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PERISTALTIC TRANSPORT OF BI-VISCOSITY FLUIDS THROUGH A CURVED TUBE: *A MATHEMATICAL MODEL FOR INTESTINAL FLOW*

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Abstract

The human intestinal tract is a long curved tube constituting the final section of the digestive system in which nutrients and water are mostly absorbed. Motivated by the dynamics of chyme in the intestine, a mathematical model is developed to simulate the associated transport phenomena via peristaltic transport. Rheology of chyme is modelled using the Nakamura-Sawada bi-viscosity non-Newtonian formulation. The intestinal tract is considered as a curved tube geometric model. Low Reynolds number (creeping hydrodynamics) and long wavelength approximations are taken into consideration. Analytical solutions of the moving boundary value problem are derived for velocity field, pressure gradient and pressure rise. Streamline flow visualization is achieved with Mathematica symbolic software. Peristaltic pumping phenomenon and trapping of the bolus are also examined. The influence of curvature parameter, apparent viscosity coefficient (rheological parameter) and volumetric flow rate on flow characteristics is described. Validation of analytical solutions is achieved with a MAPLE17 numerical quadrature algorithm. The work is relevant to improving understanding of gastric hydrodynamics and provides a benchmark for further computational fluid dynamics (CFD) simulations.

Keywords: *Peristalsis; Biviscosity fluids; Intestinal movement; Curvature; Streamlines; Analytical solutions; Mathematica; MAPLE17; bio-rheology.*

Nomenclature

Roman

a half width of curve coiled in a circle with centre, O.

A	algebraic function
B	algebraic function
b	amplitude of sinusoidal wave
c	peristaltic wave speed
e_{ij}	$(i,\ \mathrm{j})$ component of deformation rate
F	*v*olumetric flow rate in fixed frame
k	curvature parameter
$\overline{H}(\overline{X},\bar{t})$	geometric parameter defining wall displacement for flexible channel
p_{y}	yield stress of the biofluid.
R^{*}	radius of curve coiled in a circle with centre, O.
$\dfrac{\partial\overline{P}}{\partial\overline{X}}$	axial pressure gradient
$\overline{P}$	dimensional pressure
Q	volumetric flow rate in wave frame
$\overline{R}$	radial coordinate in fixed frame
$\bar{r}$	radial coordinate in wave (laboratory) frame
r	non-dimensional wave frame radial coordinate
Re	Reynolds number
$\overline{X}$	axial coordinate in fixed frame
$\bar{x}$	axial coordinate in wave (laboratory) frame
x	non-dimensional wave frame axial coordinate
$\overline{U}$	velocity component in axial $\overline{X}$ direction
$\bar{u}$	velocity component in $\bar{x}-$ direction in the wave frame.
u	non-dimensional wave frame axial velocity
$\overline{V}$	velocity component in radial $\overline{R}$ direction
$\bar{v}$	velocity component in $\bar{r}$-direction in the wave frame
v	non-dimensional wave frame radial velocity

Greek

δ wave number

ΔP dimensionless pressure rise

β apparent viscosity coefficient (Nakamura-Sawada parameter)

λ wave length.

$\tau_{\overline{R}\overline{R}}, \tau_{\overline{R}\overline{\Theta}}, \tau_{\overline{\Theta}\overline{\Theta}}$ shear stress terms

