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# Spontaneous spatial fractal patterns: Towards nonparaxial nonlinear ring cavities

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# Spontaneous spatial fractal patterns: towards nonparaxial nonlinear ring cavities

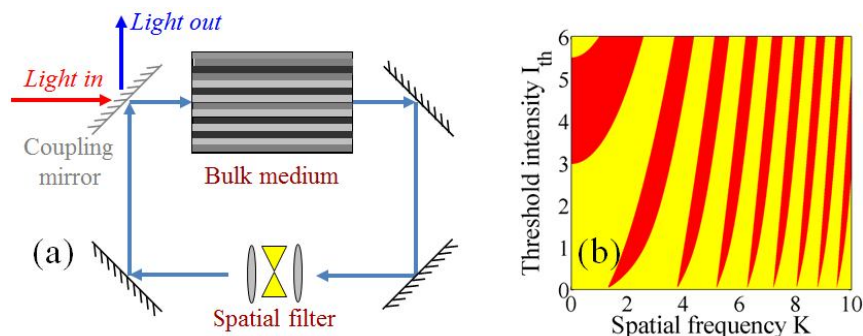
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Spontaneous pattern formation in optical ring cavities containing a nonlinear (e.g., Kerr-type) material [see Fig. 1(a)] has been studied extensively for the past three decades. A notable trend in the literature over recent years has been a shift away from the (bulk) *cavity + boundary condition* models of McLaughlin *et al.* [1] toward the (longitudinally-averaged) *mean field* descriptions of Lugiato and Lefever [2]. While this latter approach is analytically more tractable, it does not yield Turing instability spectra with the multiple-minimum characteristic proposed as necessary for spontaneous fractal (i.e., multi-scale) pattern formation [3,4] [see Fig. 1(b)]. Here, we revisit the approach taken by McLaughlin *et al.* [1] but instead allow for the full generality of Helmholtz (broadband) as opposed to paraxial (narrowband) diffraction. Such a restoration of spatial symmetry (whereby diffraction occurs in both transverse and longitudinal dimensions) allows a much more reliable description of small-scale spatial structure in the circulating cavity field. Our analysis also goes some way toward addressing the issue of fractal pattern formation in systems with finite light-medium interaction lengths [3,4]. Linear analysis has predicted the threshold condition for spatial pattern emergence, and simulations have begun to investigate these new instabilities.



**Figure 1.** (a) Schematic diagram of an optical ring cavity filled with a bulk nonlinear (e.g., Kerr) material so that light-medium interactions can take place over a finite propagation length. (b) Typical multi-Turing threshold instability curves for the thin-slice cavity system [4]. The introduction of finite medium length changes some key characteristics of the spectrum.

## References

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