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1 **A fast inverse dynamics model of walking for use in optimisation studies**

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3

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22

23 **Word count: 3810**

24 **Abstract**

25 Computer simulation of human gait, based on measured motion data, is a well-established  
26 technique in biomechanics. However, optimisation studies requiring many iterative gait cycle  
27 simulations have not yet found widespread application because of their high computational cost.  
28 Therefore, a computationally efficient inverse dynamics model of 3D human gait has been  
29 designed and compared with an equivalent model, created using a commercial multi-body  
30 dynamics package. The fast inverse dynamics model described in this paper led to an eight fold  
31 increase in execution speed. Sufficient detail is provided to allow readers to implement the  
32 model themselves.

33

34 **Keywords:** Fast inverse dynamics; Gait Simulation; Prediction

35

36 **Notation**

37	$a_i$	Acceleration of segment $i$ 's origin
38	$a_{Ci}$	Acceleration of segment $i$ 's centre-of-mass
39	$F_i$	Force applied to segment $i$ at its proximal end (origin)
40	$F_{grrk}$	Component $k$ of the (right) ground reaction force ( $k=X, Y$ or $Z$ )
41	$\sum F_{gr}$	The total ground reaction force
42	$\sum F_{grk}$	Component $k$ of the total ground reaction force ( $k=X, Y$ or $Z$ )
43	$I_i$	Moment of inertia of segment $i$
44	$m_i$	Mass of segment $i$
45	$MF$	Moment as a result of a distal force
46	$MFs$	Sum of the moments resulting from distal forces
47	$MPF$	Moment as a result of the proximal force
48	$MPFs$	Sum of the moments resulting from proximal forces
49	$n_i$	Moment applied to segment $i$ at its proximal end (origin)
50	$n_{grrk}$	Component $k$ of the (right) ground reaction moment ( $k=X, Y$ or $Z$ )
51	$\sum n_{gr}$	The total ground reaction moment
52	$\sum n_{grk}$	Component $k$ of the total ground reaction moment ( $k=X, Y$ or $Z$ )
53	$N_i$	Euler's equation for segment $i$

54	${}^j P_i$	Position of segment $i$ origin in segment $j$ 's frame
55	${}^j P_{Ci}$	Position of segment $i$ centre-of-mass in segment $j$ 's frame
56	$R_j = {}_d^{pr} R$	Joint rotation matrix (maps vectors from distal to proximal segment frames)
57	$\alpha$	Joint rotation about X axis
58	$\beta$	Joint rotation about Y axis
59	$\gamma$	Joint rotation about Z axis
60	$\omega_j$	Angular velocity of joint $j$
61	$\dot{\omega}_j$	Angular acceleration of joint $j$
62	${}^k \omega_{i/j}$	Angular velocity of segment $i$ relative to segment $j$ , written in segment $k$ 's frame
63	${}^k \dot{\omega}_{i/j}$	Angular acceleration of segment $i$ relative to $j$ , written in $k$ 's frame

64

65 **Subscripts and superscripts**

66 Note that leading superscripts before a vector indicate the frame in which that vector is written.

67  $d$  Distal segment

68  $f$  Foot

69  $gr$  Ground

70  $h$  Head

71  $l$  Left

72	<i>larm</i>	Lower arm
73	<i>p</i>	Pelvis
74	<i>pr</i>	Proximal segment
75	<i>r</i>	Right
76	<i>sh</i>	Shank
77	<i>st</i>	Stance
78	<i>sw</i>	Swing
79	<i>t</i>	Torso
80	<i>th</i>	Thigh
81	<i>uarm</i>	Upper arm

82

83

## 84 **Introduction**

85 Computer simulation of human gait (walking or running), based on measured motion data, is a  
86 well-established research technique for estimating the forces acting on the body's joints and  
87 muscles. Conversely, optimisation of gait kinematics (known as gait prediction) is a relatively  
88 new and challenging area of research, which has not yet found widespread application because of  
89 its high computational cost (Anderson & Pandy, 2001; Xiang et al., 2010).

90 Typically, gait prediction is achieved by embedding a forward or inverse dynamics model of  
91 human locomotion within an optimisation framework (henceforth referred to as the optimiser).

92 The optimiser is used to represent the coordination of the body's motions by the central nervous  
93 system (CNS) based on the assumption that we have evolved to optimise our gait in order, for

94 example, to minimise energy consumption, maximise speed or minimise pain, depending on the  
95 situation. The forward dynamics approach to gait prediction is very computationally demanding,

96 with one of the best known examples of this approach requiring 10,000 hours of CPU time to  
97 satisfy the terminal conditions (Anderson & Pandy, 2001). Although this well-known study is

98 now rather dated, based on a review of internet sources we estimate that there has been a 10 to 20  
99 fold increase in computational power over the intervening period. As this type of information is

100 very hard to find and to verify, we conservatively assume a 20 fold increase in computation

101 power. This means that the execution time quoted by Anderson & Pandy would reduce to 500

102 hours which is still very excessive. For this reason, in our previous work, we have chosen to

103 focus on the inverse dynamics approach to gait prediction (Ren, et al., 2007).

