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Numerical Study of Interface Tracking for the Unsteady Flow of Two Immiscible Micropolar and Newtonian Fluids Through a Horizontal Channel with an Unstable Interface

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The dynamics of the interaction between immiscible fluids is relevant to numerous complex flows in nature and industry, including lubrication and coating processes, oil extraction, physicochemical separation techniques, etc. One of the most essential components of immiscible flow is the fluid interface, which must be consistently monitored. In this article, the unsteady flow of two immiscible fluids i.e., an Eringen micropolar and Newtonian liquid is considered in a horizontal channel. Despite the no-slip and hyper-stick shear stress condition at the channel edge, it is accepted that the liquid interface is dynamic, migrating from one position to the next and possibly get absolute change; as a result, The CS (continuum surface) model is integrated with the single moment equation based on the VOF (volume of fluid) approach to trace the interface. The immiscible fluids are considered to flow under three applied pressure gradients (constant, decaying, and periodic) and flow is analyzed under seamless shear stress over the entire interface. The modified cubic b-spline differential quadrature method (MCB-DQM) is used to solve the modeled coupled partial differential equations for the fluid interface evolution. The advection and tracking of the interface with time, wave number, and amplitude are illustrated through graphs. It is observed that the presence of micropolar parameters affects the interface with time. The novelty of the current study is that previous studies (which considered the smooth and unstable movement of the micropolar fluid, the steady stream of two immiscible fluids, and interface monitoring through different modes) are extended and generalized to consider the time-dependent flow of two immiscible fluids namely Eringen micropolar and Newtonian with a moving interface in a horizontal channel. For the decaying pressure gradient case, which requires more time to achieve the steady-state, the peak of the waves resembles those for the constant pressure gradient case. The interface becomes steady for a more extensive time when a constant pressure gradient is applied. The interface becomes stable quickly with time as the micropolar parameter is decreased for the constant pressure gradient case i.e., weaker micropolar fluids encourage faster stabilization of the interface. With periodic pressure gradient, the interface takes more time to stabilize, and the crest of the waves is significantly higher in amplitude compared to the constant and decaying pressure cases. The simulations demonstrate the excellent ability of MCB-DQM to analyze complex interfacial immiscible flows.


1. INTRODUCTION

Many commercial and technological fluids do not function as Newtonian fluids- they exhibit non-Newtonian characteristics. Various mathematical models have therefore been developed to describe non-Newtonian fluids which include viscoelastic, and polar (couple stress) fluids. An alternative and more elegant formulation are the micropolar fluid model which has stimulated considerable interest in rheological modeling worldwide. Eringen pioneered the theory of micropolar fluids in the 1960s.1 A remarkable aspect of the Eringen model is the competency to depict the fluid microstructure, including stresses which are neglected...
in the Newtonian (Navier-Stokes classical viscous flow) model and most theoretical models. The microstructure is simulated via the microrotation vector, which is extraneous to the vorticity vector that creates unsymmetric stress. Microelements of the micropolar fluid can thus sustain gyrationary motions i.e., spin. Micropolar fluid dynamics has therefore been effectively used in various applications, including hemodynamics, nuclear reactor slurry flows, liquid crystals, lubricants, gels, greases, etc. Peddisiemon explored the micropolar model for turbulent channel flow analysis. Thermal coating flows with micropolar fluids in the presence of cross-diffusion effects were studied numerically by Bég et al.1. Micropolar blood flows were examined comprehensively by Kang and Eringen.4 Devakar and Iyerger3 studied the time-dependent flow of micropolar liquid in channels. Mekheimer and Kot6 analyzed the stenotic hemodynamic flow of micropolar fluids with applications in diseased tapered arteries. Srinivasa and Shiferaw7 used the homotopic analysis method to compute the effects of cross-diffusion on micropolar convection flow in a channel. Wang et al.8 examined the motion of micropolar fluids into a micro parallel system with applications in industrial flow separation in chemical engineering systems.