μ_β plastic dynamic viscosity

$\overline{\psi}$ dimensional stream function

$\overline{\psi}$ non-dimensional stream function

$\bar{\dot{\gamma}}$ dimensional strain rate

$\dot{\gamma}$ non-dimensional strain rate

1. Introduction

Peristaltic fluid dynamics continues to attract the attention of engineers and scientists owing to ever-growing applications in emerging technologies and a need for refining understanding of physiological mechanisms in the human body. Peristalsis is an automatic and important periodic series of muscle contractions and relaxation that occurs during movement of food bolus through the digestive system, urine flow from the kidneys into the bladder and transportation of bile from the gall-bladder into the duodenum, to name a few medical applications. The wave motion in peristalsis is usually a *circumferential progressive wave* propagating along a flexible conduit. Inherently 3-dimensional in nature, peristalsis constitutes an intriguing *moving boundary value problem* in mathematical modelling. Numerous modern mechanical devices have been designed on the principles of peristaltic pumping to transport fluids without internal moving parts. These include the heart-lung machine [**1**], dialysis machines **[2, 3]**, and blood pump machines **[4, 5]**. Mechanically since peristalsis requires no pistons, it achieves greater overall efficiency and safety (parts do not wear out or degrade leading to contamination via debris) which is crucial in the secure transportation of hazardous fluids (aggressive chemicals, slurries, corrosive and noxious fluids) and representative designs in this context include roller pumps **[6]**, rotary pumps **[7]**, and multi-actuated electrohydraulic peristaltic pumps **[8, 9]** etc. Peristalsis was first observed by Bayliss and Starling [**10**] during the study of chyme movement in intestines. Their work was largely observational. Many decades elapsed before engineering fluid mechanical investigations were reported. Defining studies in fluid dynamics of peristalsis were presented by Burns and Parkes **[11]**, Fung and Yih **[12]**, Shapiro *et al.* **[13],** Jaffrin and Shapiro **[14]**, Shukla *et al*. **[15]**, Takabatake, and Ayukawa **[16]**, Pozrikidis **[17]**. These analyses were confined to *steady* flow (owing to the intrinsic mathematical simplicity); however from a physiological point of view, the vast majority of peristaltic transport phenomena are strongly *unsteady* in nature. Early work on transient peristalsis was presented by Li and Brasseur **[18]**. These studies are limited for Newtonian fluids so further modifications for non-Newtonian fluids, MHD fluids, heat transfer and nanofluids are required. Tripathi and Bég **[19]** presented a model for peristaltic viscoelastic fluid propulsion. Tripathi and Bég **[20]** further considered peristaltic dynamics of MHD couple stress fluids. François **[21]** discussed the influence of suspended drops on peristaltic pumping. Ellahi *et al*. **[22]** reported the heat transfer analysis on peristaltic flow. Tripathi and Bég **[23]** investigated peristaltic transport of nanofluids, Kothandapani and Prakash **[24]** addressed peristaltic hydromagnetic nanofluids. Akbar *et al*. **[25]** further considered MHD nanofluid peristaltic propulsion through permeable channels. Teran *et al.* **[26]** studied peristaltic wave propagation in viscoelastic fluids, achieving excellent visualization of circumferential waves.

The above studies were largely confined to *straight* conduits. *Curvature* is however a significant characteristic of actual physiological vessels, in particular the intestinal ducts [**27**]. The intestine is fact an enormously long duct with many sub-sections including the initial stage of the small intestine (duodenum), the pyloric sphincter, the jejunum, and the the ileum (the latter two constitute in excess of 4m in length) and the large intestine (ascending, descending, transverse and sigmoid colon and cecum) and so on. The key attribute which allows the extensive intestine to occupy a small volume in the body is curvature i.e. geometric twisting and turning. In fact directly owing to this twisting and turning allows the duodenum to be located in close proximity to the head of the pancreas. It has been shown clinically that the transport of digested food (achieved with peristalsis) and the mixing (achieved via segmentation) are more efficient along lower curvature zones in the intestinal duct [**28**]. Furthermore the slow contractive waves associated with peristaltic propulsion (usually 3 waves per minute) originate in the interstitial cells of Cajal in the central section of the larger curvature (proximal corpus) and propagate distally towards the pylorus. Therefore curvature (waves move slightly quicker along the greater curve than the lesser curve) has a pivotal role to play in sustainable motility in the intestine and has indeed stimulated significant attention from engineers and medical scientists. Many regions of movement have been categorized in terms of fluid dynamics principles including transpyloric flow and retropulsive flow. Peristaltic flow with heat or mass transfer in curved vessels has also attracted some attention in recent years, largely due to the improvement in experimental and computational methods. Tharakan *et al.* [**29**] developed an experimental rig that simulates the segmentation motion occurring in the small intestine to simulate mass diffusion in the lumen, observing that glucose available for absorption may be strongly decreased by altering the lumen viscosity. Pal *et al.* [**30**] employed a Lattice Boltzmann computational solver to investigate intestinal fluid dynamics and mass transport, observing that high retrograde flow velocities arise in the narrowest antral segment inducing fast particle separation and that luminal surface motions caused by peristaltic contraction waves give rise to recirculating eddies which are strongly influenced by wall curvature. Spratt *et al.* [**31**] employed a three-stage tubular model to simulate transport phenomena in the large human intestine, although they did not consider peristaltic waves or pulsating motion, and instead focused attention on water transfer across the membrane and also demonstrated that Taylor dispersion contributes strongly to gastric mixing. A seminal analysis of the peristaltic propulsion induced by transverse deflections of the walls of a curved channel was presented by Sato *et al*. [**32**], who computed stream function, flow velocity and pressure distributions and observed that pressure-flow characteristic gradient (which is linear) is weakly enhanced with greater channel curvature. They also fund that the trapped bolus of fluid is composed of two asymmetrical sections, with the outer one growing and the inner one shrinking with greater channel curvature.

