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On generalizing the knife-edge problem for fractal waves: the Weierstrass-Lamb solution

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One of the classic configurations in electromagnetic theory is describing how a plane wave scatters from a screen of semi-infinite extent. By anticipating that the screen is perfectly conducting and has zero thickness, one can obtain an exact continuous-wave solution to the boundary-value problem that follows from Maxwell's equations. While Sommerfeld first solved the knife-edge problem towards the end of the nineteenth century [A. Sommerfeld, Math. Ann. **47**, 317 (1896)], our interest is with the subsequent and more elegant approach devised by Lamb [H. Lamb, Proc. Lond. Math. Soc. **4**, 190 (1907)] who considered the screen as the limit of a parabolic cylinder.

Here, Lamb's solution is generalized to regimes wherein the incident wave has spatial structure across many scalelengths. The Weierstrass function is one obvious candidate model for multi-scale illumination. It comprises a linear superposition of N cosine-type waves defined by a spatial-frequency spectrum whose constituents are geometrically separated (distinct from the uniformly-spaced frequency combs that tend to characterize Fourier decompositions), and in the limit $N \to \infty$ has the intriguing property of being *everywhere continuous but nowhere differentiable*. In electromagnetics, one is going to be concerned with finite-N (or "pre-fractal") scenarios since we do not expect to find an infinite number of pattern scalelengths in any physically-meaningful field; such a truncation of the Weierstrass summation helps simplify some of the subsequent mathematical and computational considerations. We will detail our recently-derived *Weierstrass-Lamb solution* to describe the scattering of a pre-fractal incident waveform. This fully-vectorial solution to Maxwell's equations captures both transverse-electric and transverse-magnetic polarization states, satisfies the boundary conditions everywhere on the screen (which are of the Dirichlet and Neumann type, respectively), and holds throughout all space. In the relatively narrow window of validity prescribed by the slowly-varying envelope approximation, a solution to the paraxial diffraction equation emerges asymptotically.