Apart from the micro-structured element, the efficiency of energy transfer in the fluid has been strengthened by the suspension of the nanoparticle. Using the magnetic field and nanofluid, Selimefendigil et al.9 improved the thermal efficiency of the separated flow. The flow behavior of hybrid nanofluids was examined by Dogonchi et al.10 in an embedded system comprising magnetic field effects. The problem of heat exchange among fluid and hot surfaces within a smooth air channel was addressed by Menni et al.11 Raza et al.12 investigated the cumulative influence of heat radiation and a magneto force on molybdenum disulfide nanofluid in a conduit with shifting boundaries. The influence of Hall and ion slip on the laminar flow of two immiscible fluids in a channel. Umavathi et al.13,14 considered polar (couple stress) and Newtonian immiscible fluids in a channel. Kumar et al.32 provided an analytical solution for two immiscible (micropolar and Newtonian) fluids in a composite porous medium inside a vertical channel. Srinivasa and Murthy34,35 further investigated the influence of entropy generation in heat transfer of two couple stress immiscible fluids. Borrelli et al.36 considered the hydromagnetic flow of two immiscible Newtonian fluids in a channel.

One of the primary problems in evaluating the behavior of two immiscible liquid flows is the presence of an unusual interface in the area under consideration. Therefore, the interface of immiscible liquids is important for the design of industrial systems (e.g., chemical separation devices, nuclear reactor transport, etc.). Tauber et al.37 addressed the deformations of a shearing immiscible fluid interface. Yih38 found that when the viscosity of one fluid shifts to another, the tangential velocity slope, causing strife at the interface. DeBar39 introduced the volume of fluid procedure, which was later improved by Youngs.40 Li41 investigated the VOF for interface development between two immiscible liquids. The VOF technique was used by Li and Renardy42 to investigate the contact between two immiscible fluids. Vinay and Wachem43 investigated the interface development of immiscible fluids including VOF.

Most of the above numerical approaches have proved to be generally reliable for simulating accurately the interface dynamics in immiscible flows. However, both artificial and normal conditions exist where the perspective of time dependence is deeply desirable. There are no real circumstances in immiscible fluid dynamics that do not feature some instability. Consequently, the study of the time-dependent flow of immiscible liquids is critical for providing a deeper appreciation of the actual flow behavior of relevance to for example industrial processes. Despite its actual existence, the time-dependent unstable fluid flow of two immiscible liquids has received relatively sparse
attention in scientific works. The unstable immiscible flow problem has been examined carefully by Tryggvason et al. via the front tracking device. Riaz et al. explored the volatility of the immiscible two-phase flow in porous media. Vajravelu et al. investigated the magnetohydrodynamic (MHD) flow of immiscible Newtonian liquids in permeable layers. Thermo molded (heat and mass transfer) in immiscible viscous liquid channel flow was explored by Umavathi et al.

A variety of linear and nonlinear fluid flow problems can be numerically solved using finite difference, finite element, and finite volume techniques. Devakar and Raje numerically explored the time-dependent unsteady flow of two immiscible fluids by the Crank-Nicolson finite difference method. To achieve an acceptable degree of accuracy, the low-order approaches do use several grid points to obtain specific outcomes at such defined points. The differential quadrature approach was proposed by Bellman to develop an efficient discretization strategy. The differential quadrature approach was developed by Bellman to develop an efficient discretization strategy to achieve concise numerical solutions with significantly reduced grid stages. Quan and Chang improved the DQM to achieve concise numerical solutions with significantly reduced grid stages. The descriptions of notations and symbols are being mentioned in the nomenclature section. As Newtonian fluid does not contain any micro rotational effect the Eq. (2) is reduced as

\[ \rho \frac{\partial \mathbf{u}}{\partial t} = \rho \mathbf{u} - \nabla p - \mu \nabla \times \mathbf{u} + (\gamma_1 + 2\mu) \nabla \cdot (\nabla \cdot \mathbf{u}) \]  

(4)

2. GOVERNING EQUATION

The micropolar fluid flow field equations are:

\[ \rho_1 \mathbf{u} + \nabla \cdot (\rho_1 \mathbf{u}) = 0 \]  

(1)

\[ \rho f = -\nabla p + \delta \nabla \times \omega - (\mu + \delta) \nabla \times \nabla \times \mathbf{u} \]  

(2)

\[ \rho \xi - 2\delta \omega + \delta \nabla \times \mathbf{u} - \lambda_1 \nabla \times \nabla \times \omega \]  

\[ \omega + (\lambda_1 + \lambda_2 + \lambda_3) \nabla (\mathbf{\nabla} \cdot \mathbf{u}) - \rho \tau \frac{\partial \omega}{\partial t} = 0 \]  

(3)

