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BIOLOGICALLY INSPIRED TRANSPORT OF SOLID SPHERICAL NANO PARTICLES IN AN ELECTRICALLY-CONDUCTING VISCOELASTIC FLUID WITH HEAT TRANSFER

by

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Bio-inspired pumping systems exploit a variety of mechanisms including peristalsis to achieve more efficient propulsion. Non-conducting, uniformly dispersed, spherical nanosized solid particles suspended in viscoelastic medium forms a complex working matrix. Electromagnetic pumping systems often employ complex working fluids. A simulation of combined electromagnetic bio-inspired propulsion is observed in the present article. Currents formation has increasingly more applications in mechanical and medical industry. A mathematical study is conducted for MHD pumping of a bi-phase nanofluid coupled with heat transfer in a planar channel. Two-phase model is employed to separately identity the effects of solid nanoparticles. Base fluid employs Jeffery’s model to address viscoelastic characteristics. The model is simplified using long wavelength and creeping flow approximations. The formulation is taken to wave frame and non-dimensionalise the equations. The resulting boundary value problem is solved analytically, and exact expressions are derived for the fluid velocity, particulate velocity, fluid-particle temperature, fluid and particulate volumetric flow rates, axial pressure gradient and pressure rise. The influence of volume fraction density, Prandtl number, Hartmann number, Eckert number, and relaxation time on flow and thermal characteristics is evaluated in detail. The axial flow is accelerated with increasing relaxation time and greater volume fraction whereas it is decelerated with greater Hartmann number. Both fluid and particulate temperature are increased with increment in Eckert and Prandtl numbers, whereas it is reduced when the volume fraction density increases. With increasing Hartmann number pressure rise is reduced.

Key words: nanosized particles, two-phase model, heat transfer, MHD, bioinspired pumps

Introduction

In recent decades, bioinspired pumping mechanisms have been increasingly deployed in industrial systems owing to their superiority in efficiency and sustainability. These systems employ complex biological characteristics including ciliated walls, variable stiffness.
and wall deformability, adaptive healing, surface tension and many other intriguing features [1]. One of the most efficient and frequently deployed mechanisms of biological transport is peristalsis. This involves the propulsion of physiological fluids via rhythmic contraction of the walls of a vessel. In particular, peristaltic pumping finds significant applications in finger pumps, roller pumps, transport of hazardous wastes and also medical drug delivery. Early studies of the fabrication and engineering performance of peristaltic pumps for Newtonian viscous fluids were conducted by Latham [2]. Numerous analytical and computational investigations of peristaltic hydrodynamics have subsequently been reported. These have also featured supplementary phenomena including non-Newtonian effects, heat and mass transfer, nanofluids, etc. Representative studies include [3-10]. These studies have generally investigated peristaltic flow with the assumption of wave on the walls of the channel has a much greater wavelength when compared with amplitude of the wave and the transmission of fluid is slow results in low Reynolds number i.e. they have employed the so-called lubrication theory. Investigations on peristaltic flow with heat transfer have been reported by Ramesh and Devakar [11] who studied the effects of heat on peristaltic flow in a porous channel with perpendicular magnetic field. Bhatti, et al. [12] analysed effects of thermal radiation on the two-phase viscous fluid with constant magnetic field. Flow was induced by metachronal wave. Further studies include Vajravelu et al. [13] Mekheimer and Abd Elmaboud [14], Srinivas and Kothandapani [15] and therein.

The MHD concern the interaction of static or alternating magnetic fields with electrically-conducting fluids. It arises in diverse applications including electromagnetics casting, plasma flows, cooling of nuclear reactors, magnetic materials processing. The MHD peristaltic flows combine the flow control features of MHD with the efficient propulsive features of peristalsis. Such flows are present in bioinspired MHD medical pumps where very effective transport of different biological fluids can be achieved. These pumps also include continuous and non-pulsating modes intrinsic to a variety of complex geometrical designs. Extensive studies of MHD peristaltic flows have been communicated and have demonstrated the excellent regulation of bolus growth which is possible with the careful implementation of magnetic fields. Examples of such studies include Misra et al. [16] for viscoelastic fluid in a channel with stretching walls and Tripathi et al. [17] for Newtonian conducting fluids.