104 In gait prediction, the joint motions can be represented in many ways and well-known curve

105 fitting functions are often chosen, such as polynomials, splines, or a combination of

106 discretisation and interpolation. However, these do not take account of the special features of

107 human walking. Firstly it is periodic and, hence, using functions that explicitly enforce  
108 periodicity will avoid having to include this as an optimisation constraint. Secondly, the  
109 fundamental frequency of human walking is of the order of 1Hz and over 99% of the power  
110 content is below 6Hz (Winter, 2009). As a result, 5<sup>th</sup> order Fourier series are likely to adequately  
111 represent walking, including enforcing periodicity, which means that each joint motion trajectory  
112 can be represented by just 11 optimisation parameters. For these reasons, several previous  
113 authors have chosen to represent the joint motions using Fourier series (Koopman et al., 1995;  
114 Ren et al., 2007). In the case of Ren et al., 2007, this allowed the prediction of a realistic gait  
115 even when the initial Fourier coefficients represented standing not walking.

116 Most previous authors have limited their gait prediction studies by using planar models, because  
117 of the complexity and corresponding computational demands of 3D inverse dynamics models. Of  
118 those that adopted 3D models, the following limitations can be identified. Koopman et al., 1995,  
119 only predicted a small number of unmeasured joint motions. Tlalolini et al., 2010, did not model  
120 finite double support periods. Kim et al, 2008, avoided solving the full inverse dynamics  
121 problem by adopting an approach that constrains the centre of pressure (COP) to be within the  
122 base of support (BOS), thus ensuring that “dynamic equilibrium” is satisfied. However, because  
123 the joint moments are not calculated, many optimisation objectives cannot be adopted (e.g.  
124 minimisation of mechanical work).

125 So it is clear that there still remains a challenge to establish a fast inverse dynamics model of 3D  
126 human gait that can be used in optimisation based studies. In this paper we describe the design of  
127 a bespoke human gait model where computational efficiency has been achieved by adopting a  
128 dedicated model structure and calculation sequence that is optimised for human gait, thus  
129 avoiding the overheads of general simulation packages that must cater for any model topology.



130 We have verified this model against an equivalent model, created using a commercial multi-body  
131 dynamics package, and compared the execution times of the two models to demonstrate the  
132 computational efficiency of our model. Sufficient detail is provided to allow readers to  
133 implement the model themselves.

## 134 **Methods**

135 Although inverse dynamics is less computationally demanding than forward dynamics, in the  
136 case of a 3D skeletal model, it is still very important to adopt an efficient solution method. The  
137 chain like structure of the model lends itself to a bespoke implementation of the iterative  
138 Newton-Euler method, which is well recognised as being particularly efficient (Craig, 2004;  
139 Featherstone, 2008; Angeles, 2014) and, therefore, we have adopted this solution approach for  
140 the inverse dynamics. This method has a computational complexity of  $O(n)$ , which means that  
141 the calculations required grow linearly with the number of degrees of freedom ( $n$ ). This  
142 compares very favourably to a computational complexity of  $O(n^4)$  for a non-iterative approach  
143 (i.e. the calculations required grow with  $n^4$ ).

144 For the reasons previously discussed, we have chosen to use Fourier series to represent the  
145 trajectories of the degrees-of-freedom driving the motions of the 3D skeletal model. This has two  
146 benefits, the first of which is that this leads to a relatively small set of optimisation variables (the  
147 Fourier coefficients), which reduces computation times. Secondly, Fourier series automatically  
148 constrain the motions to be cyclic and continuous.

149

### 150 **1. The multi-body model**

151 To maximise computational efficiency whilst maintaining reasonable accuracy in the description  
152 of gait kinematics, a compromise was adopted with regard to the number of rigid segments and  
153 degrees-of-freedom (DOF). For example, the hands were treated as part of the forearm segments.

154 Referring to *Figure 1*, the multi-body model has fourteen rigid segments including: the ground, 2  
155 feet, 2 shanks, 2 thighs, pelvis, torso, head, 2 upper-arms, and 2 forearms. Each segment has an  
156 attached coordinate frame. For the simple line segments (representing the longitudinal axis of the  
157 bone), the origin is located at the proximal end and the Z-axis is determined by the unit vector  
158 directed from the distal end to the proximal end. For the torso, the segment origin is located at  
159 the lumbosacral joint and the Z-axis is determined by the unit vector directed from the  
160 lumbosacral joint to the neck joint. For the pelvis, the segment origin is located at the  
161 lumbosacral joint and the Z-axis is determined by the unit vector directed from the mid-point  
162 between the two hip joints to the lumbosacral joint. For all segments, the Y-axis points forward  
163 when the segment is vertical (i.e. its Z-axis is vertical) and is not rotated about its Z-axis. For all  
164 segments, the X axis points to the right when the segment is not rotated about its other axes.