2.1. The Volume of Fluid Method

In the system of two immiscible liquids, the characteristics resulting in the momentum model are defined by each control volume. The following equations normalize the average density and viscosity quantities:

\[ \rho^* = \rho_1 + C_1 (\rho_2 - \rho_1) \]  

(5)

\[ \mu^* = \mu_1 + C_2 (\mu_2 - \mu_1) \]  

(6)

Here \( \rho_1, \mu_1 \) be the density and viscosity of micropolar fluid and \( \rho_2, \mu_2 \) be the density and viscosity of Newtonian fluid and \( C_1 \) is volume fraction function. The interface tracking function \( C_i \) is transported by the velocity field \( \mathbf{u}^* \):

\[ \frac{\partial C_i}{\partial t} + \mathbf{u}^* \cdot \nabla C_i = 0 \]  

(7)

The flow velocity is driven by a single mass transfer equation:

\[ \rho^* \left( \frac{\partial \mathbf{u}^*}{\partial t} + \mathbf{u}^* \cdot \nabla \mathbf{u}^* \right) = -\nabla p + \mu^* \nabla^2 \mathbf{u}^* + \rho^* g + f \]  

(8)

Here \( g \) is the gravitational acceleration, \( f \) is the body force per unit mass, and \( -\nabla p \) is the applied pressure term. The characteristics of the velocity equation are calculated with increasing governing volume in a two-phase system.

3. FORMULATION

Consider the unsteady, laminar, and unidirectional flow of two immiscible, micropolar, and Newtonian fluids undergoing shearing motions. Owing to viscosity jump primarily, an interfacial instability occurs. Both fluids are viscous and incompressible and move between two horizontal parallel non-porous plates. The plates are located in the \( X-Y \) plane as depicted in Figure 1. The maximum flow velocity is \( U_l \) in the \( x \) direction. Let \( \rho_1, \mu_1 \) be the density and viscosity of micropolar fluid which lies in \( -k \leq y \leq l_1 \) with the interface height \( l_1 \). The Newtonian fluid has the density \( \rho_2 \) and the viscosity \( \mu_2 \) occupies in \( (l_1 \leq y \leq k) \). The evolution of the interface is analysed by using horizontal attenuation in the initial interface phase. Let the initial interface be an Eigen-mode (In an oscillating interface,
a normal mode is one in which all elements of the system operate at the same frequency), say \( l_1 y_m + A_0 \cos(B_0 y) \) where \( y_m \) is the average depth of interface \( A_0, B_0 \) are the amplitude and wavenumber. The flow of the immiscible fluids is mobilized by \( x \)-direction applied common pressure gradient (either constant, decaying, or periodic). The average density \( \rho^* = \rho_1 + C_1(\rho_2 - \rho_1) \) and viscosity \( \mu^* = \mu_1 + C_1(\mu_2 - \mu_1) \).

The body force \( f \) is modeled as surface tension force by

\[
F_s = \sigma \varsigma \nabla C_1
\]

Here \( \sigma \) is the interfacial tension between two liquid phases and \( \varsigma \) is the mean free surface curvature, given by \( \varsigma = -(\nabla^2 C_1)/(\|\nabla C_1\|) \). Since the flow is assumed to be unidirectional so \( \nabla C_1 = (\partial C_1/\partial y) \) and \( \nabla^2 C_1 = (\partial^2 C_1/\partial y^2) \).

Hence Eq. (9) can be written as

\[
F_s = \sigma \frac{\partial^2 C_1}{\partial y^2}
\]

The micropolar fluid velocity \( u_1 \), Newtonian liquid velocity \( u_2 \), micro-rotation vector \( \omega \) in the region I, mean flow velocity \( u^* \), and volume fraction \( C_1 \) are mathematically denoted as \( u_1 = (u_1(y, t), 0, 0) \), \( u_2 = (u_2(y, t), 0, 0) \), \( \omega = (0, 0, C_1(y, t)) \), \( u^* = (u^*(y, t), 0, 0) \), \( C_1 = (C_1(y, t), 0, 0) \) respectively. The two-fluid flow is unstable, incompressible \( \nabla \cdot u_1 = 0 \), \( \nabla \cdot u_2 = 0 \) and induced by an \( x \)-direction pressure gradient and the microrotation vector is present only in the region-I fluid. Given this, Eqs. (2)-(3) can then be simplified to:

Region-I for Micropolar fluid \((-k \leq y \leq l_1)\)

\[
\rho_1 \frac{\partial u_1}{\partial t} = -\frac{\partial p}{\partial x} + \delta \frac{\partial C_1}{\partial y} + (\alpha_1 + \delta) \frac{\partial^2 u_1}{\partial y^2}
\]

Region-II for Newtonian fluid \((l_1 \leq y \leq k)\)

\[
\rho_2 \frac{\partial u_2}{\partial t} = \alpha_2 \frac{\partial^2 u_2}{\partial y^2} - \frac{\partial p}{\partial x}
\]

By an exchange in momentum, the fluid layers are mechanically linked by the system. We do allow for the possibility of a jump across the interface. The interface movement of immiscible fluids of various densities and viscosity is described by the volume fraction function \( C_1 \) and three conditions are used to monitor the interface. \( F_s \) Each node in the computing grid records the volume fraction of micropolar and Newtonian fluid. The concentration function is the volume fraction field at the discrete level. If a cell is occupied with the micropolar fluid then \( C_1 = 0 \). The cell does not contain any volume of micropolar fluid and occupied with Newtonian fluid as \( C_1 = 1 \). There is a fluid interface in the cell, then \( 0 < C_1 < 1 \). As per the specific value of the \( C_1 = 0.5 \), we could obtain a unique volume fraction field from an interface. Hence density value \( \rho^* = \rho_1 + 0.5(\rho_2 - \rho_1) = ((\rho_2 + \rho_1))/2 \) and viscosity value \( \mu^* = \mu_1 + 0.5(\mu_2 - \mu_1) = (\mu_2 + \mu_1)/2 \) allows jump across the interface. If \( C_1 = 0 \) and 1 then there no penetration of micropolar and Newtonian fluids in each other region.

Hence by using the Eqs. (2)-(9), the mean flow in the region \((-k \leq y \leq k)\) and interface track in \((-k \leq y \leq k)\) are governed by the following equations:

\[
\frac{\partial C_1}{\partial t} + u^* \frac{\partial C_1}{\partial y} = 0
\]

\[
\rho^* \frac{\partial u^*}{\partial t} = -\frac{\partial p}{\partial x} + (\mu^* + \delta) \frac{\partial^2 u^*}{\partial y^2} + \delta \frac{\partial C_1}{\partial y} + \rho^* g + \sigma \frac{\partial^2 C_1}{\partial y^2}
\]

The classical no-slip and hyper-stick conditions are considered at the boundaries. These conditions can be written mathematically as follows.

Initial conditions: At \( t \leq 0 \),

\[
u_1(y, t) = u^*(y, t) = C_1(y, t) = 0, \quad \text{and} \quad C_1(y, t) = l_1 y_m + A_0 \cos(B_0 y)
\]

Boundary conditions: At \( t < 0 \),

\[
u_1(-k, t) = u^*(-k, t) = C_1(-k, t) = u^*(k, t) = 0
\]
\[ u_i(t_1, t) = e, \quad C_s(t_1, t) = -\frac{1}{2} \frac{\partial u_i}{\partial t} \] (17)

It is judicious to introduce the following non-dimensional variables \( \tau = (x/k), \quad \varphi = (y/k), \quad \overline{u} = (u'/U_0), \quad \overline{C_s} = (C_s^*/C_s^0), \quad \varOmega = (tU_0/k)e \in [-A_0, A_0] \) s.t. \( (u_i(e, t) - c)^2 = 0, \quad \lambda_1 = (\mu_1 + \delta)/2\tau \) with \( \tau = k^2 \). Also, the following dimensionless numbers are invoked: Eringen micropolar parameter \( n_1 = (6/\mu_1), \quad Re = (\rho_1U_0/\mu_1) \) is the Reynolds number, \( Fr = (U_0^2)/gk \) is the Froude number. \( r_1 = (\mu_2/\mu_1) \) is the ratio of viscosity, \( r_2 = (\rho_2/\rho_1) \) is the ratio density and \( Ca = (\mu^*U_0^2)/\sigma \) is the capillary number. The pressure variation based on time is implemented from X-axis for momentum in the mean flow. Three different cases for \( Ge(t) = \nabla \rho = -\partial p/\partial x \) with \( t > 0 \) are applied to obtain numerical results.