In certain energy and medical engineering applications, two-phase fluids may also arise. These generally comprise particles suspended in a fluid. Bi-phase fluid models represent a good approximation for blood (erythrocytes in plasma), Trowbridge [18], biotechnological suspensions, Beg et al. [19] and also doped working fluids in MHD generators. Some relevant examples are included in [20-30]. Several of these studies have also considered heat transfer.

The present article investigates the biologically inspired peristaltic transport of magneto-bi-phase non-Newtonian (viscoelastic) fluid and its heat transfer through a planar channel. Generation of internal energy due to viscosity is also included. The corresponding flow equations are formulated for fluid and particulate phase. Law of conservation of mass, linear momentum (fluid and particle phases) and thermal energy along with Ohm’s law are employed. Equations are simplified with the help of lubrication theory. Exact solutions are achieved. The rise in pressure is evaluated using numerical integration in MATHEMATICA. Results are visualized through graphs for the influence of a number of emerging parameters. Streamlines are also plotted to observe boluses. The computations provide a deeper insight into the transport phenomena in magnetic biopumps using a more realistic working fluid model (viscoelastic dusty model).
Modeling and dimensional analysis

Assume, MHD peristaltic pumping of a dusty (fluid-particulate) viscoelastic fluid in a 2-D planar channel. Viscoelastic characteristics of the dusty fluid are simulated with the Jeffrey’s model. The walls of the channel are considered to be electrically-insulated and a magnetic field with constant strength $B_0$ is applied externally. We have selected rectangular co-ordinates in a way that $x$-axis is taken parallel to centre line, while, $y$-axis is normal to the flow as seen through systemic geometry displayed in fig. 1. Hall current, magnetic-induction and ion slip effects are neglected. The particles are small spherical, have uniform size and in thermal equilibrium with the viscoelastic liquid. The equation describing peristaltic wave on the wall is [31]:

$$h(x,t) = \tilde{a} \left[ 1 + \frac{2\pi}{\lambda} \left( \frac{x}{a} - \frac{t}{\lambda} \right) \right], \quad \tilde{q} = \frac{\tilde{b}}{a}$$

(1)

The governing equations for mass conservation and momentum conservation for both the liquid phase and particulate phase are formulation using continuum mechanics as Mekheimer et al. [32].

- **Fluid phase:**

$$\nabla V_f = 0$$

(2)

$$\left(1-C\right) \rho_f \frac{D V_f}{D t} = -(1-C) \nabla P + \left(1-C\right) \nabla S + \frac{C_{cd}}{\alpha_f} \left( V_p - V_f \right) + JB$$

(3)

- **Particulate phase:**

$$\nabla V_p = 0$$

(4)

$$\left(1-C\right) \rho_p \frac{D V_p}{D t} = -(1-C) \nabla P - \frac{C_{cd}}{\alpha_p} \left( V_p - V_f \right)$$

(5)

The drag coefficient of the suspension in terms of Reynolds number for slowly moving fluid can be described by the following mathematical relations [33]:

$$c_{d} = \frac{24}{Re} \left( 1 + \frac{3}{16} \frac{1}{Re} \right)$$

(6)

The tensor describing the shear and normal stresses for Jeffrey’s viscoelastic fluid model may be defined, Bhatti et al. [34]:

$$S = \frac{\mu_s}{1 + \lambda_1} \left( \dot{\gamma} + \lambda_2 \ddot{\gamma} \right)$$

(7)

Equations for heat transfer by the fluid and particles for suspension with the assumption that particles are not good conductor of heat are (Bhatti et al. [35]):

$$\left(1-C\right) \rho_f c_p \frac{D \theta_f}{D t} = \left(1-C\right) \nabla \left( k \nabla \theta_f \right) + \frac{\rho_p c_p C}{\alpha_f} \left( \theta_p - \theta_f \right) + \frac{C_{cd}}{\alpha_p} \left( \tilde{u}_t - \tilde{u}_p \right)^2 + \Phi$$

(8)
\[(1-C) \rho_p c_p \frac{D \theta_p}{Dt} = \frac{\rho_p c_p C}{\sigma_T} (\theta_p - \theta_t) \]  

where \( \Phi \) the viscous forces converted to internal energy called viscous dissipation:

\[ \Phi = (SV)V' \]  

Equations are transformed in wave frame by horizontal velocity with wave speed \( u' = \bar{u} - c \) and \( x' = \bar{x} - c \bar{t} \).

