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Summary

The spontaneous pattern-forming properties of three discrete nonlinear optical systems are investigated, including the proposal of two new physical contexts for coupled-waveguide geometries. Linear analyses predict Turing threshold instability spectra with multiple minima, and simulations demonstrate emergent static patterns.

Introduction: *simplicity and complexity*

Alan Turing's profound insight into morphogenesis, published in 1952, has provided the cornerstone for understanding the birth of pattern and form in Nature. When the uniform states of a nonlinear reaction-diffusion system are sufficiently stressed, arbitrarily-small disturbances can drive spontaneous self-organization into simple patterns with finite amplitude. Emergent structures have a universal familiarity (including hexagons, honeycombs, squares, stripes, rings, spirals, and vortices), and they are characterized by a single dominant scalelength that is associated with the most-unstable Fourier component.

Here, Turing's ideas are extended to three wave-based discrete nonlinear optical models with a wide range of boundary conditions. In each case, the susceptibility of the uniform states to vanishingly-small symmetry-breaking fluctuations is addressed and we predict a threshold instability spectrum for static patterns that comprises a multiple-minimum structure. These multi-Turing systems are also studied numerically, uncovering instances of simple and complex (i.e., fractal) pattern formation.

Cavities: *nonlinear Fabry-Pérot*

We begin by considering a thin slice of nonlinear (diffusive Kerr-type) material that is sandwiched between two partially reflecting mirrors. Light injected from an external source bounces back and forth between the mirrors, and passes through the slice on each transit. This very simple nonlinear Fabry-Pérot (FP) cavity is the epitome of a complex optical system, involving the interplay between diffraction, diffusion, counter-propagation, and cavity feedback (i.e., periodic pumping, mirror losses, interferomic mistuning, and time delays).

The Turing threshold instability spectrum [1] for the FP cavity is generally found to possess a discrete island structure as opposed to the lobes of the closely-related single feedback-mirror (SFM) system [2] [see Fig. 1(a)]. By controlling the spatial frequencies that are allowed to propagate, simulations have predicted a range of simple patterns when the cavity is initialized with a perturbed plane wave solution above threshold [see Figs. 1(b)–(e)]. We will also present evidence of a *spontaneous fractal-generating capacity* [see Figs. 1(f)–(i)]. Such multi-scale pattern formation is connected to a hierarchy of comparable minima in the threshold spectrum [3].

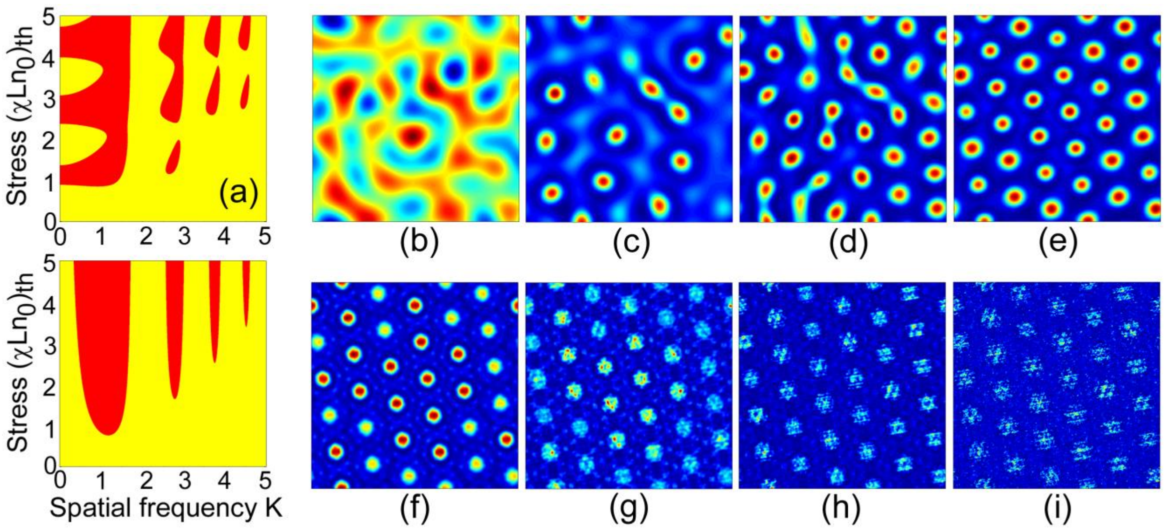

Fig. 1. (a) Multi-Turing threshold instability spectrum for an FP cavity (top) and its corresponding SFM system (bottom). Emergence of a static hexagon pattern from a perturbed plane-wave solution in the FP cavity [(b)–(e)], and its transformation towards a volume-filling fractal [(f)–(i)].

Cavities: *coupled nonlinear waveguides*

The first discrete nonlinear-Schrödinger (dNLS) context to consider involves confining a waveguide array inside a ring cavity. The complex amplitude in each channel is coupled to those in its nearest neighbours, and the host medium has a local Kerr-type response. The governing equation is also supplemented by the classic ring-resonator boundary condition. We will report on our analysis of this class of dNLS-type problem, and discuss the (periodic-in-*K*) multi-Turing threshold spectrum. Results from simulations will be presented, demonstrating simple pattern emergence in arrays with one and two transverse dimensions. Our approach goes beyond mean-field descriptions of other related dNLS-based cavity models, which are analytically more tractable at the expense of averaging propagation effects [4].

Counterpropagation: *coupled nonlinear waveguides*

We have also re-considered the fundamental optical configuration of counter-propagating (CP) laser beams [5] but within the context of nonlinear waveguide arrays. A dNLS-type model has been proposed for describing the evolution of forward- and backward-wave envelopes, which is essentially a discrete analogue of the continuum equations. The perturbative technique used to investigate the stability of the uniform states (subject to equal-intensity constant plane wave pump fields) is reminiscent of that deployed for the continuum model [6], and involves a boundary-value problem whose solution requires the exponentiation of a 4×4 matrix. We will report on the multi-Turing threshold instability spectrum for this novel discrete generalization, and present a set of simulations to illustrate pattern formation.

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