I: \( Ge(t) = Ge(\text{constant}) \)
II: \( Ge(t) = Ge* \sin(\omega t) \) (periodic with parameter \( \omega \))
III: \( Ge(t) = Ge* e^{-\lambda t} \) (decaying with parameter \( \lambda \)).

After dropping the bars and introducing the above non-dimensional parameters, appropriate initial, interfacial, and boundary conditions according to the scheme, the equations can be reduced to:

Region-I \((-k \leq y \leq 0)\) (Micro-polar fluid region):

\[
\frac{\partial u_i}{\partial t} = Ge(t) + \frac{n_1}{Re} \frac{\partial C_s}{\partial y} + \frac{(n_1 + 1)}{Re} \frac{\partial^2 u_i}{\partial y^2} \] (18)

\[
\frac{\partial C_s}{\partial t} = \frac{(n_1/2 + 1)}{Re} \frac{\partial^2 C_s}{\partial y^2} - \frac{n_1(2C_s + (\partial u_i/\partial y))}{Re} \] (19)

The mean flow in the region \((-k \leq y \leq k)\) and interface track \((-k \leq y \leq k)\) are governed by:

\[
\frac{\partial C_s}{\partial t} + u^* \frac{\partial C_s}{\partial y} = 0 \] (20)

\[
\frac{\partial u^*}{\partial t} = \frac{Ge(t)}{(1 + C_s(r_2 - 1))} + \frac{(n_1 + 1)(1 + C_s(r_2 - 1))}{Re(1 + C_s(r_2 - 1))} \frac{\partial^2 u^*}{\partial y^2} \]

4. NUMERICAL SOLUTION WITH MCB-DQM SCHEME

The differential quadrature technique is effective in the simulation of various linear and nonlinear coupled partial differential equations. The computational accuracy of DQM is very good and only a few grid points and less time are required to achieve high precision. In this problem, we started with 31 points and observe a good agreement in the computed and exact result. The DQ method’s strength lies in its ability to handle a wide range of fluid boundary conditions.

To analyze the interface tracking between micropolar and Newtonian fluids, first, the microrotation vector is
calculated in the domain \((-1 \leq y \leq 0)\) using the velocity vector of micropolar fluid, and then the flow velocity and interface vector in \((-1 \leq y \leq 1)\) is obtained. The domains \([-1, 0]\) for microrotation vector and \([-1, 1]\) for flow velocity and interface vector are equally discretized with phase length \(h\) in the spline-based differential quadrature method\(^{52,53}\) with phase length \(h\): The nodes are placed with uniform step size \(h\).

\[
\begin{align*}
\alpha = y_1 < y_2 < \cdots < y_{n-1} < y_N &= b, \text{ such that } \\
y_{i+1} - y_i &= h \text{ on the real axis}
\end{align*}
\]

After that, the approximation for 1st and 2nd order derivatives of \(u_1(y,t), C_\ast(y,t), u^\ast(y,t), \) and \(C_\ast(y,t)\) are obtained at any time on the nodes \(y_i\) as-

For \(i = 1, 2, 3, \ldots, N\) and \(j = 1, 2, \ldots, N\)

\[
\begin{align*}
\frac{\partial u_1}{\partial y} &= \sum_{j=1}^{N} Z_{i,j}^{(1)} u_1(y_j, t) \\
\frac{\partial^2 u_1}{\partial y^2} &= \sum_{j=1}^{N} Z_{i,j}^{(2)} u_1(y_j, t) \\
\frac{\partial C_\ast}{\partial y} &= \sum_{j=1}^{N} Z_{i,j}^{(1)} C_\ast(y_j, t) \\
\frac{\partial^2 C_\ast}{\partial y^2} &= \sum_{j=1}^{N} Z_{i,j}^{(2)} C_\ast(y_j, t) \\
\frac{\partial u^\ast}{\partial y} &= \sum_{j=1}^{N} Z_{i,j}^{(1)} u^\ast(y_j, t) \\
\frac{\partial^2 u^\ast}{\partial y^2} &= \sum_{j=1}^{N} Z_{i,j}^{(2)} u^\ast(y_j, t) \\
\frac{\partial C_\ast}{\partial y} &= \sum_{j=1}^{N} Z_{i,j}^{(1)} C_\ast(y_j, t)
\end{align*}
\]