After introducing non-dimensional parameters and using lubrication theory, eqs. (2)-(10) takes the form:

\[ \frac{dp}{dx} = \frac{1}{1 + \lambda_1} \frac{\partial^2 u'_p}{\partial y^2} - M^2 (u_p + 1) + \frac{NC}{(1-C)} (u_p - u_t) \]  

\[ \frac{1}{Pr} \frac{\partial^2 \theta_p}{\partial y^2} + Ec \left( \frac{\partial u_p}{\partial y} \right)^2 + \frac{Ec}{N(1-C)} \left( \frac{dp}{dx} \right)^2 = 0 \]  

\[ \frac{dp}{dx} = N (u_t - u_p) \]  

\[ \theta_t = \theta_p \]  

where it is assumed that \( x \) is of order of wave length \( \lambda \), \( y \) – equivalent to order of amplitude of wave \( a \). Similarly, horizontal velocity \( u \) proportional to speed of wave \( c \). Temperature \( \theta \) is defined as ratio of differences of temperature from any wall and between both walls. The \( Ec, N, Pr, \) and \( M \), are Eckert number, drag coefficient, Prandtl number, and Hartmann’s number, respectively.

On the wall \( y = h \) velocity and temperature take the form:

\[ \ell = -1, \quad \ell = 1 \]  

where

\[ h = 1 + \phi \sin 2\pi \]  

and at \( y = 0 \).

\[ u'_t = \theta_t = 0 \]  

**Analytical solutions**

The exact solutions of eqs. (11)-(14) is obtained under boundary conditions (15)-(17). Equation (13) is used in eq. (11) to get a uncouple both equations. Resultant equation by taking as constant with respect to \( y \) reduced to linear second order non-homogenous differential equation. Using complimentary and particular solution \( u_f \) becomes:

\[ u_f = -1 + \frac{\frac{dp}{dx} \left[ 1 - \cosh \left( M y \sqrt{1 + \lambda_1} \right) \right]}{(-1 + C) M^2} \left[ \text{sech} h M y \sqrt{1 + \lambda_1} \right] \]  

Using \( u_f \) in eq. (13) we get:

\[ u_p = \frac{1}{(-1 + C) M^2} \left[ \frac{dp}{dx} \left( -1 + c \right) M^2 \left( 1 + N \frac{dp}{dx} \right) \cosh \left( M y \sqrt{1 + \lambda_1} \right) \text{sech} h M y \sqrt{1 + \lambda_1} \right] \]
Solving eq. (12) which is also second order ODE and using eq. (17) we have:

\[
\frac{\theta_f}{\theta_p} = \left(4(-1+C)^2M^4Ny - Ec \left(\frac{dp}{dx}\right)^2Pr(h-y)\left[-N + 2hM^2((-1+C)M^2 + N)\right] + \right)
\]

\[
\left(2N + 2hM^2((-1+C)M^2 + N)\right) + \right)
\]

\[
\left(-cosh2hM\sqrt{1+\lambda_i} - EchN \left(\frac{dp}{dx}\right)^2Pr(2yM\sqrt{1+\lambda_i} + 2EchM^2N) \left(\frac{dp}{dx}\right)^2\right)\right)
\]

\[
\frac{\lambda}{8(-1+C)^2hM^4N}
\]

The volumetric flow rate is given.