165 The model has 25 DOF including: a 1-DOF rollover joint between the stance foot and the  
166 ground (the 3 ankle coordinates are functions of the foot-ground angle); 2-DOF ankle joints  
167 (dorsiflexion and eversion); 1-DOF knee joints; 3-DOF hip joints; a 3-DOF lumbosacral joint; a  
168 3-DOF neck joint; 2-DOF shoulder joints (flexion and abduction); and 1-DOF elbow joints.

169

## 170 **2. Joint motions**

171 The joint DOFs are represented by X-Y-Z sequence Euler angles. Each rotation is performed  
172 about an axis of the moving system, which is the distal (*d*) segment coordinate frame, starting

173 from an orientation aligned with the reference system, which is the proximal (*pr*) segment  
 174 coordinate frame. In this context, the pelvis is the most proximal segment and the lower arms,  
 175 head and feet are the most distal segments. Therefore, each joint's rotation matrix  $R_j$  can be  
 176 calculated from the following expression (Craig, 2004):

177  $R_j = {}^{pr}R_{XYZ}(\alpha, \beta, \gamma) = R_X(\alpha)R_Y(\beta)R_Z(\gamma)$

178 where

$$R_X(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}; \quad R_Y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}; \quad R_Z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

179

180 Then, given the rotation matrix  $R_j = {}^{pr}R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ , the joint angular velocity can be

181 calculated as follows (Craig, 2004):

182  $\omega_j = {}^{pr}\omega_{d/pr} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$

183 where  $\omega_j$  is the angular velocity of the distal (*d*) segment relative to the proximal (*pr*) segment,  
 184 expressed in the proximal segment's coordinate frame. The three components of  $\omega_j$  are given  
 185 by:

$$\omega_x = \dot{r}_{31}r_{21} + \dot{r}_{32}r_{22} + \dot{r}_{33}r_{23}$$

186  $\omega_y = \dot{r}_{11}r_{31} + \dot{r}_{12}r_{32} + \dot{r}_{13}r_{33}$

$$\omega_z = \dot{r}_{21}r_{11} + \dot{r}_{22}r_{12} + \dot{r}_{23}r_{13}$$

187 Finally, the angular acceleration vector is simply the derivative of the angular velocity vector.

188 Using the above, the corresponding expressions for each type of anatomical joint can be derived

189 and these are given in the appendix. These dedicated expressions increase computational

190 efficiency in comparison to applying the general analysis described above as is necessary in

191 general simulation packages that must cater for any model topology.

192

### 193 **3. Iterative calculation of segment kinematics**

194 The iterative Newton-Euler method has been used for the inverse dynamics calculations. The

195 first stage of this method is to calculate the segment kinematics by iteratively working outwards

196 from one segment to the next, beginning at the stationary reference segment (the ground) and

197 ending at the most distal segments (the swing foot, head and lower arms). The motion of the next

198 segment is calculated from the motion of the previous segment (already calculated) and the

199 motions of the joint DOFs connecting the two segments.

200 The exact form of the iterative Newton-Euler equations depends on whether the calculations are

201 being performed in a distal to proximal direction or vice versa. Therefore, the following sub-

202 sections deal with the different cases involved in modelling the gait cycle. In most cases (unless

203 for emphasis) the leading superscript is omitted when there is a single subscript and the

204 superscript would be the same as the subscript (e.g.  ${}^d\omega_d \equiv \omega_d$ ).

### 205 **3.1 Stance Leg**

206 For the stance leg the direction of calculation is from the ground (most distal segment) towards  
207 the pelvis (most proximal segment). Therefore, referring to *Figure 2*, the general form of the  
208 iterative calculations is as follows.

209 Segment angular velocity:

$$210 \quad {}^{pr}\dot{\omega}_{pr} = {}^dR\dot{\omega}_d + {}^{pr}\omega_{pr/d} = {}^dR\dot{\omega}_d - \dot{\omega}_j$$

211 Segment angular acceleration:

$$\begin{aligned} 212 \quad {}^{pr}\dot{\omega}_{pr} &= {}^dR\dot{\omega}_d + {}^dR\dot{\omega}_d \times {}^{pr}\omega_{pr/d} + {}^{pr}\dot{\omega}_{pr/d} \\ \rightarrow {}^{pr}\dot{\omega}_{pr} &= {}^dR\dot{\omega}_d - {}^dR\dot{\omega}_d \times \omega_j - \dot{\omega}_j \end{aligned} \quad 213$$

214 Acceleration of segment frame origin:

$$215 \quad a_{pr} = {}^dRa_d - (\dot{\omega}_{pr} \times {}^{pr}P_d) - (\omega_{pr} \times \omega_{pr} \times {}^{pr}P_d)$$

216 Acceleration of segment mass centre:

$$217 \quad a_{Cpr} = a_{pr} + (\dot{\omega}_{pr} \times {}^{pr}P_{Cpr}) + (\omega_{pr} \times \omega_{pr} \times {}^{pr}P_{Cpr})$$

218 Using the above, the corresponding equations for each segment of the stance leg can be derived.

219

### 220 **3.2 Swing leg and upper body**

221 For the swing leg and upper body the direction of calculation is from the most proximal segment  
 222 (pelvis) to the most distal segment (foot, head, or lower arm). Therefore, the general form of the  
 223 iterative calculations is as follows.