Here \(Z_{i,j}^{(1)}, Z_{i,j}^{(2)}\) are the weighting coefficients of 1st and 2nd order derivatives\(^{52,53}\) for \(n\) discrete knots \((y_1, y_2, \ldots, y_N)\) derived from the updated B-spline functions. The cubic B-spline at the knots is defined as\(^{52}\)

**Fig. 5.** 3D evolution of an interface between Micropolar and Newtonian fluids with time.

**Fig. 6.** Interface tracking with varying small time when \(Ge = 10, Re = 2, r_1 = 0.5, r_2 = 0.5, C_j = 0.5, Ca = 0.5, Fr = 2, n_1 = 0.5.\)
where \( \{\chi_0(y), \chi_1(y), \chi_2(y), \ldots, \chi_{N+1}(y)\} \) are the basis function over the region \([a, b]\). The basis function is updated as follows to get a system of equations that could be expressed by a diagonal-dominated matrix.\(^{52, 53}\)

\[
\begin{align*}
D_1(y) &= \chi_1(y) + 2\chi_0(y) \\
D_2(y) &= \chi_2(y) - \chi_0(y) \\
D_j(y) &= \chi_j, \text{ for } j = 3, \ldots, N-2 \\
D_{N-1}(y) &= \chi_{N-1}(y) - \chi_{N+1}(y) \\
D_N(y) &= \chi_N(y) + 2\chi_{N+1}(y)
\end{align*}
\]

The derivatives of the basic functions are as follows:

\[
\begin{align*}
\chi_j'(y) &= \begin{cases} 
(y - y_{j-2})^3, & y \in [y_{j-2}, y_{j-1}) \\
(y - y_{j-2})^3 - 4(y - y_{j-1})^3, & y \in [y_{j-1}, y_j) \\
(y_{j+2} - y)^3 - 4(y_{j+1} - y)^3, & y \in [y_j, y_{j+1}) \\
(y_{j+2} - y)^3, & y \in [y_{j+1}, y_{j+2}) \\
0, & \text{otherwise}
\end{cases}
\end{align*}
\]

where \( j \) is the index of the basis function. The derivatives of the basic functions are as follows:

\[
\begin{align*}
D_1'(y) &= \chi_1'(y) + 2\chi_0'(y) \\
D_2'(y) &= \chi_2'(y) - \chi_0'(y) \\
D_j'(y) &= \chi_j', \text{ for } j = 3, \ldots, N-2 \\
D_{N-1}'(y) &= \chi_{N-1}'(y) - \chi_{N+1}'(y) \\
D_N'(y) &= \chi_N'(y) + 2\chi_{N+1}'(y)
\end{align*}
\]

### Figures

**Fig. 7.** Interface tracking with varying time when \( Ge = 10, Re = 2, r_1 = 0.5, r_2 = 0.5, C_i = 0.5, Cu = 0.5, Fr = 2, n_1 = 0.5.**

**Fig. 8.** Interface tracking with varying high time when \( Ge = 10, Re = 2, r_1 = 0.5, r_2 = 0.5, C_i = 0.5, Cu = 0.5, Fr = 2, n_1 = 0.5.**
The estimate of the first-order derivative is

\[ D_k (y_i) = \sum_{j=1}^{N} Z_{i,j}^{(1)} \chi_k (y_j) \quad \text{for} \quad i = 1, 2, \ldots, N, k = 1, 2, \ldots, N \quad (28) \]

Then using \( Z_{i,j}^{(1)} \) for \( i = 1, 2, \ldots, N \) in Eq. (19) the following tri-diagonal system of equations is established as:

\[
\begin{bmatrix}
6 & 1 & 0 & 0 \\
0 & 4 & 1 & 0 \\
0 & 1 & 4 & 1 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 0 & 1 & 4 & 1 \\
0 & 0 & \ldots & 0 & 1 & 4 & 0 \\
0 & 0 & \ldots & 0 & 1 & 6
\end{bmatrix}
\]