\[
Q_t = (1-C)\int_0^h u_y dy
\]

\[
Q_p = C\int_0^h u_y dy
\]

\[
Q = Q_t + Q_p
\]

\[
Q = h\left[(1-C)M^2 + \frac{dp}{dx} + C(1-C)M^2N \frac{dp}{dx}\right] - \frac{\frac{dp}{dx} \tan h M\sqrt{1+\lambda_i}}{(1+C)M^2 \sqrt{1+\lambda_i}}
\]

Cracking eq. (24) to get \(\frac{dp}{dx}\), which is linear equation, rearranging eq. (24), by taking \(\frac{dp}{dx}\) common and obtaining a relation in terms of \(Q\):

\[
\frac{dp}{dx} = \frac{(-1+C)M^3(-h-Q)\sqrt{1+\lambda_i}}{-hM\sqrt{1+\lambda_i} - ChM^3N\sqrt{1+\lambda_i} + C^2hM^3N\sqrt{1+\lambda_i} + \tan h M\sqrt{1+\lambda_i}}
\]

Adimensional \(\Delta P\) can be calculated using relation:

\[
\Delta P = \int_0^h \frac{dp}{dx} dx
\]

Numerical Integration using MATHEMATICA 10 software is employed to obtain the results of \(\Delta P\).

**Graphs and discussion**

In this section, selected graphical results for the influence of different parameters on the fluid phase and particulate phase are elaborated. We consider the effects of the thermo-physical, non-Newtonian, and magnetic parameters on velocity and temperature profiles, pressure rise and finally bolus dynamics (peristaltic streamline plots for visualizing the trapping mechanism), respectively. Rise or drop in pressure in eq. (26) is estimated using numerical integration MATHEMATICA.
Close inspection of fig. 2 reveals that for a Newtonian fluid (vanishing viscoelastic characteristics i.e. zero relaxation time, \(\lambda_1 = 0\)) the fluid velocity declines whereas, for the non-Newtonian scenario \(\lambda_1 \neq 0\) the velocity of the fluid is enhanced with greater relaxation time, \(\lambda_1\). Axial flow is therefore, strongly accelerated with greater viscoelasticity of the working fluid. Figure 3 demonstrates that with increasing values of volume fraction density, \(C\), the axial flow is strongly decelerated i.e. fluid velocity is decreased. Figure 4 illustrates that the velocity of the fluid decreases when the Hartmann (magnetic body force) parameter increases. When magnetic field is applied it induces a retarding effect via the axial Lorentz MHD body force which serves to decelerate the axial flow. This decreases the fluid phase velocity.

**Temperature profile**

Figures 5-8 depict the variation of temperature profiles for both the fluid phase and particulate phase. From fig. 5 it is evident that temperatures are suppressed with an increment in the volume fraction density, \(C\), since greater concentration of solid particles effectively reduces thermal diffusion in the fluid phase. Figure 6 indicates that temperature increases with an increment in Eckert number since there is greater conversion of kinetic energy to thermal energy via viscous dissipation. Similarly, an increase in Prandtl number enhances temperatures in the regime, as observed in fig. 7. Figure 8 shows that temperature profile increases when the Hartmann number rises. The extra work consumed in dragging the fluid-particle suspension vs. magnetic field is released in form of internal energy. This heats the regime and lifts temperatures.

**Pumping characteristics**

Figures 9-11 illustrate the rise or drop in pressure vs. volumetric flow rate \(Q\). It can be observed using fig. 9 that pressure drops in the retrograde pumping region, whereas, the opposing response is displayed in the free pumping region and co-pumping region for various val-
The values of concentration density $C$. It is also apparent from fig. 10 that when stronger transverse magnetic field is applied, pressure drops in the co-pumping region and free pumping region, whereas it is enhanced in the retrograde pumping region. In fig. 11 we can see that pressure decreases for $\lambda_1 = 0$, i.e., Newtonian fluid, in the retrograde pumping region, whereas the opposite effect is induced in the co-pumping region and free pumping region. Viscoelastic fluids demonstrate the converse behaviour.