224 Segment angular velocity:

$$225 \quad {}^d\omega_d = {}^{pr}R\omega_{pr} + {}^d\omega_{d/pr} = {}^{pr}R(\omega_{pr} + \omega_j)$$

226 Segment angular acceleration:

$$227 \quad \begin{aligned} {}^d\dot{\omega}_d &= {}^{pr}R(\dot{\omega}_{pr} + \omega_{pr} \times {}^{pr}\omega_{d/pr} + {}^{pr}\dot{\omega}_{d/pr}) \\ \rightarrow {}^d\dot{\omega}_d &= {}^{pr}R(\dot{\omega}_{pr} + \omega_{pr} \times \omega_j + \dot{\omega}_j) \end{aligned}$$

228 Acceleration of segment frame origin:

$$229 \quad a_d = {}^{pr}R\left[ a_{pr} + \left( \dot{\omega}_{pr} \times {}^{pr}P_d \right) + \left( \omega_{pr} \times \omega_{pr} \times {}^{pr}P_d \right) \right]$$

230 Acceleration of segment mass centre:

$$231 \quad a_{Cd} = a_d + (\dot{\omega}_d \times {}^dP_{Cd}) + (\omega_d \times \omega_d \times {}^dP_{Cd})$$

232 Using the above, the corresponding equations for each segment of the swing leg and upper body  
 233 can be derived.

### 234 **3.3 Double stance**

235 To avoid kinematic redundancy during double stance, only one foot is considered to be  
 236 connected to the ground by the roll-over joint (labelled the stance foot). Therefore, in  
 237 double stance, the sequence of kinematic calculations is the same as in single stance. From

238 right heel-strike (RHS) to left heel-strike (LHS), the right foot is the stance foot. From LHS  
239 to RHS, the left foot is the stance foot.

240

#### 241 **4. Equations of motion**

242 Segment forces and moments are defined to be the forces and moments acting on a segment at its  
243 proximal joint. Therefore, from Newton's third law, the forces and moments acting at distal  
244 joints (i.e. belonging to other segments) must be multiplied by  $-1$ ; hence the minus signs shown  
245 in the free-body diagrams (*Figure 3*). Note that, in these diagrams and the accompanying  
246 equations of motion, all vectors are assumed to be written in the global frame. Therefore leading  
247 superscripts are not shown except for some position vectors. There are three different types of  
248 segments in the skeletal model as follows.

##### 249 **4.1 Standard line segment**

250 Most segments within the human body can be modelled as a simple line segment connecting a  
251 proximal and a distal joint (*Figure 3a*).

252 From Newton's second law:

253  $F_{pr} - F_d = m_{pr} a_{Cpr}$

254 From Euler's equation:

255  $n_{pr} - n_d + (-P_{Cpr} \times F_{pr}) + [({}^{pr}P_d - P_{Cpr}) \times -F_d] = [I_{pr} \dot{\omega}_{pr} + (\omega_{pr} \times I_{pr} \omega_{pr})]$

256 **4.2 Torso segment**

257 As illustrated in *Figure 3b*, the torso segment has been modelled as a quadrilateral segment with  
 258 corners at the four joints connecting it to the pelvis, head and two arms. The lumbo-sacral joint is  
 259 the proximal joint, the other joints being distal joints.

260 From Newton's second law:

261  $F_t - F_{head} - F_{uarmr} - F_{uarml} = m_t a_{ct}$

262 From Euler's equation:

263  $n_t - n_{head} - n_{uarml} - n_{uarmr} + (-P_{Ct} \times F_t) + [({}^tP_{head} - P_{Ct}) \times -F_{head}] +$   
 $[({}^tP_{uarml} - P_{Ct}) \times -F_{uarml}] + [({}^tP_{uarmr} - P_{Ct}) \times -F_{uarmr}] = [I_t \dot{\omega}_t + (\omega_t \times I_t \omega_t)]$

264 **4.3 Pelvis segment**

265 As illustrated in *Figure 3c*, the Pelvis segment has been modelled as a triangular segment with  
 266 corners at the three joints connecting it to the torso and two legs. Being the most proximal  
 267 segment in the body, all of the joints are distal joints and hence all of the joint forces and  
 268 moments have minus signs associated with them.