Solving the above system the weighting coefficients \( \{Z_{i,1}, Z_{i,2}, \ldots, Z_{i,N}\} \), \( \{Z_{1,i}^{(1)}, Z_{2,i}^{(1)}, \ldots, Z_{N,i}^{(1)}\}, \ldots, \{Z_{1,i}^{(2)}, Z_{2,i}^{(2)}, \ldots, Z_{N,i}^{(2)}\}\) of first order derivatives of linear and angular velocities are obtained and then the value of \( Z_{i,j}^{(2)} \) for \( i = 1, 2, 3, \ldots, N, j = 1, 2, 3, \ldots, N \) is calculated as follows

\[
Z_{i,j}^{(2)} = 2Z_{i,j}^{(1)} \left( Z_{i,j}^{(1)} - \frac{1}{y_i - y_j} \right) \quad \text{for} \quad i \neq j \quad (30)
\]

Substituting the estimation of the 1st and 2nd order derivatives by the MCB-DQM scheme, Eqs. (19)–(23) can be written as:

Region-I \((-k \leq y \leq 0\) (Micro-polar fluid region):

\[
u_{11} = Ge (t) + \frac{n_1}{Re} \left( \sum_{j=1}^{N} Z_{i,j}^{(1)} C_s (y_j, t) \right) + \frac{(n_1 + 1)}{Re} \left( \sum_{j=1}^{N} Z_{i,j}^{(2)} u_i (y_j, t) \right) \quad (31)
\]
Fig. 11. 3D evolution of an interface between Micropolar and Newtonian fluids with varying wavenumber.

\[
C_{yj} = \frac{(n_i + 2)}{2Re} \left( \sum_{j=1}^{N} Z_{i,j}^{(2)} C_s(y_j, t) \right)
\]

\[
-u^*(y_j, t) \left( \sum_{j=1}^{N} Z_{i,j}^{(1)} C_s(y_j, t) \right)\]

In the region \((-k \leq y \leq k)\):

\[
C_{it} = -u^*(y_j, t) \left( \sum_{j=1}^{N} Z_{i,j}^{(1)} C_s(y_j, t) \right)
\]

\[
u^*_t = \frac{Ge(t)}{r_2} + \frac{(n_i + 1)(1 + C_s(r_2 - 1))}{Re(1 + C_s(r_2 - 1))} \left( \sum_{j=1}^{N} Z_{i,j}^{(2)} u^*(y_j, t) \right)
\]

\[
\frac{1 + C_s(r_2 - 1)}{Re\alpha\sigma(1 + C_s(r_2 - 1))} \left( \sum_{j=1}^{N} Z_{i,j}^{(2)} C_s(y_j, t) \right) + \frac{1}{Fr}
\]

Thus, (31)–(34) is a reduced system of ordinary differential equations in time, that is, represented as for \(i = 1, 2, 3 \ldots, N\).

\[
U_i = R(u_{i0}, C_{s0}, C_{i0}, u^*_{i0})
\]

The system is solved by the following strong stability b-spline RK43 scheme.\(^{52,53}\)

\[
U_i = U_{i0} + \frac{\Delta t}{2} * R(u_{i0}, C_{s0}, C_{i0}, u^*_{i0})
\]

\[
U_i = \frac{U_{i0} + U_i}{3} + \frac{\Delta t}{6} * R(u_{i2}, C_{s2}, C_{i2}, u^*_{i2})
\]

\[
U_i = \frac{U_{i0} + U_i + \Delta t}{3} + \frac{\Delta t}{2} * R(u_{i3}, C_{s3}, C_{i3}, u^*_{i3})
\]

5. RESULTS AND DISCUSSION

According to the time-dependent pressure gradient, the unstable flow of two immiscible micropolar and Newtonian fluids with mutual momentum transfer and viscosity jump across the interface is considered. The traditional no-slip and hyper-stick conditions on the boundaries of the channel were presumed to describe the solution. Single mean flow problems with moving interface have been addressed, and the interface tracking profiles have been acquired by MCB-DQM. The ratio of viscous drag forces and surface tension forces acting at an interface between two immiscible fluids is known as the capillary number \((Ca = (\mu U_0)/\sigma)\). The current numerical technique is validated by comparing the approximated numerical result for a single non-micropolar Newtonian fluid \((r_1 = 1, r_2 = 1, n_i = 0)\) with the exact solution. Note that if \(r_1 = r_2 = 1\), then the interfacial force \(\sigma = 0\) and hence \((1/Ca) = 0, (1/Fr) = 0\). Figures 2, 3 shows the comparative analysis of calculated and exact solutions for velocity profiles at different times and discretized nodes. It is noticed that the applied numerical technique results achieve very good corroboration with the exact solution. The root means square error between analytical and calculated results is very less and also decreases with time and discretized node value.
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Fig. 12. Interface tracking with varying time when the flow is carried by periodic pressure gradient $Ge = 10 \times \sin(\omega t)$.