The above studies did not consider *biorheological* effects i.e. the Newtonian viscous flow model was utilized. However the *non-Newtonian* nature of gastric liquids is well known and may contribute significantly to peristaltic efficiency. Several researchers in recent years have addressed this aspect and explored a diverse range of rheological constitutive models. Narla *et al*. [**33**] presented closed-form solutions for peristaltic propulsion of a viscoelastic fluid (fractional second grade model) in a curved channel observing that pressure-flow relationships are linear and that pressure-flow function is reduced with greater values of fractional viscoelastic parameter, curvature parameter and amplitude ratio whereas the reverse effect is induced with increasing relaxation time. Kalantari [**34**] used a finite difference technique to compute Weissenberg effects on peristaltic curved channel flow with the Phan-Thien-Tanner elasto-viscous model. Kalantari *et al*. [35] investigated bolus dynamics in peristaltic flow of Giesekus rheological liquids in curved channels. Very recently Ali *et al.* [**36**] used a finite difference method and Chebyschev spectral algorithm to investigate peristaltic motion of a non-Newtonian Carreau fluid through a curved conduit. They showed for the first time in the literature, that for weakly shear-thinning fluids, an increase in non-Newtonian parameter i.e. Weissenberg number, opposes the effects of curvature and serves to enforce symmetry in the velocity profiles. They also observed that for strong shear-thinning fluids (power-law index near to zero) with larger relaxation times, the gradient of velocity sharply changes near the channel walls generating a thin boundary layer. Additionally an increase in Weissenberg number was shown to produce a small eddy in the vicinity of the lower wall of the channel, which was enhanced with further increase in Weissenberg number. These studies [**33-36**] did not consider the *Nakamura-Sawada bi-viscosity model*, which has also shown very good correlation with intestinal transport experiments [**37, 38**]. Introduced in the late 1980s [**39**], originally for industrial suspensions, the Nakamura-Sawada model has been successfully implemented in hemodynamic [**40**] and gastric fluid mechanics modelling [**41**]. Essentially a modified Casson model, the yield stress of the biviscosity model is however significantly higher than for Casson fluids. It is also superior to the Ostwald-deWaele power-law model since it avoids the pitfall of predicting for a pseudoplastic fluid (power law index less than unity), a diverging apparent viscosity diverges at zero flow rate. The Nakamura-Sawada model achieves a finite apparent viscosity coefficient even in the low shear rate region and is particularly amenable to mathematical simulations. It has been deployed in a number of biofluid dynamic simulations including pulsatile magneto-hemodynamics and pharmacological diffusion [**42**], magnetized heatconducting blood flows in tissue [**43**] and biopolymer transport in filtered media [**44**]. Akbar and Nadeem [**45**] have used this model also quite recently in studying magnetohydrodynamic peristaltic flow in an endoscope. Further investigations include Eldabe *et al.* [**46**] who considered peristaltic flow of an incompressible, magnetic bi-viscosity fluid through an axisymmetric non-uniform tube with a sinusoidal wave. These studies have generally found that with increasing rheological (apparent viscosity) parameter in the bi-viscosity model, flow is decelerated.

In the present article, for the first time, we consider the peristaltic flow of a bi-viscosity non-Newtonian gastric fluid in a curved channel. Closed form solutions are derived for the non-dimensionalized boundary value problem. Verification of solutions is achieved with a numerical method. Streamline flow visualization is presented via the Mathematica symbolic software to highlight peristaltic pumping and bolus trapping. The effects of the curvature parameter, apparent viscosity coefficient (rheological parameter) and volumetric flow rate parameter on peristaltic flow characteristics are elaborated.