269 From Newton's second law:



270  $-F_t - F_{th_l} - F_{th_r} = (m_p a_{cp})$

271 From Euler's equation:

272 
$$-n_t - n_{th_r} - n_{th_l} + (-P_{Cp} \times -F_t) + \left[ ({}^p P_{th_l} - P_{Cp}) \times -F_{th_l} \right] +$$

$$\left[ ({}^p P_{th_r} - P_{Cp}) \times -F_{th_r} \right] = \left[ I_p \dot{\omega}_p + (\omega_p \times I_p \omega_p) \right]$$

273

274 **5. Iterative kinetics calculations for single stance**

275 The second stage of the iterative Newton-Euler method is to calculate the segment kinetics by  
 276 iteratively working inwards from one segment to the next, beginning at the most distal segments,  
 277 furthest from the stationary reference segment (the ground), and ending at the ground. Therefore,  
 278 in single stance, the calculations begin at the swinging foot, lower arms, and head. This sequence  
 279 of calculations is executed twice: first to calculate the joint forces; and then again to calculate the  
 280 joint moments (which depend on the already calculated forces). In the first sequence of  
 281 calculations, the force acting at the joint connecting the current segment to the next segment is  
 282 calculated from the other joint forces acting on the current segment (already calculated) and its  
 283 translational acceleration. In the second sequence of calculations, the moment acting at the joint  
 284 connecting the current segment to the next segment is calculated from the other joint moments  
 285 acting on the current segment (already calculated), all of the joint forces acting on the current  
 286 segment (already calculated), and its angular motion.

287 Based on this sequence of calculations and the equations of motion presented in the previous  
 288 sub-section, the following equations can be derived.

289 **5.1 Forces**

290 For the standard line segments found in the upper body and the swing leg, the calculation  
291 sequence is from distal segment to proximal segment, leading to the following general equation:

$$292 \quad F_{pr} = {}^{pr}_d R F_d + m_{pr} a_{Cpr}$$

293 Using the above, the corresponding equations for each line segment in the upper body and the  
294 swing leg can be derived. Then the torso force can be calculated as follows:

$$295 \quad F_t = {}^{uarm_r}_t R (F_{uarm_r}) + {}^{uarm_l}_t R (F_{uarm_l}) + {}^{head}_t R (F_{head}) + m_t a_{Ct}$$

296 And the stance thigh force ( $F_{th_{st}}$ ) can be calculated as follows:

$$297 \quad F_{th_{st}} = {}^{th_{st}}_p R [ - {}^{th_{sw}}_p R (F_{th_{sw}}) - {}^p_t R F_t - m_p a_{Cp} ]$$

298 Note that “*st*” refers to the stance leg and “*sw*” refers to the swing leg.

299 For the standard line segments found in the stance leg, the calculation sequence is from proximal  
300 segment to distal segment, leading to the following general equation:

$$301 \quad F_d = {}^{d}_{pr} R (F_{pr} - m_{pr} a_{Cpr})$$

302 Using the above, the corresponding equations for each line segment in the stance leg, and also  
303 for the ground, can be derived.

304 **5.2 Moments**

305 For the standard line segments found in the upper body and the swing leg, the calculation  
306 sequence is from distal segment to proximal segment, leading to the following general equations:

$$n_{pr} = N_{pr} + {}^{pr}Rn_d - MPF_{pr} - MF_d$$

$$307 \quad MF_d = \left( {}^{pr}P_d - P_{Cpr} \right) \times -{}^{pr}RF_d$$

$$MPF_{pr} = -P_{Cpr} \times F_{pr}$$

$$N_{pr} = I_{pr}\dot{\omega}_{pr} + (\omega_{pr} \times I_{pr}\omega_{pr})$$

308 Using the above, the corresponding equations for each line segment in the upper body and the

309 swing leg can be derived. Then the torso moment can be calculated as follows:

$$n_t = {}^{uarml}Rn_{uarml} + {}^{uarmr}Rn_{uarmr} + {}^{head}Rn_{head} + N_t - MPF_t - MF_{uarml} - MF_{uarmr} - MF_{head}$$

$$MPF_t = -P_{Ct} \times F_t$$

$$310 \quad MF_{uarml} = \left[ ({}^tP_{uarml} - P_{Ct}) \times -{}^{uarml}RF_{uarml} \right]$$

$$MF_{uarmr} = \left[ ({}^tP_{uarmr} - P_{Ct}) \times -{}^{uarmr}RF_{uarmr} \right]$$

$$MF_{head} = \left[ ({}^tP_{head} - P_{Ct}) \times -{}^{head}RF_{head} \right]$$

311 Then the stance thigh moment ( $n_{th_{st}}$ ) can be calculated as follows:

$$n_{th_{st}} = {}^{th_{st}}R \left( -{}^pRn_t - {}^{th_{sw}}Rn_{th_{sw}} - N_p + MPF_p + MF_{th_{sw}} + MF_{th_{st}} \right)$$

$$MPF_p = -P_{Cp} \times -{}^pRF_t$$

$$312 \quad MF_{th_{sw}} = \left[ ({}^pP_{th_{sw}} - P_{Cp}) \times -{}^{th_{sw}}RF_{th_{sw}} \right]$$

$$MF_{th_{st}} = \left[ ({}^pP_{th_{st}} - P_{Cp}) \times -{}^{th_{st}}RF_{th_{st}} \right]$$