**Trapping phenomena**

Another interesting feature of such flows is trapping phenomena. Streamlines helps to unveil trapping i.e., the formation of an internally circulating bolus that is enclosed by stream-

The bolus designates to the volume of the fluid trapped in closed streamline. It can be observed through fig. 12 that when $C$ increases, then the number of boluses also increases whereas the bolus size demises. From fig. 13 it is observed that the number of bolus reduces with increasing values of magnetic parameter $M$. Figure 14 shows that when $\lambda_1$ (relaxation parameter) grows then the bolus count decreases.

Conclusions

Flow and coupled with heat transfer analysis in the biologically inspired peristaltic transport of an electrically-conducting magneto-bi-phase viscoelastic suspension has been studied theoretically in current manuscript. The present study has revealed interesting characteristics of dusty peristaltic bioinspired magnetic pumping systems. Further studies will address slip effects at the walls and will be communicated imminently. A few results are mentioned.
• Numerical computations have shown that velocity of the fluid decreases when the Hartmann (magnetic) number and volume fraction density increases.
• It is found that both fluid and particulate phase temperatures increases with magnetic Hartmann number, Eckert (viscous dissipation) number and Prandtl number.
• Whereas, fluid and particulate phase temperatures are reduced when the volume fraction density increases.
• Increasing viscoelastic relaxation time generates acceleration in the axial flow.
• Increasing volume fraction density results in a reduction in the trapped bolus magnitude whereas the number of boluses increases.
• The present analysis reduces to the Newtonian fluid by taking $\lambda_1 = 0$.

Nomenclature

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<tr>
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<tbody>
<tr>
<td>$a$</td>
<td>wave amplitude, [m]</td>
</tr>
<tr>
<td>$B_0$</td>
<td>magnetic field, [Nsc$^{-1}$m$^{-1}$]</td>
</tr>
<tr>
<td>$b$</td>
<td>width of the channel</td>
</tr>
<tr>
<td>$C$</td>
<td>volume fraction density</td>
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<tr>
<td>$c$</td>
<td>wave velocity, [m/s]</td>
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<tr>
<td>$c_p$</td>
<td>specific heat at constant volume, [Jkg$^{-1}$K$^{-1}$]</td>
</tr>
<tr>
<td>$Ee$</td>
<td>Eckert number, [--]</td>
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<tr>
<td>$k$</td>
<td>thermal conductivity, [Wm$^{-1}$K$^{-1}$]</td>
</tr>
<tr>
<td>$M$</td>
<td>Hartmann number, [--]</td>
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<tr>
<td>$\bar{P}$</td>
<td>pressure in fixed frame, [Pa]</td>
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<td>$Pr$</td>
<td>Prandtl number, [--]</td>
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<tr>
<td>$Q$</td>
<td>volume flow rate, [m$^3$s$^{-1}$]</td>
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<tr>
<td>$Re$</td>
<td>Reynolds number, [--]</td>
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<tr>
<td>$S$</td>
<td>drag force</td>
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<tr>
<td>$S$</td>
<td>stress tensor, [Nm$^{-2}$]</td>
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<tr>
<td>$t$</td>
<td>time, [s]</td>
</tr>
<tr>
<td>$U$, $V$</td>
<td>velocity components in fixed frame, [m/s]</td>
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Greek symbols

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<tr>
<td>$\gamma$</td>
<td>shear rate, [s$^{-1}$]</td>
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<td>dimensionless temperature, [--]</td>
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<td>$\lambda$</td>
<td>wavelength, [m]</td>
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<td>$\lambda_1$, $\lambda_2$</td>
<td>relaxation time and retardation time</td>
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<tr>
<td>$\mu$</td>
<td>viscosity of the fluid, [Ns$m^{-2}$]</td>
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<tr>
<td>$\omega$</td>
<td>thermal equilibrium time</td>
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<tr>
<td>$\omega_i$</td>
<td>relaxation time of the particle</td>
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<tr>
<td>$\rho$</td>
<td>fluid density, [kgm$^{-3}$]</td>
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<tr>
<td>$\phi$</td>
<td>amplitude ratio</td>
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Subscripts

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<td>$p$</td>
<td>particulate phase</td>
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References