2. Biviscosity Gastric Fluid Model and Flow Regime

The constitutive equations for incompressible Nakamura-Swada biviscosity fluids [**39**] are defined as follows:

$$\bar{S}_{ij} = \begin{cases} 2\left(\mu_\beta + p_y/\sqrt{2\pi}\right)e_{ij}, & \pi > \pi_c, \\ 2\left(\mu_\beta + p_y/\sqrt{2\pi_c}\right)e_{ij}, & \pi < \pi_c, \end{cases} \tag{1}$$

$\pi = e_{ij}$; e_{ij} is the $(i,\ \mathrm{j})$ component of deformation rate, which is defined as:

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial yi}\right). \tag{2}$$

In the above Eqns. (1) and (2), μ_β is plastic dynamic viscosity, p_{y} is yield stress of the biofluid. We consider a curved channel filled with an incompressible bi-viscosity fluid fluid. Let a be the half width of a curve coiled in a circle with centre, O, and radius, R^*. The flow in the channel is induced by sinusoidal waves of small amplitude, b, traveling along the flexible walls of the channel. The equations for the wall surfaces are:

$$\overline{H}\left(\overline{X},\bar{t}\right)= a+b\sin\left[\frac{2\pi}{\lambda}\left(\overline{X}-c\bar{t}\right)\right], \qquad \text{upper wall} \qquad (3)$$

$$-\overline{H}\left(\overline{X},\bar{t}\right)= -a-b\sin\left[\frac{2\pi}{\lambda}\left(\overline{X}-c\bar{t}\right)\right]. \qquad \text{lower wall} \qquad (4)$$

In the above equations c is the peristaltic wave speed, a is wave amplitude and λ denotes the wave length.

3. Mathematical Flow Model

The governing equations for unsteady, two-dimensional flow of incompressible, bi-viscosity fluids through a curved tube may be presented as follows:

$$\frac{\partial}{\partial \overline{R}}\left[\left(\overline{R}+R^*\right)\overline{V}\right]+\frac{R^*}{\overline{R}+R^*}\frac{\partial \overline{U}}{\partial \overline{X}}=0, \qquad (4)$$

$$\rho\left(\frac{\partial}{\partial t}+\overline{V}\frac{\partial \overline{V}}{\partial \overline{R}}+\frac{R^*\overline{U}}{\overline{R}+R^*}\frac{\partial \overline{V}}{\partial \overline{X}}-\frac{\overline{U}^2}{\overline{R}+R^*}\right)=-\frac{\partial \overline{P}}{\partial \overline{R}}+\frac{1}{\overline{R}+R^*}\frac{\partial}{\partial \overline{R}}\left[\left(\overline{R}+R^*\right)\tau_{\overline{R}\overline{R}}\right]$$
$$+\frac{R^*}{\overline{R}+R^*}\frac{\partial}{\partial \overline{X}}\left(\tau_{\overline{R}\overline{\Theta}}\right)-\frac{\tau_{\overline{\Theta}\overline{\Theta}}}{\overline{R}+R^*}, \qquad (5)$$

$$\rho\left(\frac{\partial}{\partial t}+\overline{V}\frac{\partial \overline{U}}{\partial \overline{R}}+\frac{R^*\overline{U}}{\overline{R}+R^*}\frac{\partial \overline{U}}{\partial \overline{X}}+\frac{\overline{U}\overline{V}}{\overline{R}+R^*}\right)=-\frac{R^*}{\overline{R}+R^*}\frac{\partial \overline{P}}{\partial \overline{X}}$$
$$+\frac{1}{\overline{R}+R^*}\frac{\partial}{\partial \overline{R}}\left[\left(\overline{R}+R^*\right)\tau_{\overline{R}\overline{\Theta}}\right]$$
$$+\frac{R^*}{\overline{R}+R^*}\frac{\partial}{\partial \overline{X}}\left[\tau_{\overline{\Theta}\overline{\Theta}}\right] \qquad (6).$$

In the above equations, $\overline{P}$ is the pressure, $\overline{V}$ and $\overline{U}$ are the velocity components in radial $\overline{R}$ and axial $\overline{X}$ directions respectively, R^* is the constant radius and the τ-terms represent the stresses. The flow phenomenon is unsteady in the *fixed* frame. To carry out a steady analysis we switch from the fixed frame to the *wave (laboratory)* frame $\left(\bar{r},\bar{x}\right)$ moving with the wave speed, c. The transformation between the two frames is given by:

$$\bar{x}=\overline{X}-c\bar{t},\bar{r}=\overline{R},$$
$$\bar{u}=\overline{U}-c,\bar{v}=\overline{V}, \qquad (7)$$

where $\bar{v}$ and $\bar{u}$ are the velocity components along $\bar{r}$ and $\bar{x}-$ directions in the wave frame. With the help of these transformations the Eqns. (4) to (6) take the form:

$$\frac{\partial}{\partial \bar{r}}\left[\left(\bar{r}+R^*\right)\bar{v}\right]+\frac{R^*}{\bar{r}+R^*}\frac{\partial \bar{u}}{\partial \bar{x}}=0, \tag{8}$$

$$\rho\left(-c\frac{\partial \bar{v}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{v}}{\partial \bar{r}}+\frac{R^*\left(\bar{u}+c\right)}{\bar{r}+R^*}\frac{\partial \bar{v}}{\partial \bar{x}}-\frac{\left(\bar{u}+c\right)^2}{\bar{r}+R^*}\right)=-\frac{\partial \bar{p}}{\partial \bar{r}}+\frac{1}{\bar{r}+R^*}\frac{\partial}{\partial \bar{r}}\left[\left(\bar{r}+R^*\right)\tau_{\bar{r}\bar{r}}\right]$$
$$+\frac{R^*}{\bar{r}+R^*}\frac{\partial}{\partial \bar{x}}\left[\tau_{\bar{r}\bar{\theta}}\right]-\frac{\tau_{\bar{\theta}\bar{\theta}}}{\bar{r}+R^*}, \tag{9}$$

$$\rho\left(-c\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{r}}+\frac{R^*\left(\bar{u}+c\right)}{\bar{r}+R^*}\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\left(\bar{u}+c\right)\bar{v}}{\bar{r}+R^*}\right)=-\frac{R^*}{\bar{r}+R^*}\frac{\partial \bar{p}}{\partial \bar{x}}$$
$$+\frac{1}{\bar{r}+R^*}\frac{\partial}{\partial \bar{r}}\left[\left(\bar{r}+R^*\right)\tau_{\bar{r}\bar{\theta}}\right] \tag{10}$$
$$+\frac{R^*}{\bar{r}+R^*}\frac{\partial}{\partial \bar{x}}\left[\tau_{\bar{\theta}\bar{\theta}}\right].$$

Introducing the following *non-dimensional* variables and velocity stream function relation:

$$x=\frac{2\pi \bar{x}}{\lambda}, r=\frac{\bar{r}}{a}, u=\frac{\bar{u}}{c}, v=\frac{\bar{v}}{c}, \psi=\frac{\bar{\psi}}{ca}, k=\frac{R^*}{a}, \mathrm{Re}=\frac{\rho ca}{\mu_0},$$
$$\beta=\mu_\beta\sqrt{2\pi_c}\,/\,p_y, \delta=\frac{2\pi a}{\lambda}, p=\frac{2\pi a^2\bar{p}}{c\lambda\mu_0}, \dot{\gamma}=\frac{a}{c}\bar{\dot{\gamma}}, \tag{11}$$

where Re is Reynolds number, δ is the wave number, k is the curvature parameter and β is the apparent viscosity coefficient (Nakamura-Sawada rheological parameter). Eqns. (9) and (10), under the conventional long wavelength and low Reynolds number approximations are expressed in the following dimensionless form:

$$\frac{dP}{dr}=0, \tag{12}$$

$$\frac{dP}{dx}=-\left(\frac{r+k}{r}\right)\left(1+\frac{1}{\beta}\right)\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}-\frac{u+1}{r+k}\right) \tag{13}.$$

The prescribed boundary conditions are:

$$u=0, \quad \text{at} \quad r=-h=-1-\varepsilon\sin(x), \tag{14a}$$
$$u=0, \quad \text{at} \quad r=h=1+\varepsilon\sin\left(x\right) \tag{14b}.$$

The volumetric flow rates in the fixed and wave frame are related by:

$$F = Q - 2 \qquad (15).$$

The momentum equation (13) for the proposed model is a *second order linear differential equation*, which readily yields the following *closed-form solution* for velocity and axial pressure gradient:

$$u(x,r) = \frac{\begin{array}{c} \beta k \frac{dP}{dx}(h-k)(h+k)(k+r)\log^2(k-h)(\log(h+k)-\log(k+r)) \\ -(h-k)\log(k-h)\left(\begin{array}{c}\beta k \frac{dP}{dx}(h+k)(k+r)\left(\log^2(h+k)-\log^2(k+r)\right) \\ +2(\beta+1)(h-r)\end{array}\right) \\ +\beta k \frac{dP}{dx}(h-k)(h+k)(k+r)\log^2(h+k)\log(k+r) \\ -(h+k)\log(h+k)\left(\begin{array}{c}\beta k \frac{dP}{dx}(h-k)(k+r)\log^2(k+r)-2(\beta+1) \\ (h+r)\end{array}\right) \\ -4(\beta+1)h(k+r)\log(k+r)\end{array}}{2(\beta+1)(h-k)(h+k)(\log(k-h)-\log(h+k))}, \qquad (16)$$