313 Note that “*st*” refers to the stance leg and “*sw*” refers to the swing leg.

314 For the standard line segments found in the stance leg, the calculation sequence is from proximal

315 segment to distal segment, leading to the following general equations:

$$\begin{aligned}
n_d &= {}^d_{pr}R [n_{pr} - N_{pr} + MPF_{pr} + MF_d] \\
MPF_{pr} &= -P_{Cpr} \times F_{pr} \\
MF_d &= ({}^{pr}P_d - P_{Cpr}) \times -{}^{pr}_dRF_d
\end{aligned}$$

Using the above, the corresponding equations for each line segment in the stance leg, and also for the ground, can be derived.

## 6. Iterative kinetics calculations for double stance

One complete gait cycle includes two double stance phases: right double stance (following RHS) and left double stance (following LHS). In walking, the duration of each of these is approximately one tenth of the gait cycle. In the double stance phases, the sequence of kinetics calculations for the upper body is the same as it is in the single stance phases. However, the division of forces and moments between the two stance legs is an indeterminate problem. To resolve this problem, first the total ground reaction force and moment are calculated by applying the Newton-Euler equations to the lower limbs as a whole. Then, to share the total ground reaction force and moment between the two feet, smooth transition assumptions are applied. The details of this process are as follows.

### 6.1 Calculating the ground reaction forces

During double stance, the sum of the ground reaction forces on both feet is calculated by summing Newton's second law for the pelvis and all segments in both legs as follows:

$$\sum F_{gr} = \{ {}^gr_fRm_f a_{Cf} + {}^gr_{sh}Rm_{sh} a_{Csh} + {}^gr_{th}Rm_{th} a_{Cth} \}_{l,r} + {}^gr_pRm_p a_{Cp} + {}^gr_tRF_t$$

$$\text{where } \sum F_{gr} = F_{grr} + F_{grl}$$

335 Then the right and left ground reaction forces ( $F_{grr}, F_{grl}$ ) can be calculated from the total ground  
 336 reaction force ( $\sum F_{gr}$ ) by applying the following smooth transition assumptions:

$$337 \frac{F_{gr(st)X}}{\sum F_{grX}} = STA(t); \quad \frac{F_{gr(st)Y}}{\sum F_{grY}} = STA(t); \quad \frac{F_{gr(st)Z}}{\sum F_{grZ}} = STA(t)$$

338 Where (st) refers to the stance foot (i.e. the heel-strike foot that has just landed). In this study, the  
 339 smooth transition assumption,  $STA(t)$ , is a linear function of time with its value changing from 0  
 340 to 1 over each double stance period.

### 341 **6.2 Calculating the joint forces in the legs**

342 Then, starting from the two supporting feet and working upwards, segment by segment, the force  
 343 at each leg joint can be calculated from the following general equation:

$$344 F_{pr} = {}^{pr}_d RF_d + m_{pr} a_{Cpr}$$

345 Using the above, for both stance legs, the corresponding equations for each segment force (i.e.  
 346 the force at the segment's proximal joint) can be derived.

### 347 **6.3 Calculating the ground reaction moments**

348 During double stance, the sum of the ground reaction moments on both feet is calculated by  
 349 summing Euler's equation for the pelvis and all segments in both legs as follows:

$$350 \sum n_{gr} = n_{grr} + n_{grl} = \sum_{i=f}^p {}^{gr} N_i + {}^{gr}_t Rn_t - {}^{gr} MFS - {}^{gr} MPFS$$

$$\begin{aligned}
& {}^{gr}MPFS = \{ {}^f R(-P_{Cf} \times F_f) + {}^{sh} R(-P_{Csh} \times F_{sh}) + {}^{th} R(-P_{Cth} \times F_{th}) \}_{l,r} + {}^p R(-P_{Cp} \times -{}^p RF_t) \\
& {}^{gr}MFS = {}^p R \left[ ({}^p P_{th_l} - {}^p P_{Cp}) \times -{}^{th_l} RF_{th_l} \right] + {}^p R \left[ ({}^p P_{th_r} - {}^p P_{Cp}) \times -{}^{th_r} RF_{th_r} \right] + \dots \\
& \{ {}^f R \left[ ({}^f P_{gr} - {}^f P_{Cf}) \times {}^f RF_{gr} \right] + {}^{sh} R \left[ ({}^{sh} P_f - {}^{sh} P_{Csh}) \times -{}^{sh} RF_f \right] + \dots \\
& {}^{th} R \left[ ({}^{th} P_{sh} - {}^{th} P_{Cth}) \times -{}^{th} RF_{sh} \right] \}_{l,r}
\end{aligned}$$

$$\begin{aligned}
& \sum_{i=f}^p N_i = \{ {}^f R [I_f \dot{\omega}_f + \omega_f \times (I_f \omega_f)] + {}^{sh} R [I_{sh} \dot{\omega}_{sh} + \omega_{sh} \times (I_{sh} \omega_{sh})] \\
& + {}^{th} R [I_{th} \dot{\omega}_{th} + \omega_{th} \times (I_{th} \omega_{th})] \}_{l,r} + {}^p R [I_p \dot{\omega}_p + \omega_p \times (I_p \omega_p)]
\end{aligned}$$

Then the right and left ground reaction moments ( $n_{grr}, n_{grl}$ ) can be calculated from the total ground reaction moment ( $\sum n_{gr}$ ) by applying the following smooth transition assumptions:

$$\frac{n_{gr(st)X}}{\sum n_{grX}} = STA(t); \quad \frac{n_{gr(st)Y}}{\sum n_{grY}} = STA(t); \quad \frac{n_{gr(st)Z}}{\sum n_{grZ}} = STA(t)$$

Where (st) refers to the stance foot (i.e. the heel-strike foot that has just landed). In this study, the smooth transition assumption,  $STA(t)$ , is a linear function of time with its value changing from 0 to 1 over each double stance period.

#### 6.4 Calculating the joint moments in the legs

Then, starting from the two supporting feet and working upwards, segment by segment, the moment at each leg joint can be calculated from the following general equation:

$$n_{pr} = N_{pr} + {}^{pr}Rn_d - MPF_{pr} - MF_d$$

$$N_{pr} = I_{pr}\dot{\omega}_{pr} + (\omega_{pr} \times I_{pr}\omega_{pr})$$

$$MPF_{pr} = -P_{Cpr} \times F_{pr}$$

$$MF_d = ({}^{pr}P_d - P_{Cpr}) \times -{}^{pr}RF_d$$

364 Using the above, for both stance legs, the corresponding equations for each segment moment (i.e.  
365 the moment at the segment's proximal joint) can be derived.

366

### 367 **Test results**

368 The inverse dynamics model described above has been verified against an identical model  
369 created using MathWorks' *Sim Mechanics* software. To achieve **model verification**, the degrees  
370 of freedom, segments, joints and motion inputs were identical. However, it was not possible to  
371 simulate double stance using *Sim Mechanics*, because of the indeterminacy problem, or to  
372 change the foot in contact with the ground during simulation. Therefore, the **model verification**  
373 applies only over one single stance phase with the right foot in contact with the ground.

374 There was excellent agreement between the two models. For example, over single stance, the  
375 positions of the segment origins were in agreement with a mean error of  $10^{-12}$  mm. The ground  
376 reaction forces and moments were in agreement with mean errors of  $10^{-14}$  N and  $10^{-14}$  Nm  
377 respectively.

378 Finally, the execution speeds of the two models were compared. This was done for single stance  
379 because the *Sim Mechanics* model could only model single stance. Both models were run on the  
380 same PC without compiling the associated MATLAB code. The *Sim Mechanics* model's

381 execution time was 8.5 seconds as compared to 1.1 seconds for the fast inverse dynamics model  
382 described in this paper.

383

## 384 **Conclusion**

385 A computationally efficient inverse dynamics model of human gait has been designed for use in  
386 optimisation based studies requiring many iterative gait cycle simulations. The model has been  
387 verified against an equivalent model, created using a commercial multi-body dynamics package,  
388 and the execution times of the two models compared. **The fast inverse dynamics model described  
389 in this paper led to an eight fold increase in execution speed.**

390 The increased computational efficiency is a result of a number of factors including the use of a  
391 bespoke model of the human gait cycle, which avoids the overheads associated with general  
392 simulation packages that must cater for any model topology. Furthermore, the chain like  
393 structure of the model lends itself to a bespoke implementation of the iterative Newton-Euler  
394 method, which is well recognised as being a particularly efficient approach.

395

## 396 **Disclosure statement**

397 To the best of the authors' knowledge, there are no conflicts of interest.



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418 **Appendix – Joint rotation matrices and angular motions**

419

420 **a) One degree of freedom joints rotating about X axis (Flexion-Extension)**

$${}^{pr}R(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_j & -\sin \alpha_j \\ 0 & \sin \alpha_j & \cos \alpha_j \end{bmatrix}$$

421  $\omega_j = {}^{pr}\omega_{d/pr}(\alpha) = \begin{bmatrix} \dot{\alpha}_j \\ 0 \\ 0 \end{bmatrix}$

$$\dot{\omega}_j = {}^{pr}\dot{\omega}_{d/pr}(\alpha) = \begin{bmatrix} \ddot{\alpha}_j \\ 0 \\ 0 \end{bmatrix}$$

422

The joints of this type are foot rollover, knee, and elbow.