Figures 4–14 shows the evolution of the interface between two liquids with varying time, amplitude, and wavenumber under the applied constant, periodic, and decaying pressure gradient. It is evident from Figures 4–9 that, in the case of applied constant pressure gradients, initially, the vertical elongation of the interface is large, and then the shape of the interface evolves with time; hence, the undulating sequence occurs faster for a more considerable time, and the interface becomes stable for high time. Note that the top fluid is even less viscous than the bottom fluid. It is also noticed that the qualitative characteristics of this flow shall be maintained, and as expected, the interface starts to shift vertically if the amplitude and wavenumber are enhanced (see Figs. 10 and 11). The two liquids do not penetrate each other, in the same way. The evolution of the interface profile for decaying pressure gradient cases is qualitatively similar, although greater elapse of time is required for it to become stable, and the crest of the waves is smaller compared to the constant pressure case (see Fig. 12). It is also worth mentioning that the evolution of the interface profile for the periodic pressure gradient case is qualitatively similar to other cases; however, it takes more time to stabilize, and the crest of the waves is large compared to the constant and decaying pressure case (see Fig. 13). It is also noticed from Figure 14 that the interface becomes stable quickly with time as the micropolar parameter is reduced i.e., weaker micropolar fluids encourage faster stabilization of the interface.

Fig. 13. Interface tracking with varying time when the flow is carried by periodic pressure gradient. $Ge = 10 \times e^{-\lambda t}$.
6. CONCLUSIONS

The unsteady flow of two immiscible (micropolar and Newtonian) fluids through a horizontal channel under a moving interface has been analyzed numerically using the method of modified Cubic B-Spline Differential Quadrature. The effects of key parameters such as time, amplitudes, wave number, and micropolar parameter on interface evolution have been addressed with three different applied pressure gradients. The core findings of the present article can be summarized in the following points:

— Initially, the vertical stretching of the interface is enormous, and afterward, its shape advances with time; subsequently, the undulating succession occurs quicker, and the interface becomes steady for a more extensive time when a constant pressure gradient is applied.

— The development of the interface profile for decaying pressure gradient cases is subjectively comparative; however, it requires more time to achieve the steady-state, and the peak of the waves resembles those for the constant pressure gradient case.

— The interface profile for periodic pressure gradient cases is qualitatively similar to the other cases; however, it takes more time to stabilize, and the crest of the waves is large compared to the constant and decaying pressure case.

— The interface becomes stable quickly with time as the micropolar parameter is curtailed for the case where the flow is induced by a constant pressure gradient.

The present computations have shown that MCB-DQM is a robust numerical approach for simulating complex interfacial immiscible flows of Newtonian and non-Newtonian fluids. The present study may, however, be extended to consider nanofluids and porous media and also more complex geometries e.g., micro-channels, and efforts in these directions are currently underway.

NOMENCLATURE

The density co-efficient kg/m³
The velocity of the fluid m/s
Microrotations of lower region fluids s⁻¹
The viscosity coefficients kg · m⁻¹ · s⁻¹
Velocities of lower and upper region fluids m/s
The density of lower and upper region fluids kg/m³
The viscosity of lower and upper region fluids kg · m⁻¹ · s⁻¹
The mean flow velocity field in the channel m/s
The average density of the flow in the channel kg/m³
Average viscosity of flow in the channel kg · m⁻¹ · s⁻¹
The gravitational acceleration m/s²
The interfacial tension between two liquid phases mN/m
The height of the interface at any point m
The average depth of the interface m
The amplitude of the fluid
The fluid pressure at any point Pa
The body couple per unit mass
The volume fraction function
The interface tracking function
Ratio of viscosities
Ratio of densities
Reynolds number
Wavenumber
Fr Froude number
\(n_1\) The microrotation parameter
\(f\) The body force per unit mass
\(\varsigma\) The mean free surface curvature
\(\tau\) The gyration co-efficient
\(\lambda_1\), \(\lambda_2\), \(\lambda_3\) The gyro-viscosity coefficients.

Ethical Compliance
This article does not contain any studies involving animals or humans performed by any of the authors.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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References and Notes