$$\frac{dP}{dx} = \frac{F-A}{\pi B}, \qquad (17)$$

where

$$A = \frac{\pi\left(8(\beta+1)h^2k - 4\beta h(h-k)(h+k)(\log(k-h)-\log(h+k)) - 4h(h-k)(h+k)(\log(k-h)-\log(h+k))\right)}{2(\beta+1)(h-k)(h+k)(\log(k-h)-\log(h+k))}, \qquad (18a)$$

$$B = \frac{\beta k(h-k)(h+k)\left(h^2+k^2\right)(\log(k-h)-\log(h+k))^2 + 2\beta hk^2(h-k)(h+k)(\log(k-h)-\log(h+k))}{2(\beta+1)(h-k)(h+k)(\log(k-h)-\log(h+k))}, \qquad (18b)$$

The *dimensionless pressure rise*, ΔP, is obtained by substituting Eqn. (17) into the following equation :

$$\Delta P = \int_0^1 \left(\frac{dP}{dx}\right) dx. \qquad (19)$$

4. Validation with MAPLE17

The linear dimensionless two-point moving boundary value problem (BVP) i.e. eqns. (12), (13) with conditions (14a, b) is easily solved using Runge–Kutta–Merson numerical quadrature. The solutions are needed to validate against the analytical solutions given earlier. A similar methodology is available in MAPLE17 software (RK45 algorithm). This approach has been extensively implemented recently in numerous multi-physical flow problems including entropy minimization in magnetic materials processing [**47**], nano-structural mechanics [**48**] and thermo-capillary biopolymer convection [**49**]. The robustness and stability of this numerical method is therefore well established- it is highly adaptive since it adjusts the quantity and location of grid points during iteration and thereby constrains the local error within acceptable specified bounds. In the current problem, the wall boundary conditions given in Eqns. (14a, b) are easily accommodated. The stepping formulae although designed for nonlinear problems, are even more efficient for any order of linear differential equation and are summarized below [**48**]:

$$k_0 = f\left(x_i, y_i\right), \tag{20}$$

$$k_1 = f\left(x_i + \frac{1}{4}h, y_i + \frac{1}{4}hk_0\right), \tag{21}$$

$$k_2 = f\left(x_i + \frac{3}{8}h, y_i + \left(\frac{3}{32}k_0 + \frac{9}{32}k_1\right)h\right), \tag{22}$$

$$k_3 = f\left(x_i + \frac{12}{13}h, y_i + \left(\frac{1932}{2197}k_0 - \frac{7200}{2197}k_1 + \frac{7296}{2197}k_2\right)h\right), \tag{23}$$

$$k_4 = f\left(x_i + h, y_i + \left(\frac{439}{216}k_0 - 8k_1 + \frac{3860}{513}k_2 - \frac{845}{4104}k_3\right)h\right), \tag{24}$$

$$k_5 = f\left(x_i + \frac{1}{2}h, y_i + \left(-\frac{8}{27}k_0 + 2k_1 - \frac{3544}{2565}k_2 + \frac{1859}{4104}k_3 - \frac{11}{40}k_4\right)h\right), \tag{25}$$

$$y_{i+1} = y_i + \left(\frac{25}{216}k_0 + \frac{1408}{2565}k_2 + \frac{2197}{4104}k_3 - \frac{1}{5}k_4\right)h, \tag{26}$$

$$z_{i+1} = z_i + \left(\frac{16}{135}k_0 + \frac{6656}{12825}k_2 + \frac{28561}{56430}k_3 - \frac{9}{50}k_4 + \frac{2}{55}k_5\right)h, \tag{27}$$

Here y denotes fourth-order Runge-Kutta phase and z is the fifth-order Runge-Kutta phase. An estimate of the error is achieved by subtracting the two values obtained. If the error exceeds a specified threshold, the results can be re-calculated using a smaller step size. The approach to estimating the new step size is shown below:

$$h_{\text{new}} = h_{\text{old}}\left(\frac{\varepsilon h_{\text{old}}}{2\left|z_{i+1} - y_{i+1}\right|}\right)^{1/4} \tag{28}$$

A comparison of the analytical and numerical quadrature solutions is documented in **Tables 1 and 2** for pressure rise and **Tables 3 and 4** for velocity distribution, respectively, with various values of the flow control parameters. Excellent correlation is achieved in all cases. Confidence in the present analytical solutions is therefore high.