423 **b) Two degrees of freedom joints rotating about X and Y axes**

$${}^{pr}R(\alpha, \beta) = \begin{bmatrix} \cos \beta_j & 0 & \sin \beta_j \\ \sin \beta_j \sin \alpha_j & \cos \alpha_j & -\sin \alpha_j \cos \beta_j \\ -\cos \alpha_j \sin \beta_j & \sin \alpha_j & \cos \beta_j \cos \alpha_j \end{bmatrix}$$

424  $\omega_j = {}^{pr}\omega_{d/pr}(\alpha, \beta) = \begin{bmatrix} \dot{\alpha}_j \\ \cos \alpha_j \dot{\beta}_j \\ \sin \alpha_j \dot{\beta}_j \end{bmatrix}$

$$\dot{\omega}_j = {}^{pr}\dot{\omega}_{d/pr}(\alpha, \beta) = \begin{bmatrix} \ddot{\alpha}_j \\ \cos \alpha_j \ddot{\beta}_j - \sin \alpha_j \dot{\alpha}_j \dot{\beta}_j \\ \sin \alpha_j \ddot{\beta}_j + \cos \alpha_j \dot{\alpha}_j \dot{\beta}_j \end{bmatrix}$$

425

The joint of this type is the shoulder.

426

**c) Two degrees of freedom joints rotating about X and Z axes**

$${}^{pr}R(\alpha, \gamma) = \begin{bmatrix} \cos \gamma_j & -\sin \gamma_j & 0 \\ \sin \gamma_j \cos \alpha_j & \cos \alpha_j \cos \gamma_j & 0 \\ \sin \alpha_j \sin \gamma_j & \sin \alpha_j \cos \gamma_j & c\alpha_j \end{bmatrix}$$

427  $\omega_j = {}^{pr}\omega_{d/pr}(\alpha, \gamma) = \begin{bmatrix} \dot{\alpha}_j \\ -\sin \alpha_j \dot{\gamma}_j \\ \cos \alpha_j \dot{\gamma}_j \end{bmatrix}$

$$\dot{\omega}_j = {}^{pr}\dot{\omega}_{d/pr}(\alpha, \gamma) = \begin{bmatrix} \ddot{\alpha}_j \\ -\cos \alpha_j \dot{\alpha}_j \dot{\gamma}_j - \sin \alpha_j \ddot{\gamma}_j \\ -\sin \alpha_j \dot{\alpha}_j \dot{\gamma}_j + \cos \alpha_j \ddot{\gamma}_j \end{bmatrix}$$

428

The joint of this type is the ankle.

429

**d) Three degrees of freedom joints rotating about X, Y and Z axes**

$${}^{pr}R(\alpha, \beta, \gamma) = \begin{bmatrix} c\beta_j c\gamma_j & -c\beta_j s\gamma_j & s\beta_j \\ s\alpha_j s\beta_j c\gamma_j + c\alpha_j s\gamma_j & -s\alpha_j s\beta_j s\gamma_j + c\alpha_j c\gamma_j & -s\alpha_j c\beta_j \\ -c\alpha_j s\beta_j c\gamma_j + s\alpha_j s\gamma_j & c\alpha_j s\beta_j s\gamma_j + s\alpha_j c\gamma_j & c\alpha_j c\beta_j \end{bmatrix}$$

430  ${}^{pr}\omega_{d/pr}(\alpha, \beta, \gamma) = \omega_j = \begin{bmatrix} \dot{\alpha}_j + s\beta_j \dot{\gamma}_j \\ c\alpha_j \dot{\beta}_j - c\beta_j s\alpha_j \dot{\gamma}_j \\ s\alpha_j \dot{\beta}_j + c\alpha_j c\beta_j \dot{\gamma}_j \end{bmatrix}$

$${}^{pr}\dot{\omega}_{d/pr}(\alpha, \beta, \gamma) = \begin{bmatrix} \ddot{\alpha}_j + c\beta_j \dot{\beta}_j \dot{\gamma}_j + s\beta_j \ddot{\gamma}_j \\ -s\alpha_j \dot{\alpha}_j \dot{\beta}_j + c\alpha_j \ddot{\beta}_j - c\alpha_j c\beta_j \dot{\alpha}_j \dot{\gamma}_j + c\alpha_j \ddot{\beta}_j + s\alpha_j s\beta_j \dot{\beta}_j \dot{\gamma}_j - s\alpha_j c\beta_j \ddot{\gamma}_j \\ c\alpha_j \dot{\alpha}_j \dot{\beta}_j - s\alpha_j c\beta_j \dot{\alpha}_j \dot{\gamma}_j + s\alpha_j \ddot{\beta}_j - c\alpha_j s\beta_j \dot{\beta}_j \dot{\gamma}_j + c\alpha_j c\beta_j \ddot{\gamma}_j \end{bmatrix}$$

431

432 The joints of this type are hip, lumbosacral and neck.

433

434 **Figure Captions**

435

436 *Figure 1: The multi-body model*

437

438 *Figure 2: Position vectors used in the iterative calculations*

439

440 *Figure 3: Free Body Diagrams: a) Standard line segment; b) Torso; c) Pelvis*

441