^2)$ describes the light-induced properties of the host material. With few exceptions [1,2] throughout nearly 50 years of literature, the first term in Eq. (1) has been routinely neglected. By retaining this otherwise-omitted term, we have found that pulse propagation problems are most transparently described with a frame-of-reference formulation. We have developed the mathematical and computational tools necessary for the full analysis of Eq. (1) and its solutions. Intriguing parallels with Einstein's special theory of relativity also emerge naturally (e.g., the velocity combination rule for pulses is akin to that for particles in relativistic kinematics) [3]. Exact bright and dark solitons (see figure 1) have been derived for a range of classic nonlinearities, and their robustness has been tested through exhaustive numerical simulations.



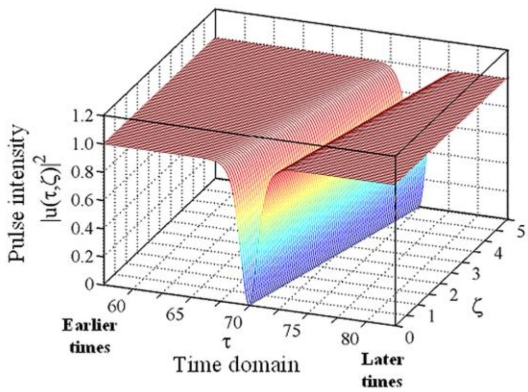


Figure 1. Simulations showing the propagation (in longitudinal space ζ) of bright (left) and dark (right) soliton pulses.

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