5. Results and Discussion

The graphical results are displayed for the influence of axial coordinate (*x*), Reynolds number (*Re*) wave amplitude (ε), wave number (δ), vessel curvature parameter (*k*) and apparent viscosity coefficient (Nakamura-Sawada rheological parameter, β) on fluid velocity profile, pressure rise, and the streamlines in **Figs. 2-7**. Colour visualization is employed to highlight more clearly the streamline plots and bolus evolution in **figs. 5-7**.

The effect of channel curvature parameter (*k*), apparent viscosity coefficient (β) and flow rate (*Q*) on flow velocity distributions are illustrated in **Figs. 2(a)-(c)**. Evidently Fig. 2(a) indicates that greater curvature parameter effectively decelerates the flow in the region $-1.5 \leq r \leq 0$; however for the region, $0.1 \leq r \leq 1.5$, a decrease in curvature induces flow acceleration. This appears to be consistent with the clinical observation in [**28**], namely that peristaltic flow is aided along low curvature sections of the intestinal tract whereas it is impeded along high curvature zones. Flow acceleration resulting from a decrease in curvature has also been reported for viscoelastic (third grade) fluids by Ali *et al.* [**35**]. Fig. 2(b) shows that velocity is enhanced with increasing volumetric flow rate, which follows logically. Increasing apparent viscosity coefficient (β), which corresponds to weaker *non-Newtonian* effect [**42-44**], effectively lowers the viscosity of the biofluid and manifests in acceleration in the flow i.e. increasing velocity, but only for a the range -1.0≤ $r \leq 0$ as observed in Fig. 2(c). The effect of β is therefore opposite to the effect of curvature parameter; velocity increases with an increase in β for the region -1.0≤ $r \leq 0$ whereas it is reduced for $r = 0.1$ onwards. This sensitivity to the value of transverse coordinate was also reported in Narla *et al*. [**33**] indicating, as also elaborated by Skalak *et al.* [**38**] for both blood flows and gastric flows, that an intricate relationship exists between geometrical location in curved vessels and rheological effects. The present simulations further warrant experimental investigations along these lines and it is hoped that readers may pursue this avenue of research.

Figs. 3(a, b) depict the response of pressure rise (ΔP) for different values of apparent viscosity coefficient, β, and curvature parameter, k, respectively. Fig. 3a shows that with an increase in value of β, there is a corresponding *decrease* in pressure rise in the peristaltic pumping region -3≤ $Q \leq 2$, whereas pressure rise is conversely enhanced in the *augmented pumping* region 2.1≤ $Q \leq 3$. Fig. 3(b) demonstrates that in the peristaltic pumping region -3≤ $Q \leq -1$ the pressure rise *increases* with *greater* curvature (k) whereas it is lowered with an increase in k in the *augmented* pumping region -0.99≤ $Q \leq 3$. Comparing Figs 3a and 3b also reveals that significantly greater magnitudes for pressure rise are computed in the former as compared with the latter. The inverse pressure rise- flow rate relationship is also confirmed in both figures, concurring with numerous other non-Newtonian curved tube peristaltic studies e.g. Narla *et al*. [**33**], Kalantari [**34**] and Kalantari *et al.* [**35**], even though these other studies utilize different rheological models.

Figs. 4(a-c) present the distributions for axial pressure gradient (dp/dx) with variation in flow rate (Q), curvature parameter (k), and upper limit apparent viscosity coefficient (β) plotted along the x-axis. It is apparent that pressure gradient exhibits a sinusoidal behavior for variation of all flow parameters. The pressure gradient magnitudes in fig. 4a are however substantially larger than in figs 4b and 4c, principally due to the low curvature (k = 0.5) and high value of rheological parameter (β = 5) in fig. 4a- this combination strongly boosts the pressure gradient. In fig. 4a even though the rheological parameter is high (i.e. weak non-Newtonian effect), the curvature is also very high (k = 2, 3, 5) and this latter effect dominates the peristaltic flow and depresses pressure magnitudes. In fig. 4c despite the low curvature of the vessel (k = 0.5), the rheological parameter is also lower than in fig. 4a which again results in a depression in pressure gradient magnitudes. The influence of increasing flow rate, Q, as computed in fig. 4a, is consistently to elevate the pressure gradient. However although in fig. 4b, increasing curvature parameter, k does increase pressure gradient, this may also be associated with weak amplitude and weaker viscosity of the biofluid, rather than purely a geometric effect due to the curved vessel. With a weak increase in rheological parameter, β, (fig. 4c) there is as expected a boost in pressure gradient; however the magnitudes attained are much lower than in fig. 4a which is associated with a much higher value of β. Again the authors encourage experimental work in this area to further elaborate the clinical implications of the present computations.

Figs. 5a-c illustrate the streamlines for various curvature parameters (k). Evidently with increasing curvature parameter, the size of the bolus is slightly increased. As with many Newtonian [**32**] and non-Newtonian [**35**] peristaltic flows, the streamlines on the centre line in the wave frame are demarcated under certain conditions in order to enclose a bolus of fluid particles circulating along closed streamlines. This phenomenon is termed trapping, which is a characteristic of peristaltic motion. In axisymmetric peristaltic flows, the positive motion displacing biofluid forwards part is of a torus shape. The bolus is trapped by the wave and therefore propagates forward with the same speed as that of the wave. In the present simulations, reflux (or retrograde flux) i.e. reversed motion of biofluid in the opposite direction opposite to the net flow (i.e. in the negative x-direction) was not observed.

Figs. 6a-c depicts the streamline plots for different values of upper limit apparent viscosity coefficient (β). It is noticed that with the increase in upper limit apparent viscosity coefficient i.e. decreasing non-Newtonian effect, the size of the bolus also slightly increases. Stronger non-Newtonian behavior (fig. 6a) therefore results in a smaller bolus size compared with weaker non-Newtonian behavior (fig. 6c). The implication for gastric phenomena is that rheological characteristics of chime actually encourage the propagation of smaller bolus sizes rather than larger boluses. This may be linked with the efficiency recorded in clinical observations wherein actual intestinal dynamics is more efficient than Newtonian models predict [**32**].

Figs.7a-c finally present the streamline visualizations for the effects of flow rate (*Q*) variation. It is seen that with an increase in flow rate (*Q*) the size of the bolus increases and this will also inevitably influence the number of boluses which are expected to decrease.

The present study, it is envisaged, will provide further stimulation for experimental investigations in curved tube peristaltic rheological propulsion and serve as a reasonable benchmark also for more advanced fluid-structure-interaction simulations of such phenomena via advanced commercial software codes. Furthermore it is relevant to electro-hydraulically driven peristaltic pumps in bio-robotics [**50**]. On another note, a very significant development in modern peristaltic hydrodynamics simulations has been *nanofluids*. Many important investigations have been communicated very recently relating to rheological and other aspects of such fluids which hold immense potential in medical engineering. A proposed future extension to the current work is therefore to explore *nanofluids and nano-particle effects* in *curved tube peristalsis*, in line with other key studies including Ahmed and Nadeem [51], Nadeem and Shahzadi [52], Nadeem *et al*. [53], and Nadeem *et al*. [54]. The inner wall of the curved channel often possesses a ciliary surface and metachronal wave beating is also of great interest in refining peristaltic flow simulations, as elaborated by Nadeem and Sadaf [55 56] and the authors are also keen to pursue studies along these lines in the future.

5. Conclusions

A mathematical model has been developed for the peristaltic propulsion of non-Newtonian gastric fluid in a curved tube intestinal geometry. The Nakamura-Sawada bi-viscosity rheological formulation has been employed. Closed-form solutions have been derived for the dimensionless “moving boundary” value problem for velocity field, pressure gradient and pressure rise. Verification of solutions has been performed with a MAPLE17 numerical quadrature solver. To visualize bolus formation, streamline plots have been presented in colour. Some interesting flow characteristics have been determined which agree quite well with clinical observations and also other computational simulations. These are summarized as follows:

(i)Increasing curvature has been found to enhance pressure rise in the peristaltic pumping region whereas the converse effect is computed in the *augmented* pumping region.

(ii)The inverse pressure rise-flow rate relationship reported in numerous other studies has also been observed.

(iii)The axial velocity is also observed to be depressed with increasing curvature parameter in a certain zone of the curved tube whereas further along the opposite effect is apparent.

(iv) Bolus size was also found to be weakly amplified with an increase in upper limit apparent viscosity coefficient i.e. reduction in the non-Newtonian